Wednesday April 6

	Room A	
8.30 – 9.00	8:30-8:35 Welcome (**Daniel Lozier**) 8:35-9:00 Very brief DLMF history (**Frank Olver**)	
9.00 – 10.00	**Roderick Wong** *Asymptotic approximations of orthogonal polynomials by using three-term recurrence relations*	
10.00 – 11.00	**Leonard Maximon** *A physicist's tribute to Frank Olver*	
	Coffee break	
	Room A	**Room B**
11.30 – 12.00	**Ranjan Roy** presented by Richard Askey *The Cotes-Newton factorization of* $x^n \pm 1$	**Barkat Ali Bhayo** *Inequalities for eigenfunctions of the* p*-Laplacian*
12.00 – 12.30	**Willard Miller, Jr** *Superintegrability as an organizing principle for special function theory*	**Avram Sidi** *Recent asymptotic results for Euler-Maclaurin expansions, Gauss-Legendre quadrature, and Legendre polynomial expansions*
	Lunch break	
14.30 – 15.00	**José L. López** *Olver's asymptotic theory, Green's functions and fixed point theorems*	**Dmitry Karp** *Log-convexity and log-concavity for series in products and ratios of rising factorials and gamma functions*
15.00 – 15.30	**Mark Dunster** *Cherry-type turning point expansions*	**Francesco Mainardi** & Rudolf Gorenflo *On the distinguished role of the Mittag-Leffler and Wright functions in fractional calculus*
15.30 – 16.00	**Chris Howls** *Truncated Pearcey functions in the post-paraxial plasmon Talbot effect.*	K. I. Gross & **Donald St. P. Richards** *On the role of the hypergeometric functions of matrix argument in the age of electronic communications*
	Tea break	
16.30 – 17.00	**Renato Spigler** *Advances on the Liouville-Green approximation for differential and difference equations*	**Karl Dilcher** *On multiple zeros of Bernoulli polynomials*
17.00 – 17.30	C. Ferreira, J.L. López & **Ester Pérez Sinusía** *Asympotic expansions of the second and third Appell's functions for one large variable*	**Pablo Sánchez-Moreno** *Rényi entropy and linearization of orthogonal polynomials*
17.30 – 18.00	**Qiu Weiyuan** *Uniform asymptotic expansions of Meixner-Pollaczek polynomials with varying weights*	**Jesus S. Dehesa** *Information-theoretic properties of special functions*

Thursday April 7

	Room A	
9.00 – 10.00	**Nalini Joshi** *Discrete and continuous Painlevé equations*	
10.00 – 11.00	**Michael Berry** *Tsunami asymptotics*	
	Coffee break	
	Room A	**Room B**
11.30 – 12.00	**Bruce Berndt** *The circle and divisor problems and Bessel function series*	A. Branquinho & **Maria das Neves Rebocho** *Difference and differential systems for Laguerre-Hahn orthogonal polynomials on the unit circle*
12.00 – 12.30	**Walter Van Assche** *Multiple orthogonal polynomials and special functions*	**Andrew E. Noble** & Nico M. Temme *Novel asymptotic expansions of hypergeometric functions enable the mechanistic modeling of large ecological communities*
	Lunch break	
14.30 – 15.00	**Natalie A. Cartwright** *Asymptotics and special functions in electromagnetic pulse propagation: a canonical problem and questions*	**Peter Clarkson** *Painlevé equations: nonlinear special functions*
15.00 – 15.30	**Diego Dominici** *Discrete analogue of an integral involving products of Bessel functions*	**Sheehan Olver** *Numerical solution of Painlevé transcendents through Riemann-Hilbert problems*
15.30 – 16.00	**Allan MacLeod** *Testing special function software*	**Xiang-Sheng Wang** *Global asymptotics of the Meixner polynomials*
	Tea break	
16.30 – 17.00	**Alfredo Deaño** & Nico M. Temme *Analytical and numerical aspects of a generalization of the complementary error function*	**Chunhua Ou** *The Riemann-Hilbert approach to global asymptotics of discrete orthogonal polynomials with infinite nodes*
17.00 – 17.30	**Rajesh K. Pandey** *New stable algorithm for Fourier Bessel transforms using hybrid of block-pulse and rationalized Haar functions*	**Howard Cohl** *Parameter differentiation for Bessel and associated Legendre functions*

Friday April 8

	Room A	
9.00 – 10.00	**William P. Reinhardt** *Orthogonal polynomials and spectral theory in the quantum theory of Schrödinger operators*	
10.00 – 11.00	**Richard Askey** *Important books on special functions*	
	Coffee break	
	Room A	**Room B**
11.30 – 12.00	**Martin E. Muldoon** *Zeros of special functions: uses of continuity and analyticity with respect to parameters*	**H. Volkmer** *Infinite divisibility of probability distributions on the nonnegative reals*
12.00 – 12.30	**Robert E. O'Malley Jr.** *Asymptotic Solutions to singular perturbation problems*	**Juri M. Rappoport** *On theory and applications of some modified Bessel functions*

Poster presentations:
Pan Jianhui *On the asymptotic expansion of discrete Chebyshev polynomials*
Yutian Li *Uniform asymptotic expansions for second-order linear difference equations with turning points*

Speakers:

Richard Askey	University of Wisconsin
Bruce Berndt	University of Illinois
Michael Berry	Bristol University, England
Barkat Ali Bhayo	University of Turku, Finland
Natalie A. Cartwright	State University of New York at New Paltz
Peter Clarkson	University of Kent, England
Howard Cohl	National Institute of Standards and Technology
Alfredo Deaño	Universidad Carlos III de Madrid, Spain
Jesus S. Dehesa	University of Granada, Spain
Karl Dilcher	Dalhousie University, Canada
Diego Dominici	State University of New York at New Paltz
Mark Dunster	San Diego State University
Chris Howls	University of Southampton, England
Pan Jianhui	City University of Hong Kong
Nalini Joshi	The University of Sydney, Australia
Dmitry Karp	Far Eastern Branch of the Russian Academy of Sciences
Yutian Li	City University of Hong Kong
José L. López	State University of Navarra, Spain
Allan MacLeod	University of the West of Scotland
Francesco Mainardi	University of Bologna, Italy
Leonard Maximon	George Washington University
Willard Miller, Jr	University of Minnesota
Martin E. Muldoon	York University, Toronto, Canada
Andrew E. Noble	University of California at Davis
Sheehan Olver	Oxford University
Robert E. O'Malley Jr.	University of Washington
Chunhua Ou	Memorial University of Newfoundland, Canada
Rajesh K. Pandey	PDPM Indian Institute of Information Technology Design & Manufacturing Jabalpur, India
Juri M. Rappoport	Russian Academy of Sciences, Moscow
Maria das Neves Rebocho	University of Beira Interior, Portugal
William P. Reinhardt	University of Washington, Seattle
Donald St. P. Richards	Pennsylvania State University
Ranjan Roy	Beloit College, Wisconsin
Pablo Sánchez-Moreno	University of Granada, Spain
Avram Sidi	Israel Institute of Technology, Haifa
Ester Pérez Sinusía	Universidad de Zaragoza, Spain
Renato Spigler	Università 'Roma Tre', Rome, Italy
Walter Van Assche	Katholieke Universiteit Leuven
H. Volkmer	University of Wisconsin, Milwaukee
Xiang-Sheng Wang	York University, Toronto, Canada
Qiu Weiyuan	Fudan University, Shanghai, China
Roderick Wong	City University of Hong Kong

Richard Askey

Important books on special functions

The NIST Digital Library of Mathematical Functions is a new version of a number of important books on special functions. Some of these books will be described and a few of the uses they have been put to will be mentioned. This summary will be inadequate because of my lack of knowledge in many areas and I hope listeners will point out other important uses.

Bruce Berndt

The circle and divisor problems and Bessel function series

A page in Ramanujan's lost notebook contains two identities for trigonometric sums in terms of doubly infinite series of Bessel functions. One is related to the famous "circle problem" and the other to the equally famous "divisor problem". These relations are discussed as well as various attempts to prove the identities. Our methods also yield new identities for certain trigonometric sums, for which analogues of the circle and divisor problems are proposed. The research to be described is joint work with Sun Kim and Alexandru Zaharescu.

Michael Berry

Tsunami asymptotics

For most of their propagation, tsunamis are linear dispersive waves whose speed is limited by the depth of the ocean and which can be regarded as diffraction-decorated caustics in spacetime. For constant depth, uniform Airy asymptotics gives a very accurate compact description of the tsunami profile generated by an arbitrary initial disturbance. Variations in depth can focus tsunamis onto cusped caustics, and this 'singularity on a singularity' constitutes an unusual diffraction problem, whose solution - a new special function - indicates that focusing can amplify the tsunami energy by an order of magnitude.

Barkat Ali Bhayo

Inequalities for eigenfunctions of the p-Laplacian

During the past decade, several authors have investigated the properties of elementary and higher transcendental functions motivated by various applications of these functions. For instance, hyperbolic functions, hypergeometric functions, and the gamma and psi functions have been extensively studied. Motivated by the work of P. Lindqvist we study eigenfunctions $\sin_p$ of one-dimensional p-Laplacian Δ_p on $(0,1)$, $p \in (1,\infty)$. The eigenvalue problem

$$-\Delta_p u = -\left(|u'|^{p-2}u'\right)' = \lambda|u|^{p-2}u,\; u(0)=u(1)=0,$$

has eigenvalues

$$\lambda_n = (p-1)(n\pi_p)^p,$$

and eigenfunctions

$$\sin_p(n\pi_p\, t),\; n\in\mathbb{N},\; \pi_p = 2\pi/(p\,\sin(\pi/p))\,.$$

We prove several inequalities for eigenfunctions and p-analogues of other trigonometric functions and their inverse functions. Similar inequalities are given also for the p-analogues of the hyperbolic functions and their inverses.

Natalie A. Cartwright

Asymptotics and special functions in electromagnetic pulse propagation: a canonical problem and questions

In the study of ultra-wideband electromagnetic pulse propagation through causal material that is both dispersive and attenuative, those methods that retain the causality property of the dielectric response of the material are usually the most accurate. In one dimension, saddle point methods have been used successfully on the integral representation of the propagated field to provide an asymptotic approximation that increases in accuracy with propagation distance. However, these successful studies are limited to causal material that is either purely dielectric or conductive, not a material that exhibits both dielectric and conductive responses. Here, we present the unsolved problem of how to obtain an asymptotic approximation to an electromagnetic field that travels through a dielectric material with static levels of conductivity.

Peter Clarkson

Painlevé equations: nonlinear special functions

In this talk I shall give an overview of the Painlevé equations, which might be thought of nonlinear special functions, and discuss some of their properties. Further I shall discuss some of the "Painlevé Challenges", i.e. open problems in the field of Painlevé equations.

Howard Cohl

Parameter differentiation for Bessel and associated Legendre functions

Many of the most commonly encountered special functions are indexed by parameters. For instance Bessel functions are indexed by a single parameter called the order and associated Legendre functions are indexed by two parameters called the degree and the order. Even though these parameters often take only integer or half-integer values, one can often treat these parameters as taking on arbitrary real or complex values. Through extension of these parameters as alternative arguments, they facilitate multiple argument generalizations for these functions. I will review and discuss progress in the evaluation of closed-form expressions for derivatives with respect to parameters for certain special functions, and especially for the parameters of Bessel and associated Legendre functions.

Alfredo Deaño

Analytical and numerical aspects of a generalization of the complementary error function

We discuss analytical and numerical properties of the function

$$V_{\nu,\mu}(\alpha,\beta,z) = \int_0^\infty e^{-zt}(t+\alpha)^\nu (t+\beta)^\mu dt,$$

with $\alpha, \beta, \operatorname{Re} z > 0$. This can be viewed as a generalization of the Kummer U function, as well as of other members of the family of confluent hypergeometric functions. For certain values of the parameters, $V_{\nu,\mu}(\alpha,\beta,z)$ appears as a first order approximation to the solution of an elliptic 3D singular perturbation problem. We consider the relation with other known special functions, and we give asymptotic expansions as well as recurrence relations in the parameters. Several methods for numerical evaluation and examples are given.

Jesus S. Dehesa

Information-theoretic properties of special functions

In this talk we will show two classes of measures to quantify the distribution of the special functions of Applied Mathematics all over its domain of definition. They are defined by means of functionals of the associated probability densities. The first class includes the information-theoretic spreading lengths of Shannon, Rényi and Fisher types, together with the well-known standard deviation; they have units of the variable. The second class includes the dimensionless measures of complexity of Fisher-Shannon, Cramér-Rao and LMC types. We will discuss their physico-mathematical relevance and their application in some quantum-physical problems. Keep in mind that the physical solutions of the Schrödinger and Dirac equations of quantum systems are often controlled by special functions and, particularly, by polynomials of various orthogonalities. To fix ideas, we will focus on the classical orthogonal polynomials. We will give explicit analytical expressions of the previous information-theoretic measures of the spreading of these polynomials all over their orthogonality interval, in terms of their degree and characteristics parameters.

Karl Dilcher

On multiple zeros of Bernoulli polynomials

Building on results of Brillhart (1969), it is shown that Bernoulli polynomials have no multiple zeros. The proof uses the theorem of von Staudt and Clausen as well as congruences for certain sums of binomial coefficients.

Diego Dominici

Discrete analogue of an integral involving products of Bessel functions

We present a series analogue of the Weber-Sonine-Schafheitlin integral. The representation has applications in finding resolutions of the Coulomb operator. We show other examples of continuous analogues of series and explore the connection between our identity and the theory of Schlomilch series.

Mark Dunster

Cherry-type turning point expansions

We consider linear second order differential equations with a large parameter and simple turning point. The classical asymptotic expansions involve Ai and its derivative Ai$'$ which were first formally developed by R. E. Langer for bounded values of the independent variable. Subsequently Frank Olver developed expansions, also involving Ai and Ai$'$, which are valid in unbounded domains, and he proved their validity rigorously by obtaining explicit error bounds. For the same problem T. M. Cherry, in 1950, obtained asymptotic expansions that involved Ai, but not Ai$'$. The Cherry-type expansions are useful in estimating the zeros of the approximated solutions. Here we obtain explicit error bounds for a Cherry-type expansion using Olver's technique. We also derive an asymptotic approximation for the derivative of the solution directly, which involves Ai$'$ but not Ai, which sharpens existing error bounds. An application to Bessel functions is given.

Chris Howls

Truncated Pearcey functions in the post-paraxial plasmon Talbot effect.

Surface plasmons are electromagnetic surface waves on metal surfaces and have been used in conjunction with engineered nanoscale features to provide analogues of optical lenses and mirrors in future plasmon-based devices. Advantages of such devices include the ability to concentrate electromagnetic energy near the metal surface with a view to realising 2-D light waveguides, or the ability to generate Raman emission from single molecules through enhancement of local field intensity by several orders of magnitude. We study the field generated by repeated monochromatic images of a grating at various characteristic distances of the image plane with respect to the grating surface in a post-paraxial approximation. The field exhibits a highly complex structure, giving way to regions of apparent simple regularity containing what looks to be an array of cusp catastrophes arising from a Pearcey function (see Chapter 36 of the DLMF). Closer inspection shows the situation to be more complicated with the the field actually being represented by a truncated Pearcey function. The truncation is determined topologically and gives rise to diffractive effects that mimic the full Pearcey behaviour. The local structure is explained by a detailed, subtle, yet elegant asymptotic analysis involving coalescences of both endpoint/saddlepoint and saddlepoint/saddlepoint type, as well as Stokes phenomena.

Pan Jianhui

On the asymptotic expansion of discrete Chebyshev polynomials

The discrete Chebyshev polynomials $t_n^N(x)$, which is also the special case of Hahn polynomial $Q_n(x, \alpha, \beta, N)$ with $\alpha = \beta = 0$, have a double integral representation. An asymptotic expansion is derived from this double integral representation. This expansion is given in terms of confluent hypergeometric functions when $x > 0$ and gamma functions when $x < 0$, for all fixed $n/N \in (0, 1)$.

Nalini Joshi

Discrete and continuous Painlevé equations

The classical Painlevé equations are now widely used as non-linear special functions. The discrete Painlevé equations are less widely known even though they possess almost all of the same properties that make the classical ODEs special. I will survey what we know about the continuous and discrete Painlevé equations, touch on some new results and point out some major open questions.

Dmitry Karp

Log-convexity and log-concavity for series in products and ratios of rising factorials and gamma functions

Consider the following general problems: under what conditions on the positive sequence $\{f_k\}_{k=0}^{\infty}$ and the numbers $a_1, \ldots, a_n$, $b_1, \ldots, b_m$ the functions:

$$\mu \to \sum_{k=0}^{\infty} f_k \frac{(a_1 + \mu)_k \cdots (a_n + \mu)_k}{(b_1 + \mu)_k \cdots (b_m + \mu)_k}$$

and

$$\mu \to \sum_{k=0}^{\infty} f_k \frac{\Gamma(a_1 + \mu + k) \cdots \Gamma(a_n + \mu + k)}{\Gamma(b_1 + \mu + k) \cdots \Gamma(b_m + \mu + k)}$$

are log-concave or log-convex? Here Γ is Euler's gamma function and $(a)_k = a(a + 1) \cdots (a + k - 1) = \Gamma(a + k)/\Gamma(a)$ is rising factorial. We give complete solution to this problem for $n + m = 1$ and nearly complete solution for $n = m = 1$. This leads to virtually unimprovable sufficient conditions for log-convexity and log-concavity of generalized hypergeometric functions as functions of parameters. A great number of known and new inequalities for Bessel, Kummer and Gauss functions are included in these results as particular cases.

Yutian Li

Uniform asymptotic expansions for second-order linear difference equations with turning points

Two linearly independent asymptotic solutions are constructed for the second-order linear difference equation

$$y_{n+1}(x) - (A_n x + B_n) y_n(x) + y_{n-1}(x) = 0,$$

where A_n and B_n have power series expansions of the form

$$A_n \sim n^{-\theta} \sum_{s=0}^{\infty} \frac{\alpha_s}{n^s}, \qquad B_n \sim \sum_{s=0}^{\infty} \frac{\beta_s}{n^s}$$

with $\theta \neq 0$ being a real number and $\alpha_0 \neq 0$. Upon a rescaling $t = n^{\theta} x$, two turning points $t_{\pm}$ are defined by $\alpha_0 t_{\pm} + \beta_0 = \pm 2$. In particular, it is shown that how the Bessel functions J_ν and Y_ν arise in the uniform asymptotic expansions around the turning point $t_- = 0$. As an illustration of the main result, a uniform asymptotic expansion is derived for the orthogonal polynomials associated with the Laguerre type weights $w(x) = x^{\alpha} \exp(-Q(x))$, where $\alpha > -1$ and Q denotes a polynomial with positive leading coefficient.

José L. López

Olver's asymptotic theory, Green's functions and fixed point theorems

We consider Olver's asymptotic method for linear differential equations of the second order with a large parameter Λ. When we add initial or boundary conditions to the differential equation, we meet an initial or a boundary value problem that may be formulated in the following form: given a linear differential equation of the second order $\mathbf{L}(y) = 0$, find a solution $y(x)$ of this equation in a certain space of functions determined by the initial or boundary conditions. We show that this problem may be solved by splitting the differential operator $\mathbf{L}$ into the difference of a 'main' operator $\mathbf{M}$ and a rest $\mathbf{N}$, that is, $\mathbf{L} = \mathbf{M} - \mathbf{N}$. Then, using the Green function of the operator $\mathbf{M}$ to compute its inverse, and a fixed point theorem, we show that sequence $y_{n+1} = \mathbf{M}^{-1}\mathbf{N}y_n$ converges to the unique solution of the problem. The sequence y_n is not only an asymptotic sequence for large Λ of a solution of the differential equation (as in Olver's expansion), but it has the additional property of being convergent. Then, we can use this technique to obtain new asymptotic as well as convergent expansions of special functions when they are solutions of linear differential equations of the second order. Moreover, we can exploit the power of this technique to extend its range of applicability to nonlinear differential equations (Painlevé equations for example).

Allan MacLeod

Testing special function software

High quality Special Function software is very difficult to construct and even harder to test effectively! The talk will survey some of the methods which have been used, illustrated with some problems found. It will also discuss the problems of testing such software embedded in user environments such as Matlab or Excel.

Francesco Mainardi

On the distinguished role of the Mittag-Leffler and Wright functions in fractional calculus

Fractional calculus, in allowing integrals and derivatives of any positive real order (the term 'fractional' is kept only for historical reasons), can be considered a branch of mathematical analysis which deals with integro-differential equations where the integrals are of convolution type and exhibit (weakly singular) kernels of power-law type. As a matter of fact fractional calculus can be considered a 'laboratory' for special functions and integral transforms. Indeed many problems dealt with fractional calculus can be solved by using Laplace and Fourier transforms and lead to analytical solutions expressed in terms of functions of Mittag-Leffler and Wright type. We outline these problems in order to single out the role of these functions. This research is carried out in the framework of the program on Fractional Calculus Modelling (htpp://ww.fracalmo.org). More details can be found in the recent book by F. Mainardi, *Fractional Calculus and Waves in Linear Viscoelasticity*, Imperial College Press, London (2010), pp. 340 (http://www.icpress.co.uk/mathematics/p614.html).

Leonard Maximon

A physicist's tribute to Frank Olver

For over two hundred years, physicists and astronomers have applied existing mathematics and created new mathematics for the solution of problems. The analysis, often generated by physical intuition, may lack the mathematical rigor sought by mathematicians. Although having a different perspective, problems of interest in new fields in physics have nonetheless often benefited by the work of pure mathematicians. This difference of approach often produces a clear distinction in the work of physicists and mathematicians. However, in bringing rigorous mathematics to the analysis central to classical physics, Frank Olver has bridged the gap, satisfying the requirements of pure mathematicians and providing tools useful to the physicist for the solution of problems. References will be given to the contrasting perspectives expressed by mathematicians and physicists during the past two centuries as well as to a number of Olver's papers of particular significance for physical problems.

Willard Miller, Jr

Superintegrability as an organizing principle for special function theory

An n-dimensional quantum Hamiltonian system $H = H_o + V$ is integrable if it admits n algebraically independent commuting symmetry operators. It is superintegrable if it is integrable and admits $2n-1$ algebraically independent symmetry operators (the maximum possible, but of course not all commuting). If the independent symmetries can all be chosen of order k or less as differential operators the system is kth order superintegrable. Superintegrability is a much stronger requirement than integrability. The operators of a quantum superintegrable system typically close under commutation to form an algebra, not usually a Lie algebra, and the representations of this algebra lead to new special functions beyond those which rise by solving the quantum eigenvalue problem $Hf = Ef$. First order superintegrable systems such as the Helmholtz, Laplace or wave equations ($V = 0$) have Lie algebras as symmetry algebras and spherical harmonics and most of the special functions of mathematical physics arise by separation of variables from such equations, characterized by commuting sets of 2^{nd} order symmetries. Eigenvalue problems for higher order superintegrable systems are associated with additional special functions, such as Painlevé transcendents. Irreducible representations of symmetry algebras lead to many new classes of special functions, including discrete orthogonal polynomials, such as multivariable Wilson polynomials. We give examples and argue for superintegrability as a useful organizing principle for the theory of special functions.

Martin E. Muldoon

Zeros of special functions: uses of continuity and analyticity with respect to parameters

With some restrictions, each zero $c_{\nu k}$ of a cylinder function $\cos\alpha\, J_\nu(x) - \sin\alpha\, Y_\nu(x)$ may be defined as a continuous function of the order ν. As early as 1950, Frank Olver showed the advantages of treating k as continuous variable. This idea was rediscovered and exploited by Á. Elbert and A. Laforgia, starting in 1984. J. Vosmanský extended the idea of continuous ranking to zeros of a class of second order ODEs in 1992. We will present some of the advantages of this approach and of other uses of continuity and analyticity, with respect to parameters, of zeros of special functions.

Andrew E. Noble

Novel asymptotic expansions of hypergeometric functions enable the mechanistic modeling of large ecological communities

Understanding the mechanisms of coexistence for competitive species in ecological communities is a central concern of mathematical ecology. Niche mechanisms tend to ignore stochastic fluctuations in community composition and require species asymmetries to stabilize coexistence. By contrast, the neutral theories of ecology, borrowing from the neutral theories of population genetics, balance speciation and extinction among functionally equivalent species to maintain an unstable coexistence of incessant species turnover. The master equation formulation of neutral theories facilitates a scaling from the mechanistic specifications of individual-level interactions to the emergent patterns of community structure. In collaboration with William F. Fagan and Timothy H. Keitt, we extended the best-studied neutral theory of community ecology, known as Hubbells neutral theory, to allow for asymmetries among species. Analytical results for the stationary distributions may provide a valuable tool for estimating departures from neutral community patterns based on empirical data. However, the statistical optimization problems are encumbered by the appearance of hypergeometric functions that become numerically intractable for large communities. To remedy this problem, a great number of asymptotic expansions were derived by identifying critical parameter values and enumerating special cases. In a few cases, the asymptotics of the hypergeometric functions can only be described by using their limits in the form of confluent hypergeometric functions. Future research is needed to develop uniform transitions between the many asymptotic cases, and we expect that the analysis of special functions will continue to forge new opportunities in the mechanistic modeling of large ecological communities.

Sheehan Olver

Numerical solution of Painlevé transcendents through Riemann-Hilbert problems

Painlevé transcendents have come to be viewed as nonlinear special functions; with important applications in nonlinear integrable systems, random matrix theory and elsewhere. To make this view true in practice, we must be able to compute them numerically. We present a new method for computing Painlevé II based on its Riemann-Hilbert formulation. By deforming the Riemann-Hilbert problem in the complex plane, we can compute Painlevé II for all values on the real line, with bounded computational cost.

Robert E. O'Malley Jr.

Asymptotic Solutions to singular perturbation problems

For many singular perturbation problems, finding solutions is easy after one guesses the right ansatz. Classical techniques are a good basis for doing this, but the search for a universal method still eludes us.

Chunhua Ou

The Riemann-Hilbert approach to global asymptotics of discrete orthogonal polynomials with infinite nodes

In this talk, we develop the Riemann-Hilbert approach to study the global asymptotics of discrete orthogonal polynomials with infinite nodes. We illustrate our method by concentrating on the Charlier polynomials $C_n^{(a)}(z)$. We first construct a Riemann-Hilbert problem Y associated with these polynomials and then establish some technical results to transform Y into a continuous Riemann-Hilbert problem so that the steepest descent method of Deift and Zhou (Ann. Math., 1993) can be applied. Finally, we produce three Airy-type asymptotic expansions for $C_n^{(a)}(z)$ in three different but overlaping regions whose union is the entire complex z-plane. When z is real, our results agree with the ones given in the literature. Although our approach is similar to that used by Baik, Kriecherbauer, McLaughlin and Miller (Ann. Math. Studies, 2007), there are crucial differences in the details. For instance, our expansions hold in much bigger regions. Our results are completely new, and one of them answers a question raised in Bo and Wong (Methods Appl. Anal., 1994). Asymptotic formulas are also derived for large and small zeros of the Charlier polynomials.

Rajesh K. Pandey

New stable algorithm for Fourier Bessel transforms using hybrid of block-pulse and rationalized Haar functions

A new stable numerical method, based on hybrid of Block-pulse and Rationalized Haar functions for numerical evaluation of Fourier Bessel (Hankel) transform is proposed in this paper. Hybrid of Block-pulse and Rationalized Haar functions are used as a basis to expand a part of the integrand, $rf(r)$, appearing in the Hankel transform integral. Thus transforming the integral into a Fourier-Bessel series. Truncating the series, an efficient, stable algorithm is obtained for the numerical evaluations of the Hankel transform of order $\nu > -1$. The method is quite accurate and stable, as illustrated by given numerical examples with varying degree of random noise terms $\epsilon\theta_i$ added to the data function $f(r)$, where θ_i is a uniform random variable with values in $[-1, 1]$

Juri M. Rappoport

On theory and applications of some modified Bessel functions

The properties of modified *Bessel* functions of the second kind with pure imaginary order $K_{i\beta}(x)$ and with complex order $K_{1/2+i\beta}(x)$ are elaborated. These functions are important as kernels of *Kontorovitch–Lebedev* and *Lebedev–Skalskaya* integral transforms. Some new representations of these functions and transforms are justified. The approximation and computation of modified *Bessel* functions of the second kind with pure imaginary order $K_{i\beta}(x)$ and with complex order $K_{1/2+i\beta}(x)$ are elaborated on the basis of several approaches. The inequalities which give estimations for their kernels - the real and imaginary parts of the modified Bessel functions of the second kind $ReK_{1/2+i\beta}(x)$ and $ImK_{1/2+i\beta}(x)$ for all values of the variables x and β are obtained. A proof of inversion formulas and *Parseval* equations for *Lebedev–Skalskaya* integral transforms is developed.

The hypergeometric type differential equations of the second order with polynomial coefficients are considered. The computational scheme of Tau method is extended for the systems of hypergeometric type differential equations.

The effective applications of the modified *Bessel* functions for the numerical solution of some mixed boundary value problems in wedge domains are given. The *Kontorovitch–Lebedev* integral transforms and dual integral equations are used. The asymptotic characteristics of dipole and quadrupole radiation are studied. The advantages of the NIST Handbook and DLMF are shown.

Maria das Neves Rebocho

Difference and differential systems for Laguerre-Hahn orthogonal polynomials on the unit circle

A sequence of orthogonal polynomials on the unit circle (OPUC), say $\{\phi_n\}$, is said to be Laguerre-Hahn if the corresponding Carathéodory function, F, satisfies a Riccati type differential equation with polynomial coefficients

$$A(z)F'(z) = B(z)F^2(z) + C(z)F(z) + D(z)\,.$$

As particular cases, some well-known families of orthogonal polynomials are obtained: the Laguerre-Hahn affine OPUC, when $B = 0$; the semi-classical OPUC, when $B = 0$ and C, D are specific polynomials.

In this talk we derive a first order differential system for Laguerre-Hahn OPUC. Such differential system will be used to construct discrete Lax equations which, in turn, lead to difference equations for the reflection parameters ($a_n = \phi_n(0)$).

Furthermore, we consider deformations in F through a t-dependence parameter, and we analyze the $t-$differential equations for the resulting reflection parameters ($a_n(t)$).

William P. Reinhardt

Orthogonal polynomials and spectral theory in the quantum theory of Schrödinger operators

Orthogonal polynomials arise in the analytic solution of essentially all soluble problems for the "bound, i.e. L^2" eigen-states of the Schrödinger operators of non-relativistic quantum theory. Following a quick overview of a few of these classic results, we move to consider the orthogonal polynomials which arise when one diagonalizes finite rank representations of Schrödinger operators which have only continuous spectra, or both discrete and continuous spectra. We ask what do the resulting discrete matrix eigenvalues have to do with the actual spectra of these operators? It is found that, properly interpreted, these matrix eigenvalues represent quadrature points in a Gaussian representation of the actual spectral resolution of the operators, and/or their resolvents. In some cases novel orthogonal polynomials arise in a natural manner: One example generalizes the Pollaczek polynomials, and relates them to the quantum Coulomb problem with its infamous essential singularities, and corresponding accumulation points in the discrete spectrum. The effect of complex dilations of the co-ordinates, $r \to re^{i\theta}$, on the spectra of these same Schrödinger operators, are then considered, as these allow discussion of complex eigenvalues which correspond to decaying states, also known as resonances. Should time permit, a discussion of the corresponding spectral distortion of continued fraction representations of the resolvent will also be discussed: this may be generalized in a manner allowing control, in a novel manner, of the branch-cut structure implicit in the continued fraction representations of arbitrary analytic functions.

Donald St. P. Richards

On the role of the hypergeometric functions of matrix argument in the age of electronic communications

The hypergeometric functions of matrix argument arose in the early twentieth century from problems in statistics, in the study of random matrices, and in pure mathematics, specifically in analytic number theory. These functions subsequently have arisen in physics, chemistry, and in engineering, and in many other areas of mathematics also. In this talk, we will describe their recent appearance in electronic communications networks, notably the multiple-input multiple-output (MIMO) models that form the basis of modern cell-phone technology.

Ranjan Roy

The Cotes-Newton factorization of $x^n \pm 1$

Newton was the first mathematician to attempt to factorize the binomial $x^n \pm 1$. It seems that Newton's motivation was to find series formulas for π, similar to the one found by Madhava and Leibniz. In his October 1676 letter to Leibniz via Oldenburg, Newton presented the interesting formula

$$\frac{\pi}{2\sqrt{2}} = 1 + \frac{1}{3} - \frac{1}{5} - \frac{1}{7} + \frac{1}{9} + \frac{1}{11} - \cdots.$$

Newton made a very cryptic remark about how he obtained this formula, but evidence seems to suggest that Leibniz did not understand the remark. Newton's notes indicate his method, suggesting that he came very close to finding Cotes's factorization of $x^n \pm 1$. Now it is not known exactly when Cotes investigated the factorization question. After Cotes's death in 1716, his cousin Robert Smith found the result among Cotes's notes and published it in the 1722 book *Harmonia Mensurarum.* In the same year, Henry Pemberton published a geometric proof. In 1730, de Moivre gave an analytic proof of a more general result based on results he had obtained many years earlier. A decade after this, Euler applied this Cotes-de Moivre factorization to compute the integrals of rational functions. In particular, he evaluated a beta integral over $(0, \infty)$. In my paper, I will give the details of all these derivations with some historical background and connections.

Pablo Sánchez-Moreno

Rényi entropy and linearization of orthogonal polynomials

The quantification of the spreading of the classical orthogonal polynomials $p_n(x)$ is investigated by means of the Rényi entropy of the associated Rakhmanov probability densities, $\rho(x) = \omega(x)p_n^2(x)$, where $\omega(x)$ is the corresponding weight function. The Rényi entropy is closely related to the L^{2q}-norm of the polynomials. It is calculated by use of some scarcely known linearization formulas of various types, which make use of generalized multivariate hypergeometric functions. Explicit applications for Hermite, Laguerre and Jacobi polynomials are presented and numerically discussed.

This is a joint work with A. Zarzo and J.S. Dehesa.

Avram Sidi

Recent asymptotic results for Euler-Maclaurin expansions, Gauss-Legendre quadrature, and Legendre polynomial expansions

In this talk, we will discuss some recent asymptotic results related to problems in numerical quadrature and approximation theory.

(1) We will present a generalization of the Euler-Maclaurin expansion for the trapezoidal rule approximation of finite-range integrals $\int_a^b f(x)\,dx$ when $f(x)$ is allowed to have arbitrary algebraic-logarithmic endpoint singularities.

(2) We will present a full asymptotic expansion (as the number of abscissas tends to infinity) for Gauss-Legendre quadrature for integrals $\int_a^b f(x)\,dx$, where $f(x)$ is allowed to have arbitrary algebraic-logarithmic endpoint singularities.

If the trapezoidal rule and the Gauss-Legendre quadrature are applied following a suitable variable transformation, their accuracy can be improved dramatically despite the fact that $f(x)$ may (and almost always will) remain singular following the variable transformation. The variable transformations involved normally have endpoint singularities that can be tuned to optimize the performance of the quadrature formulas. We will illustrate this point numerically with the Gauss-Legendre quadrature.

(3) We present full asymptotic expansions, of Legendre series coefficients $a_n = (n+\frac{1}{2})\int_{-1}^{1} f(x)P_n(x)\,dx$, as $n \to \infty$, when $f(x)$ has arbitrary algebraic-logarithmic (interior and/or endpoint) singularities in $[-1, 1]$.

These expansions are used to make statements about the asymptotic behavior, as $n \to \infty$, of the partial sums $\sum_{k=0}^{n} a_k P_k(x)$. This knowledge leads us to conclude that the Shanks transformation (or the equivalent epsilon algorithm of Wynn) and the Levin-Sidi d-transformation can be used very effectively to accelerate the convergence of the Legendre series $\sum_{k=0}^{\infty} a_k P_k(x)$ in question.

Ester Pérez Sinusía

Asympotic expansions of the second and third Appell's functions for one large variable

In this work, we consider Mellin convolution integral representations of the second and third Appell's functions and, applying the asymptotic method intro- duced in [1], we obtain new asymptotic expansions of $F_2(a, b, b', c, c'; x, y)$ and $F_3(a, a', b, b', c; x, y)$ for one large variable.

([1]) José L. López, *Asymptotic expansions of Mellin convolution integrals*, SIAM Rev., **50**, (2008), 275–293.

Renato Spigler

Advances on the Liouville-Green approximation for differential and difference equations

Theory and applications concerning the celebrated Liouville-Green (also known as WKB or WKBJ) approximation is reviewed. Applications to ordinary differential and difference equations and systems, special functions, partial differential equations, and abstract equations are shown.

Walter Van Assche

Multiple orthogonal polynomials and special functions

Multiple orthogonal polynomials are polynomials in one variable that satisfy orthogonality conditions with respect to several measures. They arise naturally as denominators in simultaneous rational approximation (Hermite-Padé approximation) to several functions. In recent years they have also emerged in random matrix theory and in the analysis of non-interesting Brownian motions. We will present some classes of multiple orthogonal polynomials for which explicit formulas are available, so that they can be considered as special functions, generalizing the classical orthogonal polynomials.

H. Volkmer

Infinite divisibility of probability distributions on the nonnegative reals

A probability distribution function $F(x)$ is called infinitely divisible if, for every $n \in \mathbb{N}$, there is a probability distribution function F_n such that F is equal to the n-fold convolution of F_n with itself. We will consider the half-Cauchy and half-student distributions. The problem whether these distributions are infinitely divisible leads to some interesting questions on special functions.

Xiang-Sheng Wang

Global asymptotics of the Meixner polynomials

Using the steepest descent method for oscillatory Riemann-Hilbert problems introduced by Deift and Zhou [Ann. Math. **137**(1993), 295-368], we derive asymptotic formulas for the Meixner polynomials in two regions of the complex plane separated by the boundary of a rectangle. The asymptotic formula on the boundary of the rectangle is obtained by taking limits from either inside or outside. Our results agree with the ones obtained earlier for z on the positive real line by using the steepest descent method for integrals [Constr. Approx. **14**(1998), 113-150]. This is a joint work with R. Wong.

Qiu Weiyuan

Uniform asymptotic expansions of Meixner-Pollaczek polynomials with varying weights

In this paper, we studied the asymptotics of Meixner-Pollaczek polynomials $P_n^{(nA)}(z;\phi)$ with varying weights $\omega(z; nA, \phi)$ where A is a positive constant. We first obtained an asymptotic expansion of the equipotential measures related to the weight functions, which is uniformly valid on the support of the measures. Then we derived two asymptotic expansions of $P_n^{(nA)}(z;\phi)$ uniformly valid, respectively, for z in two overlapping regions which cover the whole complex plane. One of the expansions is in terms of parabolic cylinder functions and the other one is in terms of elementary functions. Our approach is based on the steepest descent method for Riemann-Hilbert problems.

Roderick Wong

Asymptotic approximations of orthogonal polynomials by using three-term recurrence relations

One important property of orthogonal polynomials is that they all satisfy a three-term recurrence relation. However, constructions of even simple asymptotic formulas of classical orthogonal polynomials based on this fact do not seem to appear in the literature. In this talk, we shall present a summary of methods now available, which are based on recurrence relations and can be used to derive asymptotic approximations (expansions) for orthogonal polynomials, not only when the variable is fixed but also when it is allowed to vary in an interval containing a transition point.