Extract from OP-SF NET

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Subject: Bille C. Carlson 1924-2013

Bille Carlson, author of the DLMF chapter on Elliptic Integrals (http://dlmf.nist.gov/19), died on August 16.

I first met Bille many years ago and became somewhat familiar with his pathbreaking advances in the theory of elliptic integrals. His development of the theory brought many disparate formulas into a much more structured setting. The formulas he derived were very general, fitting the classical Legendre elliptic integrals into a more symmetric framework. By setting parameters appropriately, long lists of less general formulas of the type that are important in applications are subsumed. In principle, and perhaps to some extent in practice, his general formulas have reduced the need for massive printed compendia, such as Byrd and Friedman's *Handbook of Elliptic Integrals for Engineers and Scientists* and Gradshteyn and Ryzhik's *Tables of Integrals, Series and Products*.

More specifically, according to http://dlmf.nist.gov/about/bio/BCCarlson :
"The main theme of Carlson's research has been to expose previously hidden permutation symmetries that can eliminate a set of transformations and thereby replace many formulas by a few. In 1963 he defined the R-function, a multivariate hypergeometric function that is homogeneous in its variables, each variable being paired with a parameter. If some of the parameters are equal, then the R-function is symmetric in the corresponding variables. This symmetry led to the development of symmetric elliptic integrals, which are free from the transformations of modulus and amplitude that complicate the Legendre theory. Symmetric integrals and their degenerate cases allow greatly shortened integral tables and improved algorithms for numerical computation. Also, the homogeneity of the R-function has led to a new type of mean value for several variables, accompanied by various inequalities."

Bille's formulas are also well-suited to computation. He developed effective algorithms for computing numerical values of elliptic integrals. These were published in 1965 in *Journal of Mathematical Physics*, in 1972 in *Mathematics of Computation*, and in 1979 in *Numerische Mathematik*. In 1981, Bille and Elaine Notis co-authored *Algorithm 577: Algorithm for incomplete elliptic integrals*, published in *ACM Transactions on Mathematical Software*. Many commercial and open-source software packages quickly incorporated Bille's methods, where they remain in use today.
Bille was an outstanding contributor to the DLMF project. His chapter consists of two parts, the first on the classical Legendre theory and the second on his far-reaching

extensions and generalizations of it.

I append the following excerpt from the obituary published by Adams & Soderstrum Family of Funeral Homes, Ames, Iowa, reproduced here with permission:

"Bille Chandler Carlson was born June 27, 1924in Boston, Massachusetts, and spent his boyhood on the seashore of Cape Cod. He began studies at Harvard College, but joined the U.S. Navy after the onset of World War II and worked on the island of Guam with radar technology, which was novel at the time. After the War, he returned to Harvard and completed Bachelor's and Master's degrees in physics and mathematics. He then went to Oxford as a Rhodes Scholar and completed a doctoral degree in physics. After four years in the Physics Department at Princeton, Bille came to the Ames Laboratory and Iowa State University in 1954, where he was a Professor in the Physics and Mathematics Departments, ultimately as a Professor Emeritus.
He is known for having developed [the] Carlson elliptic integrals, some of which he described in his 1977 book, Special Functions of Applied Mathematics, and in the National Institute of Standards Handbook of Mathematical Functions (2010)."