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NUMERICAL EVALUATION OF SPECIAL FUNCTIONS

D. W. LOZIER AND F. W. J. OLVER

ABSTRACT. This document is an excerpt from the current hypertext version of an article that appeared in Walter Gautschi (ed.), *Mathematics of Computation 1943–1993: A Half-Century of Computational Mathematics*, Proceedings of Symposia in Applied Mathematics 48, American Mathematical Society, Providence, RI 02940, 1994. The symposium was held at the University of British Columbia August 9–13, 1993, in honor of the fiftieth anniversary of the journal *Mathematics of Computation*.

The original abstract follows.

Higher transcendental functions continue to play varied and important roles in investigations by engineers, mathematicians, scientists and statisticians. The purpose of this paper is to assist in locating useful approximations and software for the numerical generation of these functions, and to offer some suggestions for future developments in this field.

4.2. Error Functions, Dawson's Integral, Fresnel Integrals.

4.2.1. *Error Functions of Real Argument.*

Algorithms: [**Cle62**], [**Cod69**], [**Luk69b**], [**Luk75**], [**Sch78**], [**SL81**].

Software Packages: [**Ada69**, Algol], [**CMW63**, Algol], [**Cod90a**, Fortran], [**CT85**, Fortran], [**Hil73**, Fortran], [**SZ70**, Fortran], [**Tem94**, Pascal].

Intermediate Libraries: [**Bak92**], [**Mos89**], [**ULI90**].

Comprehensive Libraries: **IMSL**, **NAG**, **Numerical Recipes**, **Scientific Desk**, **SLATEC**.

4.2.2. *Inverse Error Functions of Real Argument.*

Algorithms: [**BEJ76**].

Software Packages: [**Cun69**, Fortran], [**HD73**, Algol].

Intermediate Libraries: [**Mos89**], [**ULI90**].

Comprehensive Libraries: **IMSL**, **NAG**, **Scientific Desk**.

4.2.3. *Integrals of the Error Function.*

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[1] /nesf/Sect0/Sect0.html
[2] /nesf/Sect4/Sect4.html
[3] /nesf/Sect4-1/Sect4-1.html
[4] /nesf/Sect4-3/Sect4-3.html
[5] ftp://gams.nist.gov/pub/lozier/nesf/Paper.ps.Z
[6] ftp://gams.nist.gov/pub/lozier/nesf/Sect4-2.ps.Z

1991 *Mathematics Subject Classification.* Primary 65D20; Secondary 33-00.
The research of the second author has been supported by NSF Grant CCR 89-14933.

Algorithms: [**Woo67**].
Software Packages: [**Gau77a**, Fortran].
Intermediate Libraries: [**Bak92**].

4.2.4. *Dawson's Integral of Real Argument.*
Algorithms: [**CPT70**], [**Hum64**].
Software Packages: [**Ryb89**, Fortran].
Intermediate Libraries: [**Bak92**], [**Mos89**], [**ULI90**].
Comprehensive Libraries: **IMSL**, **NAG**, **Numerical Recipes**, **Scientific Desk**, **SLATEC**.

4.2.5. *Fresnel Integrals of Real Argument.*
Algorithms: [**Cod68**], [**Hea85**], [**Luk69b**], [**Luk75**].
Software Packages: [**Bul67**, Algol], [**LG64**, Algol], [**Sny93**, Fortran].
Intermediate Libraries: [**Bak92**], [**Mos89**], [**ULI90**].
Comprehensive Libraries: **IMSL**, **NAG**, **Numerical Recipes**.
Interactive Systems: **Maple**.

4.2.6. *Complex Arguments.*
Algorithms: [**Luk69b**].
Software Packages: [**Gau69a**, Algol], [**Lyn93**, Fortran], [**PW90a**, Fortran], [**SZ81**, Fortran].
Intermediate Libraries: [**Bak92**].
Comprehensive Libraries: **IMSL**, **NAG**.
Interactive Systems: **Maple**, **Mathematica**.

4.2.7. *Articles.* [**BR71**], [**Cod90b**, includes survey], [**Col87a**], [**Fle68**], [**Gau70**], [**Gau77b**], [**Hen79**], [**HR72**], [**LW90**], [**LW91**], [**McC74**], [**Mor83**], [**MR71**], [**PW90b**], [**Str68**], [**vdLT84**], [**Wei94a**, includes survey], [**Wei94b**].

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