Micromagnetics on curved geometries using rectangular cells: error correction and analysis

Michael J. Donahue
Robert D. McMichael
NIST, Gaithersburg, Maryland, USA
Uniformly Magnetized Strip

Magnetization

Demag Field

Detail
Uniformly Magnetized Strip, Rotated

Detail

Magnetization

Demag Field

Detail Area

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Uniformly Magnetized Strip

Magnetization Detail

Grid Aligned

Rotated

Total charge preserved, but strength reduced, area increased.
Vortex Expulsion Test

220 nm x 220 nm x 2.5 nm Py square
Cellsize $\Delta = 2.5$ nm (cubes)

- Compute $M$ vs. $H_{\text{app}}$
- Compute expulsion field $H_c$ vs. grid angle $\theta$

$H_{\text{app}} = 0$ mT
$H_{\text{app}} = 4$ mT
$H_{\text{app}} = 8$ mT
Vortex Expulsion: Field dependence

Uncorrected, 35° rotation
No rotation (θ=0)
Vortex Expulsion: Angular dependence

Expulsion field, $\mu_0 H_c$ (mT)

Rotation angle, $\theta$ (deg)

Ideal

Uncorrected

Grid axis

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Edge mode test

\[ H_{\text{app}} = 0.1 \text{T} \]
\[ \theta = 0 \]
Corrections: Angular dependence

Uncorrected

Ideal

Rotation angle, $\theta$ (deg)

Edge mode frequency (GHz)

$H_{\text{app}} = 0.1$ T

Grid axis
Edge mode test: key points

- Edge mode sensitive only to edge effects
- Quantitative
- Robust quantity, does not involve critical field
- Experimentally accessible
Discrete demag field

In general:

\[ H_{\text{demag},i} = - \sum_{j} N_{i,j} M_{j}. \]

For uniform grid:

\[ H_{\text{demag},i} = - \sum_{j} N_{i-j} M_{j}. \]

Here FFT can be used to evaluate \( H_{\text{demag}} \).

(Note: Uniform grid; \( |M_{j}|'s \) can vary cell-to-cell.)
PROBLEM: Partially filled cell has different geometry, so FFT can’t be used to compute demag field.

Partially filled cell, height=t, $|M|=M_s$
SOLUTION: Use full cell so all cells have same geometry, but reduce $M_s$ so far-field demag is correct.

Single cell stray field

$H_{demag}/M_s$

Offset/$\Delta$

1x1x0.5 brick, $|M|=M_s$

1x1x1 cube, $|M|=M_s/2$

$0.08/x^3$

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Reduced $|M|$ approximation error

$|E|/M_s$

$0.625/x^2$
$0.05/x^5$

Relative error

Absolute error $/M_s$
$H_{\text{demag}}$ decomposition

\[
H_{\text{demag},i} = - \sum_j N_{i,j} M_j
\]

\[
= - \sum_{j \in \Omega_{\text{local}}} N_{i,j} M_j - \sum_{j \in \Omega_{\text{far}}} N_{i-j} M_j.
\]

Handle $\Omega_{\text{far}}$ via modified $M_s$ and FFT, $\Omega_{\text{local}}$ some other way.

Local field computation

Problem: Computing $H_{\text{demag}}$ on $\Omega_{\text{local}}$ not easy.

Idea: Use existing demag code on a local, refined grid.
Local field computation

- Compute $N_{i'j'}^{\text{fine}}$ for fine mesh on $\Omega_{\text{local}}$ (once)
- For $i, j$ near boundary, compute $\langle H_{\text{demag}}^{\text{fine}} \rangle_{i,j}$
- $H_{\text{demag}}^{\text{fine}} - H_{\text{demag}}^{\text{coarse}}$ define correction factors $K_{i,j}$
- NOTE: Done once during initialization!
Local field computation

During simulation run:

• Compute $H_{\text{demag}}$ as usual, with volume-modified $|M|$.

• For cells near boundary, include local corrections

$$H_{\text{corr},i} = - \sum_{j \in \Omega_{\text{local},i}} K_{i,j} M_j$$

• Correction is $O(N_{\text{boundary}})$
Local correction, pushed

\[ H_{\text{corr},i} = - \sum_{j \in \Omega_{\text{local}_i}} K_{i,j} M_j \]

\[ = \sum_{j \in \Omega_{\text{local}_i}} K_{i,j} |M_j| (m_i - m_j) - \sum_{j \in \Omega_{\text{local}_i}} K_{i,j} |M_j| m_i \]

\[ \approx -K_i M_i \quad \text{(if } |m_i - m_j| \ll 1) \]

where

\[ K_i = \sum_{j \in \Omega_{\text{local}_i}} \frac{|M_j|}{|M_i|} K_{i,j}. \]
Vortex Expulsion: Field dependence

Uncorrected, 35° rotation
No rotation (θ=0)
Far field + local subgrid, 35° rotation

\[ \frac{M \cdot H_{\text{app}}}{M_s} \]

\[ \mu_0 H_{\text{app}} \] (mT)

Applied field, \( \mu_0 H_{\text{app}} \) (mT)
Vortex Expulsion: Angular dependence

Expulsion field, $\mu_0 H_c$ (mT)

Rotation angle, $\theta$ (deg)

Ideal

Far field + local subgrid

Uncorrected

Grid axis

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Corrections: Angular dependence

Edge mode frequency (GHz)

Rotation angle, θ (deg)

Uncorrected
Ideal
Far field + local subgrid

$H_{app} = 0.1 \, T$

Grid axis

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Summary

- Staircase artifact can be significant.
- Far field (FFT) with local correction ($K_{ij}$ or $K_i$) decomposition effective and efficient.
- $K_{ij}$ terms computed once via usual demag code on local mesh.
- Edge mode frequency test quantitative and numerically robust.