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Micromagnetic Dynamics in Thin Films

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Web Pages

- Home Page:

  http://math.nist.gov/~MDonahue/

- OOMMF:

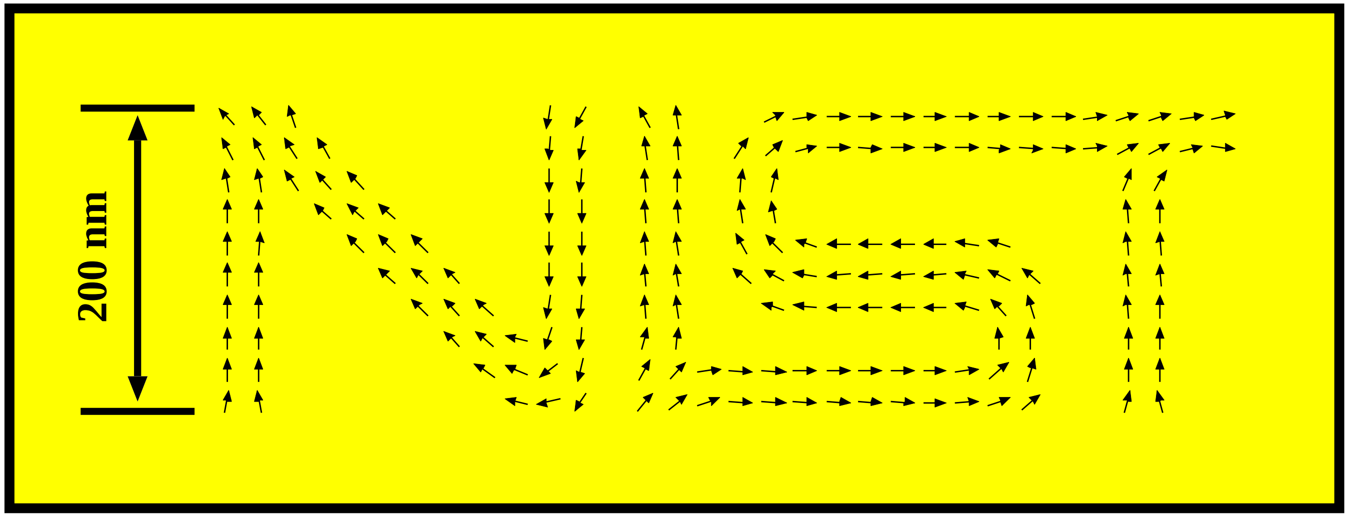
  http://math.nist.gov/oommf/

- μMAG:

  http://www.ctcms.nist.gov/~rdm/mumag.org.html

Outline

- Background
- μMAG Standard Problems
- High Speed Switching (Std. Prob. 4)
- Dynamics Near Quasi-Static Coercive Field
- Dynamics at Remanence



Micromagnetics is the study, modeling and simulation of magnetic materials and their behavior at the sub-micron scale.

Micromagnetic Equations

Landau-Lifshitz-Gilbert:

$$\frac{d\mathbf{M}}{dt} = -\gamma \mathbf{M} \times \mathbf{H}_{\text{eff}} - \frac{\gamma\,\alpha}{M_{\text{s}}} \mathbf{M} \times (\mathbf{M} \times \mathbf{H}_{\text{eff}})$$

$$\mathbf{H}_{\text{eff}} = -\frac{1}{\mu_0} \frac{\partial E_{\text{density}}}{\partial \mathbf{M}}$$

Energies:

$$E_{\text{total}} = E_{\text{exchange}} + E_{\text{anisotropy}} + E_{\text{Zeeman}} + E_{\text{demag}}$$

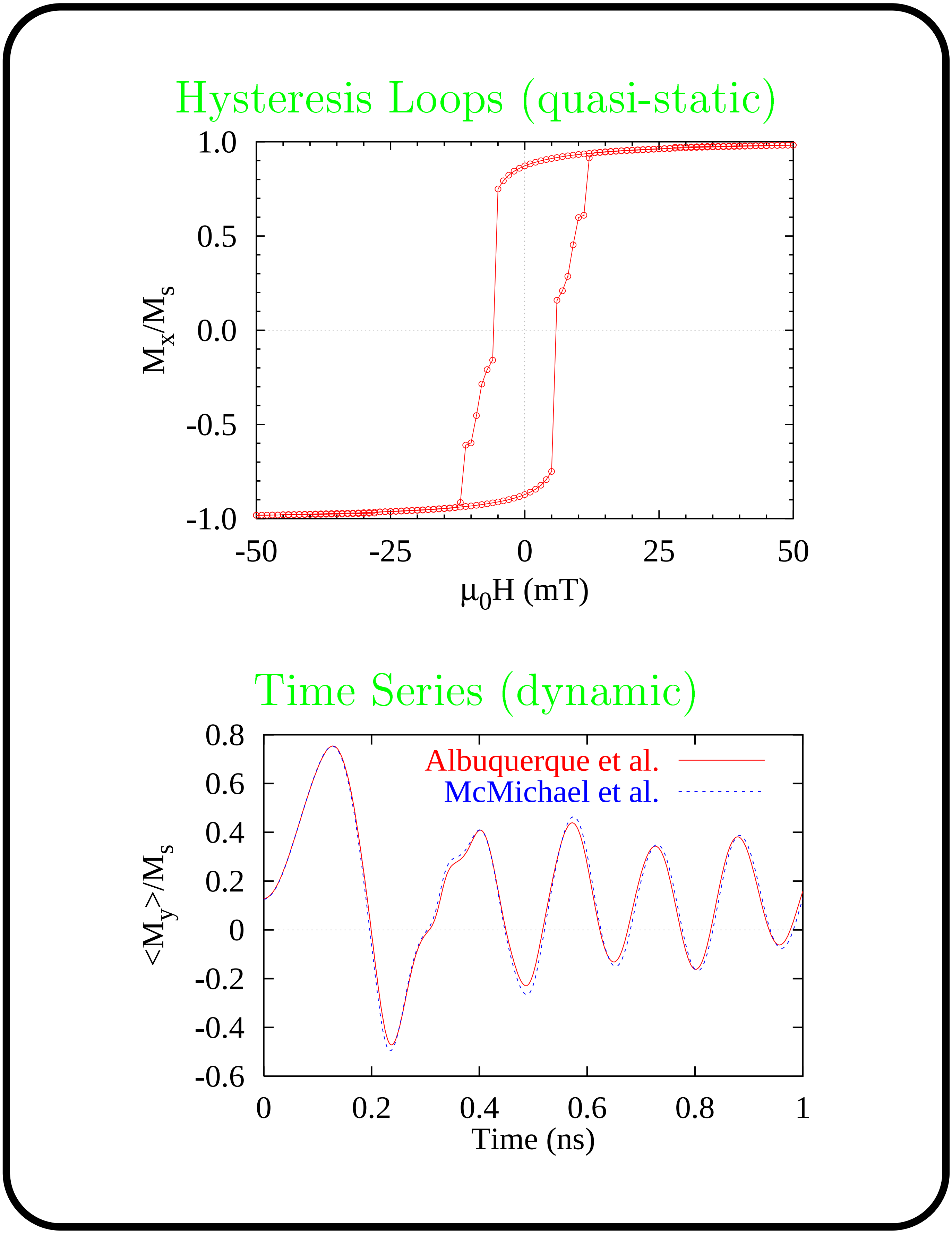
Micromagnetic Energy Equations

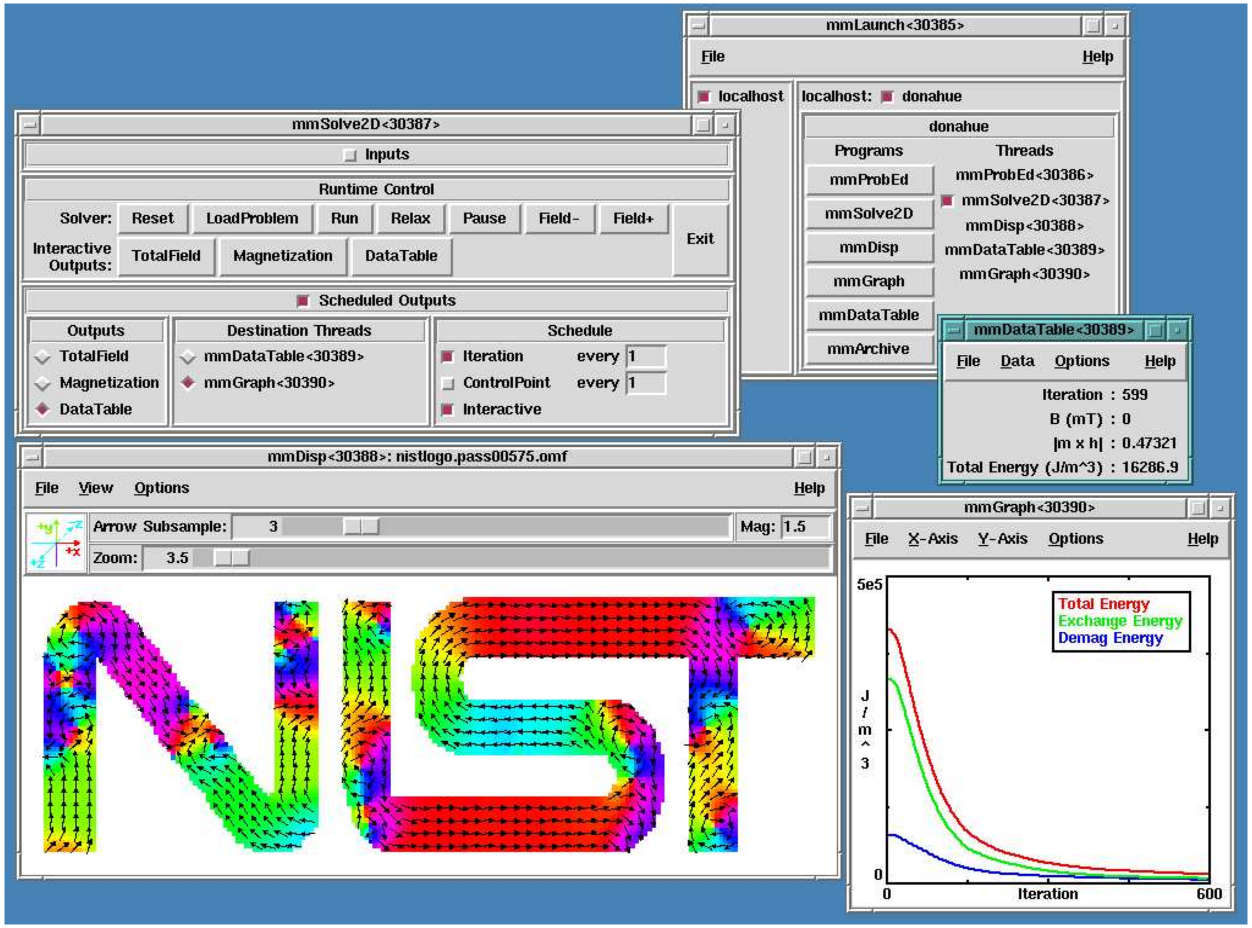
$$
\begin{aligned}
E_{\text{exchange}} &= \int_V \frac{A}{M_s^2}\left(|\nabla M_x|^2 + |\nabla M_y|^2 + |\nabla M_z|^2\right)\, d^3r \\
E_{\text{anisotropy}} &= -\int_V \frac{K_1}{M_s^2}(\mathbf{M}\cdot\mathbf{u})^2\, d^3r \\
E_{\text{Zeeman}} &= -\mu_0 \int_V \mathbf{M}\cdot\mathbf{H}_{\text{applied}}\, d^3r \\
E_{\text{demag}} &= -\frac{\mu_0}{2}\int_V \mathbf{M}\cdot\mathbf{H}_{\text{demag}}\, d^3r
\end{aligned}
$$

$$
\text{where} \qquad \mathbf{H}_{\text{demag}}(r) = -\frac{1}{4\pi}\int_V \nabla\cdot\mathbf{M}(\mathbf{r}')\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}\, d^3r' + \frac{1}{4\pi}\int_S \hat{\mathbf{n}}\cdot\mathbf{M}(\mathbf{r}')\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}\, d^2r'
$$

Comparative Material Constants

Material	M_s (kA/m)	K_1 (kJ/m^3)	A (pJ/m)	l_{ex} (nm)
Co	1400	520	30	4.9
Fe	1700	48	21	3.4
Ni	490	-5.7	9	7.7
Permalloy	860	0	13	5.3
$Nd_2Fe_{14}B$	1280	4500	13	3.5


Hysteresis Loops (quasi-static)
M_x/M_s
$\mu_0 H$ (mT)
1.0
0.5
0.0
-0.5
-1.0
-50
-25
0
25
50
Time Series (dynamic)
Albuquerque et al.
McMichael et al.
$<M_y>/M_s$
Time (ns)
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0
0.2
0.4
0.6
0.8
1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Solver:
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LoadProblem
Run
Relax
Pause
Field-
Field+
Exit
Interactive Outputs:
TotalField
Magnetization
DataTable
Scheduled Outputs
Outputs
TotalField
Magnetization
DataTable
Destination Threads
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mmGraph<30390>
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Iteration every 1
ControlPoint every 1
Interactive
mmDataTable<30389>
File Data Options Help
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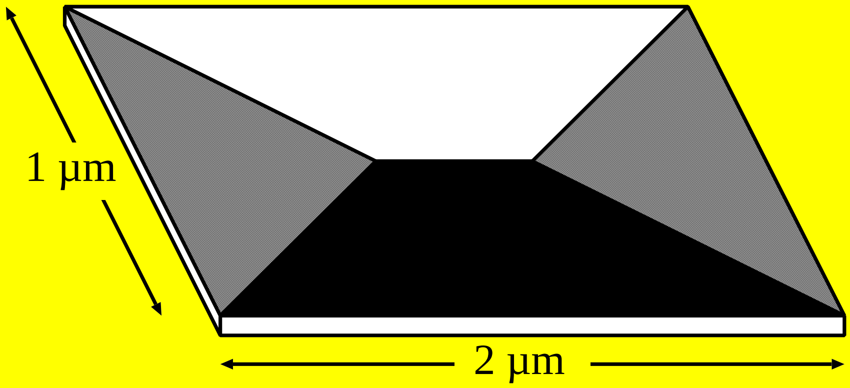
B (mT) : 0
|m x h| : 0.47321
Total Energy (J/m^3) : 16286.9
mmDisp<30388>: nistlogo.pass00575.omf
File View Options Help
Arrow Subsample: 3
Mag: 1.5
Zoom: 3.5
mmGraph<30390>
File X-Axis Y-Axis Options Help
5e5
Total Energy
Exchange Energy
Demag Energy
J/m^3
0
0
Iteration
600

Public Code

μMAG Problem #1 Specification

"Permalloy" rectangle



20 nm thick

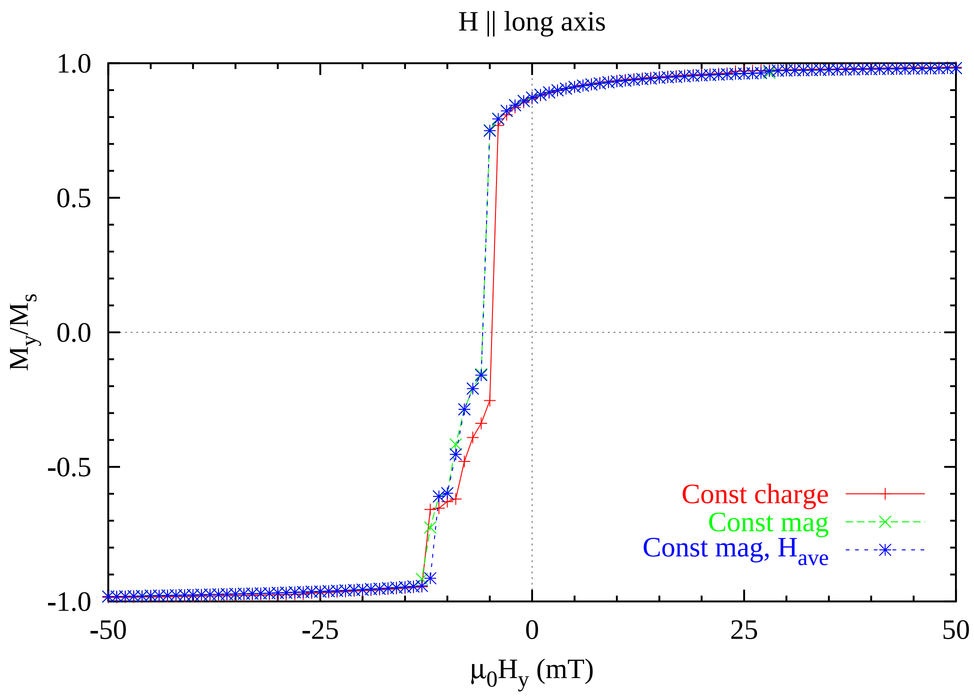
$A = 1.3\text{x}10^{-11}$ J/m ($1.3\text{x}10^{-6}$ erg/cm)

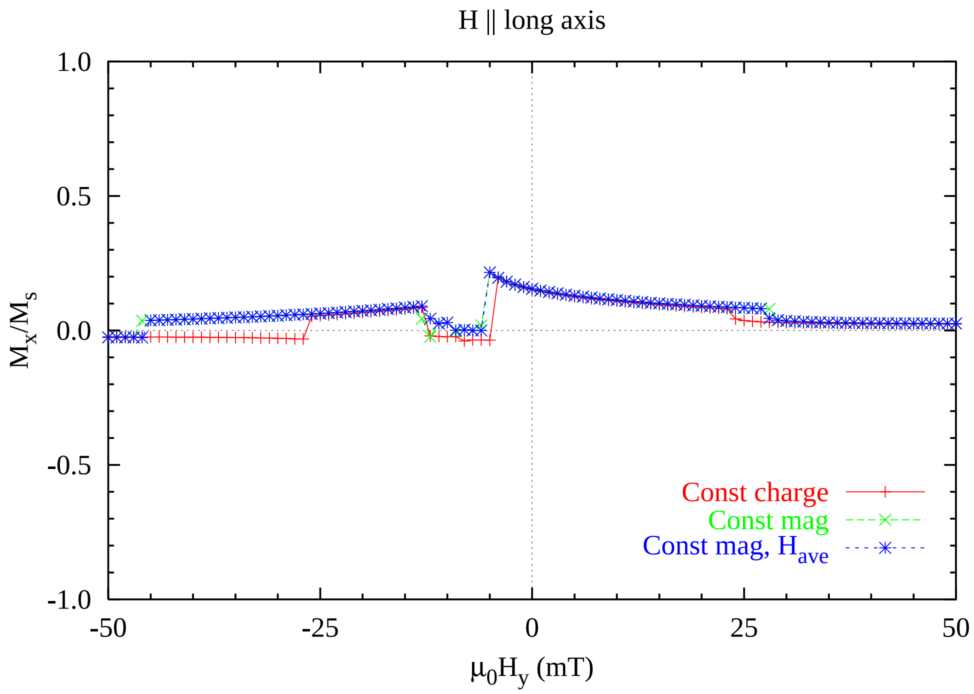
$M = 8.0\text{x}10^{5}$ A/m (800 emu/cm^3)

$K = 500$ J/m^3 (5000 erg/cm^3)

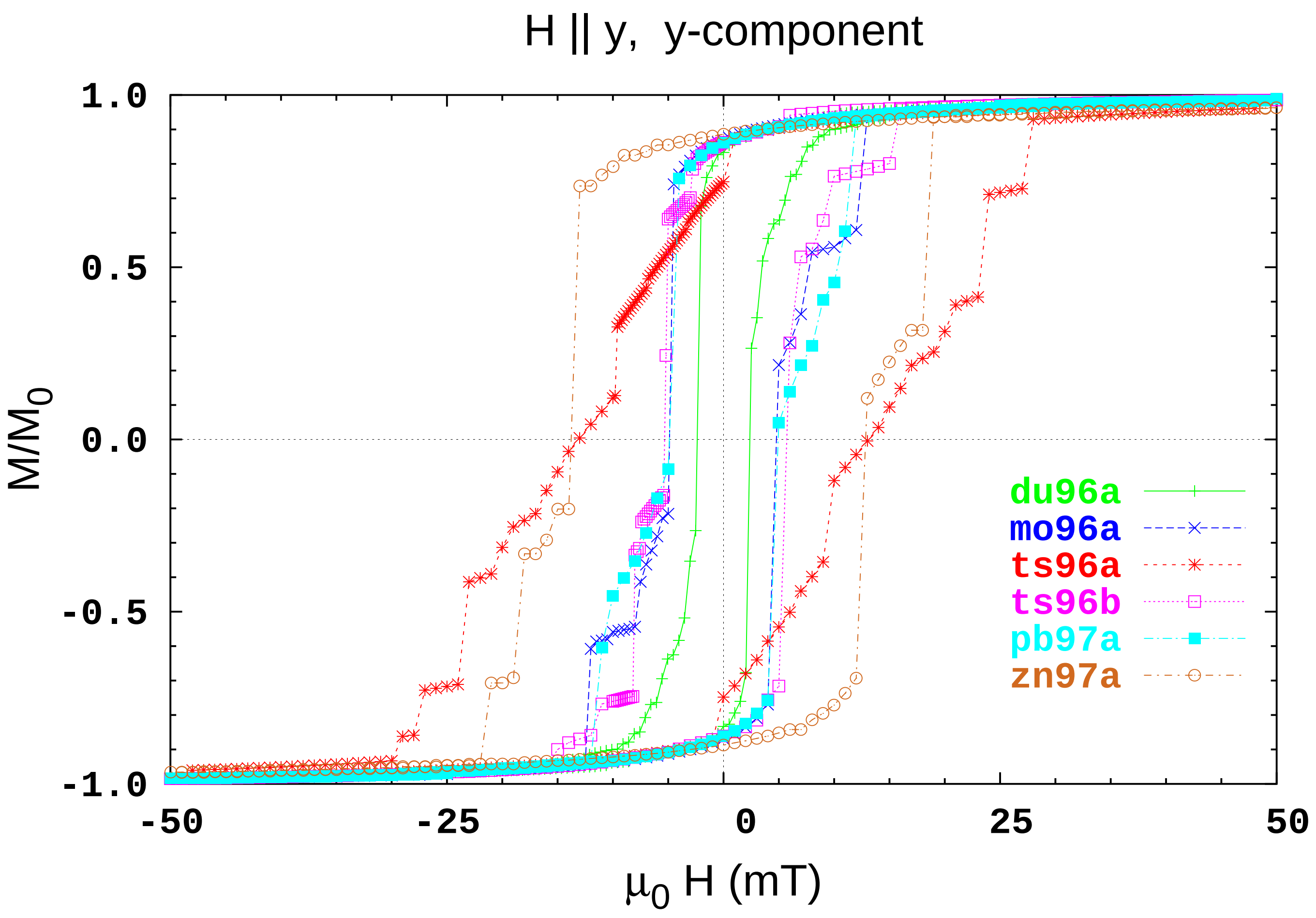
μMAG Problem #1

OOMMF Results (20 nm cells)

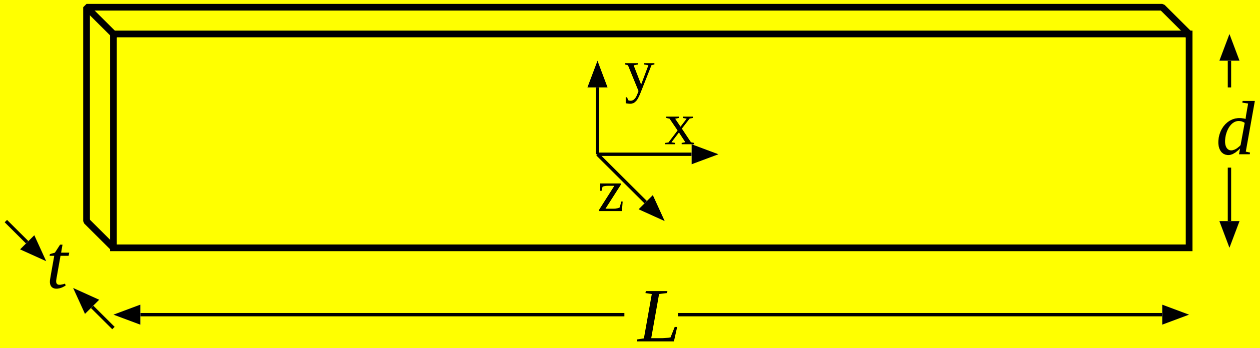




μMAG Problem #1 Results



μMAG Problem #2 Specification

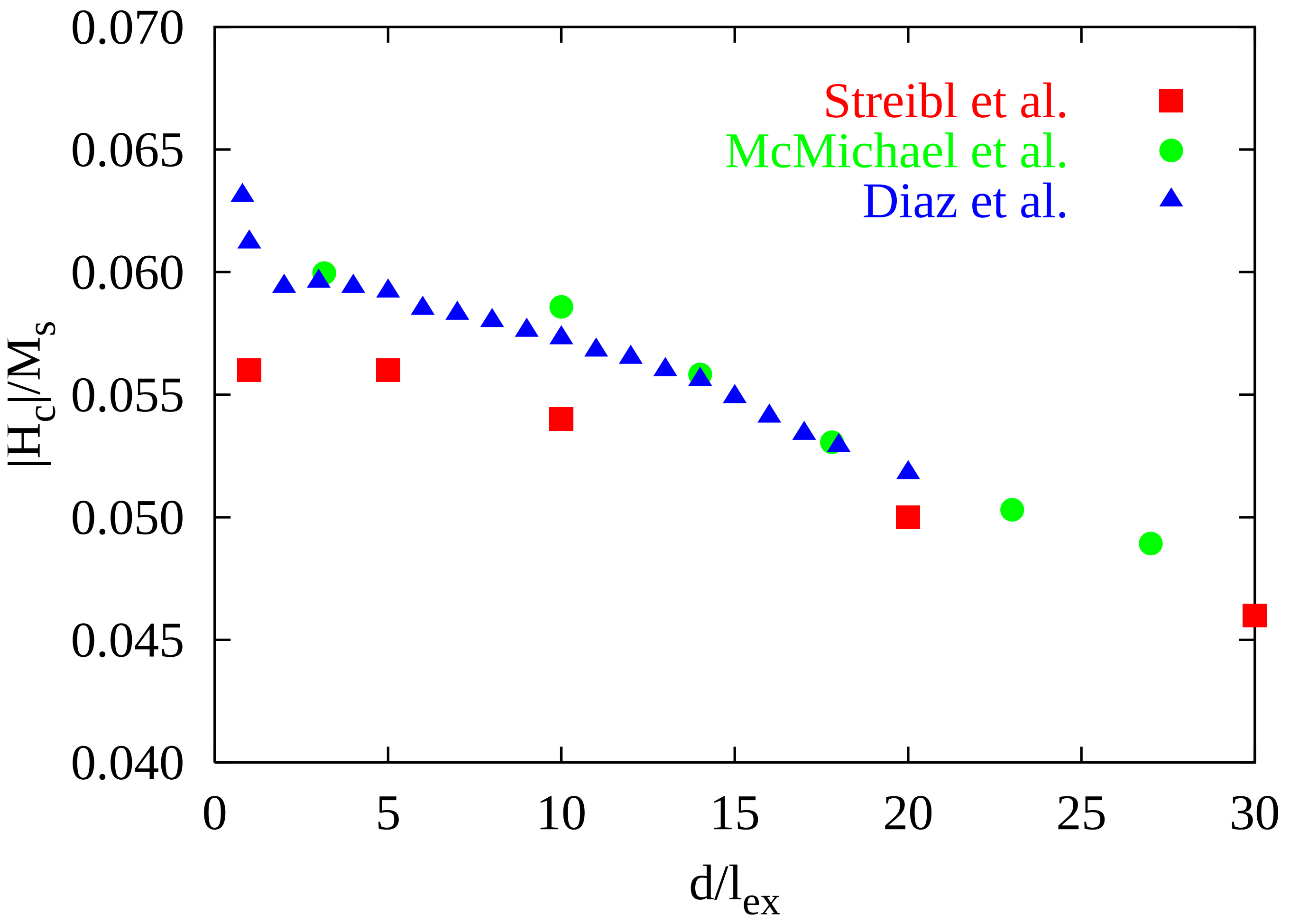


$L/d = 5.0,\ t/d = 0.1$

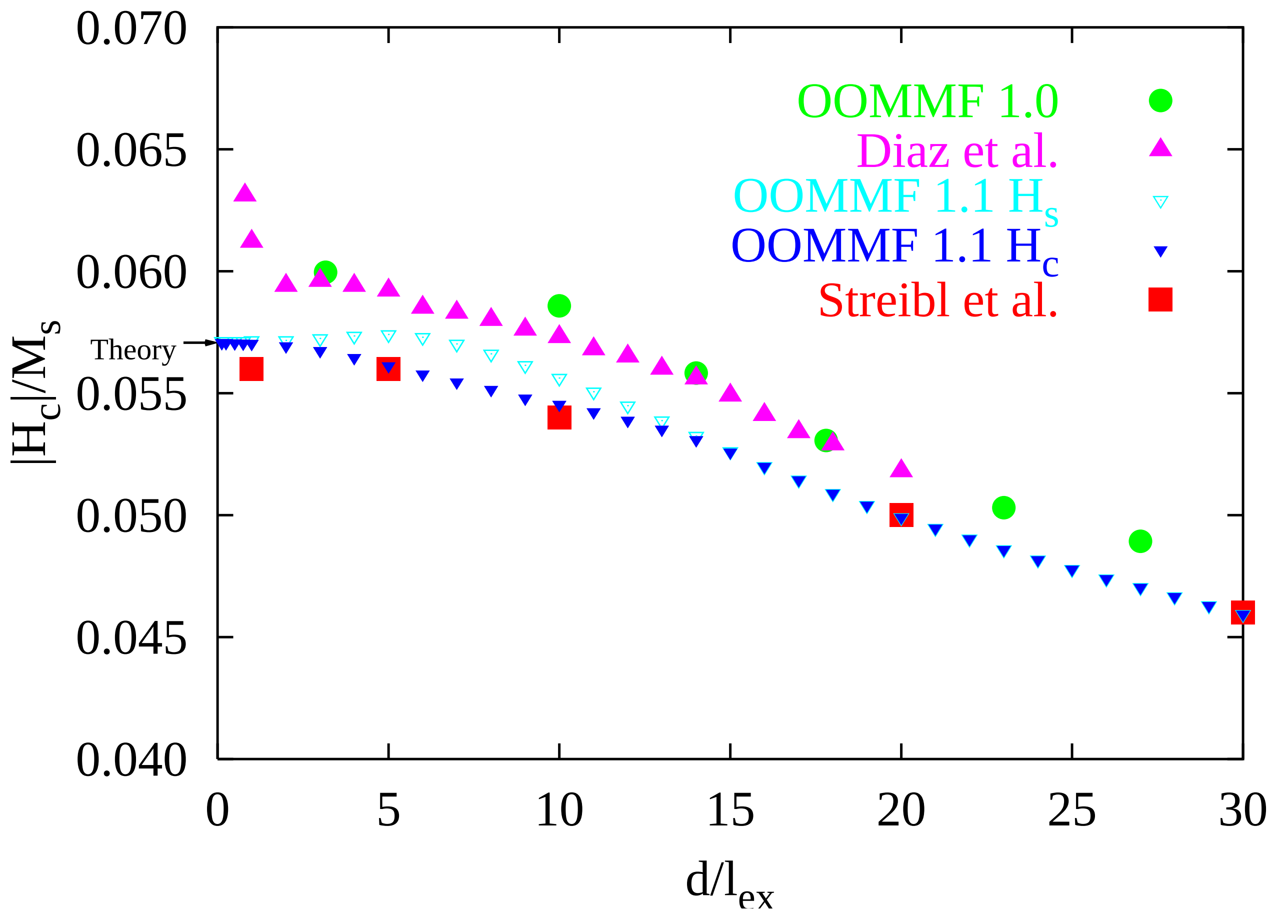
Exchange length $l_{ex} = (A/K_m)^{1/2}$

Mag. energy density $K_m = \mu_o M_s^2/2$

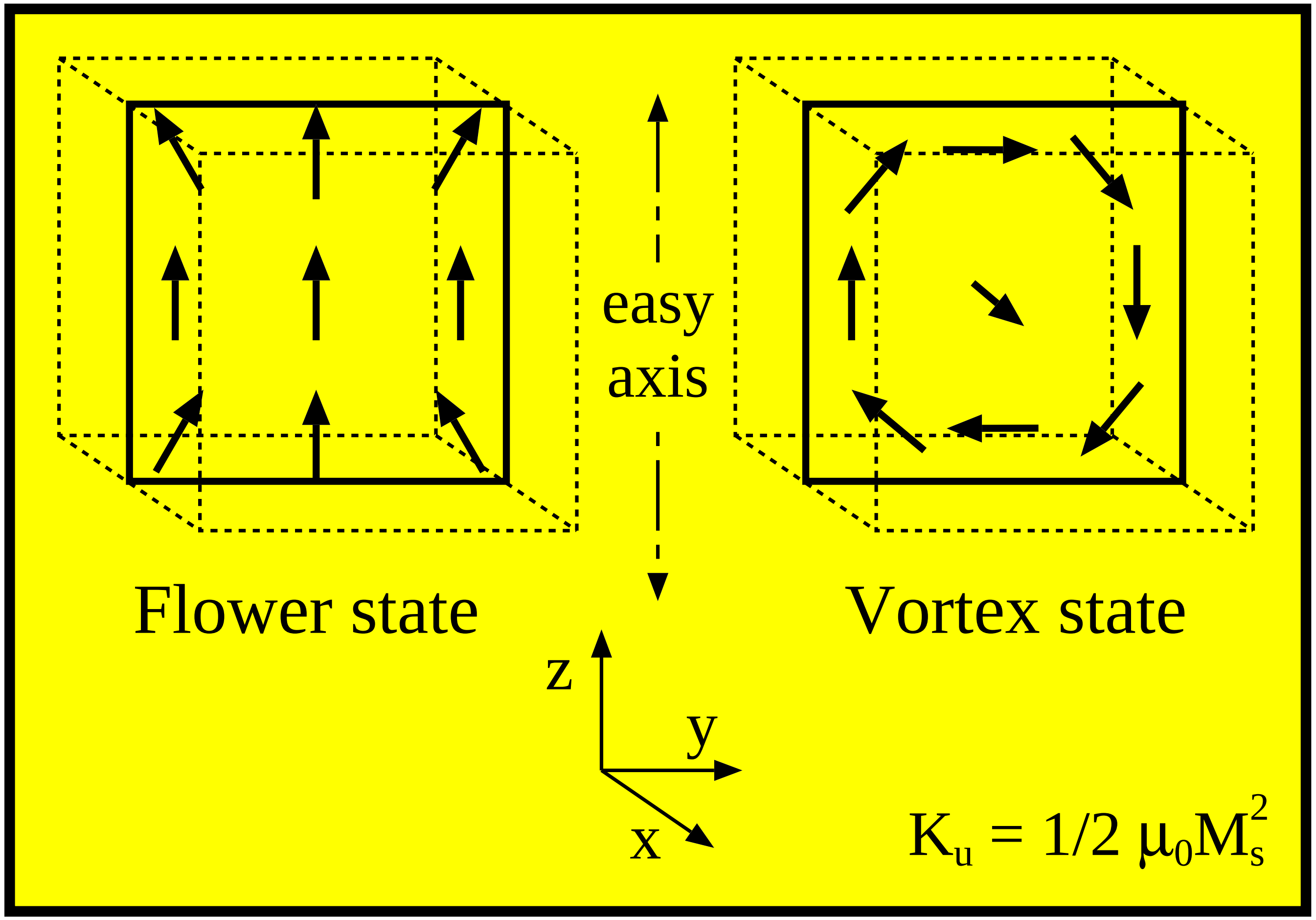
μMAG Problem #2, Coercivities



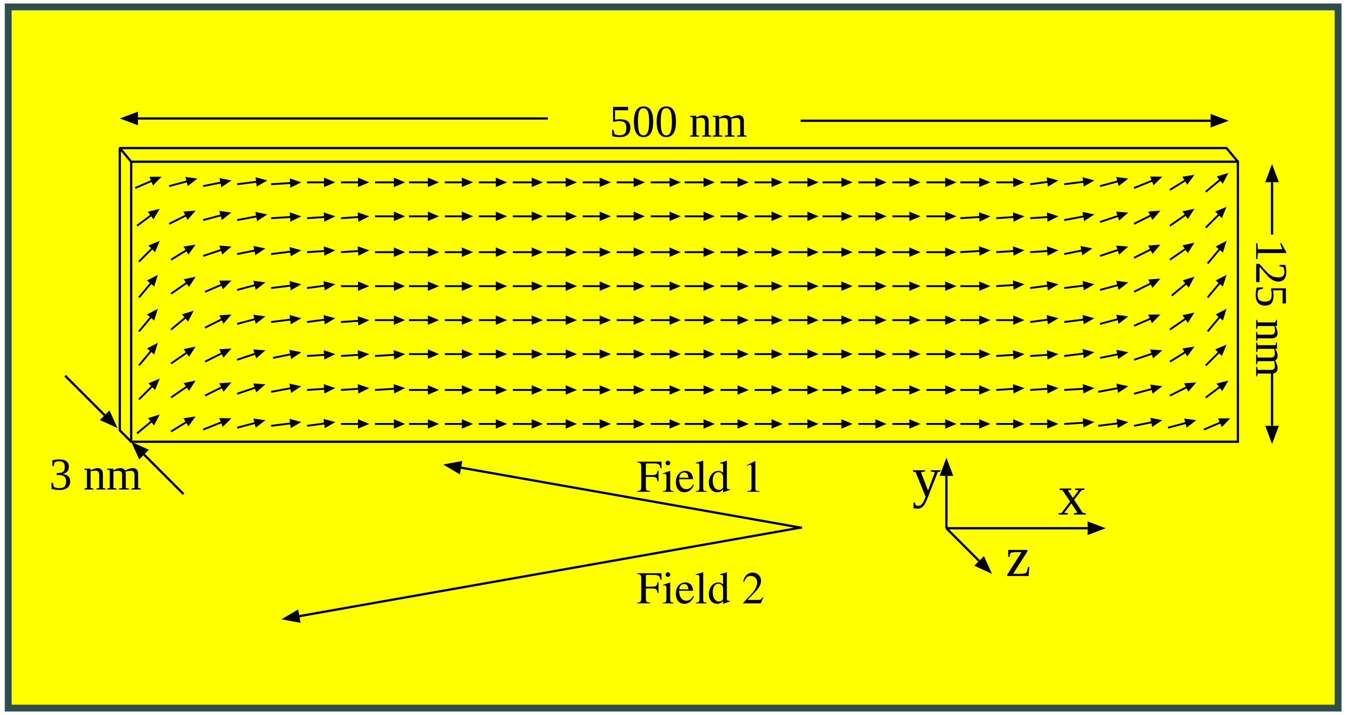
μMAG Problem #2, Coercivities



μMAG Problem #3 Specification



μMAG Problem #4 Specification

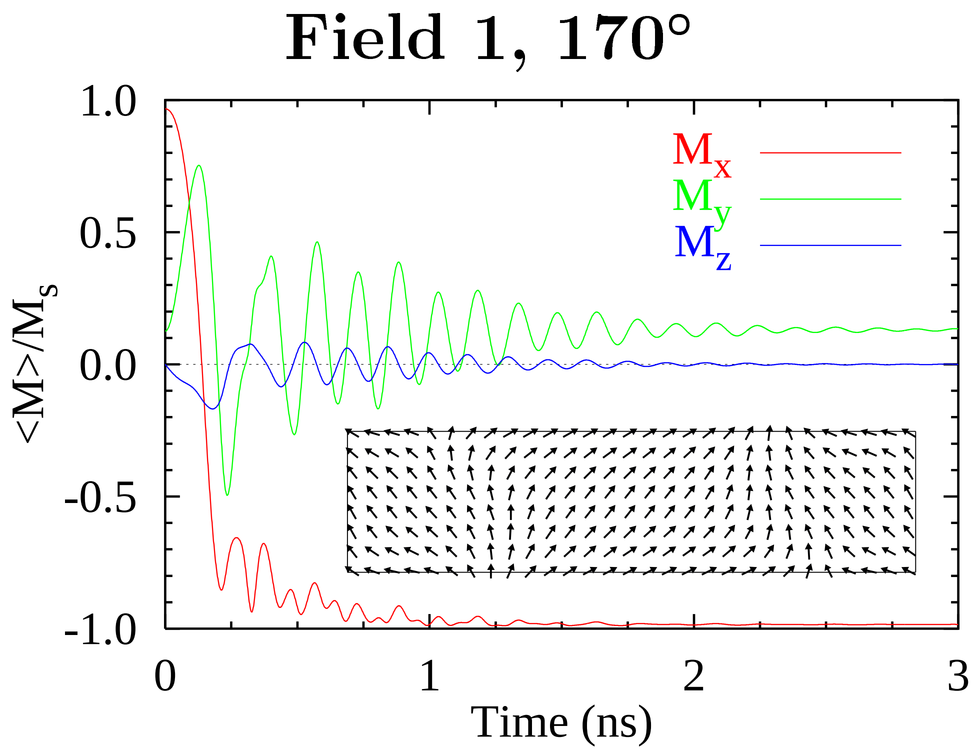


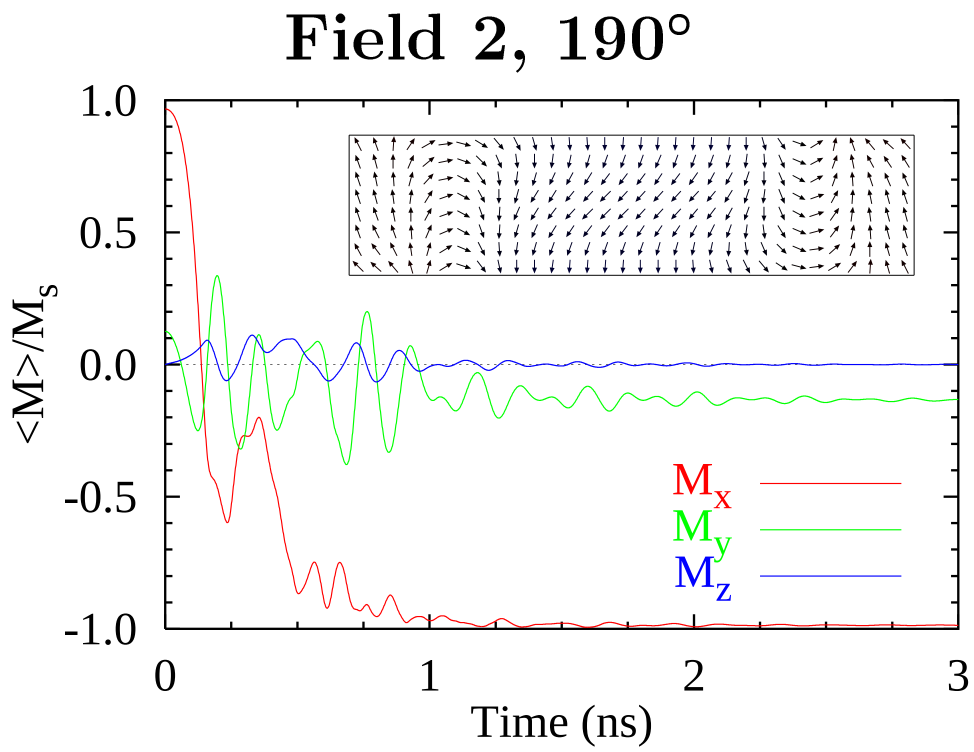
Permalloy parameters

Field 1: $\mu_0 H = 25$ mT at 170°

Field 2: $\mu_0 H = 36$ mT at 190°

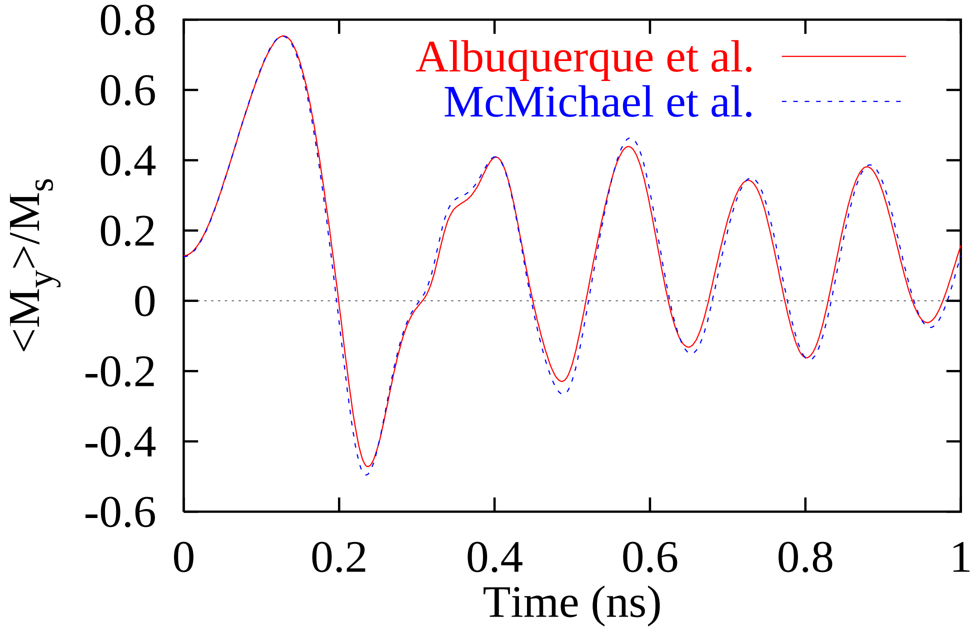
(Fields are approximately 1.5 times the quasi-static coercivity.)


Field 1, 170°
1.0
0.5
0.0
-0.5
-1.0
<M>/Ms
0
1
2
3
Time (ns)
Mx
My
Mz

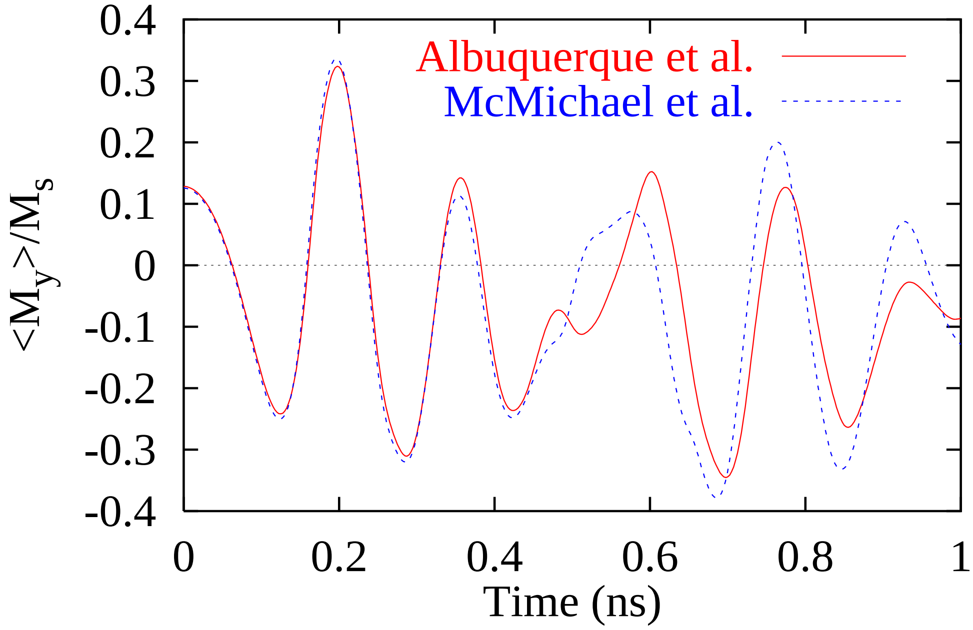

Field 2, 190°
1.0
0.5
0.0
-0.5
-1.0
<M>/Ms
0
1
2
3
Time (ns)
Mx
My
Mz

Prob. 4 Comparisons

Field 1 (170°)



Field 2 (190°)



Max angle data

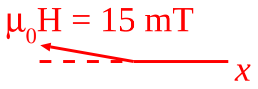
- Field 1, 2.5 nm cells, 8-dot exchange: 22.3°
- Field 2, 3.125 nm cells, 8-dot exchange: 118°
- Field 2, 2.5 nm cells, 8-dot exchange: 71.9°
- Field 2, 3.125 nm cells, 4-ang exchange: 48.1°

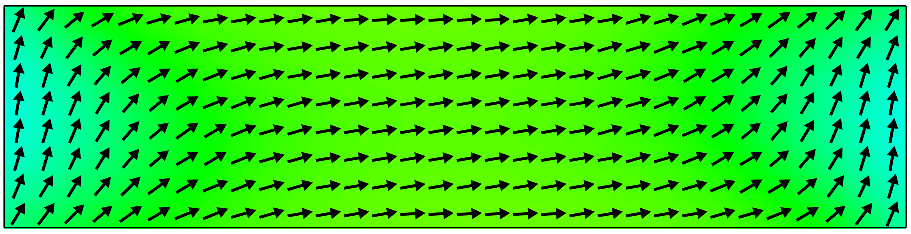
Exchange reference:

Exchange Energy Representations in Computational Micromagnetics,
M. J. Donahue and R. D. McMichael,
Physica B, **233**, pp 272-278 (1997).

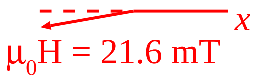
Quasi-Static Equilibria

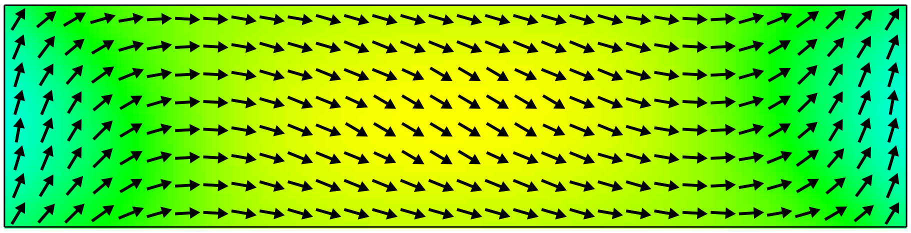
Field 1, $H_{\mathrm{app}} = 15$ mT





Field 2, $H_{\mathrm{app}} = 21.6$ mT

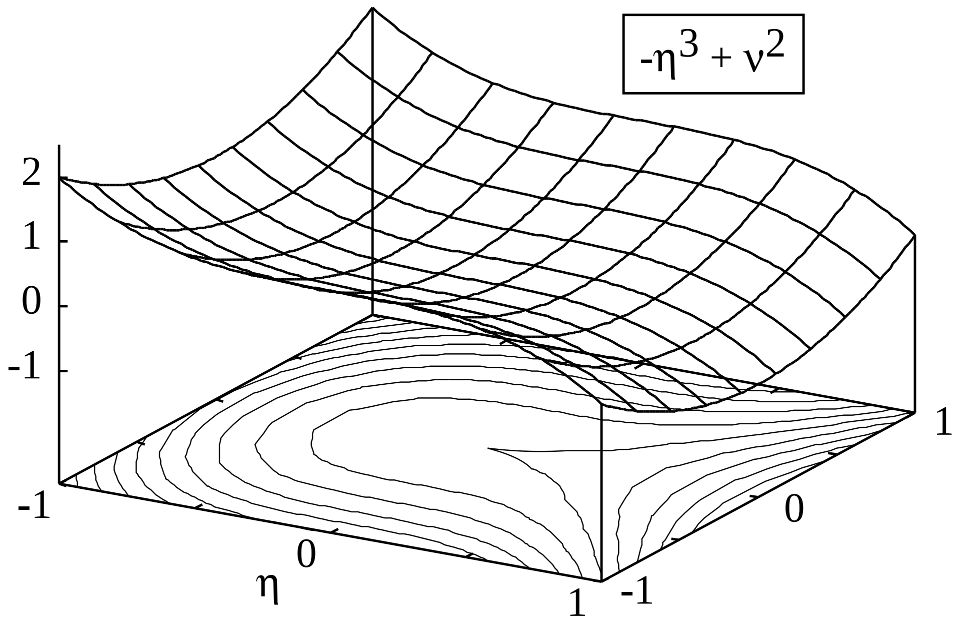




Single Spin: Energy Surface

At critical field $\mathbf{H}_c$ assume

$$E_c = -U\eta^3 + K\nu^2$$



Let $\mathbf{h} = \mathbf{H} - \mathbf{H}_c$, then

$$\begin{aligned} E &= -U\eta^3 + K\nu^2 \\ &\quad -\mu_0 M \left[h_\eta \eta + h_\nu \nu + h_\zeta \left(1 - \frac{1}{2}\eta^2 - \frac{1}{2}\nu^2 \right) \right] \end{aligned}$$

and

$$\eta_{\text{eq}} = \frac{\mu_0 M}{6U} \left[h_\zeta - \sqrt{h_\zeta^2 - \frac{12 U h_\eta}{\mu_0 M}} \right].$$

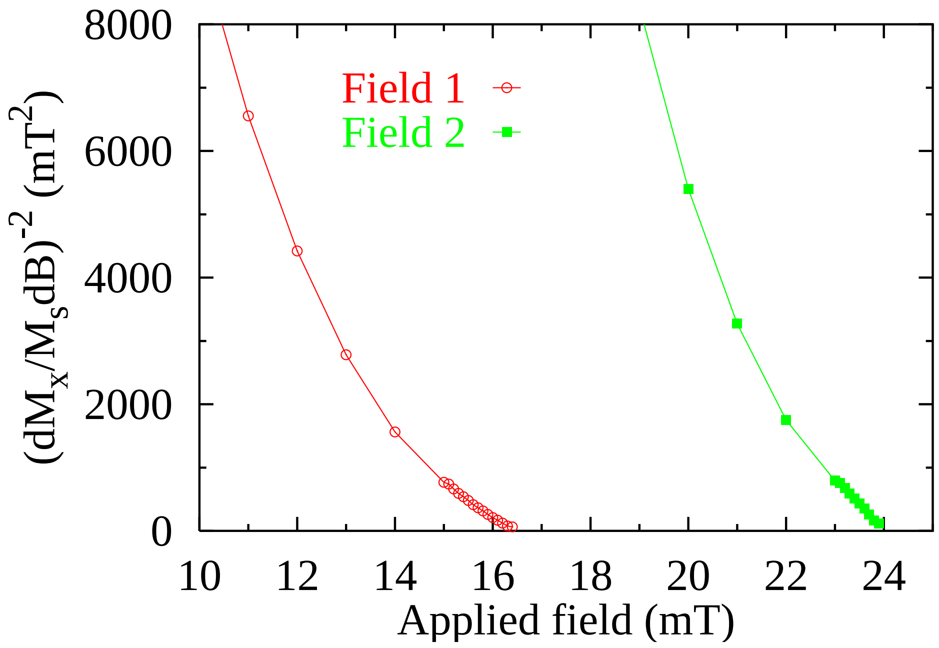
Critical Field Susceptibility

Divergent susceptibility $\chi_{\eta,\eta}$ is given by

$$\chi_{\eta,\eta} = M\frac{\partial \eta_{\mathrm{eq}}}{\partial h_\eta} = M\left[h_\zeta^2 - \frac{12U}{\mu_0 M}h_\eta\right]^{-1/2}.$$

If $h_\zeta^2 \ll 12Uh_\eta/\mu_0 M$, then

$$\chi_{\eta,\eta} \propto h_\eta^{-1/2} \qquad \text{as } |\mathbf{h}| \downarrow 0.$$



See also: A. Hubert and W. Rave, *Physica Status Solidi B*, **211**, pp 815–829 (1999).

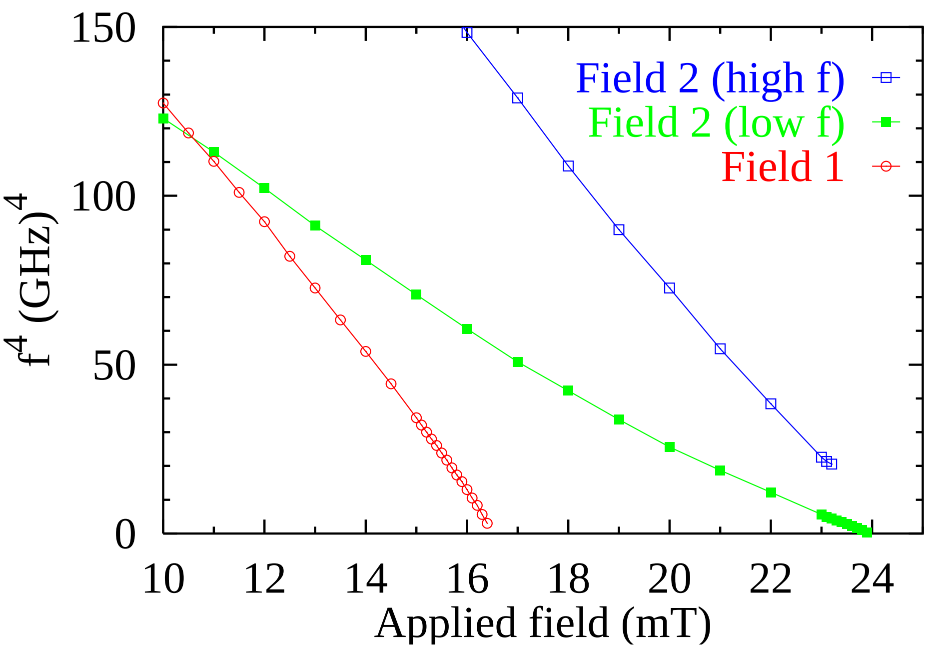
Ring Down Frequency

Precession frequency f is given by

$$\begin{aligned} f &= \frac{\gamma}{2\pi\mu_0 M}\left[\frac{\partial^2 E}{\partial\eta^2}\frac{\partial^2 E}{\partial\nu^2} - \left(\frac{\partial^2 E}{\partial\eta\partial\nu}\right)^2\right]^{1/2}\Bigg|_{\eta_{\rm eq}\nu_{\rm eq}} \\ &= \frac{\gamma}{2\pi}\left[h_\zeta + \frac{2K}{\mu_0 M}\right]^{1/2}\left[h_\zeta^2 - \frac{12U}{\mu_0 M}h_\eta\right]^{1/4}. \end{aligned}$$

If $h_\zeta^2 \ll 12Uh_\eta/\mu_0 M$, then

$$f \propto h_\eta^{1/4} \qquad \text{as } |\mathbf{h}| \downarrow 0.$$



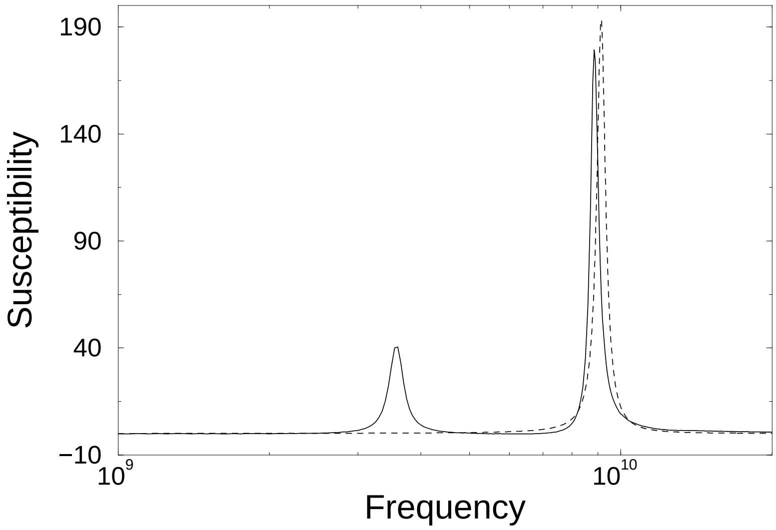
High Frequency Susceptibility

Define susceptibility χ by

$$< \mathbf{M}(t) > \cdot \mathbf{u} = \int_{-\infty}^{+\infty} \chi(t-t')[\mathbf{h}(t') \cdot \mathbf{u}]dt',$$

or in Fourier domain

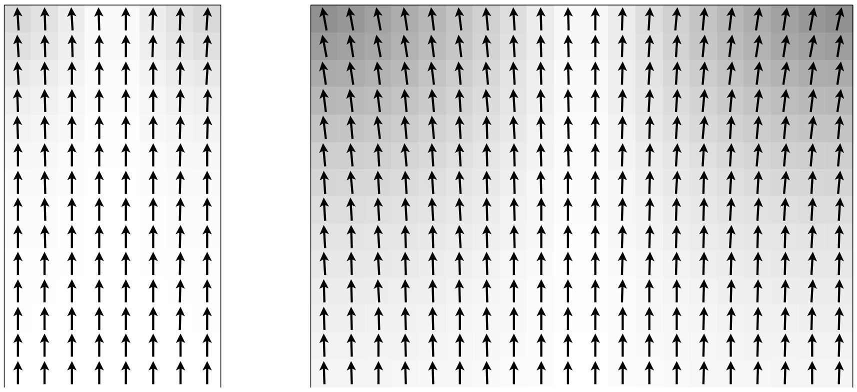
$$M(t) = \chi(t) \star h(t) \leftrightarrow M(\omega) = \chi(\omega) \cdot h(\omega).$$



Permalloy stripe, 1 μm $\times$ 50 nm $\times$ 5 nm.

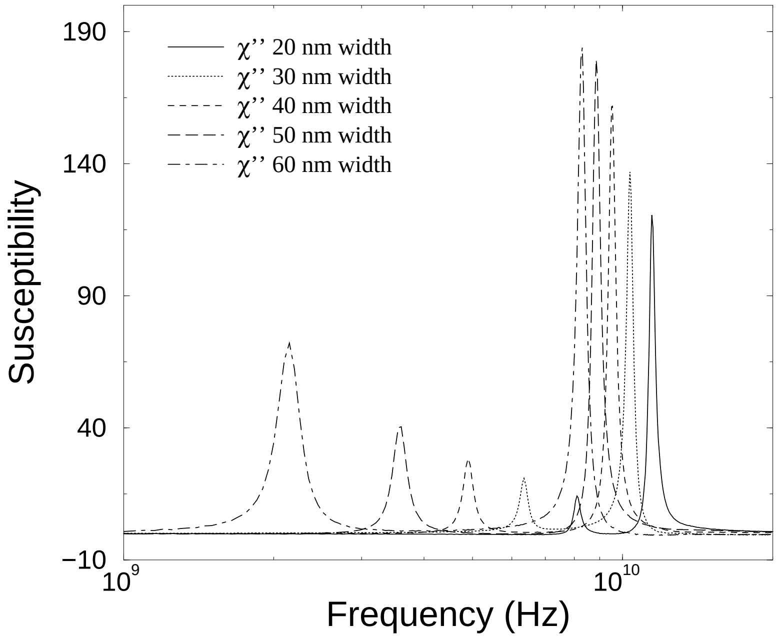
Solid line is micromagnetic simulation.

Stripe Width Effects

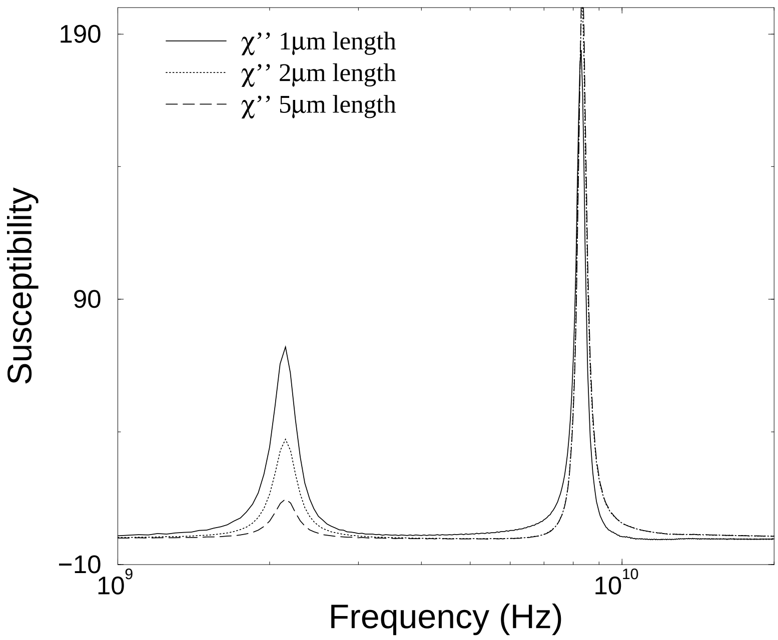


20 nm

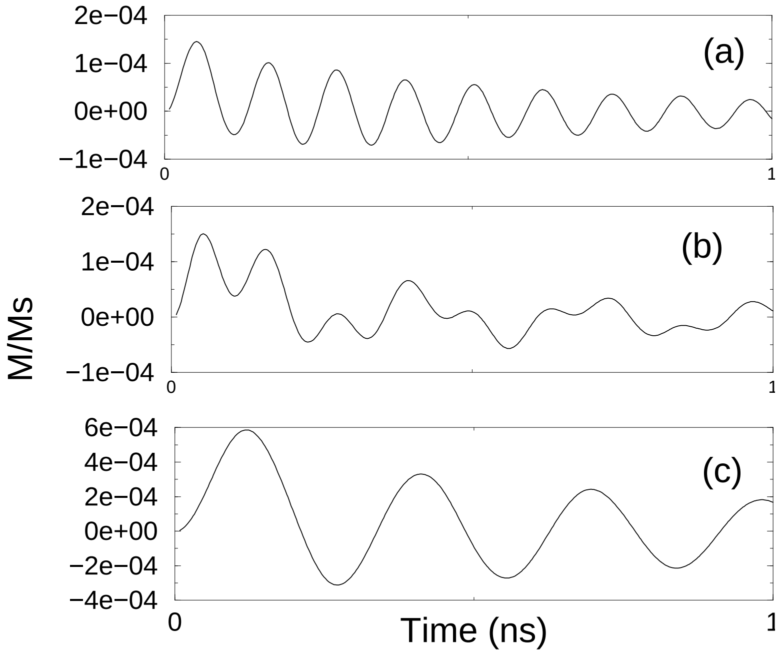
50 nm



Stripe Length Effects



Individual Spin Dynamics

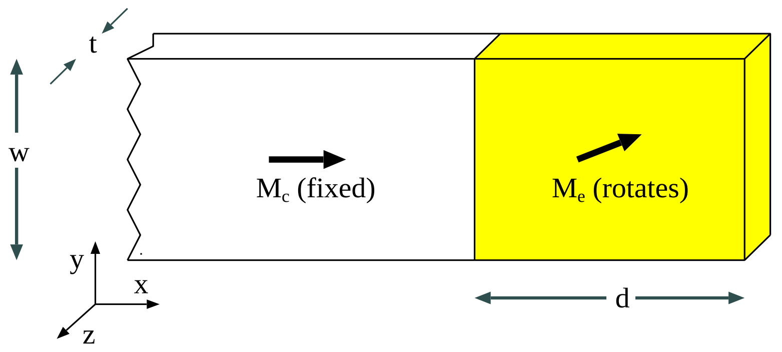


(a) Center spin

(b) Intermediate spin

(c) End spin

Simple Edge Spin Model



$$
\begin{aligned}
H_{\text{exch}} &= \left(2A/(\mu_0 M_s d^2), 0, 0\right) \\
H_{\text{demag}} &= (-N_x M_x/2, -N_y M_y, -N_z M_z)
\end{aligned}
$$

Then

$$\text{Freq} = \frac{\gamma M_s}{2\pi}\sqrt{\left(\frac{2A}{\mu_0 M_s^2 d^2} - \frac{N_x}{2} + N_y\right)\left(\frac{2A}{\mu_0 M_s^2 d^2} - \frac{N_x}{2} + N_z\right)}$$

where

$$d \approx 4\, l_{\text{ex}} = 4\sqrt{2A/\mu_0 M_s^2}$$

Simple Model Results

Part width	Frequency (GHz)
20	9.6
30	7.8
40	6.5
50	5.4
60	4.5

Conclusions

- μMAG Problem 4 exhibits 2 reversal modes:

  $\boldsymbol{H}_{\mathbf{app,170^\circ}}$**:** End domains propagate towards center.

  $\boldsymbol{H}_{\mathbf{app,190^\circ}}$**:** Counter-rotating domains, 360° walls, complex dynamics.

- Near critical field H_c:
  - $\chi \propto (H_c - H)^{-1/2}$ for small range of H.
  - $f \propto (H_c - H)^{1/4}$ for wider range of H.

- Remanence dynamics:
  - Distinct central core and end domain behavior.
  - End domain frequency especially dependent on stripe width.

References

Micromagnetics of the Dynamic Susceptibility for Coupled Permalloy Stripes, O. Gérardin, J. Ben Youssef, H. Le Gall, N. Vukadinovic, P. M. Jacquart and M. J. Donahue, to appear in *Journal of Applied Physics.*

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Additional References

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Systematic Analysis of Micromagnetic Switching Processes, A. Hubert and W. Rave, *Physica Status Solidi B*, **211**, pp 815–829 (1999).

Web Pages

- Home Page:

  http://math.nist.gov/~MDonahue/

- OOMMF:

  http://math.nist.gov/oommf/

- μMAG:

  http://www.ctcms.nist.gov/~rdm/mumag.org.html