Outgrowths of the DLMF Project: **Part 2**: NIST Digital Repository of Mathematical Formulae

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Challenges in 21st Century: Experimental Mathematical Computation

Institute for Computational and Experimental Research in Mathematics, Providence, Rhode Island

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Digital Repository of Mathematical Formulae

- **Online compendium** of **mathematical formulae**
  - **orthogonal polynomial** and **special function formulae**
- **DRMF** attempts to use **Web 2.0 technologies** to move beyond the **static presentation** of **reference data** to a **platform** that **encourages community interaction** and **collaboration**.
- **DRMF** utilizes of **DLMF LaTeX macros**
  - tie **specific character sequences** to **well-defined mathematical objects**.
  - Provides an **internet link** to **standard**, **precise** orthogonal polynomial and special function **definitions** through the DLMF and DRMF
- Uses **MediaWiki** wiki software
  - **MathML** support
  - **LaTeXML**
  - **MathJax**

DRMF goals

The **DRMF** will be designed for a **mathematically literate audience** and should:

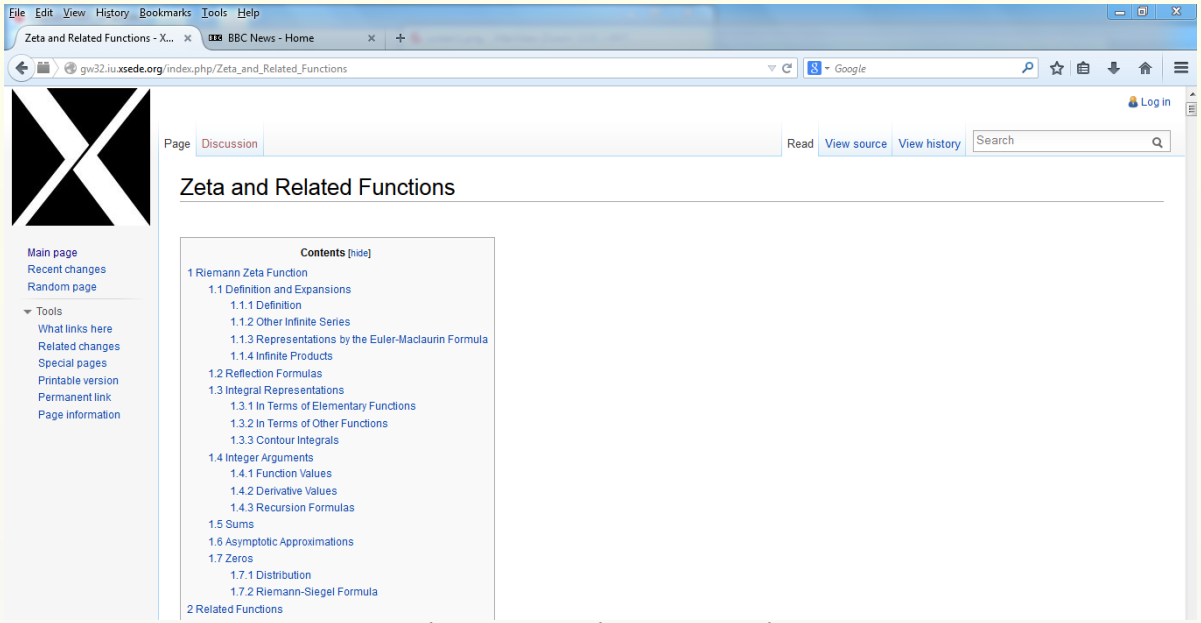
1. **facilitate interaction** among a **community of mathematicians and scientists** interested in **formulae data** related to **orthogonal polynomials and special functions** (OPSF);
2. be **expandable,** allowing the input of new formulae;
3. be **accessible as a standalone resource**;
4. have a **user friendly, consistent, and hyperlinkable viewpoint and authoring perspective**; and
5. contain **easily searchable mathematics** and take advantage of modern **MathML tools** for **easy to read**, **scalably rendered mathematics**.

DRMF Seeding Projects

Math OCR, Macro replacement, and Wikitext generation, to implement pre-existing book compendia

- **Mathematical Optical Character Recognition** project
  - **Bateman manuscript project**: Higher Transcendental Functions, Tables of Integral Transforms
  - **Byrd & Friedman**'s Handbook of Elliptic Integrals for Engineers and Scientists
- **DLMF LaTeX macro replacement** project
  - Hypergeometric Orthogonal Polynomials and Their q-Analogues – **KLS**
  - **KLS addendum** by Tom Koornwinder
  - future?: **Andrews, Askey & Roy** : Special Functions
  - future?: **Ismail** : Classical and Quantum Orthogonal Polynomials in One Variable
  - future?: etc.
- **Wikitext generation** project
  - NIST Digital Library of Mathematical Functions (ch. 25) : 170 formulas

DRMF Zeta and Related Functions Page ... DLMF Wikitext



$$\zeta(s)=\frac{(2\pi)^s e^{-s-(\gamma s/2)}}{2(s-1)\Gamma\left(\frac{1}{2}s+1\right)}\prod_{\rho}\left(1-\frac{s}{\rho}\right)e^{s/\rho}$$

Constraint(s): product over zeros ρ of ζ with $\Re\rho>0$

Reflection Formulas

$$\zeta(1-s)=2(2\pi)^{-s}\cos\left(\tfrac{1}{2}\pi s\right)\Gamma(s)\zeta(s)$$

Constraint(s): $s\neq 0,1$

$$\zeta(s)=2(2\pi)^{s-1}\sin\left(\tfrac{1}{2}\pi s\right)\Gamma(1-s)\zeta(1-s)$$

$$\xi(s)=\xi(1-s)$$

$$\xi(s)=\tfrac{1}{2}s(s-1)\Gamma\left(\tfrac{1}{2}s\right)\pi^{-s/2}\zeta(s)$$

$$(-1)^k\zeta^{(k)}(1-s)=\frac{2}{(2\pi)^s}\sum_{m=0}^{k}\sum_{r=0}^{m}\binom{k}{m}\binom{m}{r}\left(\Re\left(c^{k-m}\right)\cos\left(\tfrac{1}{2}\pi s\right)+\Im\left(c^{k-m}\right)\sin\left(\tfrac{1}{2}\pi s\right)\right)\Gamma^{(r)}(s)\zeta^{(m-r)}(s)$$

Substitution(s): $c=-\ln(2\pi)-\frac{1}{2}\pi\mathbb{I}$

Constraint(s): $s\neq 0,1$ & $k=1,2,3,\ldots$

DLMF macros provide semantic content in formulas

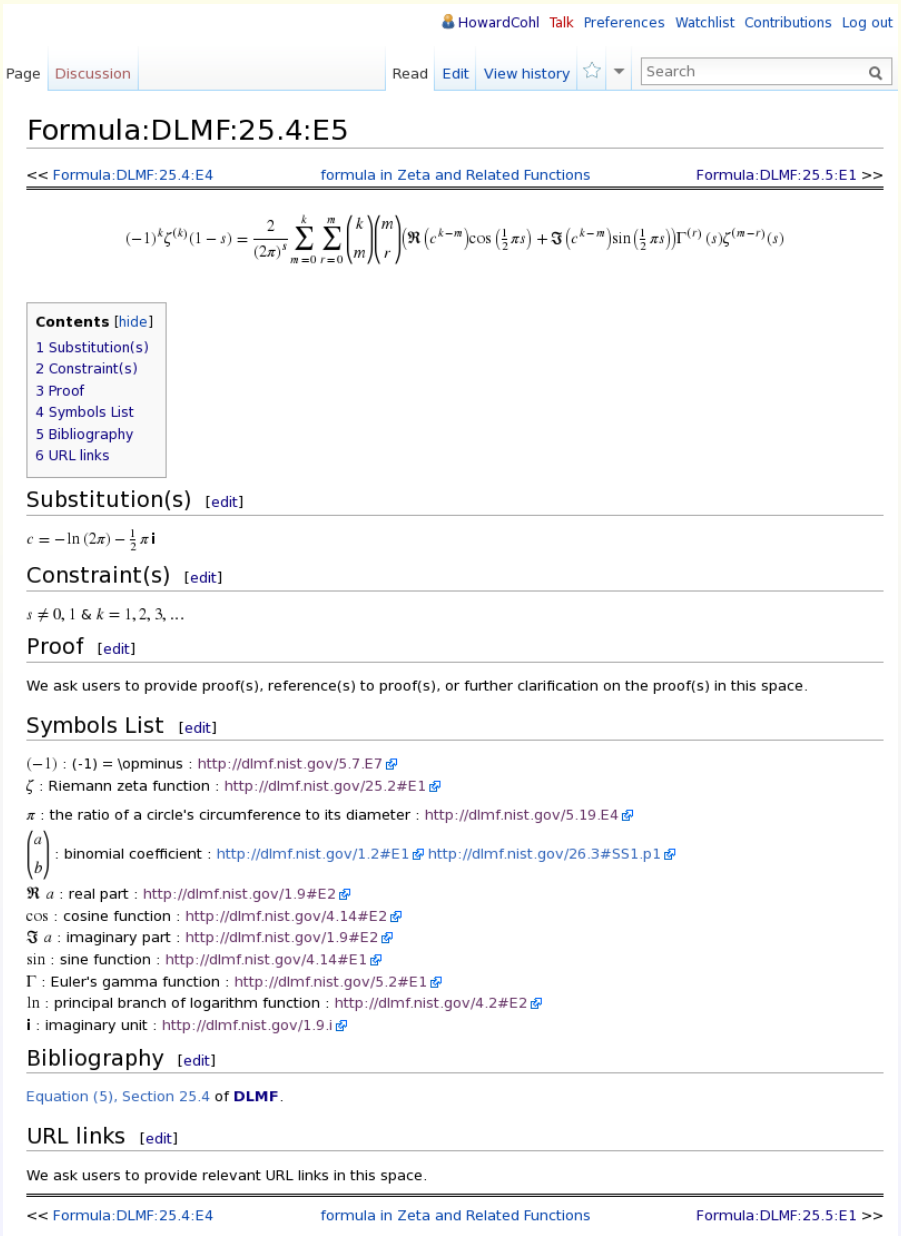
- **DLMF OPSF Macros** via **LaTeXML-server**
  - 546 **semantic DLMF** LaTeX **OPSF** macros
  - **additional** 38 **semantic LaTeX macros**
- **Objects:** `\sum,\int,\deriv{f}{x},\qderiv[n]{q}@{z}`
- **Constants:** `\expe,\iunit,\cpi,\EulerConstant`
- **Special Functions and Orthogonal Polynomials**

$\Gamma(z)$	`\EulerGamma@{z}`	`http://dlmf.nist.gov/5.30#E1`
$J_\nu(z)$	`\BesselJ{\nu}@{z}`	`http://dlmf.nist.gov/10.2#E2`
$Q^\mu_\nu(z)$	`\LegendreQ[\mu]{\nu}@{z}`:	`http://dlmf.nist.gov/14.3#E7`
$P_n^{(\alpha,\beta)}(x)$	`\JacobiP{\alpha}{\beta}{n}@{x}`	`http://dlmf.nist.gov/18.3#T1.t1.r3`

Mathematical Formulae

- Whereas **Wikipedia** and other web authoring tools **manifest notions or descriptions** as **first class objects**, the DRMF does that with **mathematical formulae**.
- **DRMF** provides for each formula, a **formula home page**:
  1. **Rendered description of the formula** (required);
  2. **Constraints** the formula must obey
  3. **Substitutions** required to understand formula;
  4. **Bibliographic citation** (required);
  5. Open section for **proofs** (required) – *DLMF*;
  6. **List of symbols** and **links** to definitions (required) – *DLMF macros*;
  7. Open section for **notes** – *connections between formulas*; and
  8. Open section for **external links** – *computer generated proofs*;

Sample formula home page



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Page Discussion Read Edit View history Search

Formula:DLMF:25.4:E5

<< Formula:DLMF:25.4:E4 formula in Zeta and Related Functions Formula:DLMF:25.5:E1 >>

$$(-1)^k\zeta^{(k)}(1-s)=\frac{2}{(2\pi)^s}\sum_{m=0}^{k}\sum_{r=0}^{m}\binom{k}{m}\binom{m}{r}\left(\Re\left(c^{k-m}\right)\cos\left(\tfrac{1}{2}\pi s\right)+\Im\left(c^{k-m}\right)\sin\left(\tfrac{1}{2}\pi s\right)\right)\Gamma^{(r)}(s)\zeta^{(m-r)}(s)$$

Contents [hide]

1 Substitution(s)
2 Constraint(s)
3 Proof
4 Symbols List
5 Bibliography
6 URL links

Substitution(s) [edit]

$c=-\ln(2\pi)-\frac{1}{2}\pi\mathbf{i}$

Constraint(s) [edit]

$s\neq 0,1$ & $k=1,2,3,\ldots$

Proof [edit]

We ask users to provide proof(s), reference(s) to proof(s), or further clarification on the proof(s) in this space.

Symbols List [edit]

(-1) : (-1) = \opminus : http://dlmf.nist.gov/5.7.E7
ζ : Riemann zeta function : http://dlmf.nist.gov/25.2#E1
π : the ratio of a circle's circumference to its diameter : http://dlmf.nist.gov/5.19.E4
$\binom{a}{b}$: binomial coefficient : http://dlmf.nist.gov/1.2#E1 http://dlmf.nist.gov/26.3#SS1.p1
$\Re\ a$: real part : http://dlmf.nist.gov/1.9#E2
$\cos$: cosine function : http://dlmf.nist.gov/4.14#E2
$\Im\ a$: imaginary part : http://dlmf.nist.gov/1.9#E2
$\sin$: sine function : http://dlmf.nist.gov/4.14#E1
Γ : Euler's gamma function : http://dlmf.nist.gov/5.2#E1
$\ln$: principal branch of logarithm function : http://dlmf.nist.gov/4.2#E2
$\mathbf{i}$: imaginary unit : http://dlmf.nist.gov/1.9.i

Bibliography [edit]

Equation (5), Section 25.4 of **DLMF**.

URL links [edit]

We ask users to provide relevant URL links in this space.

<< Formula:DLMF:25.4:E4 formula in Zeta and Related Functions Formula:DLMF:25.5:E1 >>

Further questions

- How does one **facilitate** effective **community interaction** & **contribution** with such a **resource**?
  - implement a high degree of **computer verification** of community input
  - ensure a degree of **moderation** in the wiki
- Can one build a piece of **intelligent software** which is able to
  - scan in **books**;
  - produces **LaTeX source**;
  - replaces commands for functions in the source with **semantic macros**;
  - **extracts data** from the **text** (such as constraints)
  - **associates data** with relevant formulae and **removes** text;
  - **produces Wikitext**;
  - and **uploads** Wikitext to a publicly accessible website?
- How does one **search** the resulting mathematical database?

Ongoing projects to investigate the above questions

- **Macro replacements** from well-constructed LaTeX source
- **Extraction** of **mathematical data** from **text** (keywords)
- **Wikitext generation**
- Porting the **DLMF search engine** in **MediaWiki** (DRMF)
- **Output of formula data** from **right-clickable menus** in a variety of **formats** so that formulas can be **used** and also **verified**
  - LaTeX expanded
  - LaTeX semantic
  - presentation MathML
  - content MathML
  - Mathematica
  - Maple
  - Sage

Virtual machines

Virtual Machine Instances:

- **XSEDE** project
  - 2 **XSEDE** CentOS: **demo and deployment**
  - 2 **XSEDE** Ubuntu **server**: **LaTeXML, Mathoid**
- **Wikimedia Foundation – Wikitech**
  - 4 **WMF Vagrant** instances

Acknowledgements

- **Moritz Schubotz** (TU-Berlin): **MediaWiki Math**
- **Bruce Miller** (NIST) : **DLMF Macros**
- **Janelle Williams** (VSU) : **2013 SURF student**
- **High School Students:**
  - **Jake Migdall** – MathJax menu customization
  - **Cherry Zou** – seeding/macro replacement
  - **Alex Danoff** – seeding/macro replacement
  - **Amber Liu** – MathJax menu customization
  - **Jimmy Li** – mathematical search

Poster session with website demos : Wed. evening

NIST Digital Repository of Mathematical Formulae (DRMF)

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- A **digital compendium of math formulae** for **orthogonal polynomials and special functions** and **associated math data**. Uses **Web 2.0** technologies to move beyond the **static presentation of reference data** to a **platform** that encourages **community interaction** and **collaboration.**
- **Offshoot** project of **NIST Digital Library of Math Functions (DLMF)** using **MediaWiki** and **MediaWiki Math extension**. **Math rendering** in **MediaWiki** using **MathML. Menuing** support using **MathJax**. **MediaWiki extension development** using **PHP, JavaScript** & **Java**.
- Use of **DLMF semantic LaTeX macro set** for **special functions and orthogonal polynomials** e.g.,

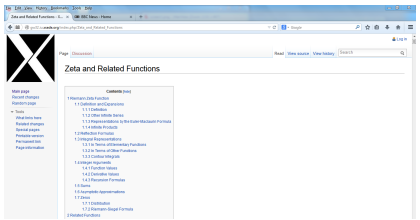
$\Gamma(z)$	`\EulerGamma@{z}`	`http://dlmf.nist.gov/5.2#E1`
$P_n^{(\alpha,\beta)}(x)$	`\JacobiP{\alpha}{\beta}{n}@{x}`	`http://dlmf.nist.gov/18.3#T1.t1.r3`

- **DRMF** treats **formulae** as **first class objects**, describing them in **formula home pages** which contain:

1. **Rendered description** of the **formula** (required);
2. **Bibliographic citation** (required);
3. Open section for **proofs** (required);
4. **List of symbols** and **links** to definitions (required);
5. Open section for **notes**;
6. Open section for **external links**;
7. **Substitutions** required to understand formula; and
8. **Constraints** the formula must obey.

- **Wikitext** generation and **semantic DLMF LaTeX macro replacement** effort using **IDL** & **Python**.
  1. **DLMF LaTeX macros** already implemented for the **DLMF Zeta** chapter;
  2. Hypergeometric **Orthogonal Polynomials** and their q-Analogues + **KLS addendum**.

Zeta and Related Functions Page



Formula Home Page ⟶

`http://gw32.iu.xsede.org/index.php/Main_Page`

13th International Symposium on Orthogonal Polynomials, Special Functions & Applications

June 1-5, 2015
National Institute of Standards and Technology
Gaithersburg, Maryland, USA

`http://www.siam.org/meetings/opsfa13`
`http://www.nist.gov/itl/math` `https://wis.kuleuven.be/events/OPSFA/Steering`

Plenary Speakers:

- Percy Deift, Courant Institute of Mathematical Sciences, New York University, USA
- Charles F. Dunkl, University of Virginia, USA
- Olga Holtz, Technische Universität Berlin, Germany
- Mourad E.H. Ismail, University of Central Florida, USA
- Teresa E. Pérez Fernández, Universidad de Granada, Spain
- Sarah Post, University of Hawaii at Manoa, USA
- Nico Temme, Centrum Wiskunde & Informatica (CWI), The Netherlands
- Craig A. Tracy, University of California Davis, USA
- Lauren Williams, University of California Berkeley, USA
- Wadim Zudilin, The University of Newcastle, Australia
- Alexei Zhedanov, Donetsk Institute for Physics and Technology, Ukraine

Themes: **Orthogonal Polynomials and Special Functions**, including aspects within:

- classical analysis
- approximation theory
- continued fractions
- potential theory
- q-calculus
- asymptotics
- Riemann-Hilbert problems
- random matrix theory
- superintegrability and supersymmetry
- integrable systems
- Painlevé equations
- orthogonal polynomials and special functions of several variables
- orthogonal polynomials associated with root systems
- spherical functions
- orthogonality on the complex plane
- multiple orthogonal polynomials
- Sobolev orthogonal polynomials
- stochastic processes

and connections to other disciplines, including:

- science and industry
- handbooks
- numerical algorithms and tables
- symbolic computation
- combinatorics
- number theory
- theoretical physics
- probability theory and statistics

Organizing Committee

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