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NUMERICAL EVALUATION OF SPECIAL FUNCTIONS

D. W. LOZIER AND F. W. J. OLVER

ABSTRACT. This document is an excerpt from the current hypertext version of an article that appeared in Walter Gautschi (ed.), *Mathematics of Computation 1943–1993: A Half-Century of Computational Mathematics*, Proceedings of Symposia in Applied Mathematics 48, American Mathematical Society, Providence, RI 02940, 1994. The symposium was held at the University of British Columbia August 9–13, 1993, in honor of the fiftieth anniversary of the journal *Mathematics of Computation*.

The original abstract follows.

Higher transcendental functions continue to play varied and important roles in investigations by engineers, mathematicians, scientists and statisticians. The purpose of this paper is to assist in locating useful approximations and software for the numerical generation of these functions, and to offer some suggestions for future developments in this field.

5.7. Incomplete Gamma Functions, Generalized Exponential Integrals.

These functions are essentially equivalent; thus $E_p(z) = z^{p-1}\Gamma(1-p,z)$. This section includes the chi-square distribution function.

5.7.1. *Real z and Integer or Half-Integer p.*

Software Packages: [**Amo80a**, Fortran], [**SP75**, Fortran], [**SZ74**, Fortran].

Intermediate Libraries: [**Mos89**], [**ULI90**].

Comprehensive Libraries: **SLATEC**.

Interactive Systems: **Maple**.

5.7.2. *Real z and Real p.*

Software Packages: [**CLM90a**, Fortran], [**CLM90b**, Fortran], [**DM87**, Fortran], [**Ful72**, Fortran], [**Gau79a**, Fortran], [**Moo82**, Fortran], [**She88**, Fortran], [**Tem94**, Pascal].

Intermediate Libraries: [**Bak92**], [**Mos89**], [**ULI90**].

Comprehensive Libraries: **IMSL**, **NAG**, **Numerical Recipes**, **Scientific Desk**, **SLATEC**.

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[1] /nesf/Sect0/Sect0.html
[2] /nesf/Sect5/Sect5.html
[3] /nesf/Sect5-6/Sect5-6.html
[4] /nesf/Sect5-8/Sect5-8.html
[5] ftp://gams.nist.gov/pub/lozier/nesf/Paper.ps.Z
[6] ftp://gams.nist.gov/pub/lozier/nesf/Sect5-7.ps.Z

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5.7.3. *Complex z and Real or Complex p.*
Interactive Systems: **Maple**, **Mathematica**.
5.7.4. *Inverse Function.*
Software Packages: [**DM87**, Fortran], [**Phi88**, Fortran].
Intermediate Libraries: [**Mos89**], [**ULI90**].
Comprehensive Libraries: **IMSL**, **NAG**, **Scientific Desk**.
Interactive Systems: **Maple**, **Mathematica**.
5.7.5. *Articles.* [**AB87a**], [**Amo80b**], [**Bar61**], [**CLM87**], [**CLM88**], [**CLM90c**], [**DM86**], [**Gau79b**], [**JT85**], [**LDP93**], [**Luk75**], [**Mar82**], [**Tem85**], [**Tem87**], [**Tem92a**].

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