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The Electronic News Net of the
SIAM Activity Group on Orthogonal Polynomials and Special Functions

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Editors:

Howard S. Cohl howard.cohl@nist.gov
Sarah Post spost@hawaii.edu

Topics:

Calendar of Events:

March 16-20, 2020—Cancelled due to COVID-19 outbreak.
IV ORTHONET School
Universidad de la Rioja, Logroño, Spain
http://euler.us.es/~orthonet/orthonet20

March 21-22, 2020—**Cancelled due to COVID-19 outbreak.**
AMS Spring Eastern Sectional Meeting
Tufts University, Medford, Massachusetts, USA
Special Session on "Quantum Probability, Orthogonal Polynomials, and Special Functions"
http://www.ams.org/meetings/sectional/2275_program.html

May 11-15, 2020
LMS-CMI Research School: *Methods for Random Matrix Theory and Applications*
University of Reading, Reading, UK
https://janivirtanen.wordpress.com/research-school-2020

May 18-22, 2020
Baylor Analysis Fest: *From Operator Theory to Orthogonal Polynomials, Combinatorics, and Number Theory*
Baylor University, Waco, Texas, USA
https://www.baylor.edu/math/conference

June 15-24, 2020
Foundations of Computational Mathematics (FoCM2020)
Workshop on Approximation Theory, June 18-20
Organized by Albert Cohen, Peter Binev and Maria Charina
Workshop on Random Matrices, June 18-20
Organized by Ionna Dumitriu and Sheehan Olver
Workshop on Special Functions and Orthogonal Polynomials, June 22-24
Organized by Ana Loureiro, Francisco Marcellán and Andrei Martínez Finkelshtein
Simon Fraser University, Vancouver, Canada
http://focm-society.org/2020/index.html

July 5-11, 2020
8th European Congress of Mathematics (8ECM)
Mini-symposium on Orthogonal Polynomials and Special Functions
Organized by Paco Marcellán, Juan J. Moreno-Balcázar and Galina Filipuk,
Portorož, Slovenia
https://www.8ecm.si/minisymposia

July 6-10, 2020
SIAM Annual Meeting, held jointly with CAIMS
(Canadian Applied and Industrial Mathematics Society)
The OPSF activity group has a track of sessions and one invited speaker (Andrei Martínez-Finkelshtein). There are minisymposia on orthogonal polynomials and asymptotic methods, random matrices, symbolic computation, integrable systems and combinatorics, basic hypergeometric series and q-orthogonal polynomials.
Sheraton Centre Toronto Hotel, Toronto, Ontario, Canada
https://www.siam.org/Conferences/CM/Main/an20

July 7-10, 2020
Functional Analysis, Approximation Theory and Numerical Analysis (FAATNA)
Matera, Italy
http://web.unibas.it/faatna20/

July 13-17, 2020
33rd International Colloquium on Group Theoretical Methods in Physics (Group33)
Cotonou, Benin
http://www.cipma.net/group33-cotonou-benin

July 13–17, 2020
XXI Lluis Santaló School *Random and Deterministic Point Configurations*
Universidad Internacional Menéndez Pelayo, Santander, Spain
https://www.ub.edu/santalo20/

July 13–18, 2020
Combinatorics around the q-Onsager algebra, celebrating the 65th birthday of Paul Terwilliger
Satellite event of the 8th European Congress of Mathematics
which will be held the prior week in Portorož, Slovenia,
Kranjska Gora, Slovenia
https://conferences.famnit.upr.si/indico/event/15/overview

August 10–14, 2020
OPSFA Summer School 2020
Radboud University, Nijmegen, The Netherlands
https://www.ru.nl/radboudsummerschool/courses/2020/opsfa-summer-school-2020

Topic #1 ——— OP – SF Net 27.2 ——— March 15, 2020

From: Peter A. Clarkson (P.A.Clarkson@kent.ac.uk)
Subject: Message from the Chair

Dear all,

First I would like to thank members of the Activity group for electing me as Chair. I am pleased to have the support of the other officers: Luc Vinet (Vice Chair), Andrei Martínez-Finkelshtein (Program Director) and Teresa Pérez (Secretary).

Despite being a year in which there is no OPSFA conference, there are several meetings planned which are devoted to various activities in which the SIAG is heavily involved. In particular, the workshop FoCM2020 in Vancouver, Canada (June 18–20), a mini-symposium at the 8th European Congress of Mathematics in Portorož, Slovenia (July 5–11) and the OPSFA Summer School 2020 at Radboud University, Nijmegen, The Netherlands. At the SIAM Annual Meeting this year, which is to be held jointly with the Canadian Applied and Industrial Mathematics Society in Toronto, Canada (July 6–10), Andrei Martínez-Finkelshtein is an invited speaker and there will be several mini-symposia organised by members of the SIAG. However due to the outbreak of the coronavirus disease (COVID-19) there has to be some doubt whether all these meetings will take place, which is unfortunate.

During the current year the SIAG has to get renewal of its charter, as it was only renewed last year for one year, rather than the usual three years. I, together with some of the other SIAG officers, past and present, are planning to attend this year's SIAM Annual Meeting in Toronto and meet with SIAM officers. Be assured the officers of the SIAG will do our best to ensure that our charter is renewed.

Peter Clarkson
School of Mathematics, Statistics & Actuarial Science,
Sibson Building, Parkwood Road, University of Kent,
Canterbury, CT2 7FS, UK
Tel: +44 (0) 1227 827781 (direct line)
Email: P.A.Clarkson@kent.ac.uk
URL: https://www.kent.ac.uk/smsas/personal/pac3/

From: Óscar Ciaurri (oscar.ciaurri@unirioja.es) and Renato Álvarez-Nodarse (ran@us.es)
Subject: Cancellation: *IV ORTHONET School*, Universidad de la Rioja, Logroño, Spain

[Taken from: http://euler.us.es/~orthonet/orthonet20/index.html]

Google-translate to English:

Subject: Cancellation of the IV ORTHONET School

On the morning of Tuesday, March 10, 2020, the Government of La Rioja has suspended any teaching activity in the autonomous community for fifteen days from March 11 as a consequence of the effects of the coronavirus in our region. This fact entails the paralysis of the University of La Rioja, in which we were going to celebrate the IV Orthonet School and, as reflected in the information available on the website of our university (see "other activities comparable to face-to-face classes are suspended: use of the library's reading rooms, activities at the University Sports Center, congresses, conferences and the like").

For this fact we are forced to cancel the ORTHONET School. We will try to organize it on other dates, when the complicated situation in which we find ourselves today (March 11, 2020) is resolved. We are very sorry for the inconvenience that such cancellation may have caused but it is not in our hands to carry it out and we appreciate the understanding of all participants. For any clarification you can contact any of the organizers.

Greetings from the organizers of the IV Orthonet School.

Óscar Ciaurri and Renato Álvarez-Nodarse.

More information by clicking HERE. In an official statement from the Government of La Rioja HERE.

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aviso original en español:

Cancelación de la IV Escuela ORTHONET

En la mañana del martes 10 de marzo de 2020 el Gobierno de La Rioja ha procedido a suspender cualquier actividad docente en la comunidad autónoma durante quince días a partir del 11 de marzo como consecuencia de los efectos del coronavirus en nuestra región. Esto hecho conlleva la paralización de la Universidad de La Rioja, en la que íbamos a celebrar la IV Escuela Orthonet y, como queda recogido en la información disponible en la página web de nuestra universidad (ver "quedan suspendidas otras actividades asimilables a las clases presenciales: uso de las salas de lectura de la Biblioteca, actividades en el Polideportivo Universitario, congresos, jornadas y similares.")

Por este hecho nos vemos obligados la cancelar de la Escuela ORTHONET. Intentaremos organizarla en otras fechas, cuando se resuelva la complicada situación en la que nos encontramos a día de hoy (11 de marzo de 2020). Sentimos mucho los inconvenientes que dicha cancelación haya podido ocasionar pero no está en nuestras manos la realización de la misma y agradecemos la comprensión de todos los participantes. Para cualquier aclaración podéis contactar con cualquiera de los organizadores.

Un saludo de los organizadores de la IV Escuela Orthonet.

Más información pinchando AQUI. En comunicado oficial del Gobierno de La Rioja AQUI.

Topic #3 ——— OP – SF Net 27.2 ——— March 15, 2020

From: Maxim Derevyagin (derevyagin.m@gmail.com) and Ambar Sengupta (MathDeptHead@uconn.edu)
Subject: Cancellation: *AMS Spring Eastern Sectional*, Tufts University, Massachusetts, USA

Dear Participant of the Special Session on Quantum Probability and Orthogonal Polynomials,

The AMS meeting has been canceled due to coronavirus concerns and you should have received an official announcement from the AMS by now. It's a pity that the situation has come up this way but c'est la vie.

We would like to thank all of you for agreeing to give a talk in our session and we apologize for any inconvenience this may have caused.

The life goes on and there will be other opportunities for all of us to get together and to exchange math ideas.

With best wishes, Maxim Derevyagin and Ambar Sengupta

Topic #4 ——— OP – SF Net 27.2 ——— March 15, 2020

From: Walter Van Assche (Walter.VanAssche@kuleuven.be) and
Tom Koornwinder (T.H.Koornwinder@uva.nl)
Subject: Obituary for **Ahmed Fitouhi** (1948–2020)

In Memoriam: **Ahmed Fitouhi**, 1948–2020.

We have learned that Prof. Emer. A. Fitouhi passed away on January 18, 2020 at the age of 72.

Picture taken from the invitation to the memorial meeting of February 26, 2020 (40 days after his death).

Ahmed Fitouhi was one of the main figures of the mathematical community in Tunisia and one of the founders of the Société Mathématiques de Tunisie. He worked during his whole career at the Faculté des Sciences de Tunis. The 12th OPSFA conference in Sousse (March 25–29, 2013) was dedicated to him and a biographical note (written by Lotfi Khériji) appeared in the abstract book. We will borrow some information about him from this note.

Ahmed Fitouhi was born on January 29, 1948 in Gafour (northern Tunisia). He received his degree in Mathematics in 1974 from the Faculty of Sciences of Tunis and started his professional career in the Department of Mathematics as Assistant Professor. He obtained a Doctorate (troisième cycle) in Mathematics in 1977 and the Doctorat d'État in 1988 with a thesis titled *Fonctions propres, équations de la chaleur et des ondes associées à un opérateur différentiel singulier sur* $(0,\infty)$ [Eigenfunctions, heat equations and waves associated to a singular differential operator on $(0,\infty)$]. In the same year (1988) he became Full Pro-

fessor. Since then he has supervised a large number of PhD theses and habilitations.

The work in Fitouhi's Thèse d'État—and in subsequent publications until 2000, including his most cited paper, *Heat "polynomials" for a singular differential operator on* $(0, \infty)$ in *Constructive Approximation* (1989)—fit into the Tunis school of harmonic analysis associated with singular differential operators which perturb the Jacobi or Bessel differential operator. After 2000, he initiated a program on harmonic analysis, special functions and q–analogues. It generated many publications, starting with an influential paper, *The* q*-*j_α *Bessel Function* in the *Journal of Approximation Theory* (2002), joint with M. M. Hamza and F. Bouzeffour, on the q–function which is now called Jackson's third q–Bessel function. For many aspects of classical harmonic analysis involving special functions, Fitouhi's team has developed q–analogues. MathSciNet lists 61 research papers authored or coauthored by Fitouhi. He had 34 coauthors. His last paper, *On some* q*-versions of the Ramanujan master theorem*, joint with K. Brahim and N. Bettaibi, appeared in *The Ramanujan Journal* (2019).

Topic #5 ——— OP – SF Net 27.2 ——— March 15, 2020

From: Howard Cohl (Howard.Cohl@nist.gov)
Subject: Eight remembrances and communications to **Richard A. Askey**

Eight remembrances and communications to Richard Allen Askey (June 4, 1933—October 9, 2019)

by Alladi, Andrews, Bendt, Ismail, Koornwinder, Nevai, Roy and Stanton

An obituary of Richard A. Askey appeared in OP–SF Net 26.6, published on November 15, 2019. Below are eight remembrances of Dick from some of his colleagues, students, and friends:

Krishnaswami Alladi, George Andrews, Bruce Berndt, Mourad E. H. Ismail,
Tom H. Koornwinder, Paul Nevai, Ranjan Roy and Dennis Stanton.

The following collection of eight individual contributions regarding Dick represent part I of a multi–part series selected from the Askey Liber Amicorum, a Friendship Book for Dick Askey. The Askey Liber Amicorum was described in OP–SF Net 27.1, published on January 15, 2020.

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Krishnaswami Alladi, University of Florida, Gainesville, Florida, USA.

My association with Richard Askey, world authority on special functions and Ramanujan's work

Richard Askey is a world leader in the field of special functions. In an illustrious career spanning six decades, he has greatly influenced the development of that subject by means of his own fundamental research and the work of numerous mathematicians he has groomed. He has also been instrumental, along with George Andrews and Bruce Berndt, in making the mathematical world aware of the wide ranging and deep contributions of Ramanujan. Although I have not collaborated with Professor Askey, I have had close interaction with him for the past four decades. Indeed, my family and I have had the pleasure and privilege of hosting him and his wife both in Madras, India, and in Gainesville, Florida, several times.

I first met him at the Summer Meeting of the AMS at the University of Michigan, Ann Arbor, in 1980. He was giving an hour lecture on the Selberg integral which I attended. I was charmed by his conversational, yet engaging, lecturing style. A vast panorama of the area of special functions unfolded in his lecture, revealing his encyclopedic knowledge of the subject. Of course, he made connections with Ramanujan's startling discoveries, and extorted everyone in the audience to study the work of the Indian genius. I was working in analytic number theory at that time, but before the end of that decade, owing to the lectures of Andrews, Askey and Berndt that I heard at the Ramanujan Centennial in India in 1987, I entered the world of q, and as Askey would say, I was smitten by the q-disease!

The Ramanujan Centennial was an occasion when mathematicians around the world gathered in India to pay homage to the Indian genius, and take stock of the influence his work has had and the impact it might have in the future. Askey was one of the stars of the centennial celebrations. I organized a one day program during a conference at Anna University, Madras, in December 1987, and he graciously accepted our invitation to inaugurate that conference. He delivered a magnificent lecture on "Beta integrals before and after Ramanujan" in my session. We were also honored to have him give a public lecture at our family home in Madras under the auspices of the Alladi Foundation that my father, the late Prof. Alladi Ramakrishnan, had created in memory of my grandfather Sir Alladi Krishnaswami Iyer.

With my research being focused in the theory of partitions and q series from 1990, we have had a series of conferences at the University of Florida emphasizing this area. Professor Askey has visited Gainesville several times both as a lead speaker at these conferences, as well as for History Lectures and talks on mathematics education during the regular academic year. I have enjoyed every one of his lectures in Gainesville and elsewhere at major meetings. I want to share with you one interesting episode.

In 1995, there was a two week meeting on special functions, q-series, and related topics, at the Fields Institute in Toronto. The first week was an instructional workshop, and the second week was a research conference. I attended the second week. The great I. M. Gel'fand was scheduled to be the Opening Speaker for the research conference. I was looking forward to Gel'fand's lecture since I had heard so much about the Gel'fand Seminar he had conducted in Moscow, and how he would dominate the seminar and cut people down to size. It turned out that Gel'fand could not come to Toronto due to ill health (he was 82 years old). So Askey got up and said that he was the one who had invited Gel'fand, and if the person you had invited is unable to come, then you should give a talk in his place. So Askey gave a masterly lecture on special functions in his imitable style that I thoroughly enjoyed.

I also want to share an episode regarding Askey in the audience for one of my lectures. This was in June 1993 and I was speaking in the Paris Number Theory Seminar at the Institute Henri Poincare on the theme "The combinatorics of words with applications to partitions". Just as I started my lecture, Askey walked in. He keenly followed the lecture, and at the end when my host Michel Waldschmidt asked if anyone had comments or questions, Askey's hand went up. He said: "You gave a very nice talk, but having said that, let me tear you apart." He then proceeded to point out where all I had erred, or could have been more accurate, with regard to historical statements as well as statements regarding the depth of various q-series identities. I had started research in the theory of partitions and q-series only in 1989, and so this was among my early talks in the field. And yes, he was right in all the comments he made.

His insight and critical comments have been immensely useful to me in various ways. Starting from the Ramanujan Centennial, I wrote articles annually for Ramanujan's birthday for *The Hindu*, India's National Newspaper, comparing Ramanujan's work with that of various mathematical luminaries in history. I benefited from Askey's comments and (constructive) criticism in preparing these articles. A collection of these articles appeared in a book that I published with Springer in

2013 for Ramanujan's 125th birthday [1].

By the time the Ramanujan 125 celebrations came around in 2012, I was firmly entrenched in the Ramanujan World, and so was involved with the celebrations in various ways. In particular, owing to my strong association with SASTRA University, I organized a conference at their campus in Kumbakonam, Ramanujan's hometown. We felt that Askey, Andrews, and Berndt, had to be recognized in a special way in Ramanujan's hometown for all they had done to help us understand the plethora of identities Ramanujan had discovered. So *The Trinity* (Askey, Andrews, and Berndt) were awarded Honorary Doctorates by SASTRA University in a colorful ceremony at the start of which they entered the auditorium with traditional South Indian Carnatic music being played on the *Nadaswaram*, a powerful wind instrument. Askey enjoyed the ceremony but felt that the music was too loud; but that is how the Nadaswaram is, since it is played in festivals attended by a thousand people or more!

There is much that can be said of Dick Askey. But I will conclude by emphasizing, that in spite of his eminence, he is a very friendly and helpful person. It is rare to find eminence combined with humanity, and Askey has this precious combination which has been beneficial to so many of us.

Krishna Alladi
Department of Mathematics
University of Florida
Gainesville, FL 32611 USA
email: alladik@ufl.edu
(Article written in Sept 2019)

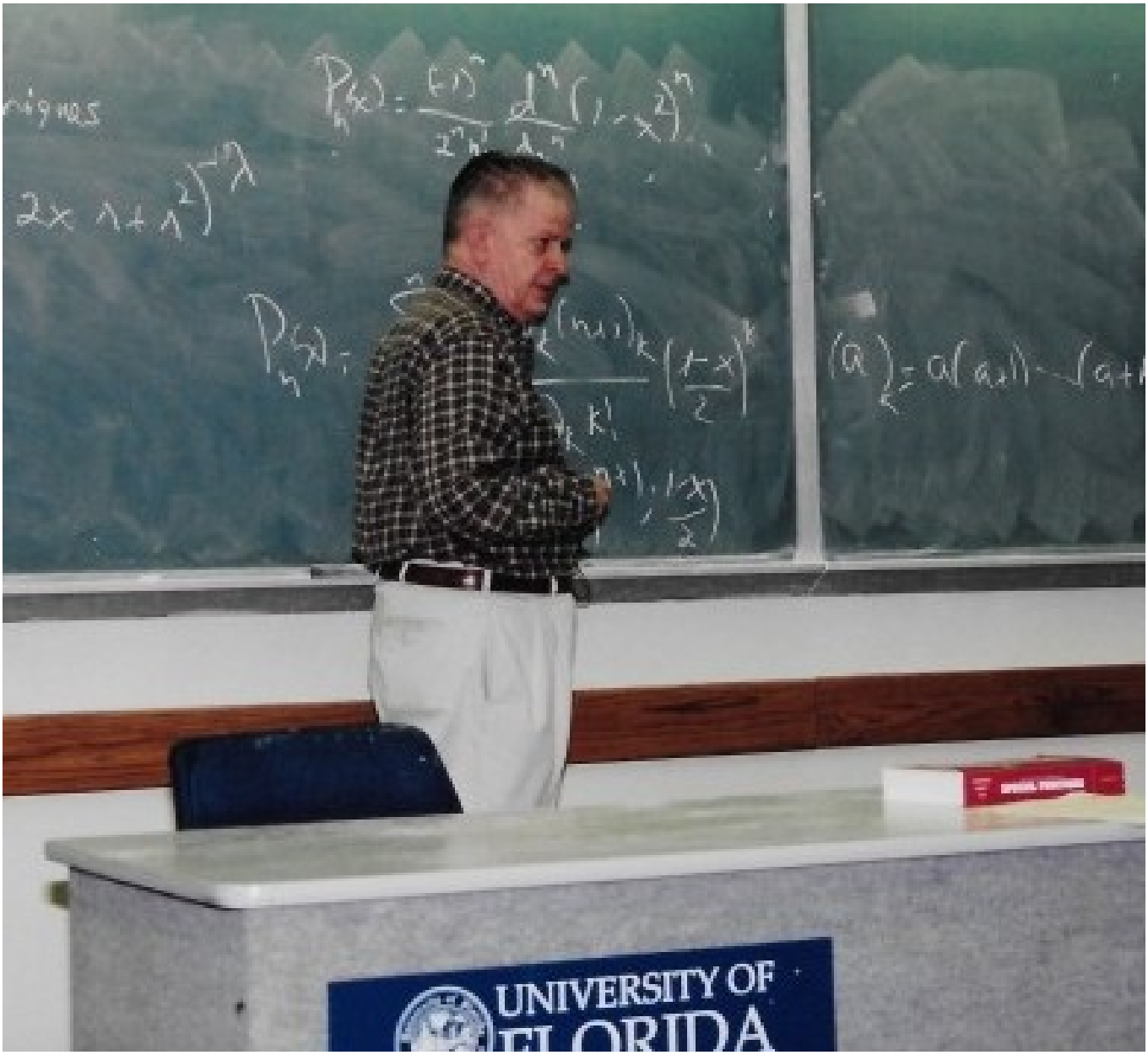
References

[1] K. Alladi. *Ramanujan's place in the world of mathematics*. Springer, New Delhi, 2013. Essays providing a comparative study, With a foreword by George E. Andrews.

Richard Askey giving a talk on Ramanujan at the Alladi's family home in Madras, India, on December 19, 1987, during the Ramanujan Centennial. The photograph behind Askey is that of Krishna's grandfather Sir Alladi Krishnaswami Iyer.

Askey responding to a question after his talk on Ramanujan at the Alladi family home in Madras, India, in December 1987. Next to Dick is Krishna's father, the late Prof. Alladi Ramakrishnan.

Askey and his wife with some delegates for the Ramanujan Centennial at the Alladi family home in Madras, India, in December 1987. From L to R: David Bressoud, Bruce Berndt, Mrs. Askey, Alladi Ramakrishnan, George Andrews, Krishna Alladi, Dick Askey, Basil Gordon, and M. V. Subba Rao.



Dick Askey giving a featured History Lecture at the University of Florida, Gainesville, Florida, on March 21, 2005.

Dick Askey in conversation with Mathura Alladi (Krishna's wife), Lalitha Ramakrishnan (Krishna's mother), and Alladi Ramakrishnan (Krishna's father), during a party in honor of Dick at the Alladi home in Gainesville, Florida, March 21, 2005.

Dick Askey at the Alladi home in Gainesville, Florida, March 21, 2005.
From L to R: Krishna Alladi, John Thompson (group theorist, Fields medalist),
George Andrews, Dick Askey, and Alladi Ramakrishnan (Krishna's father).

Richard Askey receiving an Honorary Doctorate from SASTRA University in Kumbakonam (Ramanujan's hometown), during the Ramanujan 125 celebrations, December 15, 2012.

Richard Askey speaking after receiving an Honorary Doctorate from SASTRA University in Kumbakonam (Ramanujan's hometown), during the Ramanujan 125 celebrations, December 15, 2012.

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George Andrews, Pennsylvania State University, State College, Pennsylvania, USA.

We are all grateful beyond words to have shared part of our professional lives with you, a grand mathematician and a grand man. This is especially true for me.

I think with fondness on the numerous discussions and collaborations we have had over the years. Also you have greatly inspired everyone around you with the scope and grandeur of your vision. The fact that special functions have come into such prominence owes much to your sustained efforts to promote every aspect of the subject. Decades ago you were pushing hard for a new and much grander version of the Bateman Project. Your leadership inspired many others to get involved and eventually this produced the Digital Library of Mathematical Functions.

I have many times reflected on a conversation we had many years ago in which you referred to someone as "...a very broadminded mathematician: he doesn't care what kind of eigenvalue problem you are working on." You have always epitomized the opposite of that person. You are smart, generous and dedicated to the advancement of mathematics and mathematics education at all levels.

Each of us is saddened by the great difficulties facing you. We love you and are deeply in your debt, especially me.

front row, l to r: Steve Milne, David Bressoud, Mourad Ismail and George Andrews. Second row: Dennis Stanton is behind Mourad.

Dick Askey and George Andrews sightseeing in Tianjin, China in 2004.

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Bruce Berndt, University of Illinois at Urbana-Champaign, Urbana, Illinois, USA.

When I was a graduate student at the University of Wisconsin, it took me some time to find and begin my journey into analytic number theory. The first courses in number theory that I ever took were both in modular forms, during the spring semester of my third year and the fall semester of my fourth year, with the former taught by Rod Smart and the latter taught by Marvin Knopp. With this background, my research led me to special functions. But, unfortunately, it was too late at the end of my graduate career to have the opportunity to know Richard Askey very well and to take a course in special functions taught by him. It is one of the biggest regrets in my life that I did not take such a course from Dick.

My doctoral thesis featured Bessel functions with a lot of classical complex analysis. Shortly after graduation, Askey approached me with sagacious advice; I will never forget my surprise! I had no idea that he knew anything about me and my thesis! This meeting began his life-long support of my work.

In the approximately 45 years that I have devoted my attention to Ramanujan's (earlier) notebooks [1] and his lost notebook [2], Dick, in innumerable ways, increased my understanding and offered me insights that only he could supply. In each of the Introductions of my five books on Ramanujan's notebooks [3, 4, 5, 6, 7], I express my gratitude to Dick for his careful reading of several chapters, and for his many comments and suggestions. Although I have not personally counted references to each mathematician in my five books, except for Ramanujan, I likely mentioned Askey's name more than any other mathematician.

In recent years, others have referred to me as a member of the "gang of three" for our work on Ramanujan. I could have no greater honor than to be associated in this way with Dick Askey and George Andrews in our quest to understand the myriad of beautiful ideas given to us by one of the greatest mathematicians in the history of our subject.

References

[1] S. Ramanujan. *Notebooks. Vols. 1, 2.* Tata Institute of Fundamental Research, Bombay, 1957.

[2] S. Ramanujan. *The lost notebook and other unpublished papers.* Springer-Verlag, Berlin; Narosa Publishing House, New Delhi, 1988. With an introduction by George E. Andrews.

[3] B. C. Berndt. *Ramanujan's notebooks. Part I.* Springer-Verlag, New York, 1985. With a foreword by S. Chandrasekhar.

[4] B. C. Berndt. *Ramanujan's notebooks. Part II.* Springer-Verlag, New York, 1989.

[5] B. C. Berndt. *Ramanujan's notebooks. Part III.* Springer-Verlag, New York, 1991.

[6] B. C. Berndt. *Ramanujan's notebooks. Part IV.* Springer-Verlag, New York, 1994.

[7] B. C. Berndt. *Ramanujan's notebooks. Part V.* Springer-Verlag, New York, 1998.

Bruce Berndt and Dick Askey at the *Askey 80th birthday Conference*, Madison, Wisconsin, USA in December 2013. The picture was taken by Patsy Wang-Iverson.

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Mourad E. H. Ismail, University of Central Florida, Orlando, Florida, USA.

To Dick Askey in Friendship
Mourad E. H. Ismail

It is difficult for me to write these lines knowing that Dick is not well and is in a hospice. Since 1974 when I started working with Dick, he has been the person I turn to for help, advise, or inspiration.

I first met him in 1972 when he visited Edmonton and I was a doctoral student working with Waleed Al-Salam. At the time, Waleed asked me to look at a characterization theorem that Dick suggested and it came out of a paper by Al-Salam and Chihara [1]. I could not solve the problem but I worked with Jerry Fields on another problem Dick suggested and we solved it. This resulted in my first published paper and was really a good exposure to Askey's kind of mathematics and I greatly enjoyed this work.

Both Thanaa and I attended the highly influential 1974 Conference Board of the Mathematical Sciences (CBMS) lectures at Virginia Polytechnic Institute with Dick as the principal lecturer (published in [6]). In these lectures, Dick outlined many directions for research in the field of special functions and orthogonal polynomials, and the event was an eye opener for me.

Dick kindly supported my application to spend the academic year 1974-75 at the University of Wisconsin as an assistant scientist with no teaching duties. This was an amazing year. Dick introduced me to the fascinating area of combinatorics of orthogonal polynomials. He also completely changed my outlook about what is important in Mathematics and how to do research. Through Dick, I was introduced to so many well-known mathematicians and I always appreciated this. People may not know that Dick is the reason I fell in love with Florida and its beaches. When we were working on our first combinatorics paper he suggested that I go to the combinatorics conference in Boca Raton, give a talk and ask people for references and help. The conference was 3 days long and I went to the lectures only for two days and swam in the ocean on the third day. This was in late February when Madison has a foot of snow on the ground. I did get many references on enumeration and then we did write a nice paper which appeared in the Canadian Journal of Mathematics in 1976 [5]. After we returned to Canada, I continued to go to the Boca Raton meetings and eventually we moved to Florida.

Another very memorable event happened in the summer of 1977. At the time, I was an assistant professor at McMaster University and wanted to get my brain recharged by being around Dick. I spent the whole summer in Madison. At this time, Dennis Stanton was very close to graduation, Jim Wilson had another year to go, and Dan Moak has just started. Paul Nevai was visiting the University of Wisconsin. Dick ran a seminar that met 4 times a week. Some short term visitors also attended the seminar. I remember Donald Newman and possibly others. We also had several guest speakers. George Andrews, George Gasper, and Willard Miller, Jr., also visited. There may have been others that I forgot. Many papers resulted from this seminar. Paul Nevai and I presented the Pollaczek memoir [4] and I started working with Dick on what evolved into our 1984 AMS memoir [3]. We also started the continuous q-ultraspherical polynomials paper [2]. I have never slept as little as I did during that summer.

Over the years, I continued my collaboration with Dick and I learned a lot of mathematics just from talking to him. Dick also wrote numerous letters of recommendation for me when I was applying for jobs. He also evaluated my work for promotion and tenure. Dick was always very generous with his time and ideas. He solved many important problems in special functions and orthogonal polynomials and posed many more. He attracted many good young people to the subject. He also helped many of us in our early careers to publish our papers and advance in our careers.

When the Askey–Wilson polynomials were found, Dick called them the q–Wilson polynomials. Based on many hints from Dick, Wilson identified the Wilson polynomials. I would like to claim credit for disagreeing with Dick about naming the polynomials "q–Wilson polynomials" and when I discussed the matter with Mizan Rahman and Dennis Stanton, they agreed with me. So the three of us engineered a mutiny and insisted on using "The Askey–Wilson polynomials". Eventually Dick relented. Once, I was discussing certain issues with Persi Diaconis. In the middle of the discussion he said that a hundred years from now the Askey–Wilson polynomials will be still around.

Thank you Dick for all you did for me personally and for all the mathematicians of my generation. Most importantly, thank you for making me and Thanaa part of your family.

Mourad

References

[1] W. A. Al-Salam and T. S. Chihara. Convolutions of orthonormal polynomials. *SIAM Journal on Mathematical Analysis*, 7(1):16–28, 1976.

[2] R. Askey and M. E. H. Ismail. A generalization of ultraspherical polynomials. In *Studies in pure mathematics*, pages 55–78. Birkhäuser, Basel, 1983.

[3] R. Askey and M. Ismail. Recurrence relations, continued fractions, and orthogonal polynomials. *Memoirs of the American Mathematical Society*, 49(300):iv+108, 1984.

[4] F. Pollaczek. *Sur une généralisation des polynomes de Jacobi*. Mémorial des Sciences Mathématiques, Fascicule 131. Gauthier-Villars, Paris, 1956.

[5] R. Askey and M. E. H. Ismail. Permutation problems and special functions. *Canadian Journal of Mathematics. Journal Canadien de Mathématiques*, 28(4):853–874, 1976.

[6] R. Askey. *Orthogonal polynomials and special functions*. Society for Industrial and Applied Mathematics, Philadelphia, Pa., 1975.

Dick Askey and Mourad Ismail at the *Askey 80th Birthday Conference*, Madison, Wisconsin, USA in December 2013. Photo taken by Patsy Wang-Iverson.

Dennis Stanton, Dick Askey and Mourad Ismail
at Madison, Wisconsin, USA in September 2019.

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Tom H. Koornwinder, University of Amsterdam, Amsterdam, The Netherlands.

Formulas for one-variable special functions as germs for multi-variable analogues
Tom H. Koornwinder

Subsection 1: Introduction

Dick Askey spent the academic year 1969–1970 at the *Mathematisch Centrum* in Amsterdam, where I had started working as a PhD student a little earlier. See the slides [9] of my lecture at Askey's 80th and Nico Temme's contribution to the present Liber Amicorum for an account of that year and the impact of Dick's stay. For me it meant that I specialized in orthogonal polynomials and special functions (OPSF) with group theoretic interpretation, and that I spent the next academic year at the Mittag-Leffler Institute in Djursholm, Sweden, where the whole year was devoted to noncommutative harmonic analysis. Moreover I took Dick's open problem about the addition formula for Jacobi polynomials with me to Sweden, and I solved it there.

In the fifty years since I met Dick first I heard many lectures by him. Quite often he mentioned addition formulas, maybe only in the simple case of Legendre polynomials $P_n(x)$, and he always added that there should be a dual addition formula, which leaves x fixed rather than n, and manipulates n rather than x. I always found this a strange idea which I did not take very seriously, until, in 2016, I suddenly got a brain wave when looking at a formula in a preprint by Hallnäs and Ruijsenaars and could produce the dual addition formula for ultraspherical polynomials [10]. Dick was happy when I wrote this to him.

Dick always saw the importance of extending OPSF to several variables, although he worked himself usually only in one variable. He meant of course extensions which have sufficient depth, for instance by a Lie theoretic connection, or because the more variable functions admit analogues of the famous evaluation and transformation formulas for ${}_rF_s$ and ${}_r\phi_s$ (q-)hypergeometric functions. Usually, the formulas in several variables are much more complicated than their one-variable analogues. Still a good knowledge of the one-variable case can be helpful to predict certain phenomena in the multi-variable case. I think I learnt this approach from Dick. This was for instance successful when I could obtain in 1987 the Pieri formula for (A-type) Macdonald polynomials by being aware of the duality for continuous q-ultraspherical polynomials which is obvious from one of its q-hypergeometric representations.

In this note I will present one-variable cases of A-type and BC-type interpolation polynomials, and I will discuss some perspectives for the several variable cases in their interaction with the one-variable cases.

Acknowledgment: I thank Michael Schlosser and Ole Warnaar for helpful remarks.

Subsection 2: Interpolation polynomials

(q-)Pochhammer symbols

Let $0 < |q| < 1$, $n \in \mathbb{Z}_{\geq 0}$,

$$\begin{aligned}(a)_n &:= a(a+1)\dots(a+n-1),\\ (a;q)_n &:= (1-a)(1-qa)\dots(1-q^{n-1}a),\\ (a_1,\dots,a_k;q)_n &:= (a_1;q)_n\dots(a_k;q)_n.\end{aligned}$$

These omnipresent building blocks of one-variable special functions are known to everybody. Less known or less used is that they can be characterized by an interpolation property:

1. $z(z-1)\dots(z-n+1) = (-1)^n(-z)_n$
   is the unique monic polynomial of degree n which vanishes at $z = 0, 1, \dots, n-1$.
2. $z(z-q)\dots(z-q^{n-1}) = z^n(z^{-1};q)_n$
   is the unique monic polynomial of degree n which vanishes at $z = 1, q, \dots, q^{n-1}$.
3. $\prod_{j=0}^{n-1}(z+z^{-1}-aq^j-a^{-1}q^{-j}) = \dfrac{(az,az^{-1};q)_n}{(-1)^n\,q^{\frac{1}{2}n(n-1)}\,a^n}$
   is the unique monic symmetric Laurent polynomial of degree n which vanishes at $z = a, aq, \dots, aq^{n-1}$ (and their inverses).

Subsection 3: Generalized binomial formulas

You may be surprised to see the following formula being called a binomial formula:

$$\begin{aligned}R_n(z;a,b,c,d\,|\,q) := \frac{p_n(\frac{1}{2}(z+z^{-1});a,b,c,d\,|\,q)}{p_n(\frac{1}{2}(a+a^{-1});a,b,c,d\,|\,q)} &= {}_4\phi_3\left(\begin{matrix}q^{-n},q^{n-1}abcd,az,az^{-1}\\ ab,ac,ad\end{matrix};q,q\right)\\ &= \sum_{k=0}^{n}\frac{q^k}{(ab,ac,ad,q;q)_k}\,(q^{-n},q^{n-1}abcd;q)_k\,(az,az^{-1};q)_k. \qquad (1)\end{aligned}$$

It is the well-known q-hypergeometric representation of suitably normalized Askey-Wilson polynomials. However, with z replaced by az and $a \to \infty$ it yields a q-binomial formula

$$\begin{aligned}z^n = {}_2\phi_0\left(\begin{matrix}q^{-n},z^{-1}\\ -\end{matrix};q,q^nz\right) &= \sum_{k=0}^{n}\frac{(-1)^kq^{-\frac{1}{2}k(k-1)}}{(q;q)_k}\,q^{nk}(q^{-n};q)_k\,z^k(z^{-1};q)_k \qquad (2)\\ &= \sum_{k=0}^{n}\frac{(q;q)_n}{(q;q)_k(q;q)_{n-k}}\,(z-1)(z-q)\dots(z-q^{k-1}).\end{aligned}$$

The first equality sign in (2) follows from a confluent case of the q-Chu-Vandermonde formula [4, (II.7)]. It is less evident to see in a straightforward way that $R_n(az;a,b,c,d\,|\,q) \to z^n$ as $a \to \infty$. However, by the symmetry of the Askey-Wilson polynomials in a,b,c,d we have

$$R_n(az;a,b,c,d\,|\,q) = \frac{a^n(bc,bd;q)_n}{b^n(ac,ad;q)_n}\;{}_4\phi_3\left(\begin{matrix}q^{-n},abcdq^{n-1},abz,a^{-1}bz^{-1}\\ ab,bc,bd\end{matrix};q,q\right),$$

from which the limit is clear. From (2) we obtain in the limit for $q \to 1$ the classical binomial formula

$$z^n = {}_1F_0\left(\begin{matrix}-n\\ -\end{matrix};z-1\right) = \sum_{k=0}^{n}\binom{n}{k}(z-1)^k. \qquad (3)$$

Formulas (3), (2) and (1) have d-variable analogues which are called binomial formulas for Jack polynomials [6], (A-type) Macdonald polynomials [7] and Koornwinder polynomials [8], respectively. Formulas (1) and (2) involve an expansion of the left-hand side in terms of interpolation polynomials in z, respectively given by item 3 and item 2 above. Moreover, they are as well expansions in terms of interpolation polynomials depending on q^n. This is clear for (2), while in (1) we can rewrite

$$(q^{-n}, q^{n-1}abcd; q)_k = \big(\tilde{a}\,(\tilde{a}q^n), \tilde{a}\,(\tilde{a}q^n)^{-1}; q\big)_k, \quad \text{where} \quad \tilde{a} := (q^{-1}abcd)^{\frac{1}{2}}.$$

This also implies dualities. In (2) put $z = q^m$. Then

$$q^{mn} = {}_2\phi_0\left(\begin{matrix} q^{-n}, q^{-m} \\ - \end{matrix}; q, q^{m+n}\right) = \sum_{k=0}^{\min(m,n)} \frac{(-1)^k q^{-\frac{1}{2}k(k-1)}}{(q;q)_k}\, q^{nk}(q^{-n};q)_k\, q^{mk}(q^{-m};q)_k$$

with evident $m \leftrightarrow n$ symmetry in all parts.

For (1) introduce dual parameters $\tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}$:

$$\tilde{a} := (q^{-1}abcd)^{\frac{1}{2}}, \quad \tilde{a}\,\tilde{b} = ab, \quad \tilde{a}\,\tilde{c} = ac, \quad \tilde{a}\,\tilde{d} = ad.$$

Then we have the duality relation

$$R_n(a^{-1}q^{-m}; a, b, c, d \,|\, q) = R_m(\tilde{a}^{-1}q^{-n}; \tilde{a}, \tilde{b}, \tilde{c}, \tilde{d} \,|\, q) \quad (m, n \in \mathbb{Z}_{\geq 0}),$$

since both sides are equal to

$$\sum_{k=0}^{\min(m,n)} \frac{q^k}{(ab, ac, ad, q; q)_k}\, (q^{-n}, \tilde{a}^2 q^n; q)_k\, (q^{-m}, a^2 q^m; q)_k.$$

All this generalizes to the d-variable case of Macdonald polynomials [7] and Koornwinder polynomials [8]. The d-variable analogue [8] of (1) is, in a sense, an explicit expression for the Koornwinder polynomials.

Subsection 4: Further one-variable binomial formulas

Above the q-binomial formula (2) lies a version of the q-Chu-Vandermonde formula [4, (II.7)]

$$\frac{(az;q)_n}{(a;q)_n} = {}_2\phi_1\left(\begin{matrix} q^{-n}, z^{-1} \\ a \end{matrix}; q^n az\right) = \sum_{k=0}^{n} \frac{a^k}{(a,q;q)_k}\, q^{nk}(q^{-n};q)_k\, z^k(z^{-1};q)_k. \tag{4}$$

Indeed, (4) yields (2) in the limit for $a \to \infty$. Formula (4) can be considered as a binomial formula for the interpolation polynomials in item 2 in Subsection 2. Both in the variable z and the variable q^n it expands in terms of interpolation polynomials. For $z = q^m$ we have again the $m \leftrightarrow n$ symmetry in all parts of the formula.

Above (4) lies a version of the q-Saalschütz sum [4, (II.12)]

$$\frac{(ac^{-1}z, ac^{-1}z^{-1}; q)_n}{(a, ac^{-2}; q)_n} = {}_3\phi_2\left(\begin{matrix} q^{-n}, cz, cz^{-1} \\ a, c^2a^{-1}q^{1-n} \end{matrix}; q, q\right) = \sum_{k=0}^{n} \frac{(q^{-n};q)_k q^k}{(a, c^2a^{-1}q^{1-n}, q; q)_k}\, (cz, cz^{-1}; q)_k. \tag{5}$$

Indeed, in (5) replace z by cz and let $c \to \infty$. Then we obtain (4). Formula (5) can be considered as a connection formula between interpolation polynomials for two different parameters in item 3 in Subsection 2. It is tempting to call (5) a binomial formula for these

interpolation polynomials.

Quite surprising for me (not having known this before I wrote this note), formula (5), in its turn, is a limit case of (1). In order to see this we combine (1) with the symmetry of the Askey–Wilson polynomial p_n in the parameters and use the evaluation

$$p_n(\tfrac{1}{2}(a+a^{-1}); a,b,c,d\,|\,q) = a^{-n}(ab,ac,ad;q)_n$$

to obtain

$$\begin{aligned}
&p_n(\tfrac{1}{2}(z+z^{-1}); a,b,c,d\,|\,q)\\
&= a^{-n}\sum_{k=0}^{n}\frac{q^k}{(q;q)_k}\,(abq^k,acq^k,adq^k;q)_{n-k}\,(q^{-n},q^{n-1}abcd;q)_k\,(az,az^{-1};q)_k\\
&= b^{-n}\sum_{k=0}^{n}\frac{q^k}{(q;q)_k}\,(abq^k,bcq^k,bdq^k;q)_{n-k}\,(q^{-n},q^{n-1}abcd;q)_k\,(bz,bz^{-1};q)_k.
\end{aligned}$$

Now put $c = a^{-1}q^{-n+1}$. Then

$$\begin{aligned}
&p_n(\tfrac{1}{2}(z+z^{-1}); a,b,a^{-1}q^{-n+1},d\,|\,q) = \frac{q^n(q^{-n},bd;q)_n}{a^n(q;q)_n}\,(az,az^{-1};q)_n\\
&= b^{-n}(bd;q)_n\sum_{k=0}^{n}\frac{q^k}{(q;q)_k}\,(abq^k,a^{-1}bq^{-n+k+1};q)_{n-k}\,(q^{-n},q^{n-1}abcd;q)_k\,(bz,bz^{-1};q)_k.
\end{aligned}$$

Replace a,b by ac^{-1},c. After a few manipulations we recover (5). In particular, d is no longer present in the identity. We see also that the interpolation polynomials in item 3 of Subsection 2 are specializations of Askey–Wilson polynomials. I got the idea from a similar specialization to interpolation functions of Spiridonov's multivariable biorthogonal elliptic functions, a formula due to Rains [2] and given more explicitly in [3, (1.4.21)] (there replace μ by λ on the right).

So we have a chain of limits of functions

$$p_n(z;a,b,c,d\,|\,q)\ \to\ (az,az^{-1};q)_n\ \to\ (z;q)_n\ \to\ z^n,$$

where the second and the third are interpolation functions. We have a corresponding chain of limits of "binomial formulas" (1) $\to$ (5) $\to$ (4) $\to$ (2).

Subsection 5: A few comments on analogues in several variables

I conclude this note with a few somewhat vague thoughts (to be made more concrete if a little more time would be given). For these I climb up to the realm of elliptic hypergeometric series, a nowadays very active area which has also been enthusiastically welcomed by Dick.

1. I already mentioned the specialization of Askey–Wilson polynomials to the interpolation polynomials $(az,az^{-1};q)_n$, which is given on a much higher level in [3, (1.4.21)]. Certainly, a suitable limit for $p\to 0$ of this last formula will give a specialization of Koornwinder polynomials to Okounkov's interpolation polynomials [8]. It is quite remarkable that the A-type interpolation polynomials (see for instance [7]) generalize the A-type Macdonald polynomials, while the converse is true in the BC-case.

2. Note the elliptic analogue [4, (11.4.11)] of the q-Saalschütz sum. Just as for (5) this can be rewritten as a connection formula for elliptic interpolation functions, and such

that it has (5) as a limit case for $p \to 0$. See Schlosser [5, Example 4.3]. There the elliptic interpolation functions are

$$\frac{(bz, bz^{-1}; q, p)_n}{(cz, cz^{-1}; q, p)_n},$$

in terms of elliptic analogues of Pochhammer symbols. In fact, [4, (11.4.11)] is a rewritten form of the Frenkel–Turaev sum ${}_{10}V_9$ sum [4, (11.4.1)]. A limit on a higher level for $p \to 0$ of this sum is to Jackson's ${}_8\phi_7$ sum [4, (2.6.2)]. Accordingly the elliptic interpolation functions tend to the rational interpolation functions occurring in the expansion of Rahman's biorthogonal rational functions.

3. Rosengren & Warnaar [3, (1.4.19)] give a connection formula for d–variable elliptic interpolation polynomials, which is due to Rains [2, Corollary 4.14]. Limits for $p \to 0$ of this formula, finally leading to (5) in the one–variable case, can certainly be given by using [2, Section 8] and [1].

Dick, my best wishes for you. If you are in the mood, look at the math in this note. If you are not in the mood, then just delight in knowing that you have stimulated so many young, and sometimes older, people in their mathematical development.

References

[1] E. M. Rains. BC_n–symmetric polynomials. *Transformation Groups*, 10(1):63–132, 2005.

[2] E. M. Rains. BC_n–symmetric Abelian functions. *Duke Mathematical Journal*, 135(1):99–180, 2006.

[3] H. Rosengren and S. O. Warnaar. Elliptic hypergeometric functions associated with root systems. arXiv:1704.08406v3, 2019.

[4] G. Gasper and M. Rahman. *Basic hypergeometric series*, volume 96 of *Encyclopedia of Mathematics and its Applications*. Cambridge University Press, Cambridge, second edition, 2004. With a foreword by Richard Askey.

[5] M. J. Schlosser. A Taylor expansion theorem for an elliptic extension of the Askey–Wilson operator. In *Special functions and orthogonal polynomials*, volume 471 of *Contemp. Math.*, pages 175–186. Amer. Math. Soc., Providence, RI, 2008.

[6] A. Okounkov and G. Olshanski. Shifted Jack polynomials, binomial formula, and applications. *Mathematical Research Letters*, 4(1):69–78, 1997.

[7] A. Okounkov. Binomial formula for Macdonald polynomials and applications. *Mathematical Research Letters*, 4(4):533–553, 1997.

[8] A. Okounkov. BC–type interpolation Macdonald polynomials and binomial formula for Koornwinder polynomials. *Transformation Groups*, 3(2):181–207, 1998.

[9] T. H. Koornwinder. *Dick Askey's positive addition to Amsterdam*, slides of a lecture at the conference *Dick Askey's 80th*, Madison, Wisconsin, 6–7 December 2013. https://staff.fnwi.uva.nl/t.h.koornwinder/art/sheets/askey80.pdf, 2013.

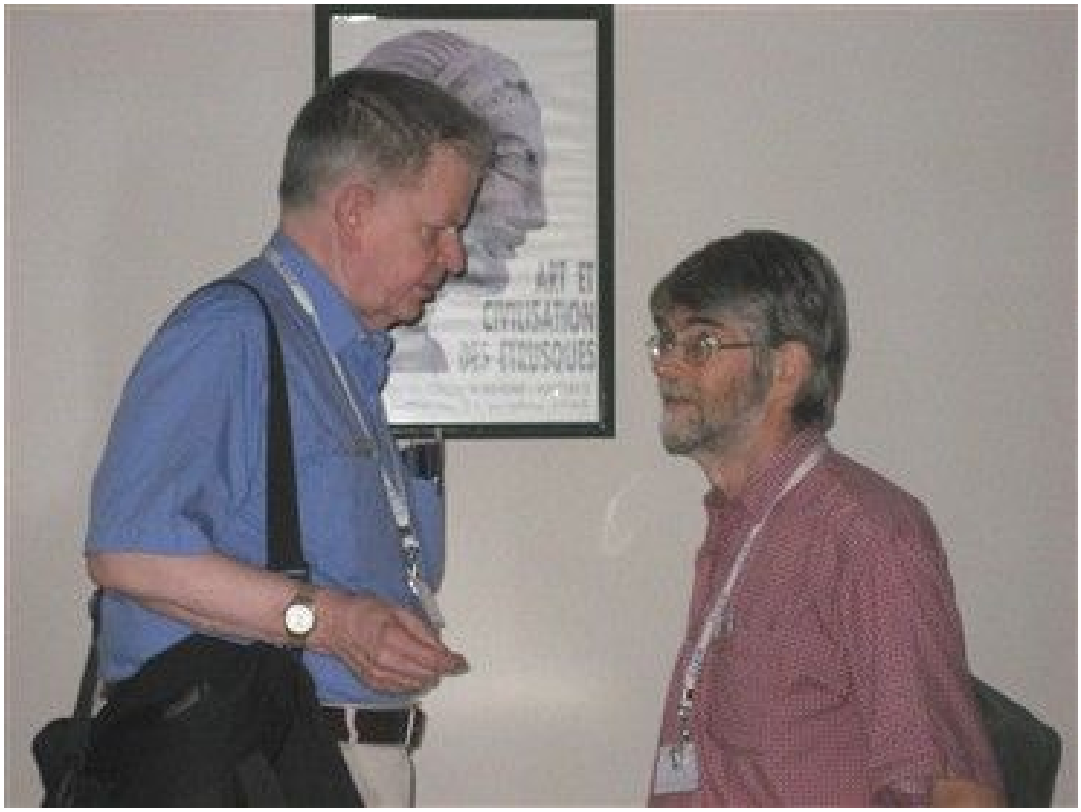
[10] T. H. Koornwinder. Dual addition formulas associated with dual product formulas. In *Frontiers in Orthogonal Polynomials and q-Series*, chapter 19, pages 373–392. World Scientific Publishing, Hackensack, NJ, 2018. Zuhair Nashed and Xin Li, editors, arXiv:1607.06053v4.

Dick Askey at Oberwolfach, Germany

Dick Askey at The Netherlands in 1970

Dick Askey in Evanston, Illinois in 1980

Dick Askey in Evanston, Illinois in 1983



Dick Askey and Tom Koornwinder at Luminy in July 2007.

Dick Askey and Tom Koornwinder at Dick Askey's *80th Birthday Conference* in Madison, Wisconsin, USA. Photo taken by Patsy Wang-Iverson.

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Paul Nevai, Ohio State University, Columbus, Ohio, USA.

Dick and I were quite close to each other. There are just way too many stories both of professional and personal nature that need to be mentioned, or else will be lost forever. Because of this, I plan to write about Dick in the Journal of Approximation Theory and I am sure this project will take a long time to complete. I did a similar project with Carl de Boor about George Lorentz and that took a couple of years to accomplish (but it was well worth the time spent on it).

Dick Askey with George G. Lorentz at USC in 1988.

Let me just mention that Dick was instrumental in my immigrating to the US. I came as a legal immigrant and had my green card even before I crossed the border. It was quite a complicated and long process 45 years ago (legal immigration is not for the faint-hearted). Dick organized almost the entire Wisconsin political apparatus to support my immigration application. Even Senator Proxmire wrote on my behalf (although he mistakenly referred to me as a physicist).

I am not sure when I met Dick first in person because it was before the era of electronic record keeping, but I know that it was when Dick gave a series of lectures on special functions at AMI (today it's called Renyi Institute). It was in 1972 or 1973 or something like that. What I definitely remember is that I was thinking that this Yankee speaks so fast that

I understand not a word. Much later I realized that Dick actually speaks at a normal speed although he retained his Missouri accent ("w" is "doubleya").

I want to mention a couple of things before they are all forgotten and lost.

First, Dick and I have a deep secret that for some reason (I don't see why) we decided to keep such. Namely, we had a good reason to believe that one of our distinguished OPs guys was a serial philanderer. No, my reader, don't panic, we won't out you; this particular person has been long dead.

Second, my first mathematical encounter with Dick, was that I discovered a serious error in one of his papers that put me on the road to OPs. I think I never mentioned this to Dick. The paper was about mean convergence of Lagrange interpolation [1] and the error was the primary reason I became involved in it (because I wanted to find a different approach). However, later I found out that what I saw was just a preprint (an MRC Technical Summary Report) and the final version [2] no longer included the erroneous conclusion. Someone had good eyes before me.

Third, it was a tremendous honor for me when I was asked to write a letter of recommendation for Dick, about 30 years ago when he was considered for membership in the National Academy of Sciences (NAS). However, and this is something I never told Dick before, I lobbied heavily in the 1990s so Dick would be elected to foreign membership of the Hungarian Academy of Sciences but I miserably failed in my mission. The only outcome was that Dick was invited to give the Turán lectures.

Another interesting story is that once Dick and I conspired in something that probably never happened in math before and will never happen again. Namely, once I wrote a paper I didn't really like and submitted it to a prestigious journal where Dick was the editor. Then I asked Dick to pick me as referee and he did and I rejected the paper. Now can anyone beat a story like this? Yes, there was a reason for this seemingly meaningless exercise, but my lips are sealed until 2068.

I love you Dick!
Paul Nevai

References

[1] R. Askey. Mean convergence of orthogonal series and Lagrange interpolation. *Math. Centrum Amsterdam Afd. Toegepaste Wisk. Rep. TW*, 113:23, 1969.

[2] R. Askey. Mean convergence of orthogonal series and Lagrange interpolation. *Acta Mathematica. Academiae Scientiarum Hungaricae*, 23:71–85, 1972.

Dick Askey sleeping in 1935.

Dick Askey getting a shoe shine in front of the Catedral Metropolitana de Santiago while posing as a "rich American capitalist" during a private tour of Santiago, Chile—while attending the *Computational Methods and Function Theory* conference in Valparaíso, Chile, March, 1989. The photo was taken by Paul Nevai.

Dick Askey sleeps during a lecture by Mourad Ismail in 1998.

Mourad Ismail, Dick Askey, Ted Chihara and Paul Nevai in 1998.

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Ranjan Roy, Beloit College, Beloit, Wisconsin, USA.

Euler's Evaluation of a Certain Hypergeometric Series
Ranjan Roy
September, 2019.

Dick Askey's main focus is and has always been the advancement of mathematics, and thus of mathematicians. He has helped us in many ways: for example, by bringing together mathematicians with complementary interests; by conducting seminars and workshops on new developments and results; or by using his unique insight to guide us in profitable mathematical directions.

Dick has also been able to teach us that a deep understanding of received mathematics can be a key step toward new discoveries. His mathematical sagacity and his fresh insight into well-known results are illustrated in his evaluation of Euler's beta integral, given on pages 5–6 of his *Special Functions* [3]. In his new and revealing evaluation, only slightly different from Euler's, Dick obtained an interesting and novel approach to a well-established topic.

There are numerous examples of Dick's mathematical perceptiveness, especially in connection with hypergeometric series; many of these are contained in the treasure trove of his unpublished lecture notes.

Last year, I saw a paper of Euler in which he had converted a certain hypergeometric series into an infinite product. I immediately wondered what Dick might have done with this example.

Euler's example occupies a paragraph in his 40-page paper, "De mirabilibus proprietatibus unciarum, quae in evolutione binomii ad potestatem quamcunqua evecti occurrunt" [2, Eu. I-15, 528–568], presented to the Petersburg Academy in 1776, when Euler was 69 years old. Euler stated the formula

$$1+\binom{n}{1}\binom{n'}{1}+\binom{n}{2}\binom{n'}{2}+\binom{n}{3}\binom{n'}{3}+\cdots=\frac{1}{n\int_0^1 x^{n'}(1-x)^{n-1}\,dx}\,. \tag{6}$$

Euler was unable to prove this formula, writing that he found it remarkable that there was no direct general proof. He verified the formula for a number of particular cases. In the case for which $n'=-n$, note that the right-hand side equals

$$\frac{1}{\Gamma(1+n)\,\Gamma(1-n)}=\frac{\sin n\pi}{n\pi}\,.$$

But Euler took a different approach to (6) for the case $n'=-n$; here note that Euler implicitly assumed that $0<n'<1$. He transformed the integral to

$$\int_0^\infty \frac{y^{n'-1}\,dy}{1+y},$$

the value of which he knew from his 1742 work on the integration of rational functions: $\frac{\pi}{\sin n'\pi}$. Thus, in Section 44 of his paper, he found that he needed to verify the formula

$$1-\frac{n^2}{1^2}+\frac{n^2(n^2-1)}{1^2\cdot 2^2}-\frac{n^2(n^2-1)(n^2-2^2)}{1^2\cdot 2^2\cdot 3^2}+\cdots=\frac{\sin n\pi}{n\pi}, \tag{7}$$

where the series on the left-hand side was obtained by taking $n'=-n$ in the series in (6). To verify (7), Euler denoted the left-hand side by S and observed that $1-n^2$ was a common factor of the sum. Dividing by this common factor, he arrived at

$$\frac{S}{1-n^2}=1-\frac{n^2}{2^2}+\frac{n^2(n^2-2^2)}{1^2\cdot 2^2\cdot 3^2}-\frac{n^2(n^2-2^2)(n^2-3^2)}{1^2\cdot 2^2\cdot 3^2\cdot 4^2}+\cdots. \tag{8}$$

One can now see that $1-\frac{n^2}{2^2}$ is a common factor of the sum in (8). So, rewrite the sum in (8) as

$$\frac{S}{(1-n^2)(1-\frac{n^2}{2^2})}=1-\frac{n^2}{3^2}+\frac{n^2(n^2-3^2)}{1^2\cdot 3^2\cdot 4^2}-\cdots,$$

where $1-\frac{n^2}{3^2}$ is clearly the common factor. Repeating this process infinitely often, Euler found that

$$\frac{S}{(1-n^2)(1-\frac{n^2}{2^2})(1-\frac{n^2}{3^2})\cdots}=1$$

and, since the product was $\frac{\sin n\pi}{n\pi}$, Euler had succeeded in verifying (6) for $n'=-n$. Of course, (6) is a particular case of Gauss's sum for $F(a,b;c;1)$, but Euler's method of verifying (7) by converting a series into a product is of great interest.

In a June 1750 letter to Goldbach [1], Euler deduced the pentagonal number theorem, starting with the observation that

$$\begin{aligned}&(1-\alpha)(1-\beta)(1-\gamma)(1-\delta)\cdots\\&\qquad=1-\alpha-\beta(1-\alpha)-\delta(1-\alpha)(1-\beta)-\gamma(1-\alpha)(1-\beta)(1-\delta)-\cdots.\end{aligned} \tag{9}$$

Euler's example of the conversion of a hypergeometric series to a product is a particular case of (9), although he started with the right-hand side. Thus, Euler's idea can be applied to series other than the hypergeometric series, in particular q-series. Gauss gave examples of this method in his work on theta functions.

As I considered Euler's examples and (9), I was inspired by Dick Askey's teachings.

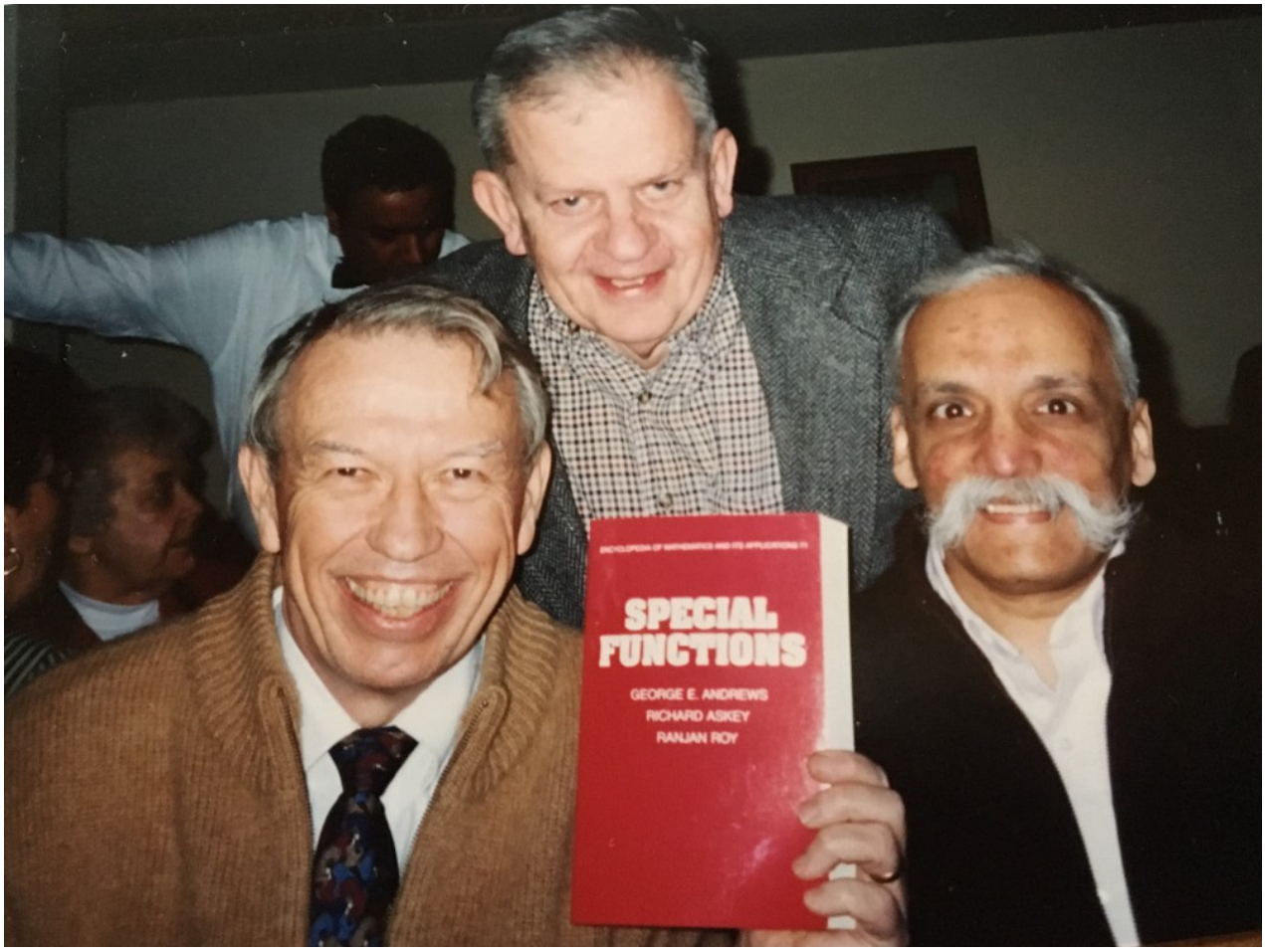
References

[1] L. Euler, June 1750. *Letter from L. Euler to C. Goldbach on June 9, 1750*.

[2] L. Euler. De mirabilibus proprietatibus unciarum, quae in evolutione binomii ad potestatem quamcunqua evecti occurrunt. *Acta Academiae Scientarum Imperialis Petropolitinae*, 5:74–111, 1784. presented to the St. Petersburg Academy on May 13, 1776.

[3] G. E. Andrews, R. Askey, and R. Roy. *Special functions*, volume 71 of *Encyclopedia of Mathematics and its Applications*. Cambridge University Press, Cambridge, 1999.

Dick and Ranjan at Beloit in 1997.



Andrews, Askey and Roy at Baltimore in 1997.

Ranjan and Dick at Madison, Wisconsin in 2019.

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Dennis Stanton, University of Minnesota, Minneapolis, Minnesota, USA.

I was in Dick Askey's Special Functions class, Mathematics 805, in Madison. His first lecture was on August 27, 1974. Also in that class were his students Dan Moak and Jim Wilson, and Mourad Ismail was there as a postdoc. (I have retained detailed notes of that course and later seminars.) At times the class was intimidating, but it was mostly exhilarating. Dick then was assimilating q-series. He did much work on q-analogues and the Askey tableau, and—when Chip Morris joined—what developed into the Macdonald conjectures. George Andrews came, and we all read Hahn's 1949 paper on orthogonal polynomials and q-difference equations. It was an exciting time to be a student.

I soon discovered that Dick was very supportive of younger people. His encyclopedic knowledge of the literature simplified our struggles, and he offered quite a bit of mathematical advice, technical and otherwise. He supported our travel to conferences, and I sometimes stayed with him in the hotel. After graduating, I lived at his house in the summer! At these conferences I saw his much larger influence on the greater mathematics community, in shaping directions of research, connecting the right people, and proposing fundamental questions. For example, much of the subsequent work on Macdonald symmetric functions was initiated by his study of multidimensional q-beta integrals. Often Dick would tell a speaker an insightful way to prove his theorem, and have many references available. Sometimes Dick did not even know that area of mathematics! It is amazing, and I thank Dick for all of his support.

Here is an anecdote. Dick gave the following advice to Jim Wilson and me. When a senior person said something could be done, you should believe him, but when he said it could not be done, be skeptical. We took his advice when we asked him about the Askey tableau. The classical summation theorems were used as measures for orthogonal polynomials, so we wanted to know if the Pfaff-Saalschütz theorem could be used. He said no it would not work, but we did not believe him. Jim Wilson found the orthogonal rational functions that did work for that sum.

l to r: Dennis Stanton, Lynne Butler and Dick Askey
at the IMA in Minneapolis, Minnesota in 1988.

Dick Askey and Dennis Stanton

Dennis Stanton presenting at the *Askey 80th Birthday Conference*, Madison, Wisconsin, USA in December 2013.

Topic #6 ——— OP – SF Net 27.2 ——— March 15, 2020

From: Paco Marcellán (pacomarc@ing.uc3m.es)
Subject: Announcement: *XXI Lluís Santaló Summer School* in Santander, Spain

XXI Lluís Santaló School: *Random and Deterministic Point Configurations* (UIMP–RSME).
Palacio de la Magdalena (Palace of La Magdalena).
Universidad Internacional Menéndez Pelayo, Santander, Spain.
Dates: Monday July 13 to Friday July 17, 2020.
Summer School Website: https://www.ub.edu/santalo20.

The summer school *Random and Deterministic Point Configurations* will take place in Santander, Spain from July 13 to July 17, 2020. Distributing points in sets (one can think on concrete spaces such as spheres or balls) is a key procedure in Approximation Theory. Classical applications of the field involve the understanding of crystalline structures, the description of microscopical objects such as pollen particles and the study of physical phenomena such as gasses consisting of charged particles. It is also frequently used to generate interpolation methods and quadrature formulas among others.

The aim of this Summer School is to introduce Master/PhD students into the field of point configurations, with wide-view courses that will include a friendly exposition of the main results in the area and its applications. The school will be of interest to any theoretical or applied scientist with a standard mathematical background. The topics presented will prove useful and self-contained on their own. Any PhD student working in related areas may benefit from the open problems session where some beautiful but challenging questions (such as the famous problem of distributing 5 spherical points that repel each other) will be discussed. Participants in the School will also be given the chance to present their work in a short communication.

There will be four courses introducing the different aspects of the subject by Dmitriy Bilyk (University of Minnesota), Alexander Bufetov (CNRS, France), Mylène Maïda (Université de Lille, France) and Joaquim Ortega-Cerdà (Universitat Barcelona, Spain). The program will be complemented with some talks by Maria Dostert (EPFL, Switzerland), Ujué Etayo (TU Graz, Austria), Adrien Hardy (Université de Lille, France) and Maurizia Rossi (Universitá degli Studi di Milano, Italy).

The directors of the School are Carlos Beltrán (Universidad de Cantabria, Spain) and Jordi Marzo (Universitat de Barcelona, Spain).

More information and instructions to participate can be found at: https://www.ub.edu/santalo20/.

Topic #7 ——— OP – SF Net 27.2 ——— March 15, 2020

From: Wolter Groenevert (W.G.M.Groenevelt@tudelft.nl)
Subject: Report by **Wolter Groenevelt**: *Matrix Valued Special Functions and Integrability*

The workshop "Matrix Valued Special Functions and Integrability" was held at the Radboud University Nijmegen in the Netherlands, from December 18–20, 2019. The workshop was organized by Erik Koelink and Walter Van Assche.

Matrix valued orthogonal polynomials were introduced in the 1940s by M. G. Krein in the study of operators with higher order deficiency indices. Since then the general theory of matrix valued orthogonal polynomials has been built up and many explicit examples have been found and studied. Nowadays they appear in many areas in mathematics and mathematical physics such as spectral theory, integrable systems, tiling theory, stochastic processes and representation theory.

On the first day of the workshop there were three talks in the afternoon, and on the last day there were two talks in the morning, which leaves only one full day of talks. A list of speakers and the titles of their talks is given below. With coffee breaks between lectures, lunches and a workshop dinner, there was plenty of time for discussions and conversations for the participants. I think this was a very nice and successful workshop.

- W. Riley Casper: Fourier Algebras
- Mirta Castro: Time-and-band limiting for matrix valued orthogonal polynomials
- Alfredo Deaño: Matrix-valued polynomials orthogonal with respect to exponential weights and their ladder relations (part 1 of 2)
- Maurice Duits: Matrix-valued orthogonal polynomials and random tilings
- Bruno Eijsvoogel: Matrix-valued polynomials orthogonal with respect to exponential weights and their ladder relations (part 2 of 2)
- Ana Foulquié Moreno: Riemann-Hilbert problem for Matrix Laguerre Polynomials defined through a given matrix Pearson equation
- María Ángeles García-Ferrero: Confluent exceptional orthogonal polynomials
- David Gómez-Ullate Oteiza: Rational solutions of Painlevé IV and its higher order generalizations: a complete classification
- Wolter Groenevelt: Stochastic duality functions and orthogonal polynomials
- Jasper Stokman: Vector-valued special functions and integrability.

Topic #8 ——— OP - SF Net 27.2 ——— March 15, 2020

From: OP-SF Net Editors
Subject: Preprints in arXiv.org

The following preprints related to the fields of orthogonal polynomials and special functions were posted or cross-listed to one of the subcategories of arXiv.org during January and February 2020. This list has been separated into two categories.

OP-SF Net Subscriber E-Prints

http://arxiv.org/abs/2001.00171
Asymptotics of the Largest Eigenvalue Distribution of the Laguerre Unitary Ensemble
Shulin Lyu, Chao Min, Yang Chen

http://arxiv.org/abs/2001.00393
Stieltjes moment sequences for pattern-avoiding permutations
Alin Bostan, Andrew Elvey Price, Anthony John Guttmann, Jean-Marie Maillard

http://arxiv.org/abs/2001.00494
Gap Probabilities in the Laguerre Unitary Ensemble and Discrete Painlevé Equations
Yang Chen, Anton Dzhamay, Jie Hu

http://arxiv.org/abs/2001.00654
Plethystic formulas for permutation enumeration
Ira M. Gessel, Yan Zhuang

http://arxiv.org/abs/2001.00875
Stahl–Totik regularity for continuum Schrödinger operators
Benjamin Eichinger, Milivoje Lukić

http://arxiv.org/abs/2001.02097
Numerical evaluation of Airy-type integrals arising in uniform asymptotic analysis
Amparo Gil, Javier Segura, Nico M. Temme

http://arxiv.org/abs/2001.02311
Dwork-type supercongruences through a creative q-microscope
Victor J. W. Guo, Wadim Zudilin

http://arxiv.org/abs/2001.03635
Further applications of the G function integral method
M. A. C. Candezano, D. B. Karp, E. G. Prilepkina

http://arxiv.org/abs/2001.03953
Asymptotic inversion of the binomial and negative binomial cumulative distribution functions
A. Gil, J. Segura, N. M. Temme

http://arxiv.org/abs/2001.04913
Observables of coloured stochastic vertex models and their polymer limits
Alexei Borodin, Michael Wheeler

http://arxiv.org/abs/2001.05637
AFLT-type Selberg integrals
Seamus P. Albion, Eric M. Rains, S. Ole Warnaar

http://arxiv.org/abs/2001.06820
Multiple orthogonal polynomials associated with confluent hypergeometric functions
Hélder Lima, Ana Loureiro

http://arxiv.org/abs/2001.06839
The Absent-Minded Passengers Problem via Computer Algebra
Shalosh B. Ekhad, Doron Zeilberger

http://arxiv.org/abs/2001.06869
Hypergeometric Integrals Modulo p and Hasse-Witt Matrices
Alexey Slinkin, Alexander Varchenko

http://arxiv.org/abs/2001.06876
On star-Moments of the compression of the free unitary Brownian motion by a free projection
Nizar Demni, Tarek Hamdi

http://arxiv.org/abs/2001.07309
Superintegrability and the dual -1 Hahn algebra in superconformal quantum mechanics
Pierre-Antoine Bernard, Julien Gaboriaud, Luc Vinet

http://arxiv.org/abs/2001.07823
Hyponormal Toeplitz Operators on Weighted Bergman Space
Brian Simanek

http://arxiv.org/abs/2001.08079
A family of q-congruences modulo the square of a cyclotomic polynomial
Victor J. W. Guo

http://arxiv.org/abs/2001.08104
WZ proofs of Ramanujan-type series (via ${}_2F_1$ evaluations)
Jesús Guillera

http://arxiv.org/abs/2001.08276
On the convergence of multi-level Hermite-Padé approximants for a class of meromorphic functions
L. G. González Ricardo, G. López Lagomasino, S. Medina Peralta

http://arxiv.org/abs/2001.08464
A proof of the Veselov Conjecture for segments
Antonio J. Durán

http://arxiv.org/abs/2001.09405
Aliasing error of the exp$(\beta\sqrt{1-z^2})$ kernel in the nonuniform fast Fourier transform
A. H. Barnett

http://arxiv.org/abs/2001.10576
Entanglement in Fermionic Chains and Bispectrality
Nicolas Crampé, Rafael I. Nepomechie, Luc Vinet

http://arxiv.org/abs/2001.10650
The entries of a refinement equation and a generalization of the discrete wave equation
Maxim Derevyagin, Jeffrey S. Geronimo

http://arxiv.org/abs/2001.10651
The image of a point charge in an infinite conducting cylinder
Matt Majic

http://arxiv.org/abs/2001.11195
The Racah algebra: An overview and recent results
Hendrik De Bie, Plamen Iliev, Wouter van de Vijver, Luc Vinet

http://arxiv.org/abs/2002.00273
A New Spectral Analysis of the Sixth-order Krall Differential Expression
K. Elliott, L. L. Littlejohn, R. Wellman

http://arxiv.org/abs/2002.00789
Diagonals of rational functions: from differential algebra to effective algebraic geometry (unabridged version)
Y. Abdelaziz, S. Boukraa, C. Koutschan, J-M. Maillard

http://arxiv.org/abs/2002.01884
On the mode and median of the generalized hyperbolic and related distributions
Robert E. Gaunt, Milan Merkle

http://arxiv.org/abs/2002.02665
Degenerate polyexponential functions and degenerate Bell polynomials
Taekyun Kim, Dae San Kim

http://arxiv.org/abs/2002.03803
Construction of potential functions associated with a given energy spectrum
A. D. Alhaidari

http://arxiv.org/abs/2002.04520
A note on new type degenerate Bernoulli numbers
Taekyun Kim, Dae San Kim

http://arxiv.org/abs/2002.04536
Quasi-birth-and-death processes and multivariate orthogonal polynomials
Lidia Fernández, Manuel D. de la Iglesia

http://arxiv.org/abs/2002.05244
Linearizable boundary value problems for the elliptic sine-Gordon and the elliptic Ernst equations
J. Lenells, A. S. Fokas

http://arxiv.org/abs/2002.05248
A Reflection Formula for the Gaussian Hypergeometric Function of Matrix Argument
Donald Richards, Qifu Zheng

http://arxiv.org/abs/2002.05722
Generating Functions for Lacunary Legendre and Legendre-like Polynomials
S. Licciardi, G. Dattoli, R.M. Pidatell

http://arxiv.org/abs/2002.06164
The positive tropical Grassmannian, the hypersimplex, and the $m = 2$ amplituhedron
Tomasz Lukowski, Matteo Parisi, Lauren K. Williams

http://arxiv.org/abs/2002.06194
Logarithmic asymptotic of multi-level Hermite-Padé polynomials
L. G. González Ricardo, G. López Lagomasino, S. Medina Peralta

http://arxiv.org/abs/2002.06245
Umbral-Algebraic Methods and Asymptotic Properties of Special Polynomials
G. Dattoli, S. Licciardi, R.M. Pidatella, E. Sabia

http://arxiv.org/abs/2002.06428
On a family of hypergeometric Sobolev orthogonal polynomials on the unit circle
Sergey M. Zagorodnyuk

http://arxiv.org/abs/2002.07430
Functional inequalities and monotonicity results for modified Lommel functions of the first kind
Robert E. Gaunt

http://arxiv.org/abs/2002.07796
Log-concavity results for a biparametric and an elliptic extension of the q-binomial coefficients
Michael J. Schlosser, Koushik Senapati, Ali K. Uncu

http://arxiv.org/abs/2002.07932
A unified construction of all the hypergeometric and basic hypergeometric families of orthogonal polynomial sequences
Luis Verde-Star

http://arxiv.org/abs/2002.09065
The Dunkl kernel and intertwining operator for dihedral groups
Hendrik De Bie, Pan Lian

http://arxiv.org/abs/2002.10978
A practical guide to Prabhakar fractional calculus
Andrea Giusti, Ivano Colombaro, Roberto Garra, Roberto Garrappa, Federico Polito, Marina Popolizio, Francesco Mainardi

Other Relevant OP-SF E-Prints

http://arxiv.org/abs/2001.00108
A complement to a recent paper on some infinite sums with the zeta values
Iaroslav V. Blagouchine

http://arxiv.org/abs/2001.00482
Collatz polynomials: an introduction with bounds on their zeros
Matt Hohertz, Bahman Kalantari

http://arxiv.org/abs/2001.00857
Sharp Hardy-Rellich Type Inequalities Associated with Dunkl Operators
Li Tang, Haiting Chen, Shoufeng Shen, Yongyang Jin

http://arxiv.org/abs/2001.01621
Zeros of p-adic hypergeometric functions, p-adic analogues of Kummer's and Pfaff's identities
Neelam Saikia

http://arxiv.org/abs/2001.01721
Green's Functions for Vladimirov Derivatives and Tate's Thesis
An Huang, Bogdan Stoica, Shing-Tung Yau, Xiao Zhong

http://arxiv.org/abs/2001.01930
On linearization coefficients of q-Laguerre polynomials
Byung-Hak Hwang, Jang Soo Kim, Jaeseong Oh, Sang-Hoon Yu

http://arxiv.org/abs/2001.01936
Parametrization of Kloosterman sets and SL_3-Kloosterman sums
Eren Mehmet Kıral, Maki Nakasuji

http://arxiv.org/abs/2001.01981
On the real and complex zeros of the quadrilateral zeta function
Takashi Nakamura

http://arxiv.org/abs/2001.02158
On asymptotics for lacunary partition functions
Alexander E. Patkowski

http://arxiv.org/abs/2001.02175
From inequalities involving exponential functions and sums to logarithmically complete monotonicity of ratios of gamma functions
Feng Qi, Bai-Ni Guo

http://arxiv.org/abs/2001.02203
Short note on a relation between the inverse of the cosine and Carlson's elliptic integral R_D
Felix Ospald, Roland Herzog

http://arxiv.org/abs/2001.02535
Discrete Hamiltonians of discrete Painlevé equations
Takafumi Mase, Akane Nakamura, Hidetaka Sakai

http://arxiv.org/abs/2001.02881
Systematic construction of non-autonomous Hamiltonian equations of Painlevé-type. I. Frobenius integrability
Maciej Błaszak, Krzysztof Marciniak, Ziemowit Domański

http://arxiv.org/abs/2001.03296
On the integrality of hypergeometric series whose coefficients are factorial ratios
Alan Adolphson, Steven Sperber

http://arxiv.org/abs/2001.03630
Reducible specializations of polynomials: the nonsolvable case
Joachim König, Danny Neftin

http://arxiv.org/abs/2001.03815
Addition formulas for the ${}_pF_p$ and ${}_{p+1}F_p$ generalized hypergeometric functions with arbitrary parameters and their Kummer- and Euler-type transformations
Krishna Choudhary

http://arxiv.org/abs/2001.03865
Riemann-Hilbert approach and N-soliton solutions for a new four-component nonlinear Schrödinger equation
Xin-Mei Zhou, Shou-Fu Tian, Jin-Jie Yang, Jin-Jin Mao

http://arxiv.org/abs/2001.03941
Proof of Sun's supercongruence involving Catalan numbers
Ji-Cai Liu

http://arxiv.org/abs/2001.04445
On the definition of Euler Gamma function
Ricardo Pérez-Marco

http://arxiv.org/abs/2001.04793
Generating functions and integral formulas the Fox-Wright function and theirs applications
Khaled Mehrez

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Neelam Saikia

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Algebraic foliations and derived geometry: the Riemann-Hilbert correspondence
Bertrand Toën, Gabriele Vezzosi

http://arxiv.org/abs/2001.05686
Divisibility Properties of the Fourier Coefficients of (Mock) Modular Functions and Ramanujan
Soon-Yi Kang

http://arxiv.org/abs/2001.06213
An explicit formula of continuant polynomials for periodic parameters
Genki Shibukawa

http://arxiv.org/abs/2001.06623
Verified computation of matrix gamma function
Shinya Miyajima

http://arxiv.org/abs/2001.06856
Anomalous relaxation in dielectrics with Hilfer fractional derivative
A. R. Gómez Plata, Ester C. A. F. Rosa, R. G. Rodriguez-Giraldo, E. Capelas de Oliveira

http://arxiv.org/abs/2001.07013
Some inequalities for Chebyshev polynomials
Geno Nikolov

http://arxiv.org/abs/2001.07085
A solvable model of the breakdown of the adiabatic approximation
Artbazar Galtbayar, Arne Jensen, Kenji Yajima

http://arxiv.org/abs/2001.07533
A rational approximation of the Fourier transform by integration with exponential decay multiplier
S. M. Abrarov, R. Siddiqui, R. K. Jagpal, B. M. Quine

http://arxiv.org/abs/2001.08114
A q-analogue of the matrix sixth Painlevé system
Hiroshi Kawakami

http://arxiv.org/abs/2001.08117
Congruence relations for p-adic hypergeometric functions $\widehat{\mathcal{F}}^{(\sigma)}_{a,\ldots,a}(t)$ and its transformation formula
Wang Chung-Hsuan

http://arxiv.org/abs/2001.08545
New expansions for $x^n \pm y^n$ in terms of quadratic forms
Moustafa Ibrahim

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Riemann-Hilbert approach to the inhomogeneous fifth-order nonlinear Schrödinger equation with non-vanishing boundary conditions
Jin-Jie Yang, Shou-Fu Tian, Zhi-Qiang Li

http://arxiv.org/abs/2001.08799
Characterizations of the Borel triangle and Borel polynomials
Paul Barry

http://arxiv.org/abs/2001.08825
Numerical Approximation of the Fractional Laplacian on $\mathbb{R}$ Using Orthogonal Families
Jorge Cayama, Carlota M. Cuesta, Francisco de la Hoz

http://arxiv.org/abs/2001.09009
Numerator polynomials of the Riordan matrices
E. Burlachenko

http://arxiv.org/abs/2001.09144
Sparse Interpolation in Terms of Multivariate Chebyshev Polynomials
Evelyne Hubert, Michael F. Singer

http://arxiv.org/abs/2001.09323
Identity for generalized Bernoulli polynomials
Redha Chellal, Farid Bencherif, Mohamed Mehbali

http://arxiv.org/abs/2001.09622
The Generalized Hypergeometric Structure of the Ward Identities of CFT's in Momentum Space in $d > 2$
Claudio Corianò, Matteo Maria Maglio

http://arxiv.org/abs/2001.10533
On an integral involving the logarithm function
Sumit Kumar Jha

http://arxiv.org/abs/2001.10846
About Convergence and Order of Convergence of some Fractional Derivatives
Sabrina Roscani, Lucas Venturato

http://arxiv.org/abs/2001.10901
q-difference equations associated with the Rubin's q-difference operator ∂_q
Meniar Haddad, Marwa Mastouri

http://arxiv.org/abs/2001.11095
Doubly periodic lozenge tilings of a hexagon and matrix valued orthogonal polynomials
Christophe Charlier

http://arxiv.org/abs/2001.11270
Monodromy in Prolate Spheroidal Harmonics
Sean R. Dawson, Holger R. Dullin, Diana M.H. Nguyen

http://arxiv.org/abs/2001.11385
Theta surfaces
Daniele Agostini, Türkü Özlüm Çelik, Julia Struwe, Bernd Sturmfels

http://arxiv.org/abs/2001.11893
Expression of the Holtsmark function in terms of hypergeometric ${}_2F_2$ and Airy Bi functions
Jean-Christophe Pain

http://arxiv.org/abs/2002.00170
Non-asymptotic behavior and the distribution of the spectrum of the finite Hankel transform operator
Mourad Boulsane

http://arxiv.org/abs/2002.00243
Non-stationary Ruijsenaars functions for $\kappa = t^{-1/N}$ and intertwining operators of Ding-Iohara-Miki algebra
Masayuki Fukuda, Yusuke Ohkubo, Jun'ichi Shiraishi

http://arxiv.org/abs/2002.00447
On Sum-Of-Tails Identities
Rajat Gupta

http://arxiv.org/abs/2002.00465
On Smoothness of the Abel Equation Solution in Terms of the Jacobi Series Coefficients
Maksim V. Kukushkin

http://arxiv.org/abs/2002.00550
A two-sided Faulhaber-like formula involving Bernoulli polynomials
Fernando Barbero G., Juan Margalef-Bentabol, Eduardo J.S. Villaseñor

http://arxiv.org/abs/2002.00888
Equal sums of two cubes of binary quadratic forms
Bruce Reznick

http://arxiv.org/abs/2002.01342
Properties of Chebyshev polynomials
N. Karjanto

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On the complex magnitude of Dirichlet beta function
Artur Kawalec

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Geno Nikolov

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Combinatorial formulas for the coefficients of the Al-Salam-Chihara polynomials
Donghyun Kim

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Orthogonal polynomial projection error in Dunkl-Sobolev norms in the ball
Gonzalo A. Benavides, Leonardo E. Figueroa

http://arxiv.org/abs/2002.01971
Impossibility of convergence of a Heun function on the boundary of the disc of convergence
Yoon-Seok Choun

http://arxiv.org/abs/2002.02073
Truncated Hilbert Transform: Uniqueness and a Chebyshev series Expansion Approach
Jason You

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Branching rules for Koornwinder polynomials with one column diagrams and matrix inversions
A. Hoshino, J. Shiraishi

http://arxiv.org/abs/2002.02633
On the extreme zeros of Jacobi polynomials
Geno Nikolov

http://arxiv.org/abs/2002.02679
The mathematical contributions by Francesco Carlini: from the asymptotic study of singular ordinary differential equations to complex roots of the equation $x^x = y$
Andrea Sacchetti

http://arxiv.org/abs/2002.03002
Bounds on the Information Divergence for Hypergeometric Distributions
Peter Harremoës, František Matúš

http://arxiv.org/abs/2002.03250
Logarithmic Integrals: A Review from Gradshteyn and Ryzhik to Recent Times
Md Sarowar Morshed

http://arxiv.org/abs/2002.03768
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George Tephnadze

http://arxiv.org/abs/2002.03918
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Milton Espinoza

http://arxiv.org/abs/2002.03994
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http://arxiv.org/abs/2002.04085
Proof of Sarkar-Kumar's Conjectures on Average Entanglement Entropies over the Bures-Hall Ensemble
Lu Wei

http://arxiv.org/abs/2002.04141
Affine Demazure crystals for specialized nonsymmetric Macdonald polynomials
Sami Assaf, Nicolle Gonzalez

http://arxiv.org/abs/2002.04191
On the Area Bounded by the Curve $\prod_{k=1}^{n} |x \sin(k\pi/n) - y \cos(k\pi/n)| = 1$
Anton Mosunov

http://arxiv.org/abs/2002.05110
Weak-Coupling, Strong-Coupling and Large-Order Parametrization of the Hypergeometric-Meijer Approximants
Abouzeid M. Shalaby

http://arxiv.org/abs/2002.05334
Generalised Hermite functions and their applications in Spectral Approximations
Sheng Changtao, Ma Suna, Li Huiyuan, Wang Li-Lian, Jia Lueling

http://arxiv.org/abs/2002.06140
Period Relations for Quaternionic Elliptic Functions
Zavosh Amir-Khosravi

http://arxiv.org/abs/2002.06270
Resurgent Trans-Series for Generalized Hastings-McLeod Solutions
Nikko J. Cleri, Gerald V. Dunne

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Keita Imaizumi

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http://arxiv.org/abs/2002.07773
The Fourier–Jacobi periods : The case of $U(n+2r) \times U(n)$
Jaeho Haan

http://arxiv.org/abs/2002.07895
Bivariate Continuous q–Hermite Polynomials and Deformed Quantum Serre Relations
W. Riley Casper, Stefan Kolb, Milen Yakimov

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Weighted Hurwitz numbers, τ–functions and matrix integrals
J. Harnad

http://arxiv.org/abs/2002.08170
Impossibility of convergence of a confluent Heun function on the boundary of the disc of convergence
Yoon–Seok Choun

http://arxiv.org/abs/2002.08615
Integral representation for Jacobi polynomials and application to heat kernel on quantized sphere
Ali Hafoud, Allal Ghanmi

http://arxiv.org/abs/2002.08703
Positivity properties of some special matrices
Priyanka Grover, Veer Singh Panwar, A Satyanarayana Reddy

http://arxiv.org/abs/2002.08814
Divisibility results concerning truncated hypergeometric series
Chen Wang, Wei Xia

http://arxiv.org/abs/2002.08830
Spectral density for the Schrödinger operator with magnetic field in the unit complex ball: Solutions of evolutionary equations and applications to special functions
Nour eddine Askour, Mohamed Bouaouid, Abdelkarim Elhadouni

http://arxiv.org/abs/2002.08903
If $L(\chi, 1) = 0$ then $\zeta(1/2 + it) \neq 0$
Sergio Venturini

http://arxiv.org/abs/2002.09819
Eisenstein series whose Fourier coefficients are zeta functions of binary Hermitian forms
Jorge Flórez, Cihan Karabulut, An Hoa Vu

http://arxiv.org/abs/2002.09906
Hyperbolicity of Appell Polynomials of Functions in the δ–Laguerre–Pólya Class
Jonas Iskander, Vanshika Jain

http://arxiv.org/abs/2002.10682
On an integral identity
Alin Bostan, Fernando Chamizo, Mikael P. Sundqvist

http://arxiv.org/abs/2002.10723
Determinantal point processes and fermion quasifree states
Grigori Olshanski

http://arxiv.org/abs/2002.10792
Symplectic quandles and parabolic representations of 2-bridge Knots and Links
Kyeonghee Jo, Hyuk Kim

http://arxiv.org/abs/2002.11094
Evaluation of exponential sums and Riemann zeta function on quantum computer
Sandeep Tyagi

http://arxiv.org/abs/2002.11240
Gaussian unitary ensemble with jump discontinuities and the coupled Painlevé II and IV systems
Xiao-Bo Wu, Shuai-Xia Xu

http://arxiv.org/abs/2002.11984
Sums of averages of gcd-sum functions II
Lisa Kaltenböck, Isao Kiuchi, Sumaia Saad Eddin, Masaaki Ueda

http://arxiv.org/abs/2002.12099
Zeta functions of periodic cubical lattices and cyclotomic-like polynomials
Yasuaki Hiraoka, Hiroyuki Ochiai, Tomoyuki Shirai

http://arxiv.org/abs/2002.12107
Explicit Formulas for General Euler Type Sums
Ce Xu

http://arxiv.org/abs/2002.12746
Shiraishi functor and non-Kerov deformation of Macdonald polynomials
H. Awata, H. Kanno, A. Mironov, A. Morozov

Topic #9 ——— OP - SF Net 27.2 ——— March 15, 2020

From: OP-SF Net Editors
Subject: Submitting contributions to OP-SF NET and SIAM-OPSF (OP-SF Talk)

To contribute a news item to OP-SF NET, send e-mail to one of the OP-SF Editors howard.cohl@nist.gov, or spost@hawaii.edu.

Contributions to OP-SF NET 27.3 should be sent by May 1, 2020.

OP-SF NET is an electronic newsletter of the SIAM Activity Group on Special Functions and Orthogonal Polynomials. We disseminate your contributions on anything of interest to the special functions and orthogonal polynomials community. This includes announcements of conferences, forthcoming books, new software, electronic archives, research questions, and job openings as well as news about new appointments, promotions, research visitors, awards and prizes. OP-SF Net is transmitted periodically through a post to SIAM-OPSF (OP-SF Talk).

SIAM-OPSF (OP-SF Talk) is a listserv of the SIAM Activity Group on Special Functions and Orthogonal Polynomials, which facilitates communication among members, and friends of the Activity Group. See the previous Topic. To post an item to the listserv, send e-mail to siam-opsf@siam.org.

WWW home page of this Activity Group:
http://math.nist.gov/opsf
Information on joining SIAM and this activity group: service@siam.org

The elected Officers of the Activity Group (2020-2022) are:
Peter Alan Clarkson, Chair
Luc Vinet, Vice Chair
Andrei Martínez-Finkelshtein, Program Director
Teresa E. Pérez, Secretary and OP-SF Talk moderator

The appointed officers are:
Howard Cohl, OP-SF NET co-editor
Sarah Post, OP-SF NET co-editor
Diego Dominici, OP-SF Talk moderator
Bonita Saunders, Webmaster and OP-SF Talk moderator

Topic #10 ——— OP - SF Net 27.2 ——— March 15, 2020

From: OP-SF Net Editors
Subject: Thought of the Month by **Richard A. Askey**

"When applications are considered, the most important special functions are the hypergeometric functions.

$$\vdots$$

$$\cos \pi x = {}_2F_1\left(\begin{matrix} x, -x \\ \frac{1}{2} \end{matrix}; 1\right).$$

The last example is particularly important, for it suggests that the parameters that occur in hypergeometric series can play a more important role in the study of hypergeometric series than that of just enabling us to distinguish between different series. Gauss was probably the first person to realize this; we will return to his results after describing some earlier work of Wallis and Euler that is necessary before we can see why this last formula holds."

Richard A. Askey, *General Editor*, Section on Special Functions,
in Foreword of *Symmetry and Separation of Variables* by Willard Miller Jr.,
Encyclopedia of Mathematics and its Applications, Addison-Wesley, Vol. 4, 1977.