FY 1999
ADVANCED TECHNOLOGY PROGRAM
INTRAMURAL FUNDING PROPOSAL
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PROJECT TITLE.

Adaptive Learning Module for Mathematical Functions

PRINCIPAL INVESTIGATOR AND RESEARCH TEAM.

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PROPOSAL CLASSIFICATION.

(1) Only NIST employees will perform all research.

HUMAN OR ANIMAL SUBJECTS.

(1) This proposal does not involve human subjects or human tissue.
(2) This proposal does not involve animal subjects.

PROPOSED RESEARCH.

Objective. Advanced technology drives the U.S. economy. Ultimately, development of advanced technology depends on the ability to exploit the vast body of already codified mathematical and scientific knowledge, and also to incorporate advances as they occur. The objective of this project is to construct a prototype Adaptive Learning Module for Mathematical Functions. This Learning Module (LM) will be intimately connected to a new framework, called the Digital Library of Mathematical Functions (DLMF), which is being planned as a fundamental new component of the professional scientific infrastructure, and as a foundation upon which to build highly interactive Web-based modules for pedagogical as well as professional purposes in a wide variety of specialized settings. The DLMF will meet the rigorous criteria for acceptance as NIST Standard Reference Data, and the LM will demonstrate some of the ways that resources like the DLMF can be used to help meet the needs of students, trainers and educators for a modern learning system in the field of mathematical functions. The LM, for which funding from the ATP is sought, will focus on the mathematical functions that are introduced in high school, elaborated in university, and used in virtually all engineering and scientific applications. We expect that it will quickly become popular as an innovative learning resource for well-motivated high-school students, university students, and technological professionals. It will provide an on-line tutorial with the ability to (i) search for and retrieve formulas and other information from the DLMF, (ii) generate computational results on demand, (iii) generate user-controlled graphics and visualizations on demand, (iv) check the correctness of exercises completed by the student, and (v) follow links to related Web sites.

Background: The DLMF Project. In 1964 the National Bureau of Standards (NBS) published the volume *M. Abramowitz and I. A. Stegun (Editors), Handbook of Mathematical Functions, Applied Mathematics Series 55.* This classic reference, known simply as AMS 55, contains formulas, graphs and tables which characterize the elementary and higher functions of applied mathematics. These functions are used extensively in mathematical analysis in many fields, such as physics and chemistry. For example, they aid in analysis and computation in areas as diverse as astronomy, atmospheric modeling, metallurgy, ocean engineering, optical fibers, radar, statistics, underwater acoustics, and the modeling of the microstructure of materials.

As a definitive source of standard mathematical reference data, AMS 55 has had an enormous impact on scientific and technological practice. It has been available continuously in many printings from the US

Government Printing Office, and since 1965 from Dover Publications. The Dover edition, which is merely a photocopy of the government edition, is a staple of the American higher education technical curriculum. It is found in every campus bookstore, and even in many general bookstores such as Borders and Barnes & Noble. Total sales by the US GPO and Dover are numbered in the hundreds of thousands of copies. The professional impact of AMS 55 is measured by the fact that Science Citation Index (SCI) reports more than 7,000 citations in the 5-year period 1992-96. Remarkably, instead of a gradual decline over time (as might be expected), the number of citations to AMS 55 has been growing, and at a rate faster than the SCI as a whole!

AMS 55 was such a success because it (a) addressed topics that were of direct use in applications, (b) was written by highly respected contributors backed by the authority of NBS, and (c) presented data in a clear, crisp, and accessible fashion. For these reasons, AMS 55 served to standardize definitions and notations for mathematical functions, and became the principal reference for their properties.

In spite of its continued popularity, AMS 55's technical content is current only up to about 1960, and its strong emphasis on mathematical tables is no longer as relevant as before. Many mathematical and computational advances have occurred in the intervening years, and many new applications of special functions have emerged. Thus, a 'new AMS 55' needs to be constructed to meet the continuing needs of the 21st century.

While updating this classic NBS document would itself be a worthwhile endeavor, advances in information technology over the past 10 years have led to the vision of an interactive, richly linked, and network-based digital library of mathematical reference data from a variety of fields. Freely accessible Web resources of this type would provide some of the basic infrastructure needed by the technical community to more fully integrate modern information technology into its day-to-day work. The firm foundation provided by AMS 55 will provide the initial mathematical domain to be addressed. The methodologies and tools developed in the Digital Library of Mathematical Functions Project will enable the extension of this work into other technical domains.

Thirty-four chapters have been identified for the DLMF. The majority treat individual function classes, such as the elementary functions (exponential, hyperbolic and trigonometric functions, and their inverses), Airy functions, Bessel functions, and orthogonal polynomials; the remainder deal with closely associated topics such as algebraic, analytical and numerical methods. The digital library approach provides new capabilities unavailable in print media, among which are:

- downloading formulas, graphs, tables and plain text into documents, thus avoiding the tedious and error-prone process of manual copying;
- importing mathematical formulas directly into mathematical software systems (such as Mathematica, Maple and MATLAB) for further analysis and processing;
- on-demand generation of tables of numerical values, intended primarily for use in constructing and testing mathematical software;
- on-demand generation of graphs, including still images, movies, and interactive graphics, intended for scientific visualization and pedagogical applications;
- advanced search capabilities, with the ability to locate complex mathematical formulae and tables;
- recommended or certified algorithms or software for computing mathematical functions;
- scientific application modules in selected fields, in which detailed accounts of the role played by special functions in the applications field are provided; and
- continuous updating to incorporate corrections, additions and extensions.

A sample Web site (http://www.nist.gov/DigitalMathLib) has been constructed that demonstrates some of these capabilities for the chapter on Airy Functions. (Such a chapter did not exist in AMS 55 but needs to be included now; it was written recently by Prof. F. Olver, one of the principal investigators on the DLMF Project, as a model for the other chapters.)

While the World Wide Web has now reached a certain level of maturity, it remains difficult to use it to adequately represent and deliver mathematical data. Community standards and industry practice for required technologies vary from emerging to nonexistent. Thus, the DLMF will necessarily be a pioneer in such developments. In particular, the project will

- develop techniques for importing mathematical reference data specified in mathematical typesetting systems such as TeX into the DLMF (so as to be able to import data from DLMF authors into our system);
- work with the OpenMath Consortium, the WWW Consortium, and other organizations to develop and test standards for machine representation of mathematical data;
- develop techniques for downloading formulas from the DLMF into word processors and mathematical analysis systems;
- investigate the use of Java and related technologies for computing mathematical functions inside Web browsers;
- investigate interactive techniques for visualizing complex mathematical functions using Web-based graphics;
- develop search techniques applicable to mathematical data; and
- develop techniques for reliable generation of accurate tables of functions on demand.

Several categories of personnel and a solid management plan are necessary to carry out an undertaking of a size that eclipses the original AMS 55 project. Project personnel include principal editors, associate editors, authors, validators, and NIST project staff.

The principal editors are *Dr. Daniel Lozier*, NIST (general editor), *Prof. Frank Olver*, University of Maryland (mathematics editor), *Dr. Charles Clark*, NIST (physical sciences editor), and *Dr. Ronald Boisvert*, NIST (information technology editor). They will act as executive committee for the project, bearing responsibility for its successful completion.

Nine very distinguished researchers have volunteered their time to serve as a technical board of associate editors, and one or two more are being sought. This board will provide critical advice on all technical aspects of the project, such as reviewing subject coverage, identifying prospective authors and validators, and providing general oversight and guidance on matters of format and presentation. The complete editorial board consisting of the principal and associate editors will meet at least once per year for the duration of the project.

As was the case with AMS 55, the chapters will be written by individual authors, or in some cases several authors, drawn from the USA and abroad. The responsibilities of the authors will include research of the literature of the past 40 years and of recent software packages. Each chapter will also be assigned an independent validator to check the accuracy of its content. The authors and validators will be engaged under contract to NIST with final payment contingent upon satisfactory completion of all contractual obligations.

The principal editors will be assisted by NIST project staff in the Information Technology Laboratory, the Physics Laboratory, and possibly other NIST organizational units. The current NIST project staff consists of *Dr. Bruce Fabijonas*, ITL, *Ms. Marjorie McClain*, ITL, *Dr. Bruce Miller*, ITL, *Dr. Peter Mohr*, PL, *Dr. David Penn*, PL, *Dr. Bonita Saunders*, ITL, and *Ms. Qiming Wang*, ITL.

The bulk of funding for the DLMF Project since its inception in the first quarter of FY 1997 has been provided by the Information Technology Laboratory. Additional funding was received from the SRD (Standard Reference Data) and SIMA (Systems Integration for Manufacturing Applications) Programs in the Office of Measurement Services and the Manufacturing Engineering Laboratory, respectively. Funding for the contracts to authors and validators is estimated to cost approximately $3M over a 48-month period. This amount is being sought in proposals to the National Science Foundation, submitted to the the Division of Mathematical Sciences, the Knowledge and Distributed Intelligence cross-cutting program, and the Digital Libraries Initiative–Phase 2. The project will also require NIST to provide approximately matching funding over the same period to coordinate the project, edit the technical data received, convert it to a standardized format, develop associated application modules, and implement a variety of techniques for searching, displaying and exporting DLMF data. Part of this funding is assured from ITL, and part is expected from the SRD and SIMA Programs under proposals that are currently in process.

The Learning Module Project. The DLMF is envisioned not only as a basic resource for scientific professionals but also as a foundation for innovative, discipline-specific 'application modules' that can, for example, eliminate some of the vexing variations that occur in the use of mathematical functions in different application areas. Some of these variations are merely notational but most are related to the fact that real-world applications involve physical constants, normalization conventions, and special conditions that have no place in a purely mathematical treatment. Since mathematical functions are intrinsic in so many

different fields, no attempt can be made to cover all fields within the DLMF project. Thus the DLMF will contain only a very restricted set of 'sample applications' in all chapters where it is appropriate. However, one of the oustanding benefits of a richly interactive and interlinked Web site is the opportunity it presents to construct associated Web sites that are tailored to specific application areas but which call upon the DLMF as the repository of core information. The DLMF project intends to provide two such application modules, one for quantum mechanics and one for electromagnetism.

The DLMF provides an opportunity to construct 'learning modules' in exactly the same way. This is the objective of our proposal to the Advanced Technology Program. Our proposal is threefold:

- to research the resources that are presently available for interactive learning, whether based on Web sites, CD-ROM's, or other electronic technologies;
- to develop a prospectus for an Adaptive Learning Module for Mathematical Functions, and a prototype Web site implementation of it; and
- to contribute to the DLMF Project in those areas where more work is needed to support the requirements of innovative learning systems, especially software to generate numerical values and a corresponding interface for accessing this software directly from Web browsers.

The LM (Learning Module) will provide the ability for users to:

- locate mathematical formulas with sophisticated tools for searching;
- retrieve formulas and insert them into technical word processors or mathematical software packages;
- generate numerical and other computational or graphical results on demand;
- validate solutions of exercises posed by the LM; and
- follow links to Web sites that provide access to additional or related information, such as computer software and documentation.

The LM project has many points of contact with the Adaptive Learning Systems Focused Program:

High-Risk Research: It is costly to develop basic mathematical and scientific reference information, especially if quality control is taken seriously, and the market for such information is limited. Wolfram Research found this out and reduced greatly their inhouse effort. In fact, it can really only be done by an academic or government institution.

National Learning Infrastructure: The DLMF through its important mathematical and scientific content and its delivery of this content via advanced Web technologies will contribute to this goal. Initially the content will be pitched toward the needs of researchers, engineers, educators, and students through standardized handbook-style presentation of basic information and tutorial-style application modules in selected subfields of physics. This information will be presented with builtin interactive features such as the ability to perform sharply focused search and retrieval of mathematical formulas and computational procedures, rich crosslinking to related parts of the DLMF and relevant external Web sites, and advanced ways of visualizing functions dynamically with color graphics. The DLMF will serve as a prototype for similar digital libraries in other fields of applied mathematics, such as numerical analysis, and the application modules will serve as prototypes for other fields of science. All of this can be extended downward to accommodate training of workers who need to know technical information based on mathematics and science.

Content: The core content of the DLMF is unavailable elsewhere except in out-of-date handbooks such as AMS 55 (published in 1964), advanced textbooks, research monographs and journals. Related content is present in computer algebra systems such as Mathematica or Maple but it is limited in scope, usability and accessibility.

Delivery: Much effort is being expended to employ the latest methods of delivery using advanced information technology. Mathematical content is one of the most difficult subject areas to represent syntactically and semantically in computer databases, to display typographically, and to manipulate using computer database technology.

Search: Search and retrieval depends upon having available a suitable technology for representing semantic mathematical information. This difficult problem has not been solved, though serious research is being performed. We will make use of research results as they become available, and we will contribute our own efforts toward this research area.

Quality of Service: NIST already has an enviable record in providing standardized reference data via the Internet and other electronic media. This is essential to the NIST mission, of course, and not many other institutions (even government institutions) have the same mission and commitment to quality.

Broad Utility: Mathematics is basic to technology, and higher mathematical functions as a subarea of applied mathematics is central to in all scientific and engineering fields.

Demonstration Project: The DLMF project has already attracted many indications of interest and support from the mathematical and scientific community that have been received by email and personal communications at conferences and workshops. Another indication is an article by one of the SIAM science writers that was published in SIAM News (vol. 31, no. 2, March 1998). We expect the Web site will quickly become extremely popular after it is released to the general public. This expectation is supported in part by the impressive citation record of AMS 55, which can be considered the predecessor of, and model for, the DLMF.

Relation to NIST Mission. The DLMF project fits squarely into the NIST mission of strengthening the nation's scientific infrastructure through the provision of standard reference data. The LM project matches this long-standing imperative to the newer mission of addressing the national need to improve the overall educational and training system while reducing its unacceptably high cost.

Dissemination Strategy. The DLMF and LM Web sites will be made accessible from NIST servers. Much prior experience and success in providing this kind of service is possessed by NIST, particularly in ITL, PL, and the SRD and SIMA Programs. In addition, traditional means of information dissemination, such as publications and presentations, will be used.

Approach. Details of the approach to the LM project are implicit in the Technical Milestones listed below.

Teaming Arrangements. There are no teaming arrangements with individuals not employed by NIST in the research and development of the Adaptive Learning Module for Mathematical Functions.

Technical Milestones.

FY 1999.

1. Locate electronic media resources (such as Web sites, CD-ROM's and software packages) that offer to provide pedagogical assistance related to elementary and higher mathematical functions, and examine emerging metadata for educational resources;
2. prepare a report cataloging the resources found;
3. begin development of a Web interface for interactively generating numerical values of mathematical functions for use in visualization and problem solving (this will be a component of the LM, and also of the DLMF);
4. in collaboration with NIST DLMF project staff, develop benchmark algorithms and software for selected functions for future support of the Web interface described in (3) above.

FY 2000.

1. Select and critically examine several of the cataloged electronic media resources;
2. develop a prospectus for a prototype LM, taking into account existing electronic media resources and expected DLMF capabilities;
3. continue development of the Web interface for generating mathematical function values by providing an easy-to-use menu of choices for users;
4. in collaboration with NIST DLMF project staff, develop benchmark algorithms and software for selected functions.

FY 2001.

1. Begin development of the prototype LM based on the completed prospectus;
2. prepare a report containing the prospectus and describing the prototype LM;
3. continue development of the Web interface for generating mathematical function values by incorporating available benchmark software;
4. in collaboration with NIST DLMF project staff, develop benchmark algorithms and software for selected functions.

FY 2002.

1. Complete the prototype LM, incorporating the Web interface for generating function values;
2. incorporate additional benchmark software as it becomes available;
3. release the prototype LM for public review;
4. write a paper for publication describing the prototype LM;
5. in collaboration with NIST DLMF project staff, develop benchmark algorithms and software for selected functions.