The NIST Digital Repository of Mathematical Formulae and Scalable Math Search

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Collecting information on special functions at NIST

- 1964: Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables (AMS 55)
  - Milton Abramowitz and Irene Stegun (A&S), editors
  - 1064 Pages (book), most highly cited NIST (NBS) publication
  - definitions, identities, approximations, plots and numerical tables
- 2010: Digital Library of Mathematical Functions
  - NIST Handbook of Mathematical Functions as successor of A&S
  - F. W. J. Olver, D. W. Lozier, R. F. Boisvert, and C. W. Clark, editors.
  - 2012 US Department of Commerce Gold Medal (Bruce Miller, Bonita Saunders, Marje McClain, Abdou Youssef, Brian Antonishek)
  - 968 Pages (printed version), HTML version
  - Links, Math Search, Info boxes, interactive 2D and 3D graphics
- 2013: Digital Repository of Mathematical Formulae

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NIST Digital Repository of Mathematical Formulae Goals

DRMF compendium of individual formulae for a mathematically literate audience

1. OPSF **community interaction** for mathematicians and scientists
2. **expandability** from the literature (support the DLMF)
3. context-free full **semantic** information
4. **user friendly** perspective
5. **searchable** mathematics
6. modern **MathML** tools

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First three DRMF seeding projects

- **2013** 167 DLMF Formulae Home Pages
  - 533 semantic DLMF Macros (developed by Bruce Miller)
  - Content MathML with LaTeXML's implicit Content Dictionaries
  - estimated effort 10 min / Formula Home Page
- **2014** 1469 additional KLS (OP) Formulae Home Pages
  - Additional 153 semantic DRMF macros
  - estimated effort 5 min / Formula Home Page
- **2015** ~5000 eCF, BMP Formula Home Pages
  - Non LaTeX input
  - desired effort 1 min / Formula Home Page

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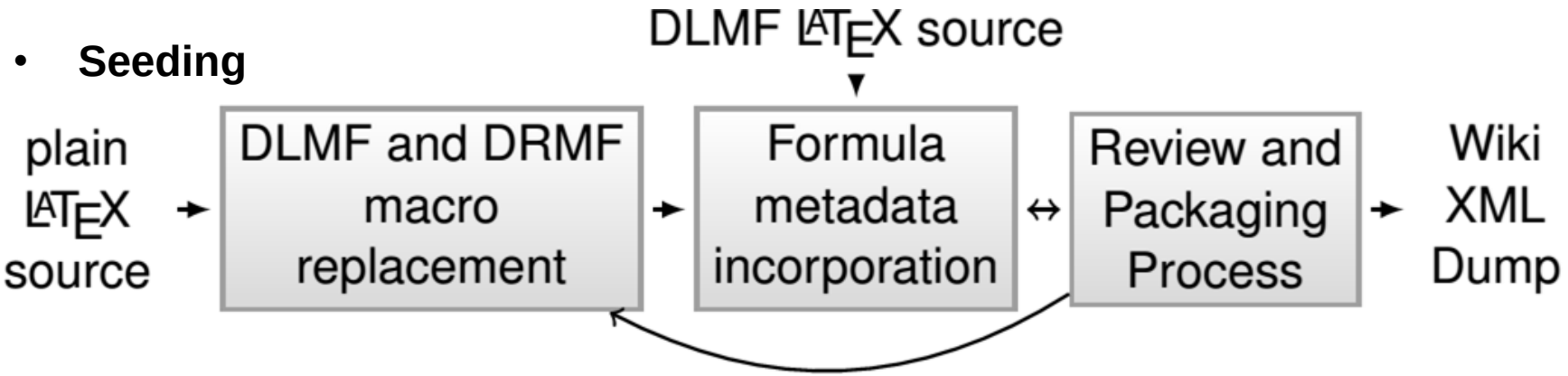
Current DRMF Implementation

- **Platform**: MediaWiki with Math and MathSearch extensions
  - 2 Table of Contents Pages
  - 76 Lists of Formulas Pages
  - 124 Definition Pages for DRMF macros
  - 1636 Formula Home Pages
- **Seeding**



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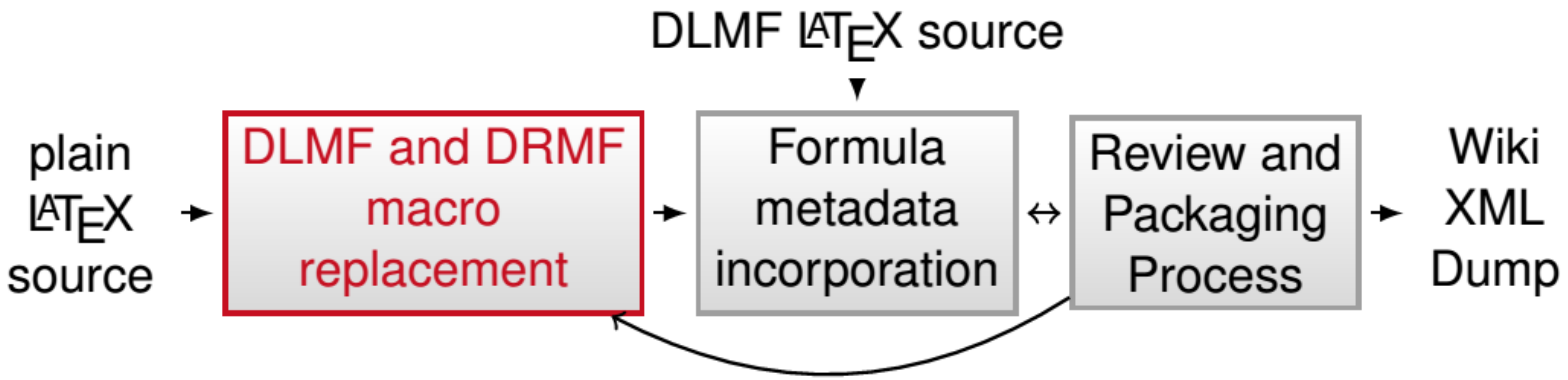
Macro replacement



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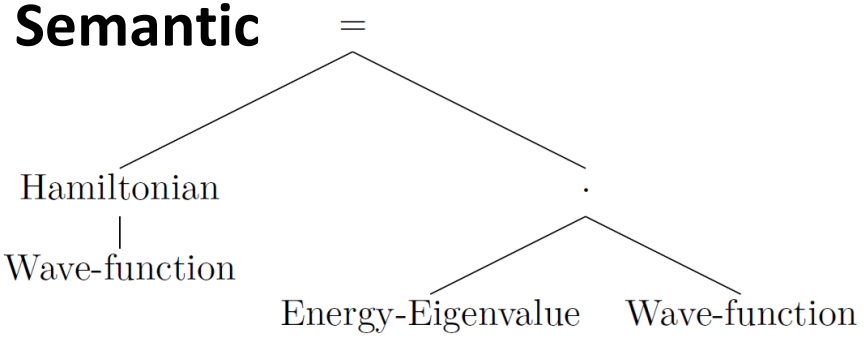
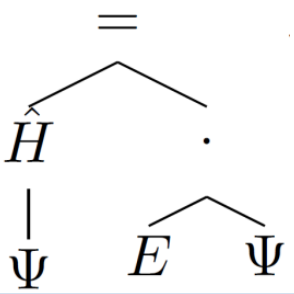
Levels of Abstraction for Mathematical Formula

Semantic



Content



Presentation

$$\hat{H}\Psi = E\Psi$$

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(Bruce Miller) LaTeXML (DLMF) Semantic LaTeX macros

Name	Type	Rendering	LaTeX	semantic LaTeX
Trigonometric sine	f	$\sin z$	`\sin z`	`\sin@@{z}`
Euler gamma	f	$\Gamma(z)$	`\Gamma(z)`	`\EulerGamma@{z}`
Jacobi polynomial	p	$P_n^{(\alpha,\beta)}(x)$	`P_n^{(\alpha, \beta)}(x)`	`\Jacobi{\alpha}{\beta}{n}@{x}`
little q-Laguerre polynomial	p	$p_n(x;a\|q)$	`p_n(x;a|q)`	`\littleqLaguerre{n}@{x}{a}{q}`

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Semantic macro **breakdown**

- **689 semantic LaTeX macros (unpublished)**
  - 533 macros from DLMF
  - 156 additional DRMF macros using the LaTeXML framework
- **Semantic LaTeX macro properties**:
  - lengths between 1 and 26 characters (median length is 8 characters)
  - Strict Naming conventions
    - individual's names capitalized
    - abbreviations utilized
    - macro names correspond with object names
    - Roman numerals

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Semantic LaTeX Macro Glossary – csv format

For each macro we store:

- Example Macro calling sequence
- Name of object described by macro
- Object description
- Brief summary and description of calling options
- Link to URL giving precise definition

Glossary.csv used in generation of symbols lists within Wikitext and for statistical purposes.

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Macro Replacements for Generic LaTeX Source Datasets

For the 3 chapters of KLS (OP) as well as the KLSadd LaTeX source

- **89** semantic macros were replaced
- a total of **3308** times
- represented by **774** (LOC) of regular expressions written by Cherry Zou

Currently the six most common replacements are:

1. *q*-Pochhammer symbol – replaced **659** times
2. Euler gamma function – replaced **266** times
3. *q*-hypergeometric function – replaced **237** times
4. Pochhammer symbol – replaced **205** times
5. Racah polynomial – replaced **117** times
6. cosine function – replaced **82** times

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Example of complexity of the problem – KLSadd dataset

- After processing of the LaTeX input, only formulas remain.
- Current metadata: are constraint and substitution annotations.
- Future metadata
  - bibliographic metadata
  - references to KLS formulae
  - errata information
  - formula comments and notes
  - symmetries in parameters
  - proofs, etc...

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Ongoing NIST Student Summer Projects

- **Yusuf Ameri** - DRMF Standard MediaWiki Text Search Integration
- **Akash Shah** - DRMF Formula Search MediaWiki Frontend
- **Shraeya Madhu** - DRMF DLMF Seeding Project
- **Claude Zou** - DRMF Semantic LaTeX Mathematical Operators
- **Jimmy Li** - DRMF Formula Search Backend & NTCIR-12 Competition
- **Moritz Schubotz** - What is "good" content MathML?

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SCALABLE MATH SEARCH

Motivation for MathSearch

Proposition 2 (expectation value of waiting time times tunnel rate) *For every PDF* $f(x)$ *in means of definition* (0.1) *the inequality*

$$\langle x \rangle \left\langle \frac{1}{x} \right\rangle \geq 1 \tag{0.10}$$

is valid.

26th of February 2011

I. POSITIVITY OF FANO FACTOR PARAMETERS

For every convex function $f(x)$, we have according to the Jenson inequality

$$f(\langle x \rangle) \leq \langle f(x) \rangle \tag{1}$$

That means that

$$\langle x \rangle^{-k} \leq \langle x^{-k} \rangle. \tag{2}$$

Especially $k = 1$ leads to the fact that $\alpha \geq 0$. 28th of March

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Example $\frac{1}{\langle x \rangle} \leq \left\langle \frac{1}{x} \right\rangle$

1. Different forms e.g.

   $\langle x \rangle \left\langle \frac{1}{x} \right\rangle \geq 1$

2. Different notations e.g.

   $\int_X f(x) x dx = \langle x \rangle$

3. Exact match seldom
4. Ambiguity in syntax e.g.

   $E\Psi = \hat{H}\Psi$

5. No TeX-function mean

```
$\frac 1 \mean ?x \le
\mean \frac 1 ?x$
<apply>
  <leq/>
  <apply>
    <divide/>
    <cn type="integer">1</cn>
    <apply>
      <mean/>
      <qvar>x</qvar></apply></apply>
  <apply>
    <mean/>
      <apply>
        <divide>
        <cn type="integer">1</cn>
       <qvar>x</qvar></apply></apply>…
```

NTCIR-11 Math-2 WMC-D1

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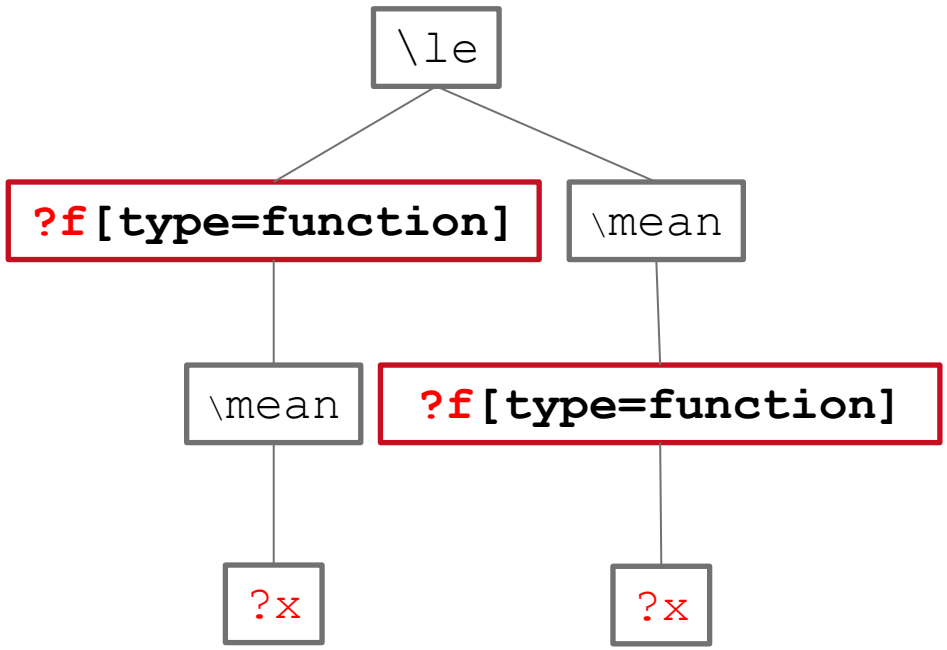
  <apply>
    <mean/>
      <apply>
        <divide>
        <cn type="integer">1</cn>
       <qvar>x</qvar></apply></apply>…
```

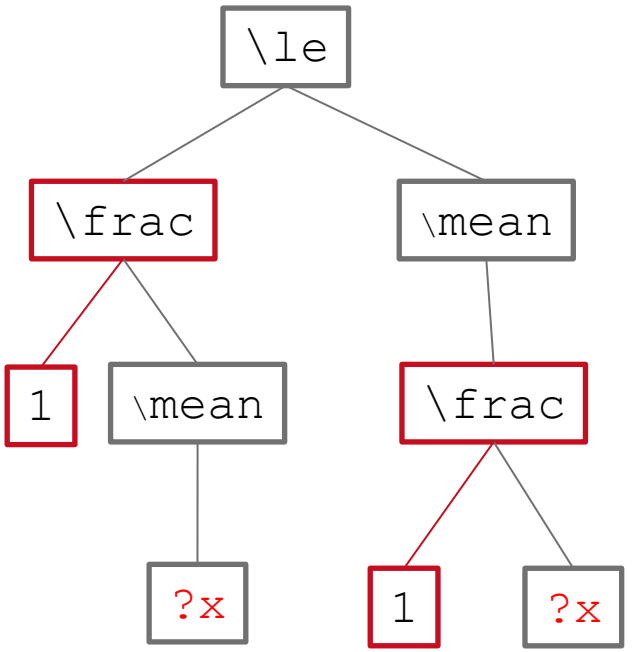
NTCIR-11 Math-2 WMC-D1

Result 1: $\varphi(\mathbb{E}[X]) \leq \mathbb{E}[\varphi(X)]$

```
$?f[type=function] \mean ?x \le
\mean ?f[type=function] ?x $
```

```
$\frac 1 \mean ?x \le
\mean \frac 1 ?x$
```





Result 1: $\varphi(\mathbb{E}[X]) \leq \mathbb{E}[\varphi(X)]$

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```
$?f[type=function] \mean ?x \le
\mean ?f[type=function] ?x $
```

```
<apply>
  <leq/>
  <apply>
    <qvar type="functi
    <apply>
      <mean/>
      <qvar>x</qvar></
  <apply>
    <mean/>
      <apply>
        <qvar type="function">f</qvar>
        <qvar>x</qvar></apply></apply>…
```

```
$\frac 1 \mean ?x \le
\mean \frac 1 ?x$
```

```
<apply>
  <leq/>
  <apply>
    <apply>
      <mean/>
      <apply></apply>
  <apply>
    <mean/>
      <apply>
        <divide>
        <cn type="integer">1</cn>
        <qvar>x</qvar></apply></apply>…
```

$\frac 1$ -> $?f[type=function]$

Not trivial

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Solution inexact matches

Refined query:

```
$\superconceptOf[
  orderby = editdistance ]{
  \frac 1 \mean ?x \le
  \mean \frac 1 ?x
}$
```

- Computational complexity
- Restriction of the search space
- Check most likely solutions at first

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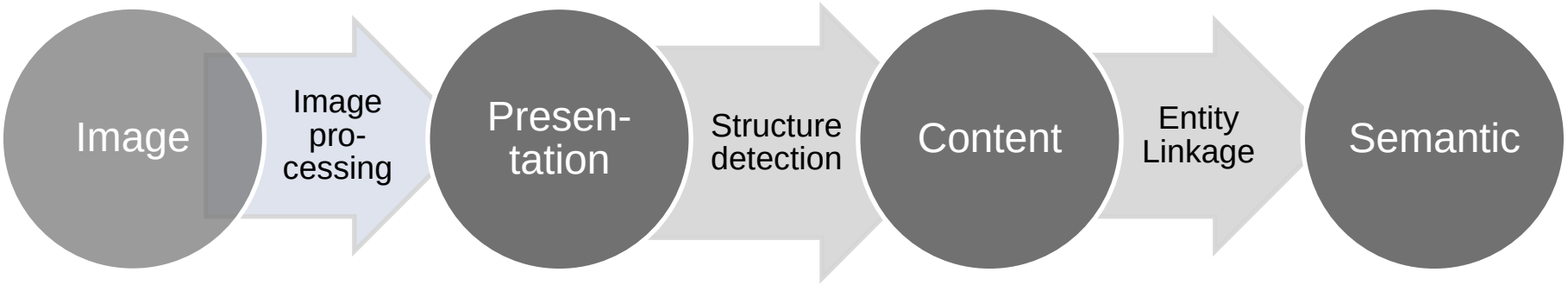
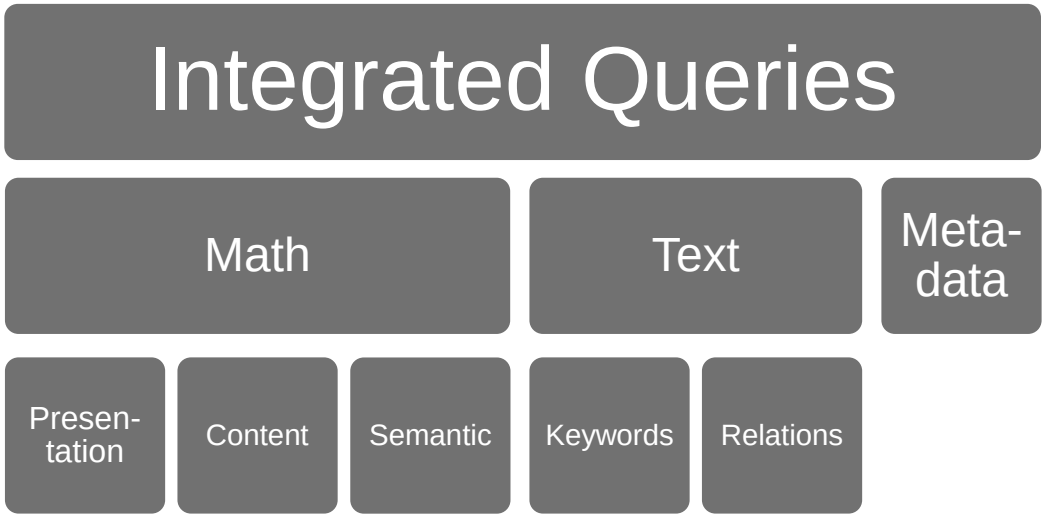
Challenges





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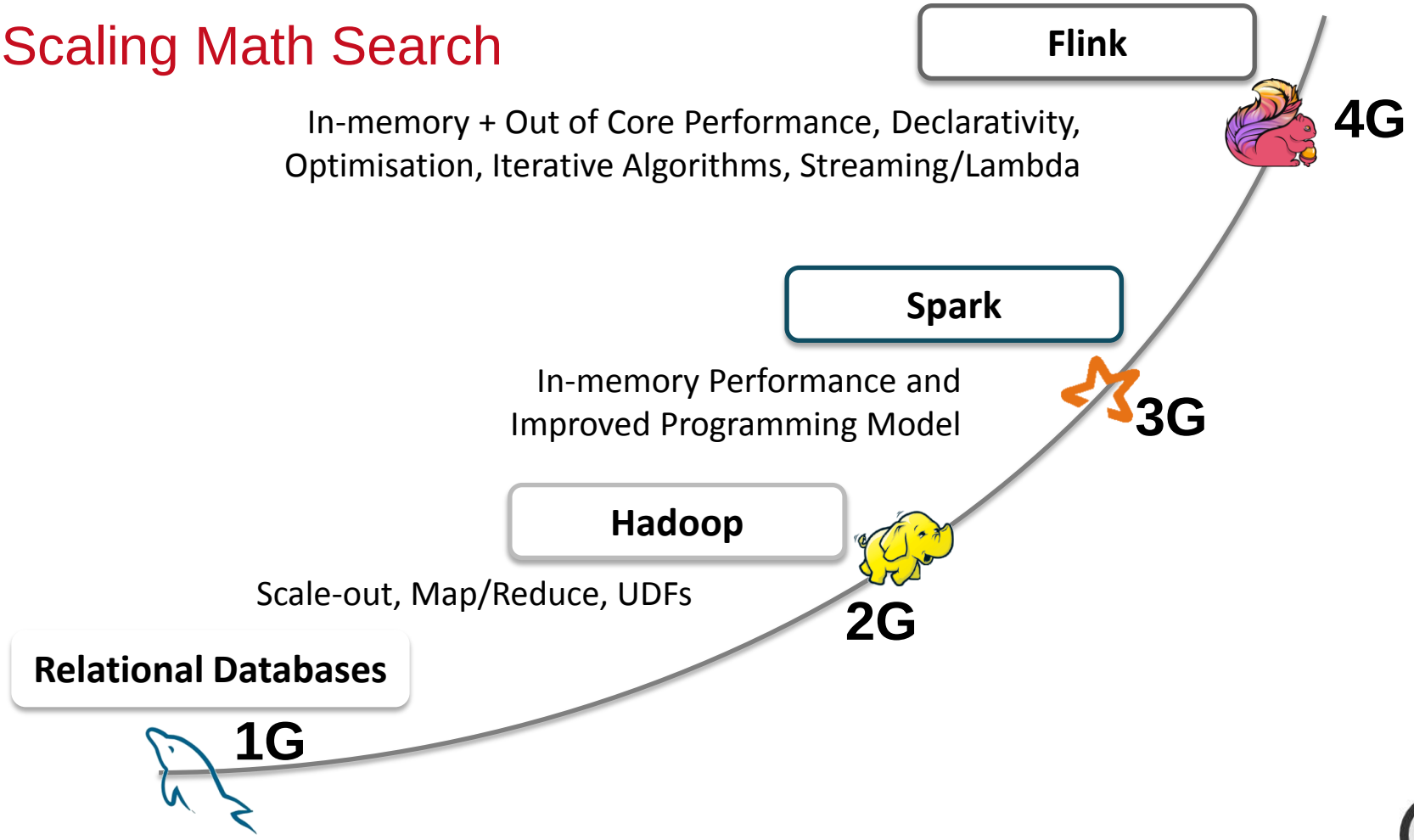
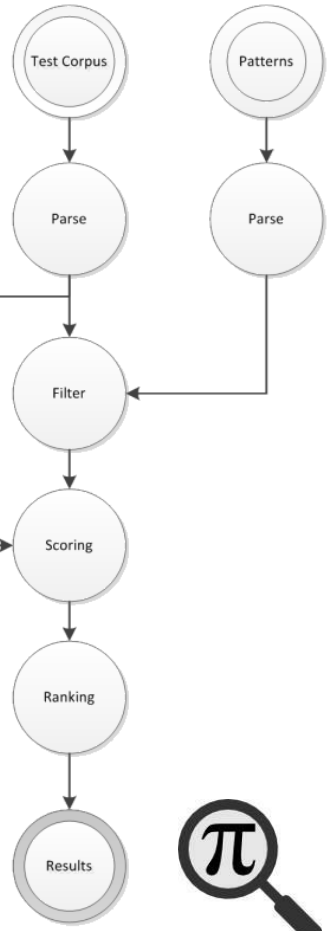
Scaling Math Search



Mathosphere: Using Apache Flink for scalable Math Search

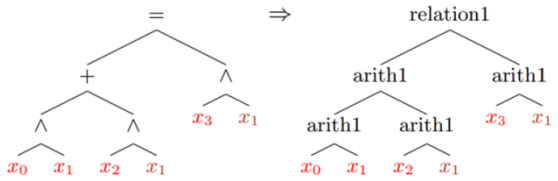


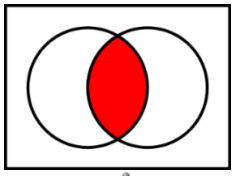
Evaluation

- CICM 2012
  (2 Participants)
- NTCIR 2013 (pilot)
  (6 Participants)
- NTCIR 2014

Evaluation of Similarity Measures Developed by Zhang & Youssef

Measure	Specificity
1 Taxonomic Distance	.91
2 Data Type	.91
3 Match Depth	(level 10) .87 … .93 (level 1)
4 Query Coverage	>0.7 (linear)
5 Formula vs. Equation	.26





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Conclusion and Summary

- Online Compendium of Mathematical Formulae using MediaWiki, DRMF
- Successes at seeding this compendium with Generic LaTeX sources
- Context free semantics for individual formulae
- Macro replacements
- Implementing Math Searchable archives
- NTCIR 12
- Ongoing: Different quality levels of Mathematical content
- DRMF Demo at http://drmf.wmflabs.org