Outgrowths of the DLMF Project: Part 2:
NIST Digital Repository of Mathematical Formulae

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Challenges in 21st Century:
Experimental Mathematical Computation

Institute for Computational and Experimental Research in Mathematics, Providence, Rhode Island

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Digital Repository of Mathematical Formulae

- Online compendium of mathematical formulae
  - orthogonal polynomial and special function formulae
- DRMF attempts to use Web 2.0 technologies to move beyond the static presentation of reference data to a platform that encourages community interaction and collaboration.
- DRMF utilizes of DLMF \( \LaTeX \) macros
  - tie specific character sequences to well-defined mathematical objects.
  - Provides an internet link to standard, precise orthogonal polynomial and special function definitions through the DLMF and DRMF
- Uses MediaWiki wiki software
  - MathML support
  - \( \LaTeX \)XML
  - MathJax
The DRMF will be designed for a mathematically literate audience and should:

1. facilitate interaction among a community of mathematicians and scientists interested in formulae data related to orthogonal polynomials and special functions (OPSF);
2. be expandable, allowing the input of new formulae;
3. be accessible as a standalone resource;
4. have a user friendly, consistent, and hyperlinkable viewpoint and authoring perspective; and
5. contain easily searchable mathematics and take advantage of modern MathML tools for easy to read, scalably rendered mathematics.
DRMF Seeding Projects
Math OCR, Macro replacement, and Wikitext generation, to implement pre-existing book compendia

- **Mathematical Optical Character Recognition** project
  - **Bateman manuscript project**: Higher Transcendental Functions, Tables of Integral Transforms
  - **Byrd & Friedman**'s Handbook of Elliptic Integrals for Engineers and Scientists

- **DLMF LaTeX** macro replacement project
  - Hypergeometric Orthogonal Polynomials and Their $q$-Analogues – **KLS**
  - **KLS addendum** by Tom Koornwinder
  - **future**?: **Andrews, Askey & Roy**: Special Functions
  - **future**?: **Ismail**: Classical and Quantum Orthogonal Polynomials in One Variable
  - **future**?: etc.

- **Wikitext generation** project
  - NIST Digital Library of Mathematical Functions (ch. 25) : 170 formulas
Zeta and Related Functions

1. Riemann Zeta Function
   1.1 Definition and Properties
   1.2 Other Infinite Series
   1.3 Representations by the Euler-Mascheroni Constant
   1.4 Infinite Products
   1.5 Reflection Formulas

2. Related Functions
   2.1 Generalized Zeta Function
   2.2 Dirichlet L-Functions

Reflection Formulas

\[
\zeta(1 - s) = 2(2\pi)^{s - 1/2} \Gamma\left(\frac{1}{2}\right) \zeta(s)
\]

\[
\zeta(2) = 2\pi^2/6
\]

\[
\zeta(4) = \pi^4/90
\]

\[
\zeta(6) = \pi^6/945
\]

\[
(-1)^k \zeta(2k) = 2(2\pi)^{2k} \Gamma(k) \zeta(2k)
\]

Constraint(s): product over areas $\rho$ of $\zeta$ with $\Re \rho > 0$

Constraint(s): $s \neq 0, 1$

\[
\zeta(s) = \frac{1}{\Gamma(s)} \int_0^\infty \frac{x^{s-1}}{e^x - 1} dx
\]

\[
\zeta(s) = \lim_{n \to \infty} \sum_{k=1}^n \frac{1}{k^s}
\]

Substitution(s): $c = -\ln(2\pi) - \frac{1}{2} \ln \Gamma(s)$

Constraint(s): $s > 0, \Im c > 0$
DLMF macros provide semantic content in formulas

- **DLMF OPSF Macros via \LaTeX XML-server**
  - 546 semantic DLMF \LaTeX OPSF macros
  - additional 38 semantic \LaTeX macros

- **Objects:** \texttt{\sum, \int, \deriv{f}{x}, \qderiv[n]{q}@{z}}

- **Constants:** \texttt{\expe, \iunit, \cpi, \EulerConstant}

- **Special Functions and Orthogonal Polynomials**

  \[
  \begin{align*}
  \Gamma(z) & \quad \EulerGamma@{z} & \text{http://dlmf.nist.gov/5.30#E1} \\
  J_\nu(z) & \quad \BesselJ{\nu}@{z} & \text{http://dlmf.nist.gov/10.2#E2} \\
  Q^{\mu}_\nu(z) & \quad \LegendreQ[\mu]{\nu}@{z}: & \text{http://dlmf.nist.gov/14.3#E7} \\
  P^{(\alpha,\beta)}_n(x) & \quad \JacobiP{\alpha}{\beta}{n}@{x} & \text{http://dlmf.nist.gov/18.3#T1.t1.r3}
  \end{align*}
  \]
Whereas Wikipedia and other web authoring tools manifest notions or descriptions as first class objects, the DRMF does that with mathematical formulae.

DRMF provides for each formula, a formula home page:

1. **Rendered description of the formula** (required);
2. **Constraints** the formula must obey;
3. **Substitutions** required to understand formula;
4. **Bibliographic citation** (required);
5. Open section for **proofs** (required) – *DLMF*;
6. **List of symbols** and **links** to definitions (required) – *DLMF macros*;
7. Open section for **notes** – *connections between formulas*; and
8. Open section for **external links** – *computer generated proofs*;
\[
(-1)^{\nu} \zeta(1-s) = \frac{2}{(2\pi)^s} \sum_{n=0}^{\infty} \left(\sum_{m=0}^{\infty} \frac{1}{m+1} \right) \left(\sum_{r=0}^{\infty} \frac{1}{r+1} \right) \left(\frac{1}{2}\right)^{r+\nu} \left(\frac{1}{s}\right)^{r+\nu} \left(\zeta^{(r+\nu)}(s) \zeta^{(r)}(1-s) \right)
\]

Substitution(s) [edit]
\[
\nu = \ln \left(\frac{2\pi}{1}\right) - \frac{1}{2} \pi
\]

Constraint(s) [edit]
\[
s \neq 0, 1 \text{ and } k = 1, 2, 3, ...
\]

Proof [edit]
We ask users to provide proof(s), reference(s) to proof(s), or further clarification on the proof(s) in this space.

Symbols List [edit]
\[
\begin{align*}
\lambda & : \text{logarithm} \quad \text{http://dlmf.nist.gov/5.7.E7} \\
\zeta & : \text{Riemann zeta function} \quad \text{http://dlmf.nist.gov/25.2.E1} \\
x & : \text{the ratio of a circle's circumference to its diameter} \quad \text{http://dlmf.nist.gov/5.19.E4} \\
\binom{n}{k} & : \text{binomial coefficient} \quad \text{http://dlmf.nist.gov/1.2.E1, http://dlmf.nist.gov/26.3#SS1} \\
\Re & : \text{real part} \quad \text{http://dlmf.nist.gov/1.9.E2} \\
\cos & : \text{cosine function} \quad \text{http://dlmf.nist.gov/4.14.E1} \\
\Im & : \text{imaginary part} \quad \text{http://dlmf.nist.gov/1.9.E2} \\
\sin & : \text{sine function} \quad \text{http://dlmf.nist.gov/4.14.E1} \\
\Gamma & : \text{Euler's gamma function} \quad \text{http://dlmf.nist.gov/5.2.E1} \\
\ln & : \text{principal branch of logarithm function} \quad \text{http://dlmf.nist.gov/4.2.E1} \\
i & : \text{imaginary unit} \quad \text{http://dlmf.nist.gov/1.9.E3}
\end{align*}
\]

Bibliography [edit]
Equation (5), Section 25.4 of DLMF.

URL links [edit]
We ask users to provide relevant URL links in this space.
Further questions

- How does one facilitate effective community interaction & contribution with such a resource?
  - implement a high degree of computer verification of community input
  - ensure a degree of moderation in the wiki

- Can one build a piece of intelligent software which is able to
  - scan in books;
  - produces \texttt{\LaTeX} source;
  - replaces commands for functions in the source with semantic macros;
  - extracts data from the text (such as constraints)
  - associates data with relevant formulae and removes text;
  - produces Wikitext;
  - and uploads Wikitext to a publicly accessible website?

- How does one search the resulting mathematical database?
Ongoing projects to investigate the above questions

- **Macro replacements** from well-constructed \( \LaTeX \) source
- **Extraction of mathematical data** from text (keywords)
- **Wikitext generation**
- Porting the DLMF search engine in MediaWiki (DRMF)
- **Output of formula data** from right-clickable menus in a variety of formats so that formulas can be used and also verified
  - \( \LaTeX \) expanded
  - \( \LaTeX \) semantic
  - presentation MathML
  - content MathML
  - Mathematica
  - Maple
  - Sage
Virtual Machine Instances:

- **XSEDE** project
  - 2 XSEDE CentOS: demo and deployment
  - 2 XSEDE Ubuntu server: \LaTeX\XML, Mathoid

- Wikimedia Foundation – Wikitech
  - 4 WMF Vagrant instances
Acknowledgements

- Moritz Schubotz (TU-Berlin): MediaWiki Math
- Bruce Miller (NIST): DLMF Macros
- Janelle Williams (VSU): 2013 SURF student

High School Students:
- Jake Migdall – MathJax menu customization
- Cherry Zou – seeding/macro replacement
- Alex Danoff – seeding/macro replacement
- Amber Liu – MathJax menu customization
- Jimmy Li – mathematical search
Poster session with website demos: Wed. evening

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- A digital compendium of math formulae for orthogonal polynomials and special functions and associated math data. Uses Web 2.0 technologies to move beyond the static presentation of reference data to a platform that encourages community interaction and collaboration.
- Use of DLMF semantic \LaTeX{} macro set for special functions and orthogonal polynomials e.g., \begin{equation} \Gamma(z) \end{equation} \begin{equation} \text{EulerGamma}(z) \end{equation} http://dlmf.nist.gov/5.2#E1
- DLMF \LaTeX{} macro set already implemented for the DLMF Zeta chapter.
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Zeta and Related Functions Page

http://www.siam.org/meetings/opsfa13

Plenary Speakers:
- Percy Deift, Courant Institute of Mathematical Sciences, New York University, USA
- Charles F. Dunkl, University of Virginia, USA
- Olga Holtz, Technische Universität Berlin, Germany
- Mourad E.H. Ismail, University of Central Florida, USA
- Teresa E. Pires Fernandes, Universidade deGranada, Spain
- Sarah Post, University of Hawaii at Manoa, USA
- Nino Temme, Centrum Wiskunde & Informatica (CWI), The Netherlands
- Craig A. Tracy, University of California Davis, USA
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Themes: Orthogonal Polynomials and Special Functions, including aspects within:
- classical analysis
- approximation theory
- continued fractions
- potential theory
- asymptotics
- Riemann-Hilbert problems
- random matrix theory
- superintegrability and supersymmetry
- and connections to other disciplines, including:
  - science and industry
  - handbooks
  - numerical algorithms and tables
  - symbolic computation

Organizing Committee:
- Diego Dominici, State University of New York at New Paltz, USA
- Daniel W. Lozier, National Institute of Standards and Technology, USA

Scientific Committee:
- Richard A. Askey, University of Wisconsin, USA
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- Aubrey Terras, University of California San Diego, USA
- Walter Van Assche, Katholieke Universiteit Leuven, Belgium
- Luc Vinet, University of Montreal, Canada

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Formula Home Page

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