Motion of magnetic domain walls in thin, narrow strips

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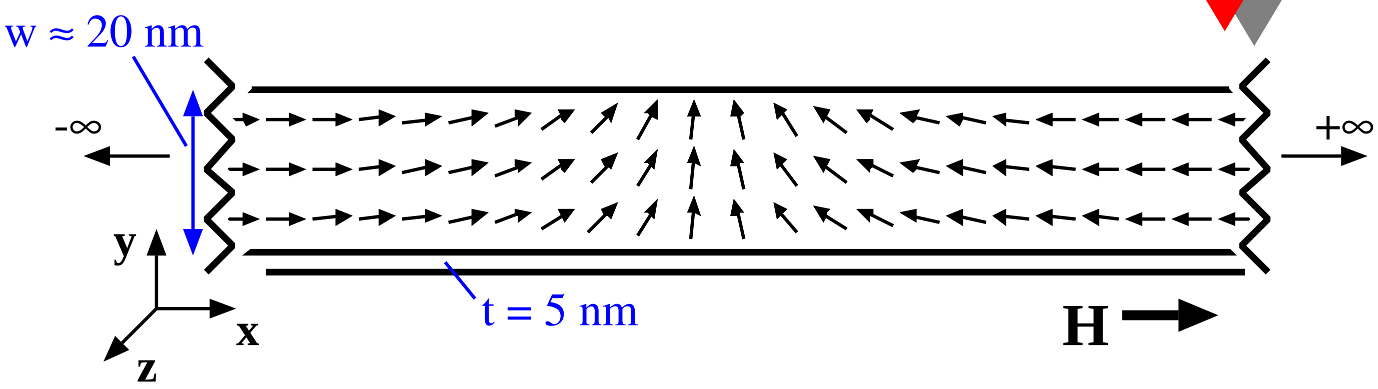
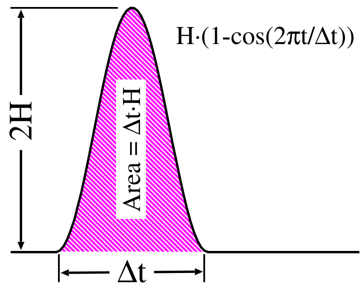
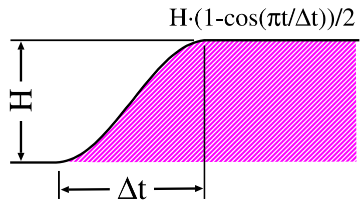


- Finite field pulse
  - $\alpha = 0$
  - $\alpha \ll 1$

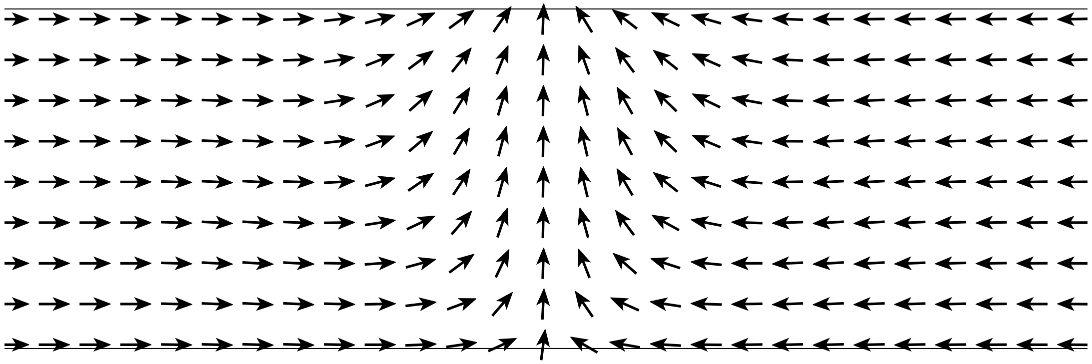


- Field step
  - $\alpha = 0$
  - $\alpha \ll 1$

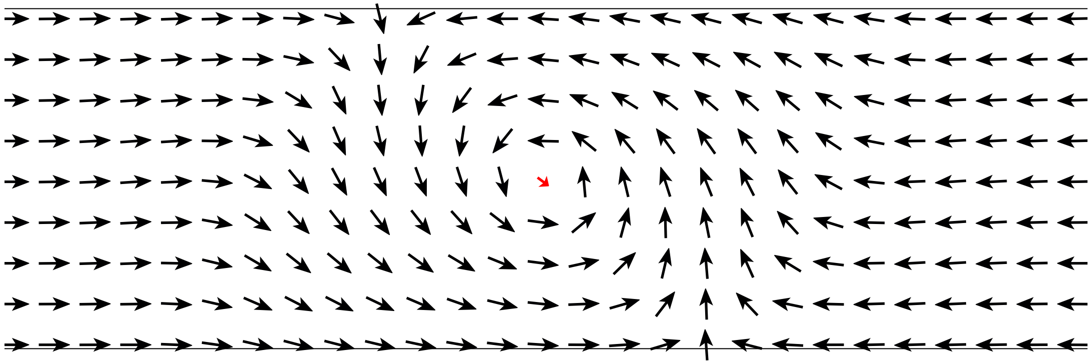


$\Rightarrow$ micromagnetics + analysis (no experiment)

Domain wall types



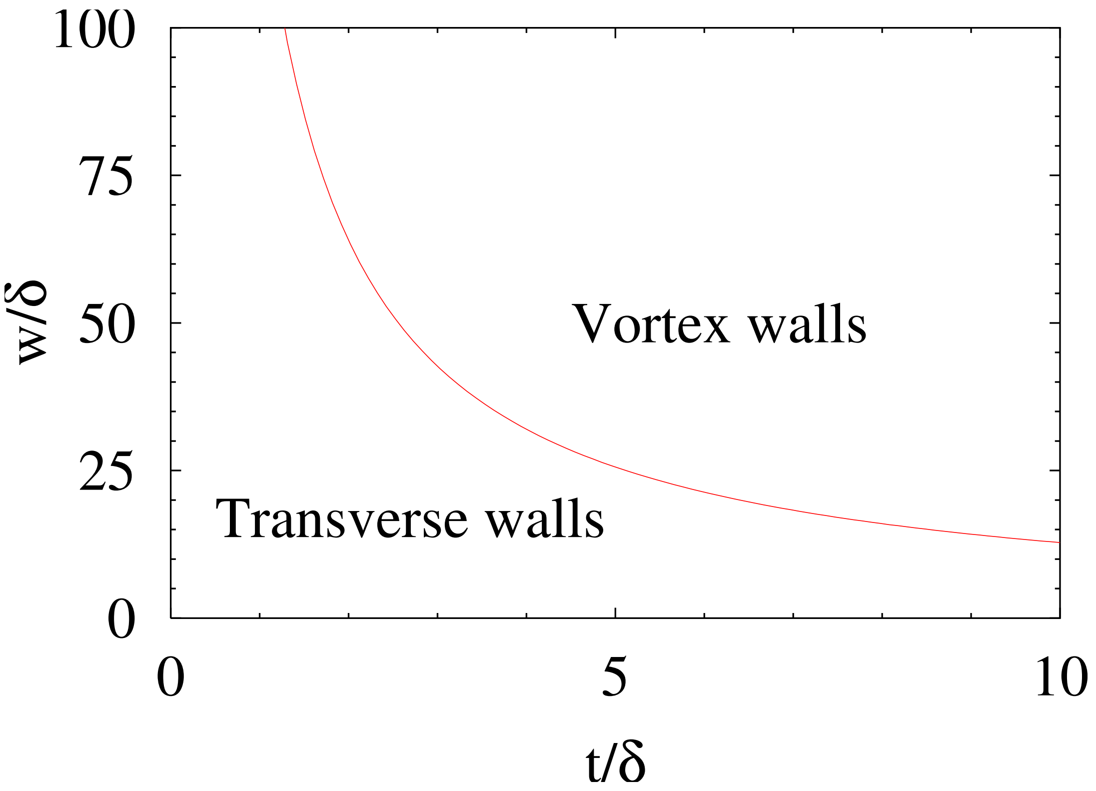
Transverse wall



Vortex wall

See L. Lopez-Diaz, J. Sanchez, et al., “Computational study of domain wall mobility in nanowires of rectangular cross section,” unpublished.

Wall phase diagram



R. D. McMichael and M. J. Donahue, *IEEE Trans. Magn.*, **33**, 4167–4169 (1997).

Constitutive Equations

Energies:

$$E_{\text{exchange}} = \int_V \frac{A}{M_s^2}\left(|\nabla M_x|^2 + |\nabla M_y|^2 + |\nabla M_z|^2\right)\, d^3r$$

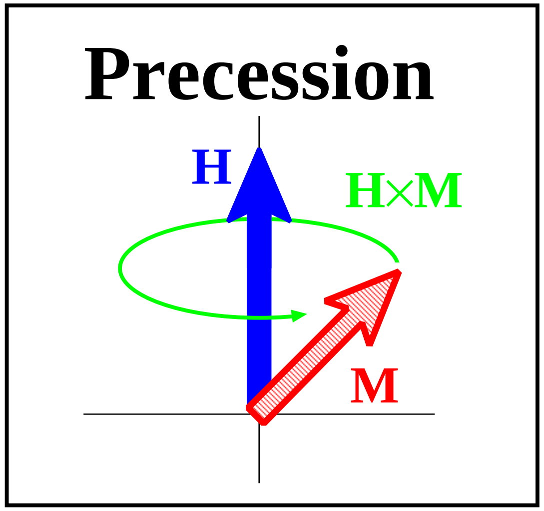
$$\begin{aligned} E_{\text{demag}} &= \frac{\mu_0}{8\pi}\int_V \mathbf{M}(r)\cdot\left[\int_V \nabla\cdot\mathbf{M}(\mathbf{r}')\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}\, d^3r' \right. \\ &\qquad \left. - \int_S \hat{\mathbf{n}}\cdot\mathbf{M}(\mathbf{r}')\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}\, d^2r'\right] d^3r \end{aligned}$$

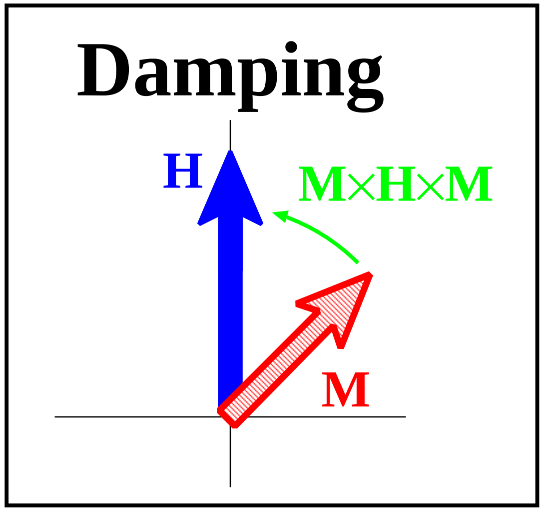
$$E_{\text{Zeeman}} = -\mu_0 \int_V \mathbf{M}\cdot\mathbf{H}_{\text{applied}}\, d^3r$$

Magnetization Dynamics

Landau-Lifshitz-Gilbert:

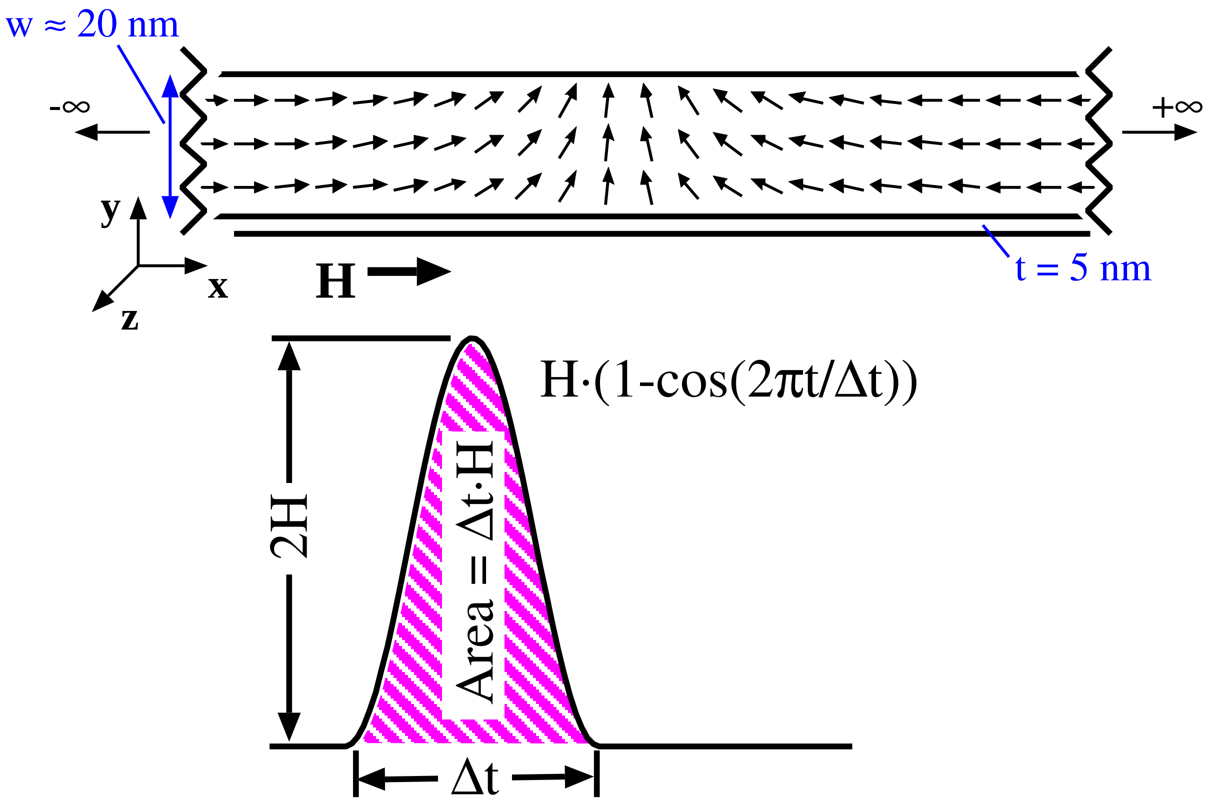
$$\frac{d\mathbf{M}}{dt} = \frac{|\omega|}{1+\alpha^2}\mathbf{H}_{\text{eff}} \times \mathbf{M} + \frac{\alpha\,|\omega|}{(1+\alpha^2)M_{\text{s}}}\mathbf{M} \times \mathbf{H}_{\text{eff}} \times \mathbf{M}$$





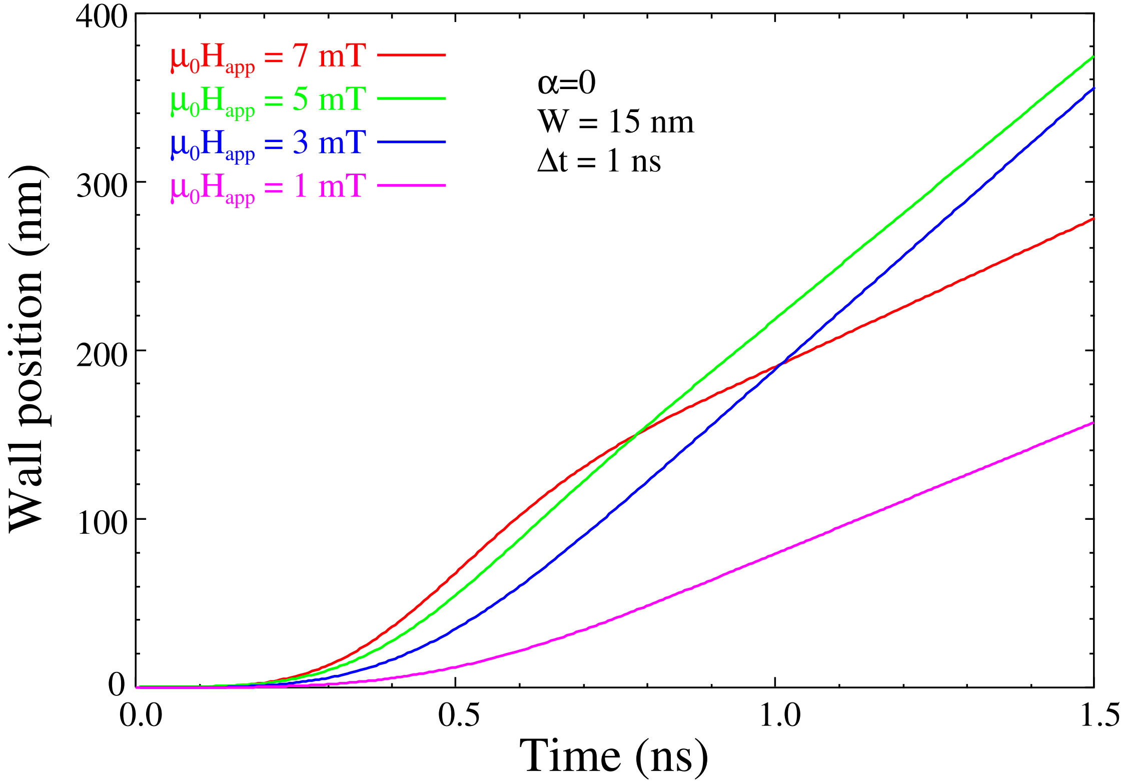
$$\mathbf{H}_{\text{eff}} = -\frac{1}{\mu_0}\frac{\partial E}{\partial \mathbf{M}}$$

Applied Field Pulse



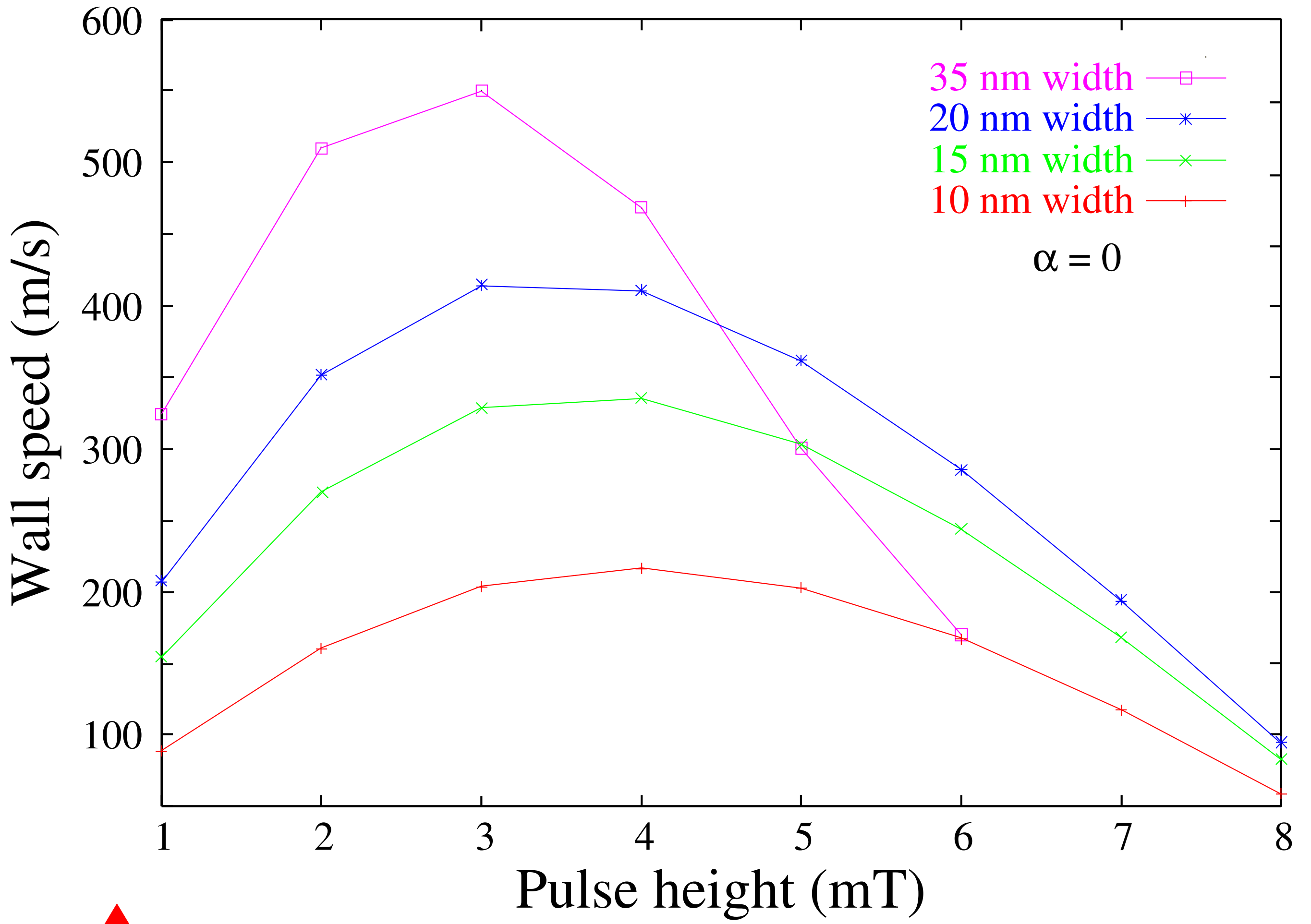
- $\Delta t = 1$ ns.

Pulse-Driven Wall Motion

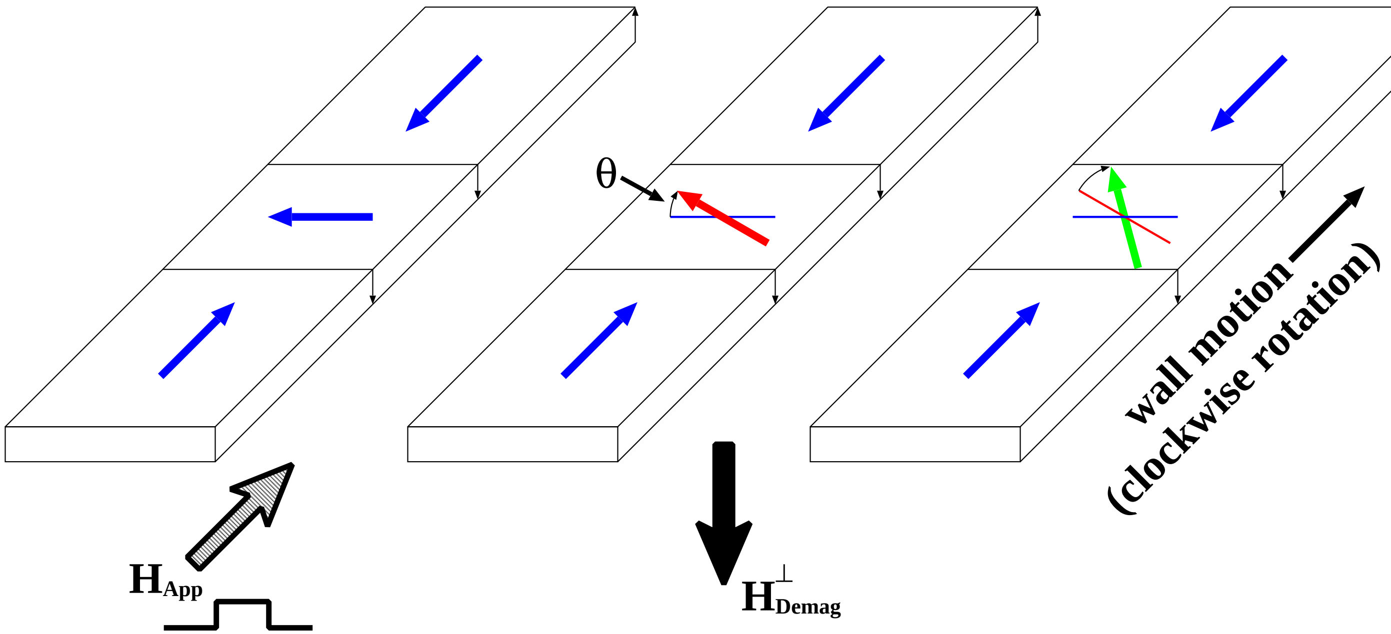


- Wall momentum: motion continues after pulse
- Velocity non-monotonic in H_{applied}

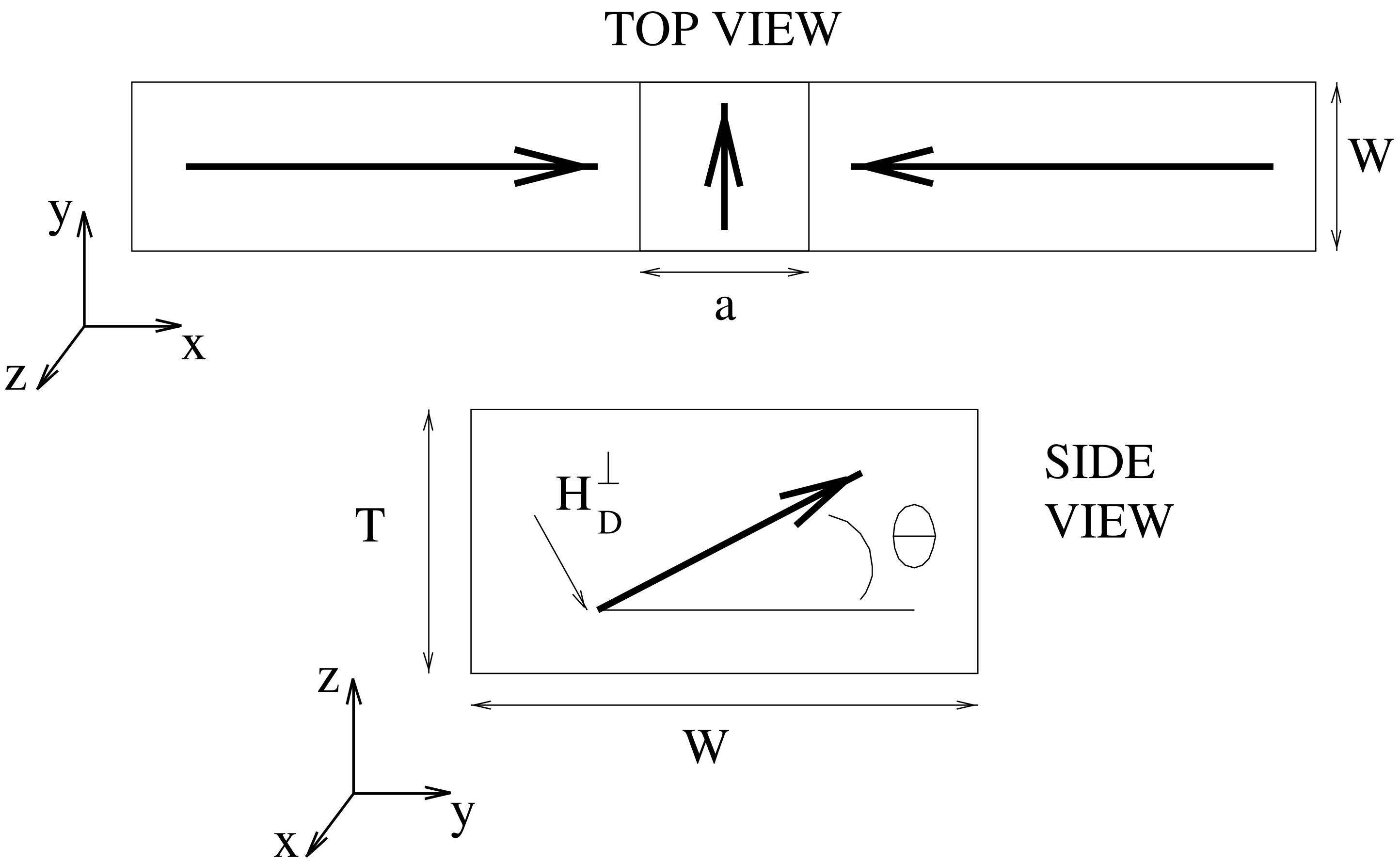
Pulse-Driven Wall Velocity



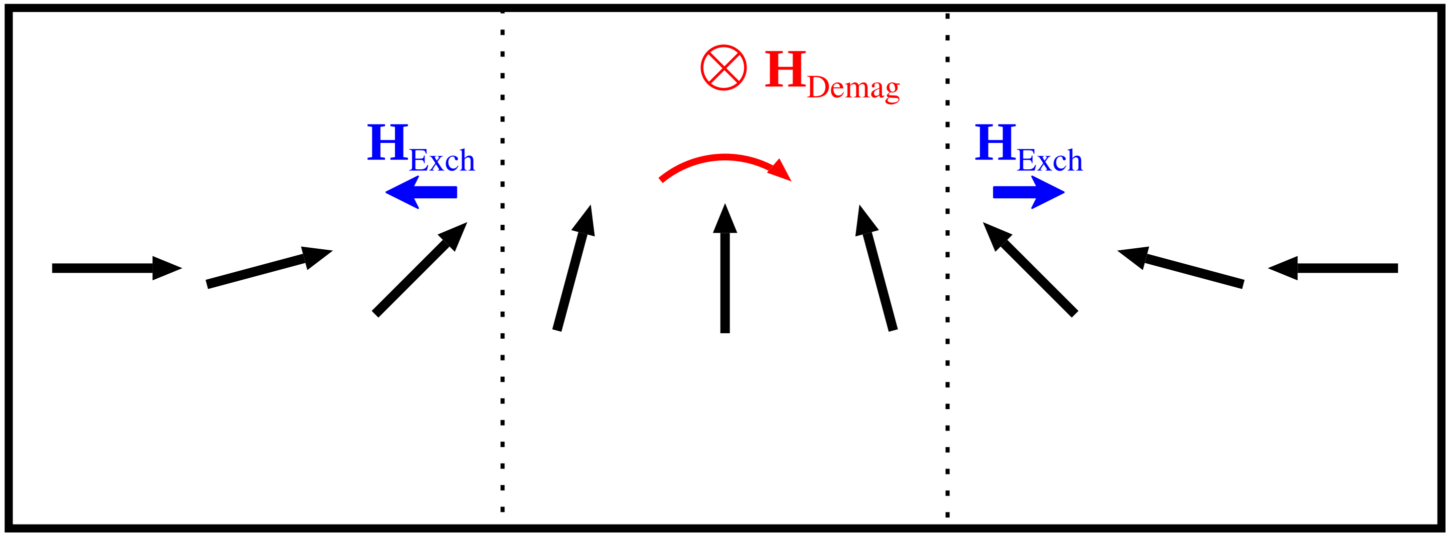
Wall Motion: $\mathbf{H}_{\text{applied}} + \mathbf{H}_{\text{demag}}$



Three Spin Model



Wall Motion: Exchange

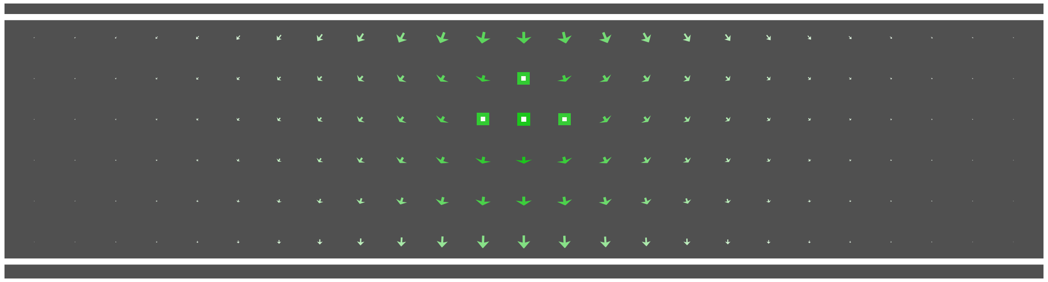


$\mathbf{H}_{\text{Exch}} \times \mathbf{M}$ **= into plane**

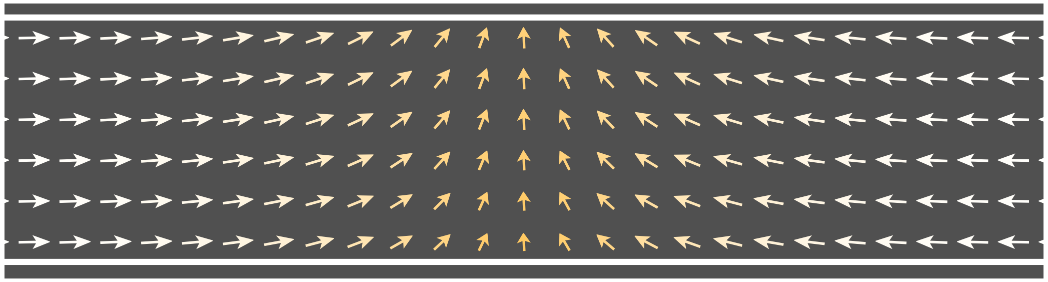
$\mathbf{H}_{\text{Exch}} \times \mathbf{M}$ **= out of plane**

Wall Motion Snapshots

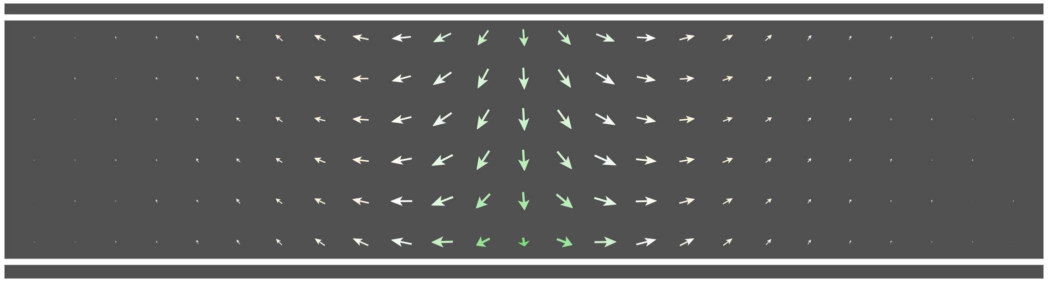
Demag field
Green: into plane



Magnetization
Orange: out of plane



Exchange field



Domain Wall Width

- Minimize exchange + demag energy:

$$a = a(\theta) = 1.15\pi\sqrt{\frac{A}{K_m(\theta)}}$$

$$K_m(\theta) = \frac{\mu_0 M_S^2}{2}\{f(\frac{W}{T})\cos^2\theta + f(\frac{T}{W})\sin^2\theta\}$$

$$f(\sigma) = 1 - \frac{2}{\pi}\tan^{-1}(\sigma) + \frac{1}{2\sigma\pi}\log(1+\sigma^2) - \frac{\sigma}{2\pi}\log(1+\sigma^{-2})$$

Theory: Equations of Motion

- Tilt angle θ of wall wrt xy-plane

$$\frac{d\theta}{dt} = |\gamma| \left(H_{\mathrm{app}} - \alpha H_{\mathrm{D}}^{\perp}\right) / \left(1 + \alpha^2\right)$$

- Demag field due to θ (N_y, N_z: demag factors of wall region)

$$H_{\mathrm{D}}^{\perp} = M_{\mathrm{s}} \left(N_z - N_y\right) \cos\theta \sin\theta$$

- Wall velocity: precess about demag + damp toward applied

$$v = v(\theta) = \gamma \left(a/\pi\right) \left(H_{\mathrm{D}}^{\perp} + \alpha H_{\mathrm{app}}\right) / \left(1 + \alpha^2\right)$$

Earlier work

- A. Thiaville, J. M. García, and J. Miltat, *J. Magn. Magn. Mater.*, **242**, 1061–1063, 2002.
- L. R. Walker, Bell Telephone Laboratories memorandum, 1956, unpublished.

Consequences (geometry)

$$H_{\mathrm{D}}^{\perp} = M_{\mathrm{s}}\,(N_z - N_y)\cos\theta\sin\theta$$

For square cross section:

$$N_z = N_y \implies H_{\mathrm{D}}^{\perp} = 0$$

$\implies$ No precessional motion*

$$\implies v = \alpha\gamma\,(a/\pi)\,H_{\mathrm{app}}/\left(1+\alpha^2\right)$$

(*Well, almost. . .)

Consequences (H_{app})

$$\frac{d\theta}{dt} = |\gamma| \left(H_{\mathrm{app}} - \alpha H_{\mathrm{D}}^{\perp} \right) / \left(1 + \alpha^2 \right)$$

leads to

$$\frac{d\theta}{dt} = 0 \iff H_{\mathrm{app}} = \alpha H_{\mathrm{D}}^{\perp}$$

$$\begin{aligned} \alpha H_{\mathrm{D}}^{\perp} &= \alpha M_{\mathrm{s}} \left(N_z - N_y \right) \cos\theta \sin\theta \\ &\le \alpha M_{\mathrm{s}} \left(N_z - N_y \right) / 2 \end{aligned}$$

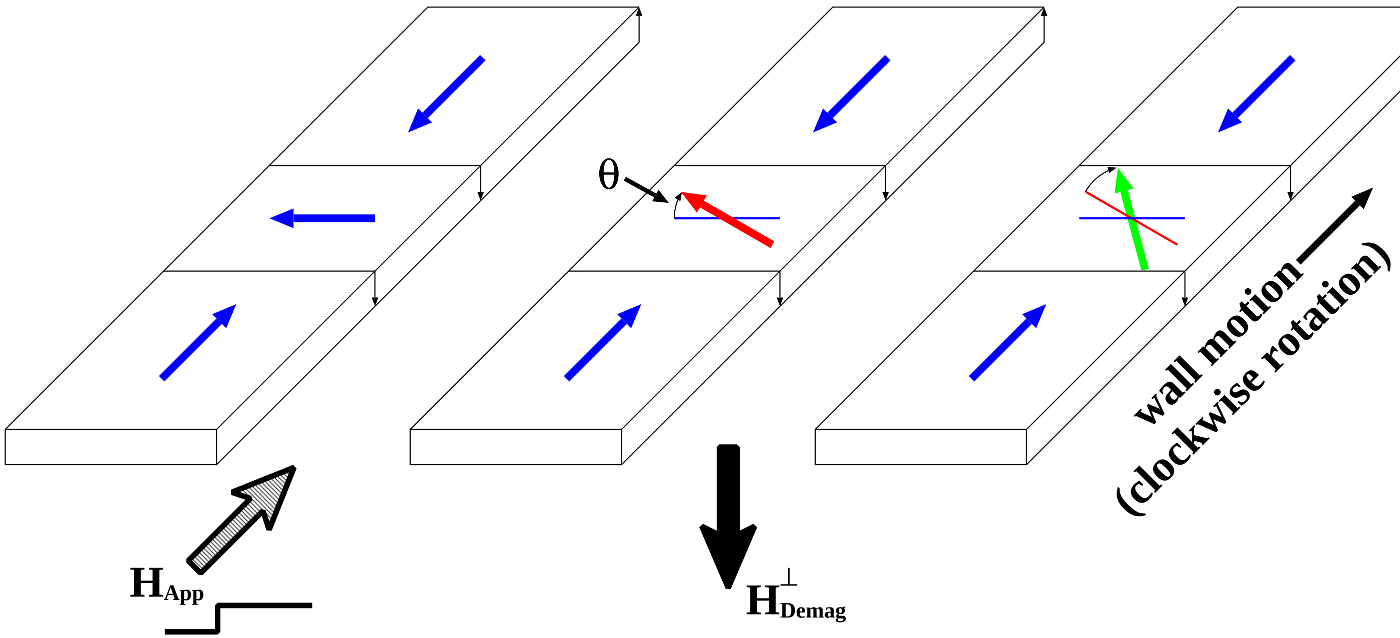
The “Walker field.”

Consequences (H_{app})

$$H_{\mathrm{app}} \quad > \quad \alpha M_{\mathrm{s}} \left(N_z - N_y\right)/2$$

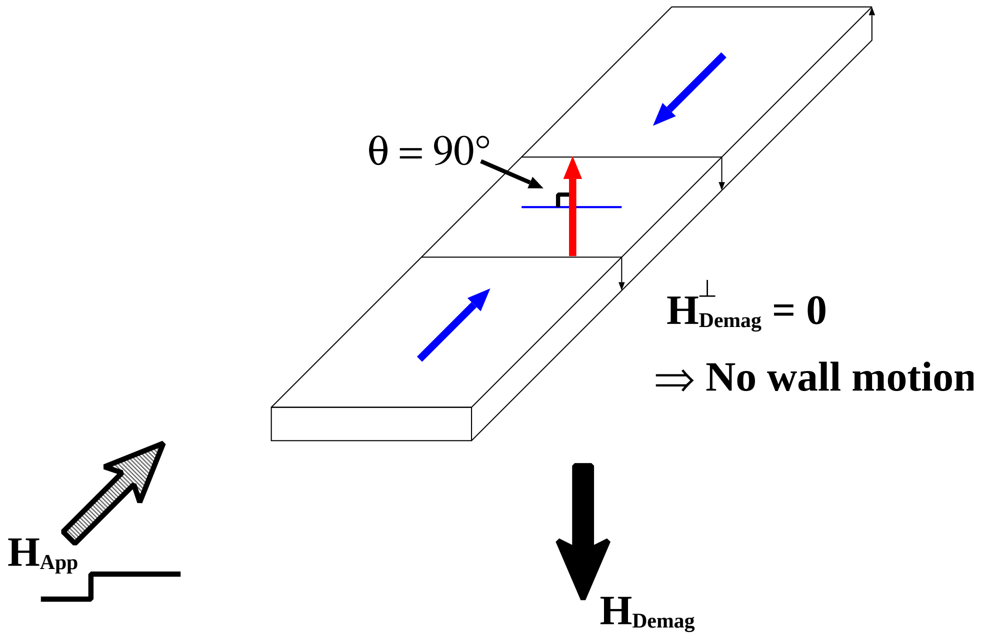
$$\Longrightarrow \quad \frac{d\theta}{dt} > 0 \quad \text{for all } t$$

Large θ

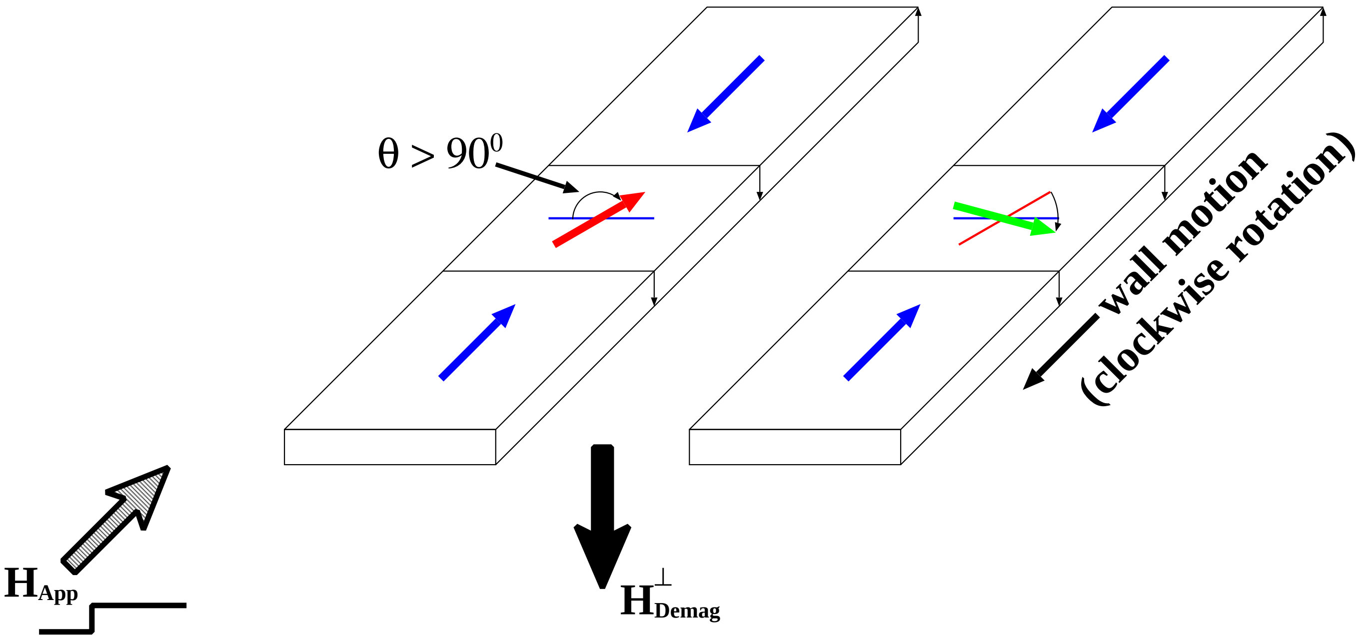


θ
wall motion
(clockwise rotation)
$\mathbf{H_{App}}$
$\mathbf{H^{\perp}_{Demag}}$

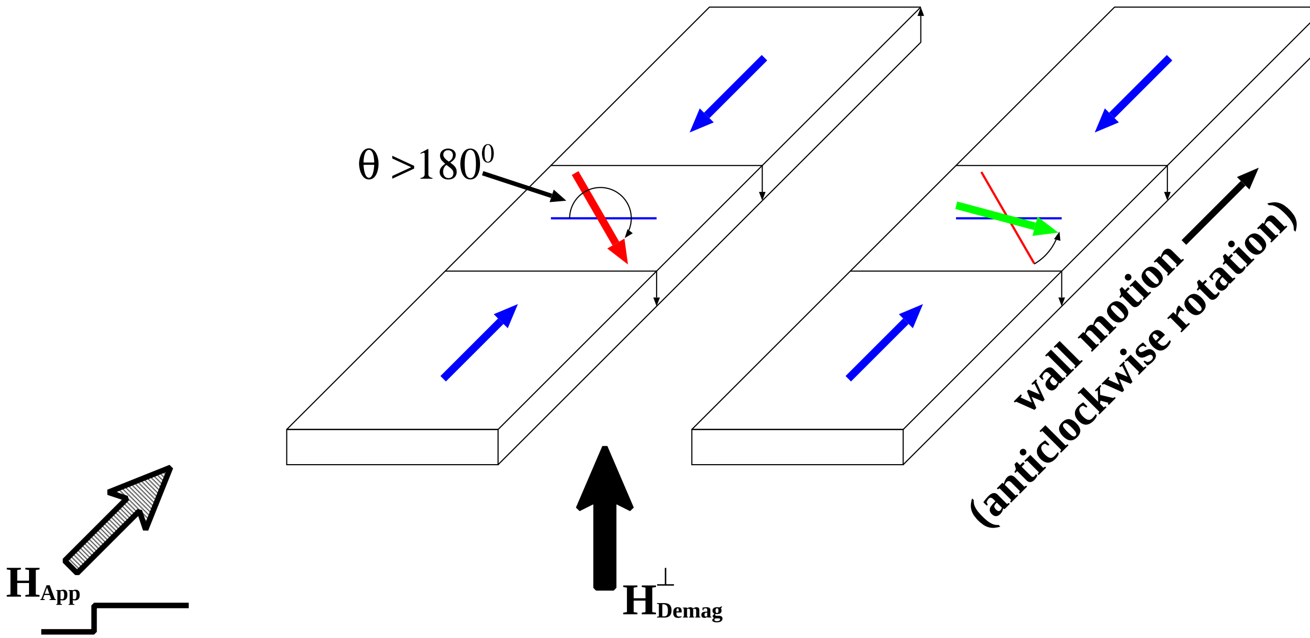
$\theta = 90°$



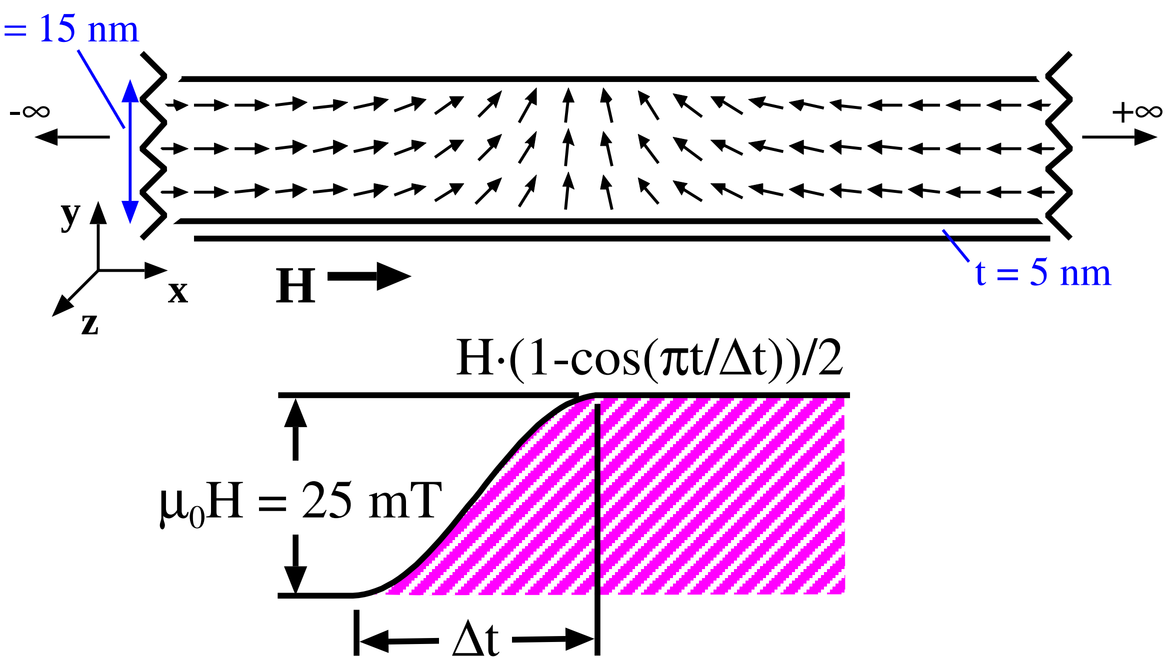
Retrograde motion ($\theta > 90^\circ$)



Retrograde motion ($\theta > 180°$)

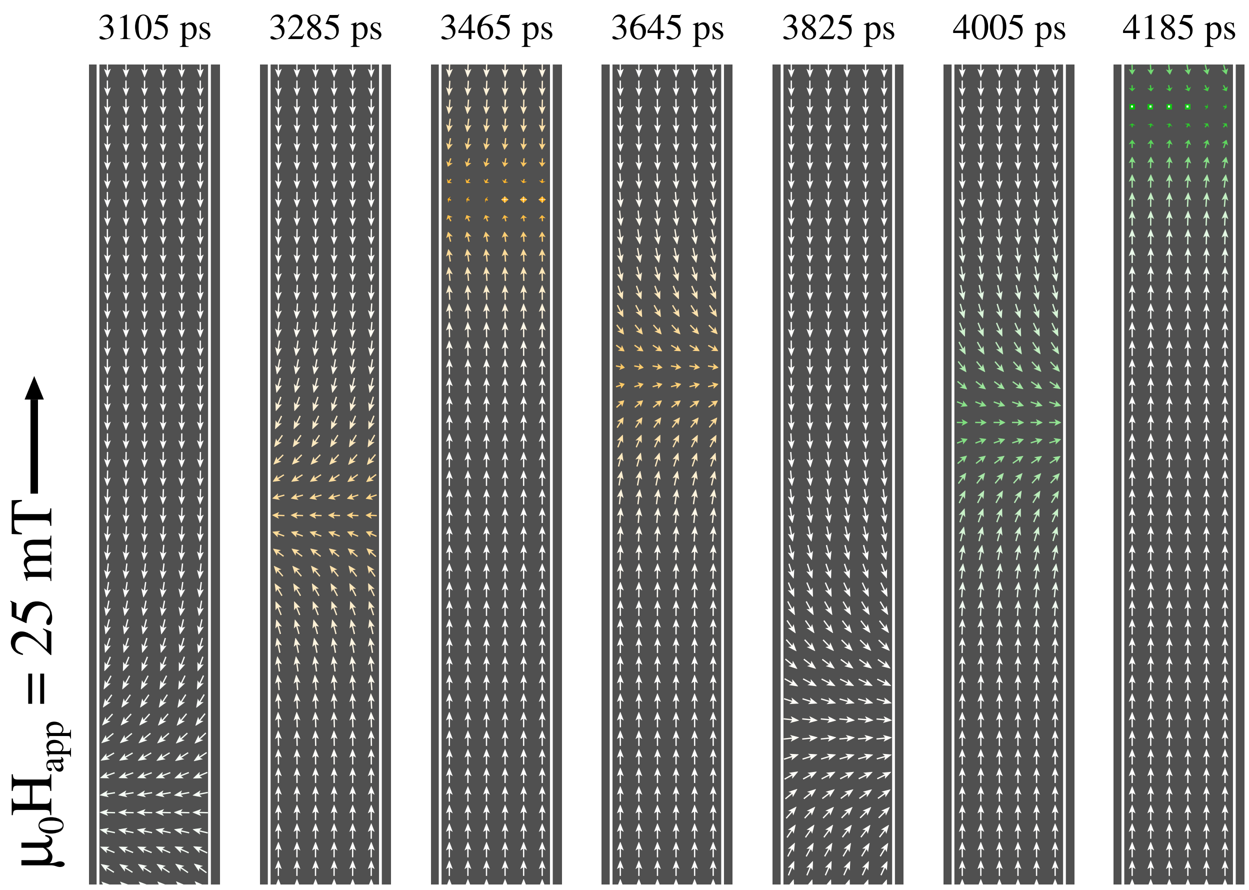


Applied Field Step

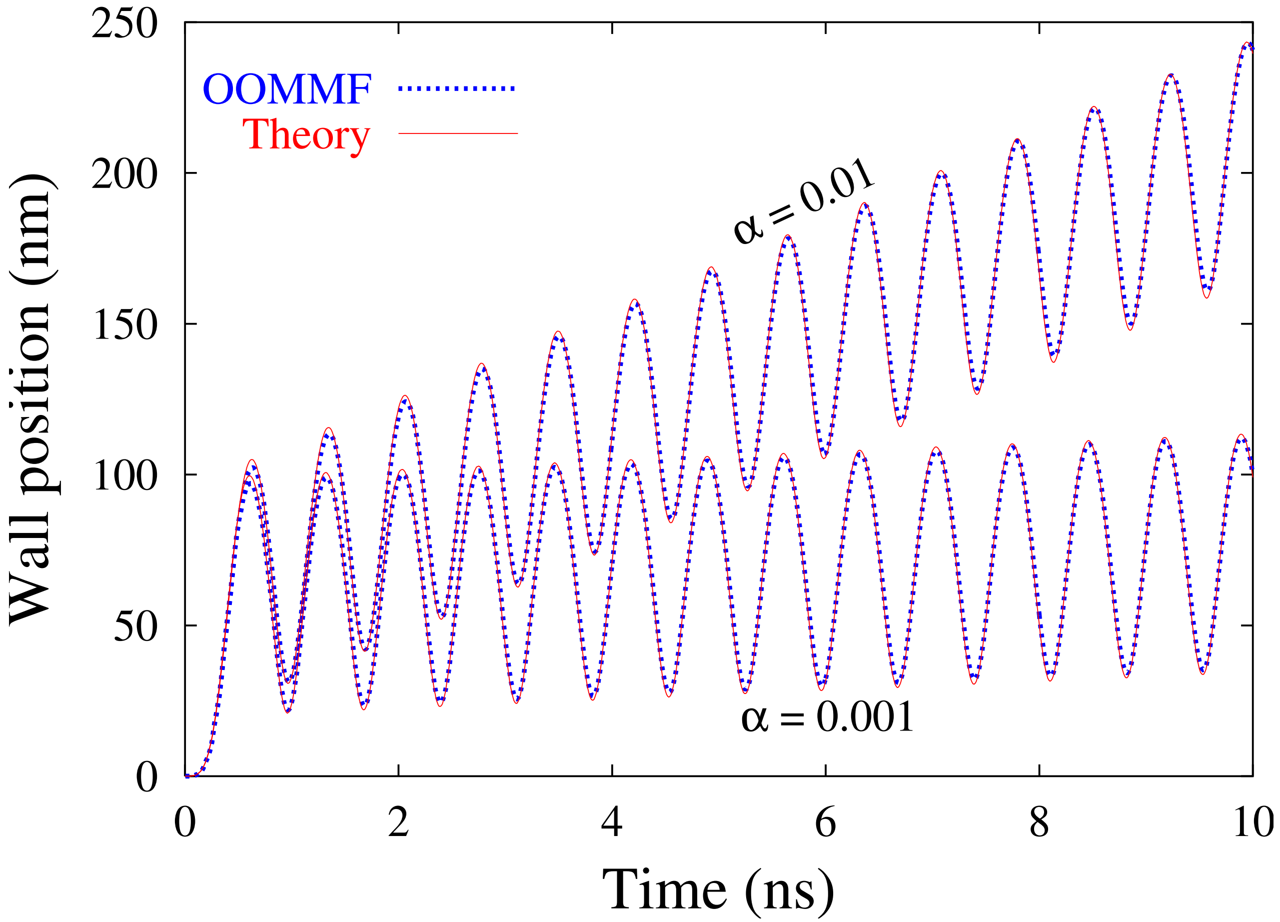


- $\Delta t = 0.5$ ns.

Retrograde Motion, $\alpha = 0.01$

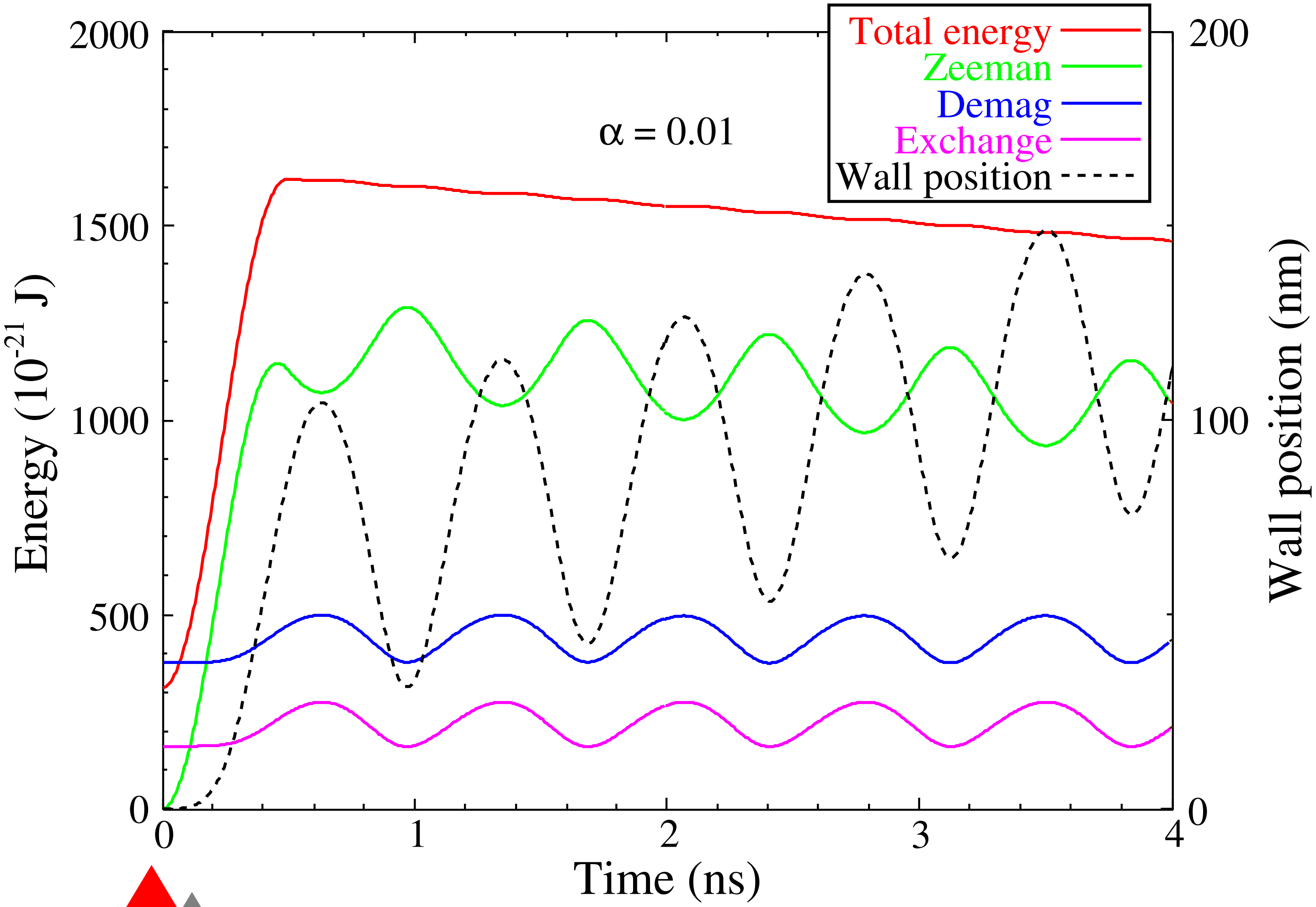


Retrograde Wall Motion



- $\mu_0 H = 25$ mT; $W = 15$ nm; rise time $\Delta t = 0.5$ ns.

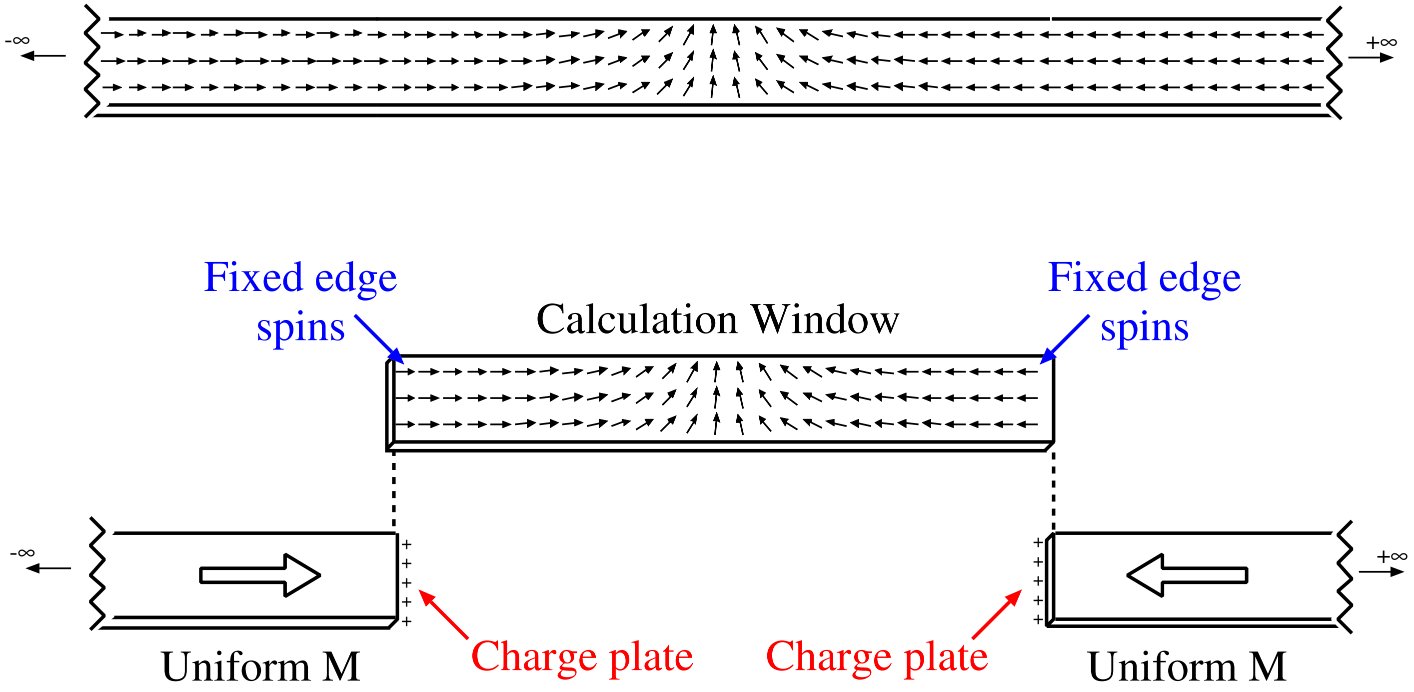
Component Energies



Summary

- Transverse wall motion predominately precession about $\mathbf{H}_{\mathrm{demag}}^{\perp}$.
- Velocity depends on aspect ratio and wall tilt angle θ.
- Retrograde motion occurs if $\mathbf{H}_{\mathrm{applied}} >$ Walker field.
- Simple analytic model agrees quite well with full micromagnetic results.
- `http://math.nist.gov/oommf`
- D. G. Porter and M. J. Donahue, “Velocity of transverse domain wall motion along thin, narrow strips,” to appear in *J. Appl. Phys.*

Infinite strips



References

1. D. G. Porter and M. J. Donahue, “Velocity of transverse domain wall motion along thin, narrow strips,” to appear in *J. Appl. Phys.*
2. A. Thiaville, J. M. García, and J. Miltat, *J. Magn. Magn. Mater.*, **242**, 1061–1063, 2002.
3. L. Lopez-Diaz, J. Sanchez, L. Torres, et al., “Computational study of domain wall mobility in nanowires of rectangular cross section,” unpublished.
4. L. R. Walker, Bell Telephone Laboratories memorandum, 1956, unpublished.