# O P-S F N E T - Volume 29, Number 3 - May 15, 2022 

The Electronic News Net of the
SIAM Activity Group on Orthogonal Polynomials and Special Functions
http://math.nist.gov/opsf

OP-SF Net is distributed to OPSF Activity Group members and non-members alike through the OP-SF Talk listserv.
If you are interested in subscribing to the Newsletter and/or OP-SF Talk, or if you would like to submit a topic to the Newsletter or a contribution to OP-SF Talk, please send an email to the OP-SF Net Editors.

## Editors:

Howard S. Cohl howard.cohl@nist.gov
Sarah Post
spost@hawaii.edu

## Topics:

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2. Announcement: Article on the History of the Relativistic Schrödinger Equation
3. Book Description for: Theta functions, elliptic functions and $\pi$ (2020) by Heng Huat Chan
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5. Preprints in arXiv.org
6. Submitting contributions to OP-SF NET and SIAM-OPSF (OP-SF Talk)
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## Calendar of Events:

May 23-27, 2022
Baylor Analysis Fest: From Operator Theory to Orthogonal Polynomials, Combinatorics, and Number Theory
Baylor University, Waco, TX, USA
https://tinyurl.com/BAFconference
May - November, 2022
Symmetries: Algebras and Physics
Thematic Semester, includes the following workshops:

## May 23-June 10, 2022

Non-commutative algebras, representation theory and special functions
July 25-August 19, 2022
Graph theory, Algebraic combinatorics and mathematical physics
September 12-October 7, 2022
Integrable systems, exactly solvable models and algebras

Centre de Recherches Mathématiques, Montréal, Quebec, Canada
http://www.crm.umontreal.ca/2022/Symmetries22/index_e.php

June 13-17, 2022
OPSFA-16
Centre de Recherches Mathématiques, Montréal, Quebec, Canada
http://www.crm.umontreal.ca/2022/OPSFA22/index_e.php
July 3-8, 2022
Complex Analysis, Spectral Theory and Approximation meet in Linz
Johannes Kepler Universität, Linz, Austria
https://www.jku.at/institut-fuer-analysis/konferenzen/complex-analysis/
July 5-8, 2022
Functional Analysis, Approximation Theory and Numerical Analysis (FAATNA)
Matera, Italy
http://web.unibas.it/faatna20/
August 8-12, 2022
OPSF-S9: Radboud OPSFA Summer School
Nijmegen, The Netherlands
https://www.ru.nl/radboudsummerschool/courses/2022/opsfa-summer-school/

## Topic \#1 _ OP - SF Net $29.3 \quad$ _ May 15, 2022

From: Benjamin Eichinger (benjamin.eichinger@tuwien.ac.at)
Subject: Announcement: Complex Analysis, Spectral Theory and Approximation meet in Linz

Dear colleagues!
The conference "Complex Analysis, Spectral Theory and Approximation meet in Linz" aims to bring together experts in Complex Analysis, Spectral Theory and Approximation and provide a framework for scientific exchange related to those topics. The event, which was initially planned in 2020 and postponed due to the Covid pandemic will eventually take place in a hybrid format through Johannes Kepler Universität, Linz.

Conference dates: July 3-8, 2022
Conference homepage: link.
Registration Deadline for on-site participation: June 15
The conference program consists of invited talks and a poster session.
The list of plenary speakers is:

- Alexander Aptekarev
- Roman Bessonov
- Jacob Christiansen
- David Damanik
- Sergey Denisov
- Iryna Egorova
- Alexandre Eremenko
- Jake Fillman
- Fritz Gesztesy
- Alexander Kheifets
- Aleksey Kostenko
- Stanislas Kupin
- Milivoje Lukic
- Barry Simon
- Mikhail Sodin
- Gerald Teschl
- Sergey Tikhonov
- Alexander Volberg
- Harald Woracek

Sincerely, Benjamin Eichinger
Topic \#2 _ OP - SF Net $29.3 \quad$ May 15, 2022

From: Sergei Suslov (sergei@asu.edu)
Subject: Announcement: Article on the History of the Relativistic Schrödinger Equation

## Article:

Discovery of the Relativistic Schrödinger equation
by Kamal Barley, José Vega-Guzmán, Andreas Ruffing, and Sergei K. Suslov
Journal:
Physics-Uspekhi, IOP Publishing, January 2022, Volume 65, Number 1, Pages 90-103


#### Abstract

: We discuss the discovery of the relativistic wave equation for a spin-zero charged particle in the Coulomb field by Erwin Schrödinger (presumably during the Christmas holidays 1925-26). However, in this new approach, an essential discrepancy was found with the fine structure formula for the energy levels already obtained by Sommerfeld in the framework of the 'old' quantum mechanics. As a result, Schrödinger had to withdraw the original 'relativistically framed' article, a draft of which has never been found, from a journal and start all over with his centennial article on the nonrelativistic stationary Schrödinger equation. Our goal here is to follow the original 'relativistic idea' from a modern mathematical viewpoint and elaborate on why Schrödinger didn't publish it. We hope that this consideration will encourage the readers to study quantum physics starting from one of the crucial moments of its creation.


## Contents:

1. Introduction
2. Introducing the relativistic Schrödinger equation
3. Solving the relativistic Schrödinger equation
4. Further treatment: nonrelativistic approximation
5. Semiclassical approximation: Wentzel-Kramers-Brillouin method for Coulomb fields
6. Erwin Schrödinger - 60 years later
7. Appendices

- Summary of Nikiforov-Uvarov method
- Evaluation of the integral
- Laplace method
- Letter from Schrödinger to Weyl

8. Acknowledgments
9. References

## Goal:

We hope that this consideration, despite potential imperfections, will encourage the readers to study quantum physics starting at one of the crucial moments of its creation and draw their own conclusions.

This article has appeared on the IOP website:
https://iopscience.iop.org/journal/1063-7869
and
https://iopscience.iop.org/article/10.3367/UFNe.2021.06.039000
in the history of physics series.
This article made it on the top of the most read list with a total 579 downloads after just four weeks online.

The History of Physics series had in the past great authors, such as
P.A.M. Dirac:
https://iopscience.iop.org/article/10.1070/PU1979v022n08ABEH005593
and F.J. Dyson:
https://iopscience.iop.org/article/10.3367/UFNe.0180.201008f.0859
among others.

## Topic \#3 _ OP - SF Net $29.3 \quad$ _ May 15, 2022

From: OP-SF Net Editors
Subject: Book Description for: Theta functions, elliptic functions and $\pi$ (2020) by Heng Huat Chan
It may be of interest to some members of the OPSFA mailing list to know about the following recently published book:
"Theta functions, elliptic functions and $\pi$ ", in the De Gruyter Textbook series: https://doi.org/10.1515/9783110541915.
Heng Huat Chan, Department of Mathematics, National University of Singapore, Singapore, Republic of Singapore

This book presents several results on elliptic functions and $\pi$, using Jacobi's triple product identity as a tool to show surprising connections between different topics within number theory such as theta functions, Eisenstein series, the Dedekind delta function, and Ramanujan's work on $\pi$. The included exercises make it ideal for both classroom use and self-study.

- A pedagogical presentation of elliptic functions, modular forms and Ramanujan's work on $\pi$.
- Connects several parts of number theory through Jacobi's triple product identity.
- Includes exercises, making it also suitable for self-study.


Figure 1: Book cover

The following is the foreword written by Bruce Berndt, reprinted with permission.

When one peruses the offerings of mathematics book publishers, one finds a large variety of text books at the undergraduate level, and also at the advanced level for upper-level graduate students and researchers. However, few books are "in between." Theta functions, elliptic functions and $\pi$ falls "in between." This book, focusing on certain classical topics related to number theory, provides a stepping stone to both the past and future. The topics are chosen both for their elegance and their usefulness. The ubiquitous theta functions play the leading role. They form relationships to elliptic functions, sums of squares, partitions, hypergeometric functions, $q$-series, and infinite series representations for $1 / \pi$. This book is valuable, because some of its topics do not appear in any of the courses taught by large, major universities. But it is even more valuable because it is inspirational. As you read it, you will exclaim, "What a beautiful theorem!" or "What an elegant proof!" or "What an interesting problem!" (and, indeed, there are many challenging exercises). Read with enjoyment!

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If you are interested in purchasing the books, then navigate here:
https://www.degruyter.com/document/doi/10.1515/9783110541915/html

The following topic is dedicated to two the AMS Mathematical Reviews and zbMATH Open reviews for this book.

## Topic \#4 _ OP - SF Net $29.3 \quad$ May 15, 2022

From: OP-SF Net Editors
Subject: Book Reviews for: Theta functions, elliptic functions and $\pi$ (2020) by Heng Huat Chan
The following are reviews of the book Theta functions, elliptic functions and $\pi$ (with a foreword by Bruce Berndt) by Heng Huat Chan, National University of Singapore, Singapore \& National Changhua University of Education, Taiwan. It was published by De Gruyter, Berlin in 2020.

This book review by Daniele Ritelli was originally published by the American Mathematical Society (AMS) as the review MR42028036 in Mathematical Reviews/MathSciNet. It is reprinted here by permission of the AMS.
$\star \star \star$

The main objective of this textbook is to provide an accessible introduction to the Theta functions, which constitute a fundamental chapter in the theory of elliptic functions and, more generally, of special functions, usually mastered by specialists in the field and this is evidenced by the fact that the reference texts in this field are starting from the classic and monumental manual of Whittaker and Watson's A Course of Modern Analysis of which it is worth mentioning the recent republication [MR4286926], Pi and the AGM of the Borwein brothers [MR1641658] and finally the monograph Ramanujan's theta functions [MR3675178] of Shaun Cooper are, inevitably, extremely specialized level. On the other hand, the author proposes to approach these topics, quote from the introduction, by bridging, preparing the reader by gradually introducing him to hypergeometric series, theta functions, elliptic functions and modular forms.

My opinion is that the aim of the text is fully successful, allowing the reader to appreciate the many identities presented in the text, starting with Ramanujan's famous series for $1 / \pi$, and at the same time to continue his or her journey by approaching the aforementioned specialised texts.

This book review by Franz Lemmermeyer was originally published by zbMATH Open as the review an:1458.11002 which is available under the CC BY-SA 4.0 license.

This book covers contributions by Euler, Jacobi, Eisenstein, Gauss and Ramanujan: Euler's special case of the $q$-binomial theorem, Jacobi's triple product formula, Jacobi's theta functions, Jacobi elliptic functions, Eisenstein series, Ramanujan's series for $\frac{1}{\pi}$, and Gauss's theory of the arithmeticgeometric mean. The choice of topics is guided by the aim of making the book J. M. Borwein and $P$. B. Borwein [Pi and the AGM. A study in analytic number theory and computational complexity. New York, NY: John Wiley (1987; Zbl 0611.10001)] accessible to students familiar with basic complex analysis. The present book is similar in spirit to M. D. Hirschhorn's equally charming [The power of q. A personal journey. Cham: Springer (2017; Zbl 1456.11001)] - I highly recommend both books.

## Topic \#5 _ OP - SF Net $29.3 \quad$ May 15, 2022

From: OP-SF Net Editors
Subject: Preprints in arXiv.org

The following preprints related to the fields of orthogonal polynomials and special functions were posted or cross-listed to one of the subcategories of arXiv.org during March and April 2022. This list has been separated into two categories.

## OP-SF Net Subscriber E-Prints

http://arxiv.org/abs/2203.00243
Lattice paths, vector continued fractions, and resolvents of banded Hessenberg operators Abey López-García, Vasiliy A. Prokhorov
http://arxiv.org/abs/2203.00498
Eigenfunctions of the van Diejen model generated by gauge and integral transformations F. Atai, M. Noumi
http://arxiv.org/abs/2203.01204
Generalised symmetries and bases for Dunkl monogenics
Hendrik De Bie, Alexis Langlois-Rémillard, Roy Oste, Joris Van der Jeugt
http://arxiv.org/abs/2203.01419
Electrostatic partners and zeros of orthogonal and multiple orthogonal polynomials
Andrei Martínez-Finkelshtein, Ramón Orive, Joaquín Sánchez-Lara
http://arxiv.org/abs/2203.01739
Indefinite $q$-integrals from a method using $q$-Ricatti equations
G. E. Heragy, Z. S. I. Mansour, K. M. Oraby
http://arxiv.org/abs/2203.02273
The eigenvector-eigenvalue identity for the quaternion matrix with its algorithm and computer program
Yuchao He, Mengda Wu, Y-H. Xia
http://arxiv.org/abs/2203.02675
An alternate proof for a case of a Malmsten integral
Abdulhafeez A. Abdulsalam
http://arxiv.org/abs/2203.03039
Landau-Ginzburg mirror, quantum differential equations and qKZ difference equations for a partial flag variety
Vitaly Tarasov, Alexander Varchenko
http://arxiv.org/abs/2203.03318
Higher-order recurrence relations, Sobolev-type inner products and matrix factorizations
Carlos Hermoso, Edmundo J. Huertas, Alberto Lastra, Francisco Marcellán
http://arxiv.org/abs/2203.03589
Euler-Kronecker constants for cyclotomic fields
Letong Hong, Ken Ono, Shengtong Zhang
http://arxiv.org/abs/2203.03777
A class of Bernstein-type operators on the unit disk
Marlon J. Recarte, Misael E. Marriaga, Teresa E. Pérez
http://arxiv.org/abs/2203.03855
A new Approach to fully degenerate Bernoulli numbers and polynomials
Taekyun Kim, Dae San Kim
http://arxiv.org/abs/2203.04966
Linear-Time and Constant-Space Algorithms to compute Multi-Sequences that arise in Enumerative Combinatorics (and Elsewhere)
Shalosh B. Ekhad, Doron Zeilberger
http://arxiv.org/abs/2203.05202
Schmidt-type theorems for partitions with uncounted parts
George E. Andrews, William J. Keith
http://arxiv.org/abs/2203.05631
On the general family of third-order shape-invariant Hamiltonians related to generalized Hermite polynomials
Ian Marquette, Kevin Zelaya
http://arxiv.org/abs/2203.06134
Gegenbauer expansions and addition theorems for a binomial and logarithmic fundamental solution of the even-dimensional Euclidean polyharmonic equation
Howard S. Cohl, Jessie E. Hirtenstein, Jim Lawrence, Lisa Ritter
http://arxiv.org/abs/2203.07132
Szegő condition, scattering, and vibration of Krein strings
R. Bessonov, S. Denisov
http://arxiv.org/abs/2203.07863
An asymptotic approximation for the Riemann zeta function revisited
R. B. Paris
http://arxiv.org/abs/2203.09421
Point Source Equilibrium Problems with Connections to Weighted Quadrature Domains
Peter D. Dragnev, Alan R. Legg, Edward B. Saff
http://arxiv.org/abs/2203.09426
D-brane masses at special fibres of hypergeometric families of Calabi-Yau threefolds, modular forms, and periods
Kilian Bönisch, Albrecht Klemm, Emanuel Scheidegger, Don Zagier
http://arxiv.org/abs/2203.10025
Sharp estimates for the hypergeometric functions related to root systems of type $A$ and of rank 1 Piotr Graczyk, Patrice Sawyer
http://arxiv.org/abs/2203.10526
Hankel Determinant and Orthogonal Polynomials for a Perturbed Gaussian Weight: from Finite $n$ to Large $n$ Asymptotics
Chao Min, Yang Chen
http://arxiv.org/abs/2203.10955
Chebyshev polynomials in the 16th century
Walter Van Assche
http://arxiv.org/abs/2203.11348
Openness of Regular Regimes of Complex Random Matrix Models
Marco Bertola, Pavel Bleher, Roozbeh Gharakhloo, Kenneth T-R McLaughlin, Alexander Tovbis
http://arxiv.org/abs/2203.12475
A universal lower bound for certain quadratic integrals of automorphic $L$-functions Laurent Clozel, Peter Sarnak
http://arxiv.org/abs/2203.13578
Oscillatory banded Hessenberg matrices, multiple orthogonal polynomials and random walks Amilcar Branquinho, Ana Foulquié-Moreno, Manuel Mañas
http://arxiv.org/abs/2203.14344
On generalizations of discrete and integral Cauchy-Bunyakovskii inequalities by the method of mean values. Some applications
S. M. Sitnik
http://arxiv.org/abs/2203.14391
Bailey pairs and strange identities
Jeremy Lovejoy
http://arxiv.org/abs/2203.14605
New orthogonality relations for super-Jack polynomials and an associated Lassalle-Nekrasov correspondence
Martin Hallnäs
http://arxiv.org/abs/2203.14837
Christoffel functions for multiple orthogonal polynomials
Grzegorz Świderski, Walter Van Assche
http://arxiv.org/abs/2203.15942
Combinatorics of Triangular Partitions
François Bergeron, Mikhail Mazin
http://arxiv.org/abs/2203.16016
Global asymptotics of the sixth Painlevé equation in Okamoto's space
Viktoria Heu, Nalini Joshi, Milena Radnović
http://arxiv.org/abs/2203.16889
Exactly solvable anharmonic oscillator, degenerate orthogonal polynomials and Painlevé II Marco Bertola, Eduardo Chavez-Heredia, Tamara Grava
http://arxiv.org/abs/2203.17231
Progressive approximation of bound states by finite series of square-integrable functions
A. D. Alhaidari
http://arxiv.org/abs/2204.00105
Refinements of Beck-type partition identities
Tewodros Amdeberhan, George E. Andrews, Cristina Ballantine
http://arxiv.org/abs/2204.00756
A generalization of certain associated Bessel functions in connection with a group of shifts
J. Choi, I. A. Shilin
http://arxiv.org/abs/2204.00962
Telescoping continued fractions for the error term in Stirling's formula
Gaurav Bhatnagar, Krishnan Rajkumar
http://arxiv.org/abs/2204.01021
Malmsten's integral and some related results: A different approach with Special functions Abdulhafeez A. Abdulsalam
http://arxiv.org/abs/2204.01045
When does a hypergeometric function ${ }_{p} F_{q}$ belong to the Laguerre-Pólya class $L P^{+}$? Alan D. Sokal
http://arxiv.org/abs/2204.02535
Schmidt Type Partitions
Runqiao Li, Ae Ja Yee
http://arxiv.org/abs/2204.04625
Gap probability for the hard edge Pearcey process
Dan Dai, Shuai-Xia Xu, Lun Zhang
http://arxiv.org/abs/2204.04706
Moment sequences and difference equations
Paweł J. Szabłowski
http://arxiv.org/abs/2204.05505
Lattice paths and negatively indexed weight-dependent binomial coefficients
Josef Küstner, Michael J. Schlosser, Meesue Yoo
http://arxiv.org/abs/2204.05647
Combinatorial identities and hypergeometric series
Enno Diekema
http://arxiv.org/abs/2204.05696
Positive definite functions on a regular domain
Martin Buhmann, Yuan Xu
http://arxiv.org/abs/2204.05801
Generalized quadratic commutator algebras of PBW-type
Ian Marquette, Luke Yates, Peter Jarvis
http://arxiv.org/abs/2204.06220
Product Inequalities for Multivariate Gaussian, Gamma, and Positively Upper Orthant Dependent Distributions
Dominic Edelmann, Donald Richards, Thomas Royen
http://arxiv.org/abs/2204.08228
Human and automated approaches for finite trigonometric sums
Jean-Paul Allouche, Doron Zeilberger
http://arxiv.org/abs/2204.09305
Recent Advances in Asymptotic Analysis
R. Wong, Yu-Qiu Zhao
http://arxiv.org/abs/2204.09306
On the $\nu$-zeros of the Bessel functions of purely imaginary order
R. B. Paris
http://arxiv.org/abs/2204.09887
Two General Series Identities Involving Modified Bessel Functions and a Class of Arithmetical Functions
Bruce C. Berndt, Atul Dixit, Rajat Gupta, Alexandru Zaharescu
http://arxiv.org/abs/2204.09910
Some remarks and conjectures about Hankel determinants of polynomials which are related to Motzkin paths
Johann Cigler
http://arxiv.org/abs/2204.11242
Parameter and $q$ asymptotics of $\mathfrak{L}_{q}$-norms of hypergeometric orthogonal polynomials
Nahual Sobrino, Jesús Sanzhez-Dehesa
http://arxiv.org/abs/2204.11528
Multiple orthogonal polynomials, $d$-orthogonal polynomials, production matrices, and branched continued fractions
Alan D. Sokal
http://arxiv.org/abs/2204.13278
Sums of Distances on Graphs and Embeddings into Euclidean Space
Stefan Steinerberger
http://arxiv.org/abs/2204.13729
Quasi-polynomial representations of double affine Hecke algebras
Siddhartha Sahi, Jasper Stokman, Vidya Venkateswaran
http://arxiv.org/abs/2204.14098
Characterization of Orthogonal Polynomials on lattices
D. Mbouna, Juan F. Mañas-Mañas, Juan J. Moreno-Balcázar

## Other Relevant OP-SF E-Prints

http://arxiv.org/abs/2203.00009
A geometrical point of view for branching problems for holomorphic discrete series of conformal Lie groups
Quentin Labriet
http://arxiv.org/abs/2203.00207
Generalized hypergeometric $G$-functions take linear independent values
Sinnou David, Noriko Hirata-Kohno, Makoto Kawashima
http://arxiv.org/abs/2203.00223
From boxes to polynomials: a story of generalisation
Gypsy Akhyar, Yifan Guo, Lihexuan Yuan
http://arxiv.org/abs/2203.00248
Extension of Irreducibility results on Generalised Laguerre Polynomials $L_{n}^{(-1-n-s)}(x)$
Saranya G. Nair, Tarlok Nath Shorey
http://arxiv.org/abs/2203.00264
On minima of difference of theta functions and application to hexagonal crystallization Senping Luo, Juncheng Wei
http://arxiv.org/abs/2203.00777
Apéry-Type Series with Summation Indices of Mixed Parities and Colored Multiple Zeta Values, II Ce Xu, Jianqiang Zhao
http://arxiv.org/abs/2203.00856
E-Polynomials of Generic $\mathrm{GL}_{n} \rtimes<\sigma>$-Character Varieties: Unbranched Case Cheng Shu
http://arxiv.org/abs/2203.02070
Computing zeta functions of algebraic curves using Harvey's trace formula
Madeleine Kyng
http://arxiv.org/abs/2203.02254
A real variable calculus for planar orthogonal polynomials
Haakan Hedenmalm, Aron Wennman
http://arxiv.org/abs/2203.02278
Analytic expressions for some Mellin transforms with their application to prime counting function and interpolation formulas for the zeta function
Omprakash Atale
http://arxiv.org/abs/2203.02509
Double Exponential method for Riemann Zeta, Lerch and Dirichlet $L$-functions
Sandeep Tyagi
http://arxiv.org/abs/2203.02647
Rational solutions of Painlevé-II equation as Gram determinant
Liming Ling, Bing-Ying Lu, Xiaoen Zhang
http://arxiv.org/abs/2203.02949
Random walks on crystal lattices and multiple zeta functions
Takahiro Aoyama, Ryuya Namba
http://arxiv.org/abs/2203.02973
Mixed-norm of orthogonal projections and analytic interpolation on dimensions of measures Bochen Liu
http://arxiv.org/abs/2203.03052
Cone Vertex Algebras, Mock Theta Functions, and Umbral Moonshine Modules Miranda C. N. Cheng, Gabriele Sgroi
http://arxiv.org/abs/2203.03242
Product formulas for hypergeometric functions over finite fields
Noriyuki Otsubo, Takato Senoue
http://arxiv.org/abs/2203.04126
Some multivariable Rado numbers
Gang Yang, Yaping Mao, Changxiang He, Zhao Wang
http://arxiv.org/abs/2203.04590
Two conjectures for Macdonald polynomials: The stretching symmetry and Haglund's conjecture Seung Jin Lee, Jaeseong Oh, Brendon Rhoades
http://arxiv.org/abs/2203.04917
Anosov Flows and Dynamical Zeta Functions (Errata)
Paolo Giulietti, Mark Pollicott, Carlangelo Liverani
http://arxiv.org/abs/2203.05083
Skew-invariant curves and the algebraic independence of Mahler functions
Alice Medvedev, Khoa Dang Nguyen, Thomas Scanlon
http://arxiv.org/abs/2203.05302
A Riemann-Hilbert approach to the modified Camassa-Holm equation with step-like boundary conditions
Iryna Karpenko, Dmitry Shepelsky, Gerald Teschl
http://arxiv.org/abs/2203.05350
A family of orthogonal polynomials corresponding to Jacobi matrices with a trace class inverse Pavel Stovicek
http://arxiv.org/abs/2203.05432
Estimates for the largest critical value of $T_{n}^{(k)}$
Nikola Naidenov, Geno Nikolov
http://arxiv.org/abs/2203.05529
Commentary on $S p(6)$ hypergeometric groups
Jitendra Bajpai
http://arxiv.org/abs/2203.05533
Lee-Yang zeroes of the Curie-Weiss ferromagnet, unitary Hermite polynomials, and the backward heat flow
Zakhar Kabluchko
http://arxiv.org/abs/2203.06236
Euler-MacLaurin summation formula on polytopes and expansions in multivariate Bernoulli polynomials
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The eighth moment of the Riemann zeta function
Nathan Ng, Quanli Shen, Peng-Jie Wong

## Topic \#6 _ OP - SF Net 29.3 __ May 15, 2022

From: OP-SF Net Editors
Subject: Submitting contributions to OP-SF NET and SIAM-OPSF (OP-SF Talk)
To contribute a news item to OP-SF NET, send e-mail to one of the OP-SF Editors howard.cohl@nist.gov, or spost@hawaii.edu.

Contributions to OP-SF NET 29.4 should be sent by July 1, 2022.
OP-SF NET is the electronic newsletter of the SIAM Activity Group on Special Functions and Orthogonal Polynomials (SIAG/OPSF). We disseminate your contributions on anything of interest to the special functions and orthogonal polynomials community. This includes announcements of conferences, forthcoming books, new software, electronic archives, research questions, and job openings as well as news about new appointments, promotions, research visitors, awards and prizes. OPSF Net is transmitted periodically through a post to OP-SF Talk which is currently managed and
moderated by Howard Cohl (howard.cohl@nist.gov). Anyone wishing to be included in the mailing list (SIAG/OPSF members and non-members alike) should send an email expressing interest to him. Bonita Saunders also posts the Newsletter through SIAM Engage (SIAG/OPSF) which is received by all SIAG/OPSF members.

OP-SF Talk is a listserv associated with SIAG/OPSF which facilitates communication among members, non-members and friends of the Activity Group. To post an item to the listserv, send e-mail to howard.cohl@nist.gov.

WWW home page of this Activity Group:
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The elected Officers of the Activity Group (2020-2022) are:
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Teresa E. Pérez, Secretary and SIAM Engage (SIAG/OPSF) moderator
The appointed officers are:
Howard Cohl, OP-SF NET co-editor
Sarah Post, OP-SF NET co-editor
Bonita Saunders, Webmaster and SIAM Engage (SIAG/OPSF) moderator

## Topic \#7 _ OP - SF Net 29.3 __ May 15, 2022

From: OP-SF Net Editors
Subject: Thought of the Month by Vladimir Arnold
"Not even mentioning the relative character of initial axioms, one cannot forget about the inevitability of logical mistakes in long arguments (say, in the form of a computer breakdown caused by cosmic rays or quantum oscillations). Every working mathematician knows that if one does not control oneself (best of all by examples), then after some ten pages half of all the signs in formulae will be wrong and twos will find their way from denominators into numerators. "

Vladimir Arnold (1937-2010), in On teaching mathematics, Uspekhi Mat. Nauk 53 (1998), no. 1, 229-234; English translation: Russian Math. Surveys 53 (1998), no. 1, 229-236.

Contributed by Daniel Lichtbau

