- Grandparent: 0. Numerical Evaluation of Special Functions[1]
- Parent: 5. Functions of Two or More Variables[2]
- Previous: 5.7. Incomplete Gamma Functions, Generalized Exponential Integrals[3]
- Next: 5.9. Mathieu, Lamé, and Spheroidal Wave Functions[4]

Press here[5] to get the full document in PostScript format.
Press here[6] to get this subdocument in PostScript format.

NUMERICAL EVALUATION OF SPECIAL FUNCTIONS

D. W. LOZIER AND F. W. J. OLVER

ABSTRACT. This document is an excerpt from the current hypertext version of an article that appeared in Walter Gautschi (ed.), *Mathematics of Computation 1943–1993: A Half-Century of Computational Mathematics*, Proceedings of Symposia in Applied Mathematics 48, American Mathematical Society, Providence, RI 02940, 1994. The symposium was held at the University of British Columbia August 9–13, 1993, in honor of the fiftieth anniversary of the journal *Mathematics of Computation*.

The original abstract follows.

Higher transcendental functions continue to play varied and important roles in investigations by engineers, mathematicians, scientists and statisticians. The purpose of this paper is to assist in locating useful approximations and software for the numerical generation of these functions, and to offer some suggestions for future developments in this field.

5.8. Legendre and Associated Legendre Functions. This section includes the conical and toroidal functions. See also hypergeometric functions (§5.5) and orthogonal polynomials (§5.10).

5.8.1. *Real Argument and Parameters.*

Software Packages: [**Bra73**, Fortran], [**Del79**, Fortran], [**Gau65**, Algol], [**LS81**, Fortran], [**OS83**, Fortran].

Intermediate Libraries: [**Bak92**].

Comprehensive Libraries: **Numerical Recipes**, **SLATEC**.

5.8.2. *Complex Argument and/or Parameters.*

Algorithms: [**Kol81**, conical with real argument].

Intermediate Libraries: [**Bak92**].

Interactive Systems: **Mathematica**.

5.8.3. *Articles.* [**CM78**], [**CM79**], [**EWB84**], [**Fet70**], [**SOL81**].

---

[1] /nesf/Sect0/Sect0.html
[2] /nesf/Sect5/Sect5.html
[3] /nesf/Sect5-7/Sect5-7.html
[4] /nesf/Sect5-9/Sect5-9.html
[5] ftp://gams.nist.gov/pub/lozier/nesf/Paper.ps.Z
[6] ftp://gams.nist.gov/pub/lozier/nesf/Sect5-8.ps.Z

1991 *Mathematics Subject Classification.* Primary 65D20; Secondary 33-00.
The research of the second author has been supported by NSF Grant CCR 89-14933.

References

[Bak92] L. Baker, *C mathematical function handbook*, McGraw-Hill, Inc., New York, 1992, includes diskette.

[Bra73] W. J. Braithwaite, *Associated Legendre polynomials, ordinary and modified spherical harmonics*, Comput. Phys. Comm. **5** (1973), 390–394.

[CM78] J. N. L. Connor and D. C. Mackay, *Accelerating the convergence of the zonal harmonic series representation in the Schumann resonance problem*, J. Atmospheric Terrestrial Phys. **40** (1978), 977–980.

[CM79] J. N. L. Connor and D. C. Mackay, *Calculation of angular distributions in complex angular momentum theories of elastic scattering*, Molecular Phys. **37** (1979), 1703–1712.

[Del79] G. Delic, *Chebyshev expansion of the associated Legendre polynomial* $P_L^M(x)$, Comput. Phys. Comm. **18** (1979), 63–71.

[EWB84] A. S. Elder, J. N. Walbert, and E. C. Benck, *Calculation of Legendre functions on the cut for integral order and complex degree by means of Gauss continued fractions*, Tech. Report ARBRL–MR–03335, U. S. Army Armament Research and Development Center, Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, 1984, copies obtainable from National Technical Information Service, U. S. Dept. of Commerce, Springfield, VA 22161.

[Fet70] H. E. Fettis, *A new method for computing toroidal harmonics*, Math. Comp. **24** (1970), 667–670.

[Gau65] W. Gautschi, *Algorithm 259. Legendre functions for arguments larger than one*, Comm. ACM **8** (1965), 488–492, for remark see ACM Trans. Math. Software, v. 3 (1977), pp. 204-205.

[Köl81] K. S. Kölbig, *A program for computing the conical functions of the first kind* $P^m_{-\frac{1}{2}+i\tau}(x)$ *for* $m = 0$ *and* $m = 1$, Comput. Phys. Comm. **23** (1981), 51–61.

[LS81] D. W. Lozier and J. M. Smith, *Algorithm 567. Extended-range arithmetic and normalized Legendre polynomials*, ACM Trans. Math. Software **7** (1981), 141–146.

[OS83] F. W. J. Olver and J. M. Smith, *Associated Legendre functions on the cut*, J. Comput. Phys. **51** (1983), 502–518.

[SOL81] J. M. Smith, F. W. J. Olver, and D. W. Lozier, *Extended-range arithmetic and normalized Legendre polynomials*, ACM Trans. Math. Software **7** (1981), 93–105.

Applied and Computational Mathematics Division, National Institute of Standards and Technology, Gaithersburg, Md 20899

E-mail address: dlozier@nist.gov

Institute for Physical Science and Technology, University of Maryland, College Park, MD 20742

E-mail address: olver@bessel.umd.edu