Inferences Drawn From Atmospheric CO₂ Data

BERT W. RUST, RALPH M. ROTTY, AND GREGG MARLAND

Institute for Energy Analysis, Oak Ridge Associated Universities, Oak Ridge, Tennessee 37830

Detailed analyses of the atmospheric carbon dioxide concentration measurements from Mauna Loa Observatory have produced a model which describes most of the variability in the data. The final model is
\[ y(t) = a + bt + \sum_{i=1}^{4} A_i \sin \left( \frac{2\pi}{T_i} (t + \phi_i) \right) + \sum_{j=1}^{9} B_j \sin \left( \frac{2\pi}{T_j} (t + \theta_j) \right) \]
where \( a \) is the preindustrial value, \( \alpha \) is the exponential long-term growth rate, \( t \) represents the annual cycle, and \( j \) represents longer-term periodicities. The exponential growth rate is nearly identical to that calculated for the production of CO₂ from fossil fuel burning. A long-period component of about 44 months correlates with the southern oscillation index. Another long-period oscillation—about 142 months—has tentatively been identified with the solar activity cycle and complicates the determination of \( a \) because the two sources of curvature interact in the fitting procedure, and the data record is not long enough to specify both of them independently. The generality of the model has been demonstrated by testing it against shorter data sequences from the South Pole and Australia.

**INTRODUCTION**

There can no longer be any doubt that our industrial society is introducing significant perturbations on the global carbon cycle. Monitoring records show clearly that the concentration of CO₂ in the atmosphere is increasing, and current climate models suggest that this increase is likely to cause major changes in the global climate [Manabe and Wetherald, 1975]. If the predictions are correct, a fundamental remodeling of our industrial system may be required, and the basic changes associated with only the first question. The challenge facing us now is to determine unambiguously whether or not drastic climate changes are imminent and to do so sufficiently far in advance that we can have some options in dealing with the associated problems.

This suggests two fundamental questions: How is atmospheric CO₂ changing? and, What climate changes are likely as a function of atmospheric CO₂ changes? This paper is concerned with only the first question.

Acknowledging that our goal is to achieve the best possible understanding of atmospheric CO₂ changes at the earliest possible date, we are attempting to extract as much information as possible from the data available now. A thorough understanding of the CO₂ fluxes to and from the atmosphere and their changes with time is required. There are a variety of data that bear on the question of CO₂ fluxes, but among the most useful for understanding the atmospheric fluxes are the actual measurements of the atmospheric CO₂ concentration—particularly the 18-year record of monitoring at Mauna Loa Observatory. This paper is devoted to extracting from that data, and to a lesser extent from South Pole and Australian data, as much insight as possible from a statistical and mathematical analysis. Our principal starting point is the monthly averaged Mauna Loa data shown in Figure 1, drawn from Keeling et al. [1976a]. It is difficult to describe the extraordinary care and scientific acumen that went into the effort that finally produced the time series shown here. The measurement precision level is estimated to be 0.1 ppm [Keeling et al., 1976a].

The most obvious properties of the Mauna Loa data set are an annual cycle and a long-term increasing trend [Machta, 1973]. The purpose here is to determine the functional form of the long-term trend, identify any other components that may be interacting with the annual cycle and the general trend, and, if possible, relate all these components to physical factors and forcing functions. For example, Bacastow [1976] has suggested a relationship between the southern oscillation and the atmospheric carbon dioxide observations.

Most of the carbon dioxide literature of recent years has emphasized modeling of the CO₂ fluxes between the atmosphere and other reservoirs. Olson [1970], Baes et al. [1976], Lieth and Whittaker [1975], and Bolin [1977] have all addressed the terrestrial ecosystem-atmosphere fluxes. The ocean fluxes and oceanic CO₂ have been modeled by Bacastow and Keeling [1973], Broecker et al. [1971], Stuiver [1975], and Oeschger et al. [1975], among others. Our approach here is significantly different. We are attempting to learn from mathematical analyses of the carefully obtained data of Figure 1 what types of physical processes might be influencing the observations.

**ANALYSIS**

In our initial effort we used the method of linear least squares to fit the Mauna Loa data with a model consisting of a linear trend and sinusoidal components for the annual cycle and its first four harmonics. That is,
\[ y(t) = a + bt + \sum_{i=1}^{4} A_i \sin \left( \frac{2\pi}{T_i} (t + \phi_i) \right) \]
where \( T_1 = 12, T_2 = 6, T_3 = 4, T_4 = 3, \) and \( T_5 = 2.4 \) months. The time \( t \) was measured in months with the zero point taken to be the beginning of 1958. The constants \( a, b, A_1, A_2, A_3, A_4, A_5, \) and \( \phi_i \) were determined by linear least squares. Only the first two of the sinusoidal terms, i.e., \( T_1 = 12 \) and \( T_2 = 6, \) contributed significantly to the fit. A plot of the residuals (data points minus best fit) versus time (Figure 2) shows that this model fails to capture the long-term trend adequately and that some fundamental form with concave upward curvature is required.

The principal anthropogenic perturbation of the natural CO₂ cycle (the probable forcing function for the long-term increasing trend) is generally acknowledged to be the combustion of fossil fuels with a contribution likely from forest clearing, particularly in the tropics. Rotty [1977] has shown that the annual CO₂ production from fossil fuels and cement
closely follows a straight line relation when plotted on a log
scale (Figure 3), and we find by a least squares fit that the
carbon in fossil fuel burned annually since 1950 is closely
approximated by the expression
\[ X(t) = d'e^{\alpha t} \]  
where \( \alpha' = 0.0443 \pm 0.008 \text{ yr}^{-1} \) or \( 0.00369 \text{ month}^{-1} \pm 0.0007 \) (Figure 4). The zero point for \( t \) was taken to be 1950.0. The
value of \( \alpha' \) is independent of this choice and in fact has
remained almost constant since 1860 except for the three short
periods of major global upheaval—two world wars and the
great depression (see Figure 3). Consequently, to provide for
the concave curvature suggested in Figure 2, we selected an
exponential base line and postulated for the Mauna Loa data a
model of the form
\[ y(t) = a + d' e^{\alpha t} + \sum A_i \sin \left( \frac{2\pi}{T_i} (t + \phi_i) \right) \]
where the zero point for \( t \) was again taken to be 1958.0 and the
constants \( a, d', \alpha, A_i, \phi_i, A_0 \) and \( \phi_0 \) are parameters determined
by nonlinear least squares fitting. If atmospheric CO₂ growth
is largely a consequence of fossil fuel burning and an ex-
ponential forest clearing, we would expect \( a \) to be the pre-
industrial concentration and \( \alpha \) to be the growth rate, hopefully
near the 0.00369 month\(^{-1}\) suggested by the fossil fuel data.
Also, we have observed that the harmonics of the annual cycle
beyond the first are negligible, so only the 12- and 6-month
period components were retained in the analysis. The uncon-
strained best fit to the data was achieved with \( a = 306.4 \pm 0.9 \)
and \( \alpha = 0.0050 \pm 0.0003 \), but a plot of the residuals (Figure 5)
suggests that there is a long-term periodicity in the data which
is not accounted for in the model, and this could affect the
value of \( \alpha \). Others [Keeling et al., 1976b] have suggested that
the low values of CO₂ concentration centered on 1968 are a
singularity related perhaps to the March 1963 eruption of
Mount Agung. The resulting abnormally low solar radiation
reaching the earth could have resulted in cooler surface ocean
water being a stronger than usual sink for CO₂. However,
spectral analysis shows that the perturbation also can be
treated mathematically as a long-term periodicity, and we
suggest the possibility that there might be long-period physical
phenomena which could have impacts on the atmospheric CO₂
concentration.

Both Fourier and maximum entropy spectral analysis were
applied to the detrended data of Figure 5 to identify regular-
ities remaining in the residuals. Maximum entropy spectral
analysis (Mesa) is in many ways superior to the classical
Fourier techniques because (1) it uses all the measured data
without alterations (e.g., no tapering windows are used) and
(2) it minimizes the implicit assumptions about the unavailable
data (i.e., the assumptions about the time series outside the
data window). The reward for this more scrupulous treatment
of the data is a much enhanced frequency resolution. This
improvement is especially noticeable at lower frequencies.
Mesa is capable of resolving cycles with periods comparable to
the length of the time series record [Ulrych, 1972]. Kirk et al.
[1979] describe a Fortran program for computing Mesa
spectra and also provide an extensive bibliography on the
method. One disadvantage of Mesa is that peak heights in a
spectrum do not give accurate estimates of the amount of
variation or power associated with that frequency. The oper-
ating philosophy that we have adopted for isolating cyclic
components in time series data is to use Mesa to estimate
frequencies and the Fourier spectrum to estimate the associ-
ated amplitudes.

Figure 6 is the Mesa spectrum of the residual data from
Figure 5. It is dominated by two peaks with periods of 44.4
months and approximately 109 months. The 44.4-month pe-
period corresponds closely to a periodicity in the southern oscillation index identified by Bacastow [1976]. The longer-period peak, which appears in the Fourier spectrum with about 3 times the power of the 44-month peak, must somehow be taken into account in order to determine accurately the exponential rate constant $\alpha$. To demonstrate this fact, we selected a series of reasonable estimates of the preindustrial concentration $\alpha$ and for each one fitted the model of (3) to the data with $\alpha$ held fixed. For each fit we calculated Fourier and Mesa spectra of the residuals. The results of this exercise are shown graphically in Figure 7. The top graph gives the variation of $\alpha$ with $\alpha$. The solid circles represent the individual fits with $\alpha$ held fixed. The open circle represents the unconstrained fit, i.e., the fit in which $\alpha$ is allowed to vary also. The bottom graph demonstrates why higher than expected values were obtained for $\alpha$ and $\alpha$ in the unconstrained fit. The plot gives the square of the amplitude of the long-term cycle (estimated from the Fourier spectra) as a function of the value of $\alpha$. In the unconstrained fit the values of $\alpha$ and $\alpha$ are adjusted to minimize the power or variance remaining in the data and hence in the long-term cycle. Thus it seems likely that in the unconstrained fit some of the variation that properly belongs in the long-term cycle, which was not included in the model of (3), was transferred by the fitting procedure into the exponential term in an effort to minimize the total squared variation. This could produce anomalously high values of both $\alpha$ and $\alpha$. The middle graph of Figure 7 gives the Mesa-determined period of the long-term cycle as a function of $\alpha$. We note that $\alpha = 300$ gives a period of about 124 months.

The model suggested by the analysis thus far is

$$y(t) = \alpha + d e^{\alpha t} + \sum_{j=1}^{2} A_j \sin \left( \frac{2\pi}{\tau_j} (t + \phi_j) \right) + \sum_{j=1}^{2} F_j \sin \left( \frac{2\pi}{\tau_j} (t + \theta_j) \right)$$

(4)

where the two new harmonic terms have periods $\tau_j$ which must be determined by the fit. This model has only three nonlinear parameters: $\alpha \geq 0.0036$ month$^{-1}$, $\tau_1 \geq 124$ months, and $\tau_2 \geq 44$ months, but attempting to fit it to the data as it stands yields a very ill conditioned problem, i.e., a problem in which the estimated parameter values are extremely sensitive to very small perturbations in the data. As a result of this ill conditioning, it is possible to find an infinite number of sets of parameter estimates (with each parameter spanning a wide range), each of which 'solves' the fitting problem equally well, i.e., which minimizes the sum of squares of the residuals (Table 1). The situation is not hopeless, however, because the parameters are all highly correlated with one another; e.g., the

![Fig. 3. Carbon dioxide production from fossil fuels and cement [from Rotty, 1977].](image)

![Fig. 4. Total carbon in fossil fuel burned plus carbon released from cement manufacture, by year. Data are fitted by the model given in (2).](image)

![Fig. 5. Residual variability when Mauna Loa CO$_2$ data are fitted with the model defined by (3) with $\alpha = 306.4$ ppm and $\alpha = 0.0050$ month$^{-1}$.](image)
Loa was different from that in northern Europe. Probably the
within +2 or 3%. Thus while we should resist putting too much
over northern Europe was approximately 298 ppm (+4, -6)--
gested by Callendar, the diurnal and annual variations, and
about 5 ppm to Bray’s average provides a number that recog-
Callendar’s average is increased by about 5 ppm. Adding
between his daytime and nighttime values and weights them so as
to correct for the preponderance of daytime measurements,
and it is therefore to be preferred. The obvious bias which
remains in both studies arises from the diurnal cycle. Cal-
lendar did present a daytime mean for his ‘preferred’ studies,
plus enough data to see clearly that his overall mean was
dominated by daytime measurements. If one distinguishes be-
tween his daytime and nighttime values and weights them so as
to correct for the preponderance of daytime measurements,
Callendar’s average is increased by about 5 ppm. Adding
about 5 ppm to Bray’s average provides a number that recog-
nizes both the annual and diurnal variations. The error sug-
gested by Callendar, the diurnal and annual variations, and
the variations among the various estimates are all constrained
within ±2 or 3%. Thus while we should resist putting too much
credence in the early data, the CO₂ content of the atmosphere
over northern Europe varies by 15 ppm over the annual cycle, we can get
additional insight from the compilations of Callendar and
Bray.

It appears that Bray’s slightly higher average is a con-
sequence of more conscious considerations of the annual cycle,
and it is therefore to be preferred. The obvious bias which
remains in both studies arises from the diurnal cycle. Cal-
lendar did present a daytime mean for his ‘preferred’ studies,
plus enough data to see clearly that his overall mean was
dominated by daytime measurements. If one distinguishes be-
tween his daytime and nighttime values and weights them so as
to correct for the preponderance of daytime measurements,
Callendar’s average is increased by about 5 ppm. Adding
about 5 ppm to Bray’s average provides a number that recog-
nizes both the annual and diurnal variations. The error sug-
gested by Callendar, the diurnal and annual variations, and
the variations among the various estimates are all constrained
within ±2 or 3%. Thus while we should resist putting too much
credence in the early data, the CO₂ content of the atmosphere
over northern Europe was approximately 298 ppm (+4, -6)—
indicating a wider error band on the low side than on the high
side.

We might further inquire if the preindustrial value at Mauna
Loa was different from that in northern Europe. Probably the
best that can be done is to select the earliest of the modern
Swedish aircraft measurements [Bolin and Bischof, 1970] and
compare them with the earliest Mauna Loa data [Keeling et
al., 1976a]. Hence we observe that in 1960 the Mauna Loa
average was less than the Swedish aircraft average by 0.26
ppm, i.e., no significant difference.

Table 1 shows the variation of the other parameters over
this range of a. In the bottom row the expected small variation
of the sum of squares of the residuals is given. Note that the
amplitudes of the shorter-period harmonic terms Aₐ, Aₐₐ, and
F₂ are quite insensitive to changes in a. The period of the long-
period term τₐ, its amplitude Fₐ, and the amplitude of the
exponential rate factor a are strongly dependent on a. At a =
300 the exponential rate is very close to the growth rate derived
for fossil fuel production of CO₂.

Figure 8 illustrates the quality of the fit when a is specified to
be 298, and Figure 9a presents the residuals after the ex-
ponential, seasonal, and 45- and 142-month terms are re-
moved. Less than 3% of the 216 values are different from the
predicted value by more than ±0.6 ppm. Table 2 gives an
analysis of the variance. The sums of squares for the terms in
the model were calculated from the reductions of the residual
sum of squares as the terms were successively added to the
model, with the order of inclusion being the same as the
ordering of the terms in the table. The terms are not com-
pletely independent, but the final results do not vary much
when the ordering is perturbed. The order chosen for the table
reflects the relative strengths of the corresponding effects in
the data. The important point is that the model accounts for 99.6%
of the variance. This does not mean that the remaining 0.4%
corresponds to the true noise level. Inspection of the residuals
in Figure 9a reveals systematic runs of positive and negative
values. A cumulative periodogram for these residuals is given
in Figure 9b. If the residuals were truly white noise, this
function should remain inside the 95% confidence band de-
scribed by the two diagonal lines. A Fourier amplitude spec-
trum of the residuals (Figure 9a) is given in Figure 9c. There
are six low-frequency peaks whose amplitudes exceed the esti-
mated precision level of the measurements (i.e., whose peak-

---

**Fig. 6.** Maximum entropy spectrum of the residual variability shown in Figure 5.

---

**Fig. 7.** Parametric study to illustrate interrelationships among exponential growth rate, period of an apparent long-term sinusoid, the amplitude of this sinusoid, and base line CO₂ concentration. The unconstrained best fit, i.e., when the base line CO₂ concentration a is allowed to vary also, is shown by open circles.
TABLE 1. Solutions for the Fit of Mauna Loa CO₂ Data by the Model Given by (4) as a Function of the Preindustrial CO₂ Concentration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>a = 292</th>
<th>a = 295</th>
<th>([a = 298])*</th>
<th>a = 300</th>
<th>a = 302</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>0.00264</td>
<td>0.00293</td>
<td>([0.00329])</td>
<td>0.00358</td>
<td>0.00394</td>
</tr>
<tr>
<td>d</td>
<td>22.6</td>
<td>19.7</td>
<td>16.8</td>
<td>14.9</td>
<td>13.0</td>
</tr>
<tr>
<td>(F_r)</td>
<td>150</td>
<td>147</td>
<td>([142])</td>
<td>139</td>
<td>134</td>
</tr>
<tr>
<td>(F_e)</td>
<td>0.709</td>
<td>0.678</td>
<td>0.641</td>
<td>0.614</td>
<td>0.585</td>
</tr>
<tr>
<td>(r_2)</td>
<td>45.0</td>
<td>44.9</td>
<td>([44.9])</td>
<td>44.8</td>
<td>44.7</td>
</tr>
<tr>
<td>(F_n)</td>
<td>0.261</td>
<td>0.262</td>
<td>0.263</td>
<td>0.265</td>
<td>0.266</td>
</tr>
<tr>
<td>(A_1)</td>
<td>2.55</td>
<td>2.55</td>
<td>2.55</td>
<td>2.55</td>
<td>2.55</td>
</tr>
<tr>
<td>(A_2)</td>
<td>0.691</td>
<td>0.691</td>
<td>0.691</td>
<td>0.690</td>
<td>0.690</td>
</tr>
<tr>
<td>(\sum (\text{resid.})^2)</td>
<td>22.72</td>
<td>22.77</td>
<td>22.85</td>
<td>22.96</td>
<td>23.14</td>
</tr>
</tbody>
</table>

Preindustrial CO₂ concentration \(a\) was held fixed in each fit. The annual components are \(T_1 = 12\) months and \(T_2 = 6\) months.

*The preferred value of \(a\) is shown to be 298 ppm.

to-peak variation is more than twice the precision level). These peaks occur at frequencies which could be associated with harmonics of the 142-month cycle.

The weaker of the two long-period sinusoids discovered in the data, i.e., the one at 44.9 ± 0.7 months \((a = 298)\), seems to represent the correlation with the southern oscillation index suggested by Bacastow [1976]. Analysis of the data for atmospheric pressure difference between Easter Island and Darwin, Australia (as suggested by Quinn and Burt [1972]), from 1962 to 1975 yielded an oscillation with a period of 44.7 ± 1.1 months. An examination of the phases of the two cycles confirmed that the maxima in the CO₂ oscillation lag that in the \(\Delta P\) record by about 2½ months, as previously found by Bacastow. This periodicity in the CO₂ data has a peak-to-peak range of approximately 0.53 ppm, well above the measurement error; thus the correlation appears to be real.

We do not wish to speculate very far at this time about the apparent long-period oscillation, but we offer Figure 10, the detrended Mauna Loa data when all components other than the 142-month periodicity have been removed. Superimposed is the 142-month cycle with peak-to-peak range of 1.3 ppm, as obtained from the analysis when \(a = 298\). This amplitude is about one fourth that of the annual cycle and about 3 times that of the 44-month cycle. In Figure 11 we have plotted the rate of change for the 142-month cycle part of the CO₂ concentration, obtained by differentiating the curve of Figure 10, superimposed on a plot of monthly sunspot numbers, obtained from World Data Center A for Solar-Terrestrial Physics, NOAA, Boulder, Colorado.

We recognize that a 216-month record is not long enough to attempt a definitive analysis of periods as long as 142-months—scarcely \(\frac{1}{2}\) cycles. However, the correlations are so striking that we feel these observations should be noted. We do not suggest that the atmospheric CO₂ concentration is directly coupled to sunspot activity, although an indirect relationship is not beyond possibility. The possibility remains that this long periodicity is an artifact produced by a single (or few) random occurrence with an effect decaying over a 5- or 6-year time
TABLE 2. Analysis of Variance for the Model Given in (4) When \( a = 298 \) ppm

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Degrees of Freedom</th>
<th>Sum of Squares</th>
<th>Percent Variance Explained</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model component</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( de )</td>
<td>1</td>
<td>5240</td>
<td>86.4</td>
</tr>
<tr>
<td>Annual cycle</td>
<td>1</td>
<td>704</td>
<td>11.6</td>
</tr>
<tr>
<td>( A_1 \sin \left( \frac{2\pi}{12}(t + \phi_1) \right) )</td>
<td>1</td>
<td>51</td>
<td>0.8</td>
</tr>
<tr>
<td>Total</td>
<td>2</td>
<td>755</td>
<td>12.4</td>
</tr>
<tr>
<td>142-month cycle</td>
<td>1</td>
<td>40</td>
<td>0.7</td>
</tr>
<tr>
<td>44.9-month cycle</td>
<td>1</td>
<td>8</td>
<td>0.1</td>
</tr>
<tr>
<td>Model (total)</td>
<td>5</td>
<td>6043</td>
<td>99.6</td>
</tr>
<tr>
<td>Error (residuals)</td>
<td>2</td>
<td>23</td>
<td>0.4</td>
</tr>
<tr>
<td>Corrected total</td>
<td>215</td>
<td>6066</td>
<td>100</td>
</tr>
</tbody>
</table>

span, e.g., the Mount Agung eruption, and consequent cooling of atmosphere and ocean surface water. Neither do we offer this example as an explanation for the data, only as a possibility.

To demonstrate the technique further and to test the model developed from the Mauna Loa data, we analyzed a time series consisting of 58 monthly average atmospheric CO₂ measurements obtained by Pearman [1977] and his colleagues at the Council for Scientific and Industrial Research. The measurements were taken from aircraft at altitudes of 3-5 km over southeastern Australia. They cover the months March 1972 to December 1976, overlapping the last quarter cycle shown in Figure 10. We fit the data with two models:

\[
y(t) = a + de^{\beta t} + \sum_{i=1}^{5} A_i \sin \left( \frac{2\pi}{T_i} (t + \phi_i) \right) \tag{5}
\]

\[
y(t) = a + de^{\beta t} + \sum_{i=1}^{5} A_i \sin \left( \frac{2\pi}{T_i} (t + \phi_i) \right) + F_1 \sin \left( \frac{2\pi}{\tau_1} (t + \theta_1) \right) \tag{6}
\]

In both fits we kept the values \( a = 298 \) ppm and \( \alpha = 0.00329 \) month⁻¹ fixed—the latter value obtained from the Mauna Loa data. In (6) we also kept the Mauna Loa values for \( T_1 \) and \( \phi_1 \) and determined only the amplitude \( F_1 \) from the fit. The residuals for the two models are shown in Figure 12. Residuals using (5) clearly display a systematic trend which is eliminated when the 142-month sine wave is added. Furthermore, the value for \( F_1 (0.76 \pm 0.09) \) is comparable to the amplitude obtained at Mauna Loa.

The second longest continuous record of carefully determined monthly average values of atmospheric CO₂ is that from the South Pole [Keeling et al., 1976]. When the South Pole data were fitted by an exponential growth curve with yearly harmonics but no long-term periodicities (equation (3)), the unconstrained best fit was obtained with \( a = 301.9 \) and \( \alpha = 0.0038 \). Holding \( a \) fixed at 300, the model gave \( a = 0.00348 \pm 0.00008 \) (Figure 13). It has been obvious for some time that the amplitude of the annual cycle in the South Pole data is much less than that at Mauna Loa, and this is evident in the relative amplitudes \( A_i \). The amplitude of the 12-month cycle is 2.55 at Mauna Loa and 0.453 at the South Pole. (Actually, all harmonics other than the 12-month period itself are nearly negligible at the South Pole.) A plot of the residuals (Figure 14) clearly reveals a periodic variation with about the 44-month wavelength as identified in the Mauna Loa data, but the continuous record from the South Pole is too short to define a cycle of 140 ± 1 month periodicity.

An important question raised by comparison of the analyses is whether the Mauna Loa and the South Pole CO₂ concentra-

Fig. 10. Residual variability when all elements of the model in (4), except the approximately 142-month period, are subtracted from the data. The 142-month cycle \((a = 298 \text{ ppm})\) is superimposed.

Fig. 11. Rate of change of 142-month cycle part of CO₂ concentration at Mauna Loa \((a = 298)\) superimposed on a plot of monthly sunspot numbers.

Fig. 12. Residual variability when Australian CO₂ data [Pearman, 1977] are fitted by two models (equations (5) (6)) with \( a = 298 \text{ ppm} \) and \( \alpha = 0.00329 \text{ month}^{-1} \).
tions are experiencing exponential growth with different rate constants. A plot of the difference between the Mauna Loa and the South Pole values (Figure 15) does suggest that the Mauna Loa observations are increasing more rapidly than those at the South Pole. Interruptions in the record at the South Pole make it difficult to compare with the 18-year record at Mauna Loa, and this led us to examine the Mauna Loa data in parts. That is, since the South Pole data cover a shorter time, how might the Mauna Loa data look if it had been restricted to a shorter period? It turns out that the Mauna Loa data, fitted by (3) (i.e., without the long-period cyclic components), does give different values of $a$ for different data periods. Holding $a$ at 300 for the early interval 1958–1968, we obtain $a = 0.0030$; for a middle interval of 1961–1971, $a = 0.0036$; and for 1965–1975, $a = 0.0039$. Figure 10 shows that this is the kind of variation in $a$ that should be expected when the long-period cycle is omitted from the model. The conclusion is that only over a long data period are we able to discern and separate long-period components from the general exponential growth trend. For the South Pole data this is not yet possible and even for the Mauna Loa data the separation must be considered somewhat tentative and our specification of $a$ correspondingly uncertain. If there is embedded in the South Pole data a 140 ± 1 month periodicity (or alternatively, some mid-1960 singularity) having a smaller amplitude in the CO$_2$ observations than at Mauna Loa, the apparently different growth rates could perhaps be explained.

**REFERENCES**


(Received February 6, 1978; revised July 3, 1978; accepted July 18, 1978.)