WASHINGTON UNIVERSITY
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PHYSICALLY BASED MODELS FOR MAGNETIC RECORDING CHANNELS

by

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Prepared under the direction of Dr. J.A. O'Sullivan

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ABSTRACT

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Magnetic recording systems may be viewed as communications systems. Information theoretic analysis requires a model of the medium as a channel. Conventional models separate the physics of the medium from information storage analysis by several layers of abstraction. The applied physics literature includes detailed models used to represent magnetic microstructures. We present a model which shares many features with these models, including exchange, anisotropy, and demagnetization effects. Preliminary applications to information storage analysis are presented.

Table of Contents

Acknowledgments **v**

1 Introduction **1**

Magnetic Recording as an Information Channel 2

Magnetic Recording as a Physical Process 8

2 Micromagnetic Properties of Magnetic Recording Media **11**

Anisotropy Effects 12

Dipole-Dipole Interactions 18

Exchange Interactions 18

Magnetostriction 19

3 A Micromagnetic Model for Magnetic Recording Media **20**

Description of the Model 21

Similarity to Neural Networks 26

Limitations of the Model 28

4 A Channel Model Based on the Physical Medium Model **31**

Defining a "Use" of the MRC 31

Medium Storage Using Stable States 34

5 Conclusions **39**

A Derivation of Dipole-Dipole Interaction Coefficients **42**

B Bibliography **47**

C Vita **50**

List of Figures

1.1 A Communication System . . . 2

1.2 An Information Channel . . . 2

1.3 Magnetic Recording Channel . . . 4

2.1 Reference Framework for Anisotropic Grain . . . 13

2.2 Energy Versus Angle for Stoner-Wohlfarth Particle . . . 14

2.3 The Stoner-Wohlfarth Astroid . . . 17

2.4 Simple Illustration of Exchange Interaction . . . 19

3.1 Medium Model Geometry . . . 21

3.2 Simulated "Hysteresis" Loop . . . 29

4.1 Histogram of Number of Unstable Grains . . . 36

4.2 Gaussian Density Matched to Data . . . 37

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PHYSICALLY BASED MODELS FOR MAGNETIC RECORDING CHANNELS

1. Introduction

Magnetic recording systems store information. Their information storage capability may be analyzed by viewing them as information channels. This identification of a magnetic recording system with an information channel requires the selection of a channel model to represent the storage medium and the recording and reading processes. Once an appropriate model is selected, problems concerning the storage capacity of a recording medium, and recording and reading processes for optimum storage may be posed in a information theory framework. To achieve the most accurate results, we seek to base our models on the underlying physics of the micromagnetic properties of the recording medium.

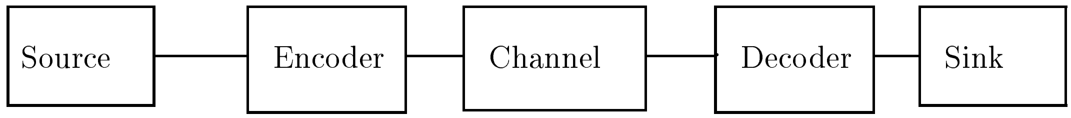

Figure 1.1: A Communication System

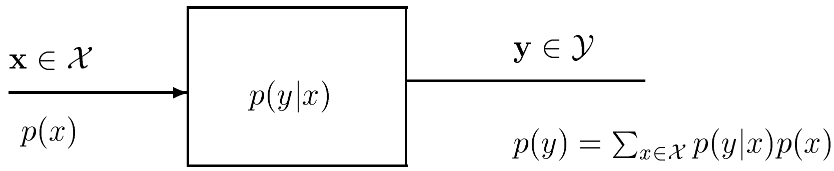

Figure 1.2: An Information Channel

1.1 Magnetic Recording as an Information Channel

Figure 1.1 is a block diagram of the conventional model of a communication system. The purpose of such a system is to convey information from the source to the sink (or destination) through the use of a channel connecting the two. A typical example of such a system is a telephone line carrying voice or data signals. Within the framework established by this block diagram, many analysis and design problems may be posed. For example, given a statistical description of the channel block, we may wish to design the encoder and decoder blocks for most reliable transmission of information. As another example, we may ask what limits exist on the amount of information that may be reliably conveyed through the channel. The study of information theory and communication theory provides the tools to approach these problems.

Figure 1.2 focuses on the middle block of Figure 1.1, providing a statistical description of the input/output relationship of the channel. A symbol $\mathbf{x}$, which takes on values from a set $\mathcal{X}$. is presented to the channel input. The channel produces as its output a symbol $\mathbf{y}$, which takes on values from a set $\mathcal{Y}$. The selection of the output symbol is governed by the transition probability function $p(y|x)$. This indicates that the output symbol is not a deterministic function of the input symbol, but it has a statistical dependence on the input symbol. Transmitting information through the channel means making use of the output symbol to estimate or otherwise characterize the input symbol with greatest reliability. Note that this view of an information channel is highly abstract. The "symbols" referred to in the diagram may correspond to sequences of what are usually thought of as symbols, e.g., the 1's and 0's of a binary data stream, or may correspond to continuous time waveforms.

Information channels are meant to convey any of several messages in an unpredictable order. If the message could be predicted, there would be no need for the communication channel. The information sink could use the prediction procedure to generate the message without any connection to the information source through the channel. To model this unpredictability, we view the input symbol $\mathbf{x}$ as a random variable drawn from $\mathcal{X}$ according to some probability function $p(x)$. This function may be a probability density function or a probability distribution function as appropriate to the type of data. Knowledge of this probability function can be used in the design of output symbol processing to recover information from the output symbol. Thus, the probability functions $p(x)$ and $p(y|x)$ characterize the function of the information channel, and hence, its ability to transmit information. Knowledge of these functions permits us to analyze the capacity of the channel, and to design strategies of input encoding and output processing to achieve reliable transmission of information. Moreover, if we have not a single channel, but an entire family of channels with probability functions indexed by a set of parameters, we may seek the best choice of parameters for maximum reliable information transmission.

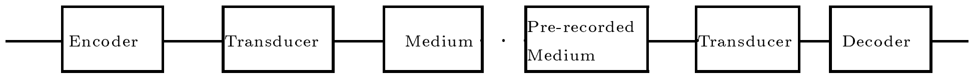

Figure 1.3: Magnetic Recording Channel

Figure 1.3 is a block diagram of a magnetic recording system. It comprises two subsystems: one for recording information, and one for reading pre-recorded information. The purpose of such a system is the reliable storage of information. Between the blocks of Figure 1.3, the information is carried by signals. These signals are of different types at different locations in the MRC. The input to the encoder is a sequence of abstract symbols, typically bits of binary data, to be stored. The signal between the encoder and the first transducer is a current waveform. The signal produced by the first transducer is a recording magnetic field that writes upon the recording medium. The second transducer reacts to the magnetization patterns on the medium, so these patterns themselves are the signal between the medium and the second transducer. The second transducer produces voltage waveforms as output. Finally, the decoder translates these voltage waveforms back into a sequence of abstract symbols.

This system has an information source and an information sink, just as the communication system does. In fact, this system can be placed in the general framework just described for an information channel. This is accomplished by linking the two subsystems, noting that after recording information on a medium, it becomes a pre-recorded medium. This view of the magnetic recording and reading processes as an information channel leads to a system we will call the magnetic recording channel (MRC).

When this connection is made, though, there remains an unanswered question: What is the channel? Shall we view the entire magnetic recording system as the channel block of a communication system? Shall we view only the medium as the channel block, identifying the rest of the magnetic recording system with other blocks of a communication system? Is some other correspondence preferable? Each choice leads to a different view of the magnetic recording channel.

If we draw our channel block boundaries symmetrically around blocks of Figure 1.3, there are three choices. At the outermost level, we may view the entire MRC as the channel block, viewing it as a system which transforms an input sequence of bits into an output sequence of bits. The next outermost level groups the medium and transducers as the channel block, viewing them as a system which transforms input current waveforms into output voltage waveforms. The transformation of input bits into current waveforms is viewed as an encoding procedure, and the transformation of voltage waveforms into output bits is viewed as a decoding procedure. The innermost level places only the medium in the channel block, viewing it as a system which transforms recording magnetic fields into magnetization patterns. The transformation function of the channel block is described by the physics of the recording medium. The encoder and decoder blocks are the entire recording and reading subsystems, respectively.

Once a choice is made, we may examine the performance of our chosen channel. To do this, we view the channel as though it were predetermined. That is, our design is limited to the parts of the communication system outside the channel block. The channel block is described by the transition probability function $p(y|x)$, and the design problem is to determine the best encoder and decoder blocks to exploit the information transmission capacity of the channel block.

As we change views from the outermost to the innermost level of description of the MRC, the description of the transformation function of the channel becomes less symbolic and abstract, and more closely related to the detailed physics of the recording medium. Also less and less of the MRC is viewed as predetermined; more of it is included in the encoder and decoder blocks, which are free for us to design. As this informal description suggests, the further inward we go in drawing the boundaries of the channel block, the greater the information capacity we can possibly achieve. A formal justification of this idea is provided by the Data Processing Inequality, a well-known result of information theory. This motivates a desire to model a magnetic recording system in the MRC framework with the innermost selection of the channel block – the channel corresponding to the recording medium itself. At this level, the storage capacity of the recording medium may be expressed in units of bits per unit area. Note that this approach does not mean we can not change the medium. Indeed one of the motivations of this research is to provide insights into how best to select recording media. It is only in the examination or testing of each medium that we hold it fixed. Then comparison of experiments involving different media may be used to compare their performance.

We may now pose the general questions applied to any information channel directly to the MRC: What is the capacity of the MRC to store and retrieve information? What is the best design of the encoder and decoder blocks , i.e., the recording and reading processes to achieve this capacity? If the transition function of the channel can be described in terms of its dependence on a set of parameters, what parameter values provide the best performance? The answers to these questions should produce techniques of analyzing magnetic recording media, motivate novel designs of magnetic recording systems, or provide guidelines for choosing materials for manufacture of recording media. If enough of the MRC is left unspecified, this analysis can include examination of adaptive recording and reading schemes.

Several examples of this view of magnetic recording systems as information channels have been published in the literature. The most abstract consider the magnetic recording channel as represented completely by its effect on a sequence of bits. An example is the bit-shift model described by Shamai and Zehavi [1]. These models do not directly model the physics of the recording media, but base their transition function on observed behavior, and on constraints imposed by the recording and reading processes which are viewed as predetermined.

At a lower level of abstraction are models which attempt to model the analog readback voltage of a magnetic recording system. These models are also based on a predetermined recording process. Properties such as intersymbol interference are visible at this level of abstraction, and a goal of reading process designs derived from these models is to overcome the effects of intersymbol interference. Partial response models have been a common approach at this level [2–4, **?**–5], and have proved useful in modeling recording levels over a wide range of bit recording densities in contemporary use. As recording densities increase, however, the applicability of these models becomes less certain. Other models directly model the readback voltage pulse observed as a Lorentzian form corrupted by magnetic and electronics noise [6, 7]. Some models include non-linear as well as linear effects by modeling the channel as a finite-state Markov process [8]. More recently, adaptive detection schemes have been proposed for such models [9, 10].

The thread common to all of these models of magnetic information storage is that they all use a "black box" approach. That is, certain input/output properties of physical magnetic recording channels are observed, and models are developed to have these properties. The models are often simple, and have proved effective in practice. However, our previous discussion indicates that there are gains to be realized by viewing the channel block of the MRC to be the innermost component of the magnetic recording system – the recording medium itself. This view treats only the medium as given, and views all transducers and processing to be designed. Also, a focus on the medium itself can provide insights into understanding effects observed in conventional recording systems such as media noise and bit shift. Thus, the capacity of this MRC is truly the capacity of the medium itself, not the capacity of any particular recording process. As performance demands grow, the need for optimal recording systems increases. The quest for optimal systems must take us inside the "black box," to models based on the physical processes involved in magnetic recording.

1.2 Magnetic Recording as a Physical Process

Magnetic recording is feasible because it is possible to induce long-term changes in magnetic materials. That is, these materials exhibit memory. In general terms, through a recording process a magnetic material can be placed into a state that can later be detected by a reading process. Furthermore, previous stable medium states can be changed to new stable medium states, providing overwrite capability. The stability of this state of the material, and the predictability of induced states, given recording process parameters, are the source of magnetic information storage capability. These physical processes provide the structure of magnetic recording to which we wish to match the function of information storage.

Experimental measurements of the fine details of the behavior of magnetic materials are becoming available. Much work has been done recently in the area of thin-film magnetic media. With the advent of these experimental results, detailed physical models of the micromagnetic properties of magnetic media have been developed.

Hoinville has reported measurements of the readback voltage of magnetic recording systems and proposed a model to describe his observations [11, 12]. Mian, *et. al.* have developed a method of estimating the transverse correlation length of thin film media [13]. This method permits non-destructive measurement of the micromagnetic properties of thin film media as it is in use. The correlation length characterizes the scale of micromagnetic features. The scale of these features is one source of the fundamental limits of the capacity of magnetic storage channels.

Computer simulations of micromagnetic behavior of magnetic recording media have been reported in the literature by Mansuripur and Giles [14] and Zhu and Bertram [15, 16]. The computer simulations focus on the physical processes at the micromagnetic level. These processes are the fundamental source of the transformation mapping from recording magnetic fields to medium magnetization patterns. The models must form the basis for a channel model with the least abstract, most physically based choice of channel block – the medium itself. However, the full detail of these models may not be necessary if simpler models can simulate the physical properties closely enough to produce similar information storage capability.

The challenge presented in this thesis is to move from these models of the physical processes of magnetic recording to a tractable model of the magnetic information channel. Such a model promises to be more accurate and more general than current models, because of its basis in the physics of the underlying magnetic processes. It is expected that properties explicitly designed into simpler channel models, such as bit error rates, bit shift, and media noise will emerge from a more physically based model.

Chapter 2 describes several properties of magnetic recording media observed at the micromagnetic scale. Chapter 3 devises a simplified model of a recording medium which exhibits the properties described in Chapter 2. Some similarities of the model to models of neural networks are noted, as are some of the limitations of the model in providing an accurate physical description of magnetic recording media. Chapter 4 discusses using this simplified medium model as the basis of a channel model of the MRC and uses an information theoretic approach to derive an upper bound on the storage capacity of a simulated medium. Chapter 5 briefly summarizes the thesis and suggests some directions for future work.

2. Micromagnetic Properties of Magnetic Recording Media

In order to create a useful model of thin-film recording media, we must first examine and describe the properties observed in such media that we wish our model to reflect. The media must be examined apart from any recording system or method, so that the properties measured are those of the media, not those of a particular recording process. Further, a model must be sufficiently detailed to accurately reflect the limits on the medium's storage capacity.

Micromagnetic models view media as large collections of magnetic particles, with interactions governed by magnetic forces. We refer to these magnetic particles as grains. We assume each grain is uniformly magnetized so that its magnetization may be represented by a single vector. Magnetization has the dimensions of magnetic dipole moment per unit volume. Thus, we may use the volume of a grain as a scale factor and view the grain as a magnetic dipole characterized completely by its magnetic dipole moment. For convenience throughout this chapter, we use the term *magnetic moment* to describe the vector which characterizes the magnetic state of a grain.

The usefulness of micromagnetic models and their simulation on computers has been established by Mansuripur and Giles [14] and Zhu and Bertram [15, 16]. Like them, we wish to base our models on the physics of the magnetic media. However, our interest is not in the physics itself, but in what the physics can tell us about the capacity of the media to store information and the mechanisms by which information is stored. Thus, we simplify their models to yield a physically-based model that is more computationally tractable.

Mansuripur and Giles model magnetic media as regular arrays of magnetic dipoles interacting through an effective magnetic field. The components of the effective field are the external field and fields derived from the anisotropy, exchange, and demagnetization energies. Our model also classifies the interactions of magnetic dipoles under these four categories, though, in the interest of computational simplicity, we use simpler formulations for some of these effects than do Mansuripur and Giles.

2.1 Anisotropy Effects

Anisotropy describes the tendency of the magnetic moment of a grain to prefer a few directions of orientation over others. The anisotropy energy which gives rise to this tendency is associated with the crystal structure and/or grain shape. The simplest form of anisotropy is uniaxial, where only one axis of orientation is preferred over all others. This preferred axis of orientation is called the *easy axis* of the grain. This effect can be modeled by a component of the magnetic field, as Mansuripur and Giles do. We take a different approach.

For a uniaxial grain which has its magnetic moment restricted to a plane, we use the Stoner-Wohlfarth model [17]. In Figure 2.1, the magnetic moment of a grain is drawn with respect to a pair of coordinate axes in the plane. The horizontal axis is the easy axis of anisotropy; the vertical hard axis is perpendicular to the easy axis in the plane. The magnetic moment $\vec{M}$ lies at an angle α with the easy axis.

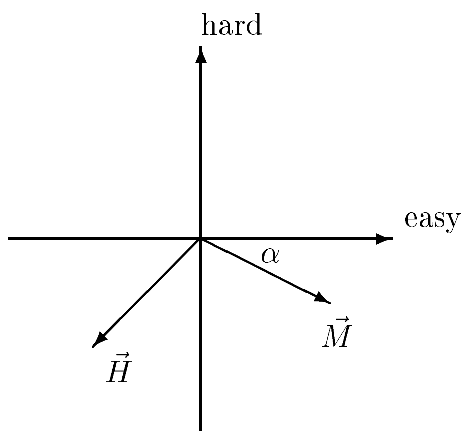

Figure 2.1: Reference Framework for Anisotropic Grain

The effective magnetic field $\vec{H}$ is also depicted in the figure. Note that since we are using the Stoner-Wohlfarth model to represent the effect of anisotropy, our effective magnetic field does not contain an anisotropy component. The precise composition of our effective magnetic field is described in the later sections of this chapter.

The energy associated with a magnetic moment $\vec{M}$ and a magnetic field $\vec{H}$ is proportional to the negative of the dot product of the two vectors. In order to include the preference of the magnetic moment $\vec{M}$ for the easy axis, we include in the energy expression an additional term representing the anisotropy energy. This term is dependent on the angle of deviation from the easy axis, α. The modified energy expression is:

$$E(\alpha) = K_A \sin^2 \alpha - MH_e \cos \alpha - MH_h \sin \alpha. \tag{2.1}$$

The first term of this expression is the anisotropy energy. The remainder is an expression of the negative dot product of $\vec{M}$ and $\vec{H}$ using subscripts to indicate the components of $\vec{H}$ along the easy and hard axes.

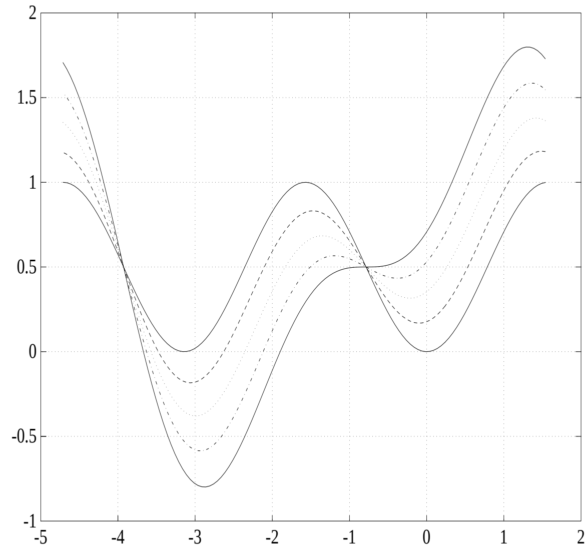

Figure 2.2: Energy Versus Angle for Stoner-Wohlfarth Particle

The grain seeks a state which minimizes energy. When the effective magnetic field, $\vec{H}$, is zero, the energy is minimized by an angle of $\alpha = 0$ or $\alpha = \pi$, that is, by an alignment of the magnetic moment with one of the two directions along the easy axis.

As the magnitude of the effective magnetic field is increased, the direction of the magnetic moment which minimizes energy changes. A particular example illustrates the general pattern. Figure 2.2 plots energy versus the angle α for several magnitudes of effective magnetic field. The magnetic field makes an angle of $-\frac{3\pi}{4}$ radians with the easy axis in this example. The constants K_A and M are set to unity for purposes of this illustration. The curves plotted in Figure 2.2 are for magnetic fields with magnitudes $0.00, 0.25, 0.50, 0.75$, and 1.00. Note that the horizontal axis includes values from $-\frac{3\pi}{2}$ to $\frac{\pi}{2}$. As the field magnitude is increased, one minimum of the energy plot becomes shallower. At the critical magnetic field magnitude of 1.00, the minimum is eliminated, and there is only one energy minimum where there had been two. If the magnetic moment was in the energy minimum which has disappeared, the magnetic moment of the grain suddenly shifts into the one remaining state which provides an energy minimum. If we then reduce the magnetic field magnitude back to zero, the vanished energy minimum reappears, but the grain's magnetic moment remains in the new direction. It does not return to its previous direction. The grain has changed state.

If the critical magnetic field magnitude had not been reached, then the magnetic moment would always return to the local minimum in which it began. The response of the grain to a changing magnetic field would be rotation through small angles away from its zero-field position on the easy axis.

The response of the grain to changing magnetic fields, then, is different depending on the magnitude of the effective magnetic field. When the magnitude exceeds a critical value, there may be discontinuous changes of state of the grain. When it stays below that critical value, the response can only be rotation through small angles, and when the field is removed, the grain returns precisely to its initial state. The key to distinguishing these two responses is to find the critical field value which separates them. This value depends on the constants K_A, M, and the angle that the effective magnetic field makes with the easy axis.

At an energy minimum the first derivative of the energy with respect to the orientation angle is zero, and the second derivative is positive. The minimum disappears when the second derivative goes to zero. In solving for the critical field value, it is convenient to normalize the quantities involved. We normalize the effective field to the *anisotropy field*, $H_A = \frac{2K_A}{M}$, yielding the normalized quantity:

$$\vec{h} = \frac{\vec{H}}{H_A}. \tag{2.2}$$

This normalized vector can be described by its components in the directions of the easy and hard axes, indicated by subscripts e and h respectively.

The normalized energy is:

$$\tilde{E}(\alpha) = \frac{1}{2}\sin^2\alpha - h_e\cos\alpha - h_h\sin\alpha. \tag{2.3}$$

which has first and second derivatives:

$$\frac{\partial \tilde{E}}{\partial \alpha} = \sin\alpha\cos\alpha + h_e\sin\alpha - h_h\cos\alpha \tag{2.4}$$

$$\frac{\partial^2 \tilde{E}}{\partial \alpha^2} = \cos 2\alpha + h_e\cos\alpha + h_h\sin\alpha. \tag{2.5}$$

Setting both of these expressions equal to zero yields a linear system of equations to solve for h_e and h_h. The solution is

$$h_e = -\cos^3\alpha \tag{2.6}$$

$$h_h = \sin^3\alpha. \tag{2.7}$$

If there is an α between $-\pi$ and π such that $\vec{h}$ is of the form indicated by this solution, then $\vec{h}$ is the critical magnetic field which may cause a change of state in the grain. This set of critical magnetic field values can be graphed in the plane by letting α vary between $-\pi$ and π, and tracing the curve described by the solutions.

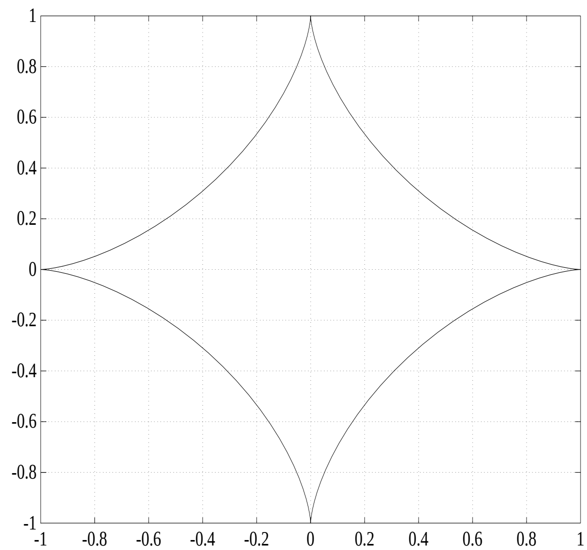

Figure 2.3: The Stoner-Wohlfarth Astroid

The resulting graph is known as the Stoner-Wohlfarth astroid, and is plotted in Figure 2.3. If we deal with the normalized magnetic field $\vec{h}$, then our discussion above lets us use the Stoner-Wohlfarth astroid to predict the response of a grain's magnetic moment. We draw the normalized magnetic field vector $\vec{h}$ on the plot of the astroid with its tail at the origin. If it lies entirely within the astroid, then there is only rotation of the magnetic moment through small angles around the easy axis. If the vector extends outside the astroid, though, then there may be a discontinuous change of state of the grain. Since a finite applied field is required to bring a permanent reversal of the grain's magnetic moment, a grain governed by the astroid exhibits coercivity.

2.2 Dipole-Dipole Interactions

We now describe the composition of the effective magnetic field. In addition to the externally applied magnetic magnetic field, several other effective field components act on the grains of an assembly.

The grains which form the basis of micromagnetic models may be viewed as magnetic dipoles, each of which generates a magnetostatic field. The dipolar interaction falls off slowly with distance, and through it, every grain in the assembly influences every other grain. The interaction complicates any micromagnetic model because as the number of grains increases, the computation required to model this effect becomes large very quickly.

Because the computation of this interaction views each grain as a magnetic dipole, the interactions are called dipole-dipole interactions. The magnetic field generated by the material is sometimes called a demagnetizing field, because its global effect in a bounded body is to oppose uniform magnetization of all the grains.

2.3 Exchange Interactions

Another magnetic interaction observed at the microscopic level of magnetic materials is the tendency of neighboring spins to align their magnetic moments parallel. This quantum mechanical effect may be described by having a large energy associated with rapid changes in the direction of the magnetic moment .

Figure 2.4 gives an idea of this effect. Because of exchange interactions, it is energetically unfavorable for a grain with “up” magnetic moment to be positioned next to a grain with “down” magnetic moment. In the absence of some other counteracting magnetic force (for example, high anisotropy energy), in order to

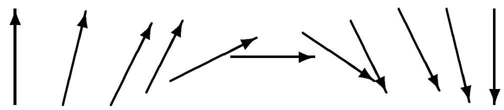

Figure 2.4: Simple Illustration of Exchange Interaction

complete a transition from a region of “up” magnetic moment to a region of “down” magnetic moment within the medium, the exchange interactions will produce a transition region with width spanning several grains over which the transition occurs. Then each step in the transition from one grain to its neighbor is small enough to produce a small exchange energy. When other forces are present, this particular effect may not be observed, depending on the relative strengths of each magnetic energy.

2.4 Magnetostriction

Another micromagnetic effect which we note here, but do not include in the model is magnetostriction. As a magnetic grain changes its magnetic properties (for example, the direction of its magnetic moment), it undergoes mechanical changes as well, including changes in size and shape. If a grain were constrained to maintain a constant size or shape, then this constraint would influence the ease with which the grain may enter certain magnetic states. This effect is called magnetostriction, and can be modeled by another energy term. We assume the effect of magnetostriction is negligible compared to other magnetic effects in the medium [18].

3. A Micromagnetic Model for Magnetic Recording Media

The preceding chapter described several of the micromagnetic interactions observed in recording media. Designing a model based on these observations requires that we choose a representation of each observation in mathematical terms. The models of Mansuripur and of Zhu, on which we base our model, make use of the Landau-Lifshitz-Gilbert (LLG) equation to relate the states of magnetic moments to the evolving values of the global magnetic field. The LLG equation at a point is:

$$\frac{d\vec{M}}{dt} = \gamma\vec{M} \times \vec{H}_{\text{eff}} + \frac{\alpha}{|\vec{M}|^2}\vec{M} \times \frac{d\vec{M}}{dt}. \tag{3.1}$$

Recall that the effective magnetic field, $\vec{H}_{\text{eff}}$, at a grain depends on the magnetic moments of all other grains, so this single equation represents an entire system of mutually dependent equations to be solved. The solution of these equations requires a great deal of computation. This computational complexity is the reason that these models are simulated on such high-powered computing machinery as the Connection Machine or a Cray supercomputer.

Our model focuses more narrowly on the aspects of medium behavior which are of interest in magnetic recording. This narrower focus allows us to simplify the model to the point that we need not solve such a complicated set of mutually dependent

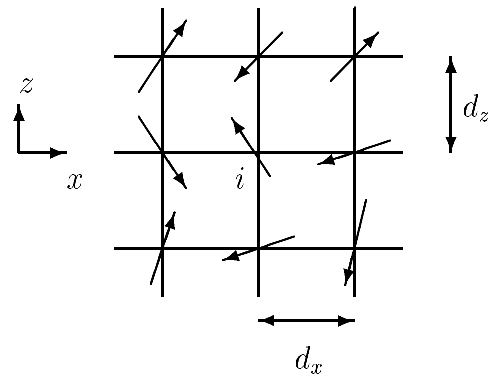

Figure 3.1: Medium Model Geometry

equations. The justification for these assumptions will be made clear as we build up the model.

3.1 Description of the Model

We represent a thin-film magnetic medium as a collection of magnetic grains interacting through magnetic forces. The grains are assumed to lie on the vertices of a regular rectangular lattice (see Figure 3.1). The rows of the lattice are separated by a distance d_x and the columns are separated by a distance d_z. The rectangular lattice determines a global x-z coordinate system in which magnetic interactions may be described. To each grain there is associated a single magnetic moment vector which completely characterizes its magnetic state. We make an assumption of identical grains in our model, so every magnetic moment vector is assumed to have the same magnitude. We use N to represent the number of grains in the medium, and index each grain and each variable associated with a grain with a subscript i which can take values in $\{1, ..., N\}$.

Following our discussion of anisotropy in the preceding chapter, we assume each grain exhibits uniaxial anisotropy, and assign to each grain an easy axis of anisotropy. When the effective magnetic field at a grain has magnitude near zero, that grain's magnetic moment will point nearly along the easy axis. When we make use of a medium to record information, there will be long periods of time between the writing of information onto the medium and the reading back of the information during which there will be no applied magnetic field present at the medium surface. During these periods, the effective magnetic field at each grain will comprise only that portion of the field due to the magnetic moments of other grains in the medium. If we assume that the anisotropy constant is large compared to exchange and dipole-dipole interactions, then the normalized effective magnetic field will be near zero during these periods and each grain's magnetic moment will be nearly aligned with its easy axis.

Based on this discussion, we will assume that the magnetic moment of every grain is always aligned with the grain's easy axis of anisotropy. Then the magnetic state of a grain is specified by the orientation angle of its easy axis, and a variable which takes one of the two values -1 or $+1$, indicating the direction along this axis in which the magnetic moment points. The expression of the magnetic moment of the i-th grain in global x-z coordinates is:

$$\vec{M}_i = M\sigma_i[\cos\theta_i, \sin\theta_i]^T, \tag{3.2}$$

where θ_i is the angle that the easy axis of grain i makes with the global x-axis, and σ_i takes on the value of either $+1$ or -1. M is a constant determining the magnitude of magnetic moment of every grain in the collection.

The angles of orientation, θ_i, determine the easy axes of the grains. We assume that these axes are fixed properties of the grains of the medium, determined in the manufacturing process. By introducing field biases in the deposition process used to manufacture recording media, a variety of probability distributions can be induced on these angles. We choose to assume that these angles are chosen randomly from a uniform distribution over 0 to π radians and are chosen independently from one grain to another. This model for easy axis orientation is chosen for its simplicity. It can be changed if knowledge of the deposition process or measurements indicate it is an unwarranted assumption.

At each grain, there is also an effective magnetic field $\vec{H}_i$. This magnetic field includes all the interactions due to magnetic forces. As described in the preceding chapter, we will consider three sources of the magnetic field: exchange interactions with neighboring grains, dipole-dipole interactions with more distant grains, and the applied magnetic field. Each of these sources may be modeled as a component of the effective magnetic field, so that the total effective magnetic field is the sum of the three component parts:

$$\vec{H}_i = \vec{H}_{e,i} + \vec{H}_{d,i} + \vec{H}_{a,i}. \tag{3.3}$$

The effective magnetic field due to exchange interactions, $\vec{H}_{e,i}$, models the tendency of the magnetic moment of a grain to align with those of its neighbors. Since magnetic dipoles tend to align with the effective magnetic field, this effect is modeled by letting the exchange field component be a magnetic field vector proportional to the magnetic moments of the neighbor grains. This leads to:

$$\vec{H}_{e,i} = K_e \sum_{j \in N(i)} \vec{M}_j, \tag{3.4}$$

where $N(i)$ is the set of grains which are neighbors of grain i, and K_e is a proportionality constant. In our model, the neighborhood consists of the four nearest grains which share one coordinate in common with a given grain; i.e., there are no "diagonal" neighbors. Also, we assume the proportionality constant is the same for all pairs of neighboring grains.

The magnetic field component due to dipole-dipole interactions, $\vec{H}_{d,i}$, models the magnetostatic interactions of a collection of magnetic dipoles. Appendix A presents a derivation from well-known magnetostatic equations in the framework of our model to the final relationship given by:

$$\vec{H}_{d,i} = \sum_{k \neq i} \mathbf{A}(i,k) \vec{M}_k, \tag{3.5}$$

where $\mathbf{A}(i,k)$ is a linear operator which determines the effect of magnetic moment $\vec{M}_k$ on the field value at grain i. When grains i and k are separated by an x displacement r_x and a z displacement r_z, and $\vec{H}$ and $\vec{M}$ are expressed in x-z coordinates, the linear operator is expressed in matrix form as:

$$\mathbf{A}(i,k) = \frac{K_d}{(r_x^2 + r_z^2)^{\frac{5}{2}}} \begin{bmatrix} 2r_x^2 - r_z^2 & 3r_x r_z \\ 3r_x r_z & 2r_z^2 - r_x^2 \end{bmatrix}, \tag{3.6}$$

where the constant K_d depends on the volume of a grain. Note that this expression implies that the operator is spatially invariant, depending only on displacements r_x and r_z.

Finally, the component of magnetic field due to an applied magnetic field does not arise from the dipole collections. It plays the role of the input variable to our information channel. Often we are interested in situations where this applied field is zero.

Now that the dipoles and their magnetic fields have been defined, all that remains is to describe their method of interaction. Rather than model the details of micromagnetic switching dynamics, we develop our model to focus on long-term stable configurations. We have already used this narrow focus to restrict a grain's magnetic moment in our model to one of two directions along its easy axis. Because of this, the evolution of the state of a grain can only take one form, a "flip" from one direction to the opposite. Thus, the interaction of a grain with the magnetic field is completely defined for our model if we determine the conditions under which a flip will occur. We refer to the value of a grain's magnetic moment as the state of the grain.

The state of a grain i is completely determined by the sign value σ_i. The interaction definition reduces to determining which value of $+1$ or -1 each σ_i will take given the field value $\vec{H}_i$ at that grain site. Since we know that magnetic dipoles tend to align with the effective magnetic field, the simplest approach is to choose the direction which is most nearly aligned with the effective magnetic field. However, this approach neglects the effect of anisotropy. To include this effect, we make use of the Stoner-Wohlfarth astroid to first determine if the effective magnetic field has great enough magnitude to overcome the coercivity of the grain. If so, then the grain changes state if it is energetically favorable to do so.

This description is complete for a single grain. However, our model consists of an entire collection of grains. When one grain changes state, the effective magnetic field seen by every other grain changes. Thus, in order to complete our specification, we must determine an update procedure for examining and possibly switching grains. The chosen update procedure is to choose grains in a random order for update. This choice guarantees that eventually a stable configuration must be reached, as discussed in the following section.

3.2 Similarity to Neural Networks

Once a fixed set of anisotropy angles $\{\theta_i\}$ is determined, the global state of the medium is completely determined by assigning the value $+1$ or -1 to each σ_i. Furthermore, if we assume the applied magnetic field is constant, the effective magnetic field value at grain i, $\vec{H}_i$, is a linear function of the set of σ_j for $j \neq i$. Thus, the simulation of one grain update may be described by the computation of a linear function of the states of the other grains (computing $\vec{H}_i$) followed by the update of value σ_i according to a non-linear function of the current σ_i value and $\vec{H}_i$. Grains are chosen at random for update according to this rule. From this perspective, our model appears very similar to the Hopfield model for a neural network [19].

Our grains correspond to the neurons, or units, of a neural network model. In particular, the linear relationship between the σ_i values and the $\vec{H}_i$ values may be described by a matrix with symmetric entries. In the realm of magnetics this signifies that magnetic forces are reciprocal; they obey Newton's third law that every action has an equal and opposite reaction. In the notation of neural networks we may write the update operation for one of our grains as:

$$\sigma_i^{(k)} = f(\sum_j w_{ij}\sigma_j^{(k-1)})$$

where f is the non-linear threshold function, the w_{ij} are the matrix entries of the linear relationship between the σ values and the normalized magnetic field values, and the k superscripts indicate iteration or time steps. In a neural network model, this property allows us to define a function of the global state of the neural network such that every step in the network's evolution causes a non-increasing change in the value of this function. In the neural network notation this function is:

$$H(\sigma^{(k)}) = -\sum_{ij} w_{ij}\sigma_i^{(k)}\sigma_j^{(k)}$$

This function is often called an energy function, but it should not be confused with the energy function used in Chapter 2 to derive the Stoner-Wohlfarth astroid.

The difference between our model and the Hopfield model is that the Hopfield model uses a simple threshold function as its non-linear operation, while our model uses a more complicated function based on the Stoner-Wohlfarth astroid. However, based on the similarities of the models we can make use of some results in the study of neural networks to examine our model.

In particular, it is a standard proof [19] in the study of Hopfield neural networks to show that the evolution of a symmetric Hopfield neural network must eventually lead to a stable state of the network, that is, a state in which no neuron will change state when it is selected for update. As mentioned above, the proof is performed by defining an energy function of the network state which is non-increasing with the evolving state of the network. The additional complication of our model, the Stoner-Wohlfarth threshold, can only prevent updates that might otherwise occur. Thus, any condition satisfied by an update in the Hopfield model must also be satisfied by an update in our model. In particular, the proof just cited is still valid; the energy function is non-increasing during the evolution of the model under the condition of constant applied magnetic field. Since there are only finitely many global states, it follows that under these conditions, the system must converge to a stable global state with probability one.

3.3 Limitations of the Model

Our model does not accurately describe all properties of a physical magnetic medium. A number of physically unrealistic assumptions have been made in an attempt to keep the complexity of the model manageable. We hope, though, that our simplified model still has characteristics similar to those observed in physical recording media.

The most restrictive simplifying assumption is that the states of the media are modeled as completely described by a sign value at each grain site. That is, the grains' magnetic moments are assumed to lie only along their easy axes of anisotropy. Physical grains obey no such restriction, but align in whatever manner yields a minimum of energy. In particular, when very large magnetic fields are applied to physical media, the grains align with the applied field, departing from their easy axes of anisotropy. Such behavior can not be observed within the constraints of our model. When we seek to observe hysteresis loops in our medium model, this

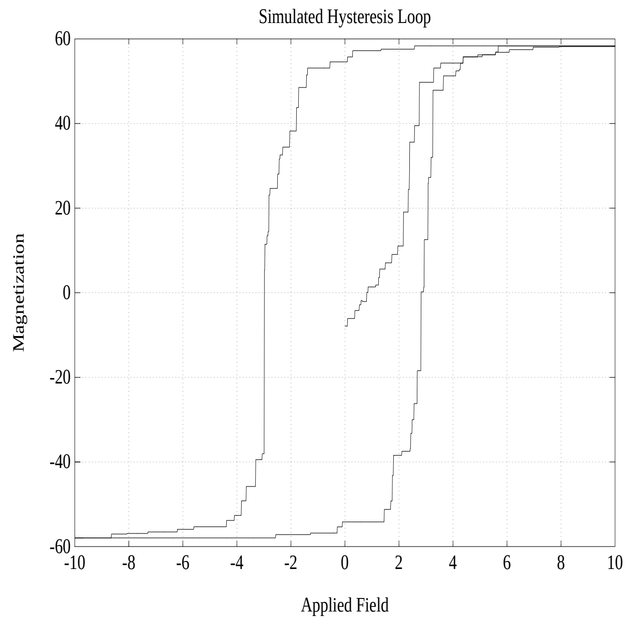

Figure 3.2: Simulated "Hysteresis" Loop

effect appears as the inability of our modeled medium to reach a full saturation magnetization.

We illustrate this shortcoming in Figure 3.2. The figure is a plot of the magnetization of a computer simulation of the physical model just described versus the applied magnetic field. The simulated medium consists of 100 grains. Only the components of applied field and magnetization along the global z-axis are plotted. The units are arbitrary for purposes of the illustration. Like a hysteresis loop, this plot demonstrates the properties of remanence and coercivity of the simulated medium. However, a true hysteresis loop would exhibit a maximum magnetization value of 100 of our arbitrary units of magnetization, as all 100 grains turn completely to align with a large applied field.

The structure of our modeled medium is also unrealistically simple. Grain sites are constrained to a rectangular lattice, completely within a plane. Such regularity is not observed in physical media. We may observe spatially periodic states or sharper transitions in the model than appear in physical media, due to the presence of this regular structure.

Several other simplifying assumptions of identical grains and interaction parameters may also not be physically realistic, but have been made to keep the model simple. The model can be improved at the cost of greater complexity if these assumptions lead to simulations in significant disagreement with physical observations.

Finally, the focus of this model is on stable magnetization states. The transient behavior of a magnetic medium is not modeled accurately. This approach has advantages of greater simplicity and a focus on the properties of the medium most relevant to long-term storage, but it also has the disadvantages of ignoring much detail that may be relevant particularly to the recording process.

4. A Channel Model Based on the Physical Medium Model

The physical model presented in the preceding chapter may be used to compute patterns of magnetization on a medium surface which form in response to applied magnetic fields. The model is not deterministic, so these computations simulate the stochastic input-to-output transformation of the medium we seek to understand. Referring back to Figure 1.2, the physical model provides the raw material needed to characterize the transition probability function $p(y|x)$. Completing this characterization will produce a physically based channel model of the magnetic recording medium.

4.1 Defining a "Use" of the MRC

An important aspect of the channel in Figure 1.2 has not yet been specified – the sets of input and output values. These sets may be restricted by physical constraints on the transducers used for reading and writing. More fundamentally, though, to specify these sets, we must determine a definition of a "use" of the channel.

Information channels are only useful if they can be used repeatedly. Capacity is usually expressed in bits transmitted per channel use. To transmit a stream of information across a channel, we must break the stream into a sequence of symbols.

Each symbol must carry an amount of information not greater than the single-use capacity of the channel. The channel is then used repeatedly to transmit the sequence of symbols. Each use of the channel is assumed to require an interval of time. So long as the sequence is generated sufficiently slowly, the channel can "keep up". This matter is the subject of the information theory study of data rates. Frequently, channel capacities are expressed in terms of a maximum rate of information flow that repeated use of the channel can achieve.

This view is problematic for the magnetic recording channel for several reasons. First, recall that the MRC is the joining together of two subsystems, a recording process and a reading process. These two processes may be separated in time by a long interval. The notion of a data rate does not match this circumstance. Instead, we are interested in capacity as a measure of storage capability. We seek to continue storing more symbols on the medium as long as we can, without interfering with the ability to recover those symbols already stored.

Second, the usual correspondence of input to output found in information channels is not present in the MRC. Data can be recorded just once, then read many times. Data bits may be read out in a different order than they are recorded because of the random access nature of many magnetic recording systems. Previously written data may be corrupted by later writes on the same medium. When reading information from a medium, the input to the channel is the entire history of applied fields at the medium surface.

We would like to make best choice, in some sense, of the definition of a use of the MRC. However, the problems just mentioned make the concept of a use of the MRC difficult to fathom. Furthermore, the sense in which a definition is "best" must be chosen. Maximizing storage capacity is of little use if the recording and reading time increases greatly as a result.

These conceptual difficulties are to some degree avoided by the standard methods of magnetic recording. Bits are recorded in a localized manner on the medium. Each bit of information to be stored has its own region on the medium devoted to its storage. Furthermore, bits are recorded in tracks composed of several bit storage regions in a sequence. Recording and reading are achieved by passing transducers over the tracks in a single direction. Thus, there is a correspondence of a sequence of input bits recorded to a sequence of bits read out, and defining a use is more straightforward. Random access is provided at a coarser scale so that blocks of bits may be called up by reference, but the bit by bit readback is sequential. The sequence read back matches the sequence recorded, so the familiar correspondence from information channels is present.

Standard methods of magnetic recording relieve us of the difficulty in defining a channel use. However, they also take us back where we started, to conventional methods with little insight into other schemes gained from the physically-based modeling. Also, the difficult details which sent us retreating back to conventional methods have not disappeared. Recording new data bits can still corrupt previously stored bits. Long-range magnetic interactions at the medium surface still occur. Conventional methods do not escape these physical effects. Conventional methods name each effect and treat each one separately: bit shift, track-edge noise, media noise, intersymbol interference, and cross-talk between adjacent tracks. The physically based media modeling considers all these effects in a unified manner.

Thus, it is still desired that we find an appropriate way to define a use of the MRC, or find another way to look at the physical medium model as an information channel. There is a great deal more work to be done on this issue.

4.2 Medium Storage Using Stable States

Fortunately, we are not at a standstill. We may look at the two subsystems of the MRC depicted in Figure 1.3 separately. Each is a perfectly good information channel on its own, with none of the difficulties that the storage device, the medium, introduces when included as the middle block. In these information channels, the medium serves as an information sink or an information source. The symbol set at the medium surface is just the set of global states of the medium.

Looking at the reading subsystem, we see that the medium is the information source of this subsystem. The amount of information generated by this information source is the entropy of the random variable representing the global medium state. This entropy is determined by the transition probability functions relating the global medium state to the entire history of recording magnetic fields presented to the medium, and by the probability distribution on those recording fields themselves. If we can determine this entropy, we will have a bound on the storage capacity of the recording medium.

A simple upper bound on this entropy serves as a first approximation to this capacity. We assume that "storage" of data refers to its preservation on the medium over a period of time when no recording field is applied to the medium. Then only those global medium states which are stable under a condition of zero-applied field may have non-zero probability when the data is read back from the medium. An estimate of the number of such stable states provides us with a bound on the storage capacity of the medium.

The entropy, expressed in bits, of a random variable which takes its values from a finite set is bounded above by the base 2 logarithm of the number of values with non-zero probability. This bound is achieved only when all of these values with non-zero probability are assumed with the same probability. That is, a uniform probability distribution over the set of possible values governs their relative probabilities.

We seek the number of possible (that is, having non-zero probability) magnetization states for long-term storage. We know from the discussion in the preceding chapter that given a long enough relaxation time during which the applied magnetic field is constant, our model of the medium guarantees that a stable global magnetization state will be reached which will not change unless the applied magnetic field is changed. For storage purposes, the long-term constant magnetic field has zero magnitude. Thus, our set of possible storage magnetization states is precisely the set of magnetization states which are stable under the condition of zero applied magnetic field.

The number of such states may be estimated by a random sampling approach. We select a magnetization state at random and use the dynamics of our model to determine whether the state is stable under a condition of zero applied magnetic field. Unfortunately, even for the modest scale of a model of 100 grains in a 10×10 grid, the number of states to consider, 2^{100}, is far too great to be tested for an accurate count of the number of stable states. When we try to test a small sample of states to estimate the proportion of stable states to the whole, we find that stable states are sufficiently rare that random sampling does not find even one stable state in 10^6 samples. We are faced with the problem of trying to estimate the probability

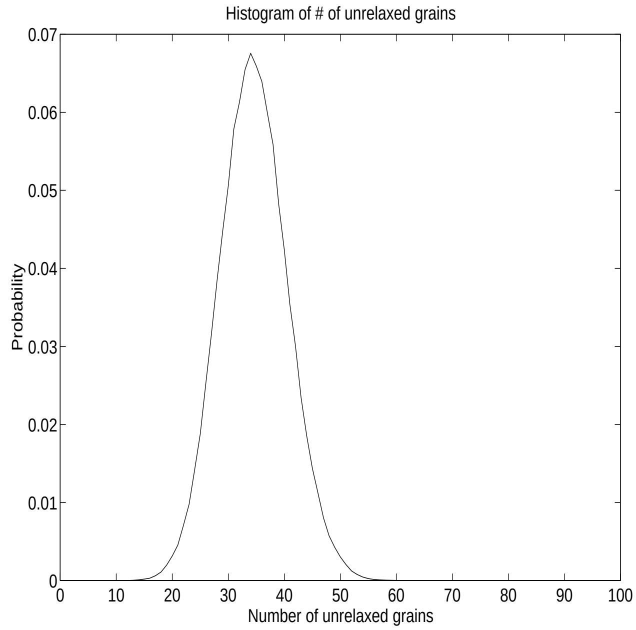

Figure 4.1: Histogram of Number of Unstable Grains

of a rare event with a limited number of samples. Direct sampling cannot provide the answer.

Another approach is suggested by recalling what it means for a state to be stable. A state is stable if every grain is in a state such that if it is selected for update, it will not change state. An unstable state has at least one grain which will change state if selected for update. We say this grain is unstable. The number of such grains in the medium is a random variable we may simulate. Using the same 10^6 randomly chosen magnetization states, we collect data on the number of unstable grains in each state. Figure 4.1 is a plot of these data showing the relative proportion of states having each number of unstable grains. The data were generated using arbitrary choices for the medium parameters. They do not represent any particular physical medium.

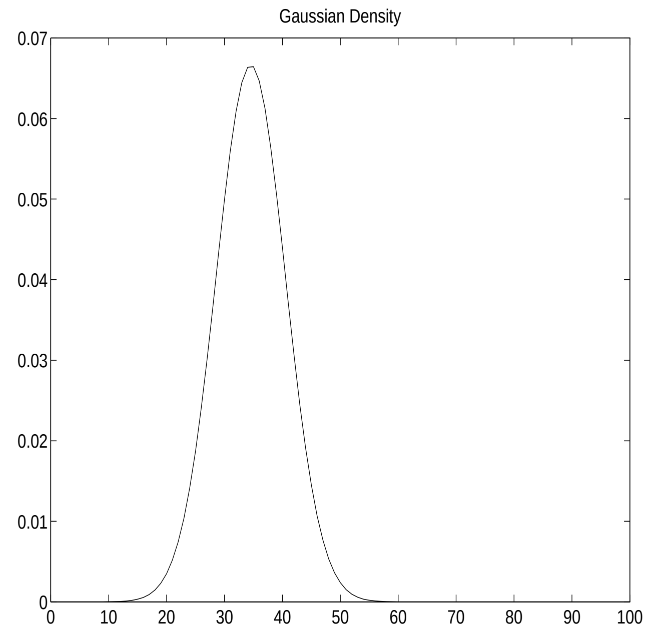

Figure 4.2: Gaussian Density Matched to Data

The smallest number of unstable grains observed in the collection of data was 12. A stable magnetization state would have zero unstable grains. In order to estimate the probability of a stable state, we wish to estimate the intercept of the histogram plot above with the ordinate axis. To do this we must extrapolate from the collected data by making some assumption about the probability distribution.

Figure 4.2 is a plot of a Gaussian probability density with mean and variance given by the sample mean and sample variance of the collected data. The values are 34.54 and 35.85 respectively. Comparison of the two plots reveals a close match over the interval of support of the data plot. Based on this favorable comparison, we extrapolate based on the Gaussian plot.

Our collected data are integer-valued and restricted to non-negative values. The Gaussian density describes a real-valued random variable with values taken from the entire real line. In matching the Gaussian density to our restricted domain, we assume that any value less than zero will be mapped to zero. Thus, in our extrapolation, we estimate the ordinate-axis intercept as corresponding with the integral of the entire Gaussian density over values less than zero:

$$\int_{-\infty}^{0} \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{(z-\mu)^2}{2\sigma^2})dz \tag{4.1}$$

Since we wish to estimate this error function quantity at a value far from the mean, we may use the well-known approximation [20, for example]:

$$\approx \frac{1}{(2\pi)^{\frac{1}{2}}} \frac{\sigma}{\mu} \exp(-\frac{\mu^2}{2\sigma^2}) \tag{4.2}$$

Multiplying this estimate of the probability of a stable state by the total number of stable states yields an estimate of the number of stable states of $2^{72.14}$. The upper bound on storage of this simulated medium is then 72.14 bits, or .7214 bits per grain.

5. Conclusions

We have presented physical descriptions of the micromagnetic characteristics of magnetic recording media. We have devised a simple model of the medium based on these physical descriptions. A discussion of placing this physical medium model into an information channel context revealed a number of difficulties in matching the MRC to the usual information channel. In spite of these difficulties, the storage capacity of a medium may still be bounded by the entropy of the medium's global state. A method for computing an upper bound on this entropy was demonstrated on a simulated medium with arbitrary parameters.

This work raises many questions to be pursued in further study. The primary quantitative result presented in this thesis is a limit to storage capacity based on an estimate of the entropy of the magnetic state of a magnetic medium. Within this work, the simulated medium has been constructed with arbitrary parameters. An important improvement will be to determine better parameter values to represent physical media in use in contemporary magnetic recording systems.

The difficulties encountered in Chapter 4 need to be addressed. Perhaps an exploration of alternative information theory models, in particular, an investigation of channels with memory, will provide a better match of the observed behavior of magnetic recording systems to a theoretical framework useful for analysis.

More work may also be done in developing a justification for the model used in this thesis, or for developing improvements to it. The most restrictive feature of the model presented in this thesis is its requirement that every magnetic moment be aligned with a single easy axis of anisotropy. The advantage of such a restriction is that it makes the number of possible magnetic states of the medium finite and easy to count. Without such restrictive assumptions, however, it may be possible to prove that the number of stable magnetization states must be finite based on more fundamental physical notions of energy minima rather than *a priori* model constraints. Work is underway on this point.

Further work needs to be done investigating the implications of this medium model beyond simple counting arguments about numbers of stable states. One simple exploration may be to repeat experiments similar to those presented in this thesis with testing of broader ranges of values for all medium parameters. This exploration should provide insights into the trends and tradeoffs involved in the choice of parameters for physical media.

Beyond this exploration, however, this model provides the opportunity to examine the behavior of magnetic media in greater simulated detail than is easily possible for physical media. For instance, the relationship between initial states and final states of a simulated medium undergoing a write process can provide insights in how to more efficiently write information for greater readback reliability. Simulating the writing of a transition between regions of magnetization on the medium may provide insight into the types of irregularities observed at transition boundaries, and a deeper understanding of the sources of media noise.

The insights gained from these studies may provide enough understanding of the relationship between the writing process and the states of the medium to allow the development of adaptive schemes of magnetic recording. For example, knowledge of the current magnetic state of a medium may be used to modify the write process to more efficiently record information. This approach can be called a pre-compensation approach. By adapting the write process on the fly to compensate for the effect of the pre-existing magnetic state, greater efficiency, reliability, and storage capacity are expected to be achievable. Other adaptive schemes may also be explored.

APPENDICES

PHYSICALLY BASED MODELS FOR MAGNETIC RECORDING CHANNELS

A. Derivation of Dipole-Dipole Interaction Coefficients

In this appendix we derive the form of the linear operator which describes the effective magnetic field at a grain i due to the dipole-dipole interaction influence of the magnetization vector at grain k.

From Cheng [21] , if we place a magnetic dipole with moment $\vec{m}$ at the origin of a spherical coordinate system aligned with the axis where $\theta = 0$, and consider a point in space described in spherical coordinates (R, θ, φ), the magnetic flux density at that point is

$$\vec{B}(R,\theta) = \frac{\mu_0 m}{4\pi R^3}(\vec{a}_R 2\cos\theta + \vec{a}_\theta \sin\theta), \tag{A.1}$$

where $\vec{a}_R$ and $\vec{a}_\theta$ are the usual unit vectors in spherical coordinates at the point in question and m is the magnitude of the magnetic dipole moment vector.

As a first step, we convert this equation to an equivalent one in an x'-y'-z' Cartesian coordinate system defined so that the z'-axis is aligned with the dipole, and the x'-z' plane is the plane of the medium in our model. The primes are used to distinguish this coordinate system from the global x-y-z coordinate system used in the definition of the model. Applying the appropriate conversions yields:

$$\vec{B}(R,\theta) = \frac{\mu_0 m}{4\pi R^3}[\vec{a}_{x'}(\pm 3\cos\theta\sin\theta) + \vec{a}_{z'}(2\cos^2\theta - \sin^2\theta)], \tag{A.2}$$

where the $\pm$ choice is made based on the location of the point in question with respect to the coordinate system. Let $\vec{R}$ be the position vector of the point in question. If $\vec{R} \cdot \vec{a}_{x'} > 0$, then choose $+$; otherwise, choose $-$.

At this point, we observe that:

$$\vec{m} \cdot \vec{R} = mR\cos\theta \tag{A.3}$$

$$|\vec{m} \times \vec{R}| = mR\sin\theta. \tag{A.4}$$

Using these equations, we may rewrite the magnetic flux density expression in a form which does not explicitly include θ:

$$\vec{B}(\vec{R}) = \frac{\mu_0}{4\pi m R^5}[\vec{a}_{x'}(\pm 3|\vec{m} \times \vec{R}|(\vec{m} \cdot \vec{R})) + \vec{a}_{z'}(2(\vec{m} \cdot \vec{R})^2 - |\vec{m} \times \vec{R}|^2)]. \tag{A.5}$$

Now, we recall that our model consists of a large number of dipoles, each with its own orientation. This magnetic flux density component is due to just one of those dipoles, and is expressed in a coordinate system with the z'-axis aligned with the source dipole. We wish to compute a total field due to all dipoles in the model, so we must describe all field components in a single global coordinate system. Since the model already defines an x-y-z coordinate system in which to place rows and columns of grains, we use it as our global coordinate system.

Let α denote the angle between the global z-axis and the local z'-axis. Since the z'-axis is aligned with the dipole moment vector $\vec{m}$:

$$\cos\alpha = \frac{m_z}{m} \tag{A.6}$$

$$\sin\alpha = \frac{m_x}{m}, \tag{A.7}$$

where $\vec{m} = \begin{bmatrix} m_x & m_z \end{bmatrix}^T$ in x-z coordinates.

Using these expressions, and applying the rotation transformation yields:

$$\vec{B}(\vec{R}) = \frac{\mu_0}{4\pi m^2 R^5} \cdot \tag{A.8}$$

$$[\vec{a}_x(\pm 3|\vec{m} \times \vec{R}|(\vec{m} \cdot \vec{R})m_z + [2(\vec{m} \cdot \vec{R})^2 - |\vec{m} \times \vec{R}|^2)]m_x) \tag{A.9}$$

$$+ \quad \vec{a}_z(\mp 3|\vec{m} \times \vec{R}|(\vec{m} \cdot \vec{R})m_x + [2(\vec{m} \cdot \vec{R})^2 - |\vec{m} \times \vec{R}|^2)]m_z)]. \tag{A.10}$$

We may also express $\vec{R}$ in terms of its x-z components:

$$\vec{R} = \begin{bmatrix} R_x & R_z \end{bmatrix}^T, \tag{A.11}$$

so that:

$$\vec{m} \cdot \vec{R} = m_x R_x + m_z R_z \tag{A.12}$$

$$\pm|\vec{m} \times \vec{R}| = m_z R_x - m_x R_z. \tag{A.13}$$

Substituting these expressions, we may write the components of the field vector as:

$$\vec{B}(\vec{R}) \cdot \vec{a}_x = \frac{\mu_0}{4\pi m^2 R^5}[3(m_z R_x - m_x R_z)(m_x R_x + m_z R_z)m_z \tag{A.14}$$

$$+ \quad 2(m_x R_x + m_z R_z)^2 m_x - (m_z R_x - m_x R_z)^2 m_x] \tag{A.15}$$

$$\vec{B}(\vec{R}) \cdot \vec{a}_z = \frac{\mu_0}{4\pi m^2 R^5}[-3(m_z R_x - m_x R_z)(m_x R_x + m_z R_z)m_x \tag{A.16}$$

$$+ \quad 2(m_x R_x + m_z R_z)^2 m_z - (m_z R_x - m_x R_z)^2 m_z]. \tag{A.17}$$

Simplifying this expression, and making use of $m^2 = m_x^2 + m_z^2$ yields:

$$\vec{B}(\vec{R}) \cdot \vec{a}_x = \frac{\mu_0}{4\pi R^5}[(2Rx^2 - R_z^2)m_x + 3R_x R_z m_z] \tag{A.18}$$

$$\vec{B}(\vec{R}) \cdot \vec{a}_z = \frac{\mu_0}{4\pi R^5}[3R_x R_z m_x + (2R_z^2 - R_x^2)m_z]. \tag{A.19}$$

When the last expression is multiplied by the appropriate constant to compute magnetic field intensity from the magnetic flux density and is put into matrix form, the result is the expression cited in Chapter 3.

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