

OP-SF NET – Volume 22, Number 3 – May 15, 2015

The Electronic News Net of the
SIAM Activity Group on Orthogonal Polynomials and Special Functions

<http://math.nist.gov/opsf>

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Calendar of Events:

June 1-5, 2015

13th International Symposium on Orthogonal Polynomials, Special Functions and Applications (OPSFA13), NIST, Gaithersburg, Maryland, USA
<http://www.siam.org/meetings/opsfa13>

June 7-11, 2015

“Asymptotics in integrable systems, random matrices and random processes and universality”, in honour of Percy Deift’s 70th birthday, Centre de Recherches Mathématiques, Montreal, Canada
http://www.crm.umontreal.ca/2015/Deift15/index_e.php

June 8-12, 2015

V Iberoamerican Workshop on Orthogonal Polynomials, Mexico City, Mexico
<http://paginas.matem.unam.mx/eibpoa2015/index.php/en>

June 10–13, 2015

AMS–EMS–SPM International meeting, with a special session on Orthogonal Polynomials and Integrable Systems, Porto, Portugal
<http://aep-math2015.spm.pt>

June 15–18, 2015

Progress on Difference Equations, Covilhã, Portugal
<http://www.pode2015.ubi.pt>

15 June–10 July, 2015

SERB (DST) – sponsored 12th SERB School on Matrix Methods & Fractional Calculus, Peechi Campus, KFRI, Peechi –680653, Kerala, India
Contact mathai@math.mcgill.ca for more information

August 9–14, 2015

Orthogonal and Multiple Orthogonal Polynomials, Oaxaca, Mexico
<http://www.birs.ca/events/2015/5-day-workshops/15w5022>

August 10–14, 2015

ICIAM 2015 (International Congress on Industrial and Applied Mathematics), Beijing, China
<http://www.iciam2015.cn>

August 26–28, 2015

Symposium “The Real World is Complex” in honour of Christian Berg, in Copenhagen, Denmark
<http://www.math.ku.dk/~henrikp/cb>

September 10–12, 2015

XIVth Annual Conference (ICSFA 2015) of Society for Special Functions & their Applications (SSFA), Amity University, Noida, India
<http://www.ssfaindia.webs.com/conf.htm>

September 28–30, 2015

International Conference on Analysis, Applications and Computations, In Memory of Lee Lorch, Fields Institute, Toronto, Canada
<http://www.fields.utoronto.ca/programs/scientific/15-16/analysisapplications>

October 4–30, 2015

International Workshop and Latin–American School on Foundations of Complexity, Nonadditive Entropies and Nonextensive Statistical Mechanics, Rio de Janeiro, Brazil
<http://www.cbpf.br/~complex>

June 27 – July 1, 2016

Abecederian of SIDE (ASIDE) 12 Summer School, Centre de Recherches mathématiques, Université de Montréal, Montréal, Quebec, Canada
<http://www.crm.umontreal.ca/2016/ASIDE2016>

July 3–9, 2016

Symmetries and Integrability of Difference Equations 12,
Hôtel Le Chanteclerc, Saint Adèle, Québec, Canada
<http://www.crm.umontreal.ca/2016/SIDE12/index.php>

July 11–15, 2016

OPSF–S6 Summer School on Orthogonal Polynomials and Special Functions,
American University, Washington D.C., USA
<https://wis.kuleuven.be/events/OPSFA>

Topic #1 ——— OP – SF Net 22.3 ——— May 15, 2015

From: Walter Van Assche (walter@wis.kuleuven.be)
Subject: CALL for OPSFA–14 (Deadline **May 31, 2015**)

The Steering Committee of the international symposia “Orthogonal Polynomials, Special Functions and Applications” has opened a call for the organization of the next international symposium on “Orthogonal Polynomials, Special Functions and Applications” (OPSFA–14), to be held preferably in 2017. Please inform Walter Van Assche (walter@wis.kuleuven.be) if you are willing to organize OPSFA–14.

The deadline is **May 31, 2015**. Please provide:

- name of the contact person;
- place where the conference will be organized; and a
- suggestion of the date.

All proposals will be evaluated by the steering committee and the final decision will be announced at the upcoming [OPSFA–13](#) meeting in Gaithersburg, USA (June 1–5, 2015).

The Steering Committee for OPSFA consists of 3 local organizers of the past 5 OPSFA meetings and a representative of the SIAM Activity Group “Orthogonal Polynomials and Special Functions” (not necessarily the chair). This Steering Committee was founded during the OPSFA–11 meeting in Leganés, Spain, in 2011. Its main task is to coordinate the international meetings in the OPSFA community, such as the biannual international symposium and summer schools. Presently the Steering Committee consists of:

Walter Van Assche (SIAG chair and OPSFA–10)
Guillermo López Lagomasino (OPSFA–11)
Mohamed Jalel Atia (OPSFA–12)
Diego Dominici (SIAG program director)

See also: <https://wis.kuleuven.be/events/OPSFA/Call>.

From: Jan Felipe van Diejen (diejen@inst-mat.utalca.cl)

Subject: Report on “Representation Theory, Special Functions and Painlevé Equations”

In the first week of March (March 3–6, 2015), a conference on “[Representation Theory, Special Functions and Painlevé Equations](#)” took place at the Research Institute of Mathematical Sciences (RIMS) of Kyoto University in Kyoto, Japan. This event, held in honor of Professor Masatoshi Noumi of Kobe University on the occasion of his 60th birthday celebration, was organized by H. Konno, Y. Masuda, M.–H. Saito, H. Sakai, J. Shiraishi, T. Suzuki and Y. Yamada. The themes of the meeting, nicely summarized in its title, were around a selection of areas where Noumi–san has made fundamental contributions: symmetric functions (Macdonald polynomials and the likes), hypergeometric transformation and summation formulas, Painlevé transcendents, elliptic integrable systems and elliptic hyper geometry, and representation theory underlying these special functions.

The conference program consisted of 60 minute talks separated by generous breaks of at least 30 minutes. On the opening day and on the closing day, only 3 afternoon and 2 morning talks were scheduled, respectively, while on the remaining two full days, the schedule consisted of 5 talks. Since the range of topics was wide, most speakers did a nice unselfish job by starting with a helpful survey of the field before thrusting towards the latest state of the art developments. This way, even for a participant mastering only a small portion of Noumi–san’s research interests (like myself), there was a lot to be captured also from talks on topics which one is less familiar with.

One of the highlights of the conference was the talk of Noumi–san himself “Special functions arising from elliptic integrable systems,” which took the audience on a fascinating tour connecting elliptic hypergeometric series and integrals, elliptic Painlevé equations, and elliptic integrable systems. Other presentations included: Yasushi Kajihara: Families of transformations for bilinear sum of (basic) hypergeometric series and multivariate generalizations; Tom Koornwinder: Some remarks about Koornwinder polynomials; Simon Ruijsenaars: Quantum integrable systems of elliptic Calogero–Moser type; Jasper V. Stokman: From Noumi’s representation to elliptic K–matrices; Frank W. Nijhoff: On elliptic Lax pairs and isomonodromic deformation problems for integrable lattice systems; Nalini Joshi: Elliptic asymptotics for discrete Painlevé equations; Kenji Kajiwara: Integrable discrete deformations of discrete curves: geometry and solitons, old and new; Philip Boalch: Non–perturbative symplectic manifolds and non–commutative algebras; Marta Mazzocco: Painlevé equations and q–Askey scheme; Tôru Umeda: Remarks on the Capelli identities for reducible modules; Anatol N. Kirillov: Plactic algebra, Cauchy kernels and plane partitions; Katsuhisa Mimachi: Monodromy representations associated with the generalized hypergeometric function ${}_{n+1}F_n$; Vyacheslav P. Spiridonov: Elliptic hypergeometric functions and superconformal indices; Jan Felipe van Diejen: Branching formula for Macdonald–Koornwinder polynomials. (Freak detail: the participants of the Japanese and Dutch nationalities each accounted for one third of the lecture program.)

Since the scientific program ended each day at 17:00 at the latest, the conference participants had plenty of opportunities to explore the many touristic attractions of Kyoto as well. While on Thursday evening, folks met at the impressive Funatsura Japanese Inn for Noumi–san’s after–lecture birthday party.

The smooth organization, top location, and many inspiring talks, certainly made this pleasant conference into a memorable one.

From: Hans Haubold (hans@neutrino.aquaphoenix.com)

Subject: An appraisal of the scientific endeavors of Prof Dr. A. M. Mathai

A. M. Mathai is Emeritus Professor of Mathematics and Statistics at McGill University, Canada. Currently he is also the Director of the [Centre for Mathematical and Statistical Sciences](#), India. His research contributions cover a wide spectrum of topics in mathematics, statistics, physics, astronomy, and biology. He is a Fellow of the Institute of Mathematical Statistics, National Academy of Sciences of India, and a member of the International Statistical Institute. He is a founder of the Canadian Journal of Statistics and the Statistical Society of Canada. He was instrumental in the implementation of the United Nations Basic Space Science Initiative (1991–2012) for the benefit of 194 Member States <http://arxiv.org/ftp/arxiv/papers/1503/1503.06654.pdf>. He is the President of the Indian Mathematical Society.

In the past there have been great relations between mathematics and physics. Mathematical formalism entered the development of theoretical physics or problems arising from physics encouraged developments in mathematics. A. M. Mathai's 80th birthday is a welcome opportunity to celebrate his contributions to the theory of special functions of mathematical physics and to note the current stormy development of the mathematics and statistics of fractional calculus and its surprising applications in physics and beyond published in a special issue [Special Functions: Fractional Calculus and the Pathway for Entropy Dedicated to Professor Dr. A. M. Mathai at the occasion of his 80th Birthday of Axioms](#).

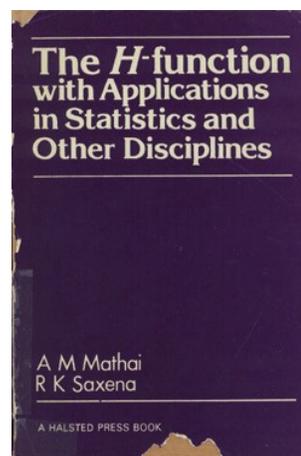
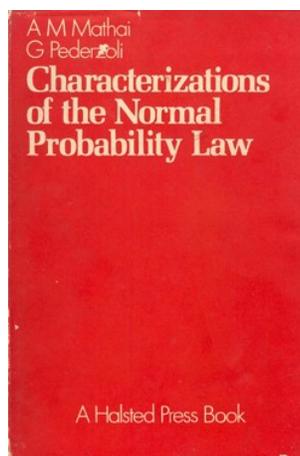
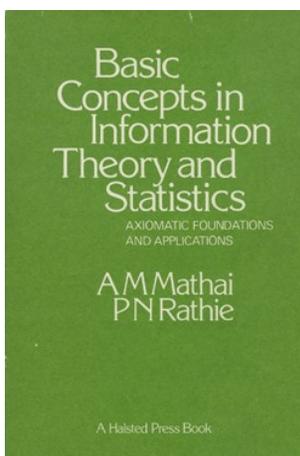
One of Mathai's early applications of the theory of special functions in (astro)physics was related to the prospective resolution of what is known in the literature as the 'solar neutrino problem'. This problem was a major discrepancy between measurements of the numbers of solar neutrinos streaming through the Earth and theoretical models of the solar interior, lasting from the mid-1960s to about 2002. The discrepancy has since been resolved by new understanding of neutrino physics, requiring a modification of the standard model of particle physics – specifically, neutrino oscillations. Essentially, as neutrinos have mass, they can change from the type that had been expected to be produced in the Sun's interior into two types that would not be caught by the detectors in use at the time. Mathai's contributions to the resolution of this problem were discussed with R. Davis Jr. who received the 2002 Nobel Prize in physics for the resolution of the solar neutrino problem. Related to this problem, in the 1980s, Mathai made important contributions to the closed-form representation of solar thermonuclear reaction rates in terms of Meijer's G-function which are a fundament of nuclear astrophysics. In 1983, W. A. Fowler received the Nobel Prize in physics for his leading developments in the field of nuclear astrophysics. Mathai's mathematics and statistics for solar neutrinos and solar nuclear astrophysics remain to be fundamental contributions to both fields of on-going research.

The two dominant examples of interaction between mathematics and physics in the twentieth century are Riemannian geometry in general relativity and the impact of quantum mechanics on development of functional analysis. Einstein finalized general relativity in 1915, while quantum field theory has been an open frontier since its foundation in 1927 by Dirac based on Heisenberg's and Schroedinger's physics. This happened almost 100 years ago. For the next fifty years there was not much interaction between theoretical

physics and mathematics. Mathematics turned to more abstract accomplishments, while quantum field theory was formulated in a rather formal way.

There are two fundamental theories in twentieth century physics: general relativity and quantum field theory. General relativity describes gravitational forces on the scale of the macrocosmos, while quantum field theory describes the interaction of elementary particles, electromagnetism, strong and weak forces at the scale of the microcosmos. There remains to be an inconsistency between the two theories. The formal quantization of general relativity leads to infinite formulas. Einstein invented general relativity to resolve an inconsistency between special relativity and Newtonian gravity. Quantum field theory was invented to reconcile Maxwell's electromagnetism and special relativity with non-relativistic quantum mechanics. But there were two basically different approaches. In Einstein's "thought experiments", which led to the discovery of general relativity, the logical framework was first. Then in Riemannian geometry of general relativity, the correct mathematical framework was found. In the development of quantum field theory on the other hand, there was no a priori conceptual basis; experimental clues played an important role, but there was no mathematical model. To date string theory is progressing in the pursuit to a formal quantization of general relativity: if accomplished it might be called the first revolution in physics in the twenty first century.

A second revolution in the making in the twenty first century might be the development of statistical mechanics beyond Boltzmann and Gibbs, taking into account the need for a physical theory for stochastic processes in non-equilibrium systems. Indications are there that the mathematics for such a theory is given with fractional calculus and Fox' H-function playing a central role in the solutions of stochastic fractional differential equations of the type of Liouville, master, Fokker-Planck, Langevin, and reaction-diffusion. However, the physical interpretation of fractional time and space derivatives and integrals has not been discovered yet and a prospective physical theory based on a "Schroedinger equation for thermodynamics", as referred to by Prigogine even if he surely had in mind a master equation, has not been discovered. Mathai's research programme from the 1970s, as outlined in three monographs, all published in the short time interval between 1875 and 1978,



does address in terms of mathematics and statistics some of the open issues for the second revolution in physics in the twenty first century: The monograph Mathai/Rathie contains a generalization of Shannon's information entropy that also supports Tsallis' successful but still hotly debated generalization of Boltzmann-Gibbs entropy for non-extensive statistical mechanics and the maximum entropy framework for non-exponential distributions as alluded to in the Mathai/Pederzoli monograph. Today the Mathai/Saxena

monograph on Fox' H-function is considered essential for many mathematicians and statisticians as the "bible" for the application of generalized hypergeometric functions in the natural sciences and central to deriving closed-form solutions of fractional master, Fokker-Planck, Langevin, and reaction-diffusion equations. Mathai is still working on the analysis of data and problems from solar neutrino observatories, operated by international teams of scientists in the US, Japan, and Europe to unfold the mystery that the solar neutrino flux seems to indicate non-Gaussian statistics showing harmonic modulation that might be modeled by a fractional dissipative first-order stochastic differential equation. This issue is not yet settled neither in (astro)physics nor in mathematics and statistics.

An updated detailed report on the status of implementation of Mathai's research programme, outlined in the above three monographs, is contained in A. M. Mathai, R. K. Saxena, and H. J. H.: *The H-function: Theory and Applications*, Springer, New York 2010.

Topic #4 ——— OP – SF Net 22.3 ——— May 15, 2015

From: Howard Cohl (howard.cohl@nist.gov)

Subject: Erratum to "Bessel Functions" (2014) by K. B. M. Nambudiripad

The following errors and misprints have been detected in the monograph of Nambudiripad (2014) *Bessel Functions*, Alpha Science International Ltd., Oxford, U.K.¹

Comments

- p. 2.4, (v), p. A.1, (iv). The outdated notation $\Pi(n)$ should be replaced with $n!$.
- p. 6.4, (6.6) & p. A.8. The term

$$\left[1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{m} + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n+m} \right]$$

in the infinite series expansion of $K_n(x)$ is incorrect. It's missing a sum contribution of 1 and is meaningless for $m = 0$. One should replace this term with a sum of harmonic numbers $[H_m + H_{n+m}]$, where $H_0 := 0$ and $H_p := \sum_{k=1}^p \frac{1}{k}$ for $p \geq 1$.

- p. 6.4, Section 6.1.2. In the string of 4 equalities evaluating $J_\nu(ix)$, the third and fourth equalities are not equal to $J_\nu(ix)$. However it is true (and follows from the derivation) that $I_\nu(x) = i^{-\nu} J_\nu(ix)$.
- p. 6.7, (6.12). Because of the previous string of sign errors in this section (see below), and a missing factor of two, one should have

$$I_\nu''(r) = \frac{1}{4}[I_{\nu-2}(r) + I_\nu] + \frac{1}{4}[I_\nu(r) + I_{\nu+2}(r)] = \frac{1}{4}[I_{\nu-2}(r) + 2I_\nu(r) + I_{\nu+2}(r)].$$

- p. 7.5, (7.13). It is not true that $-i$ can be represented as $(\sqrt{i})^2$.
- p. 7.5, between (7.16) & (7.17). The statement $i^{1/2} = (\sqrt{-1})^{1/2} = i^{3/2}$ is not true. Furthermore, $i^{1/2}$ does not equal $i^{3/2}$.

¹Errata for this book is not intended to imply recommendation or lack thereof by the National Institute of Standards and Technology. Opinions expressed are those of the author.

- p. 7.6, line -1. The parenthetical sentence “(The order, ν , is itself a fraction, in general.)” is incorrect.
- p. 9.13, line -2. Remove ‘.’ at end of line.
- p. A.9. Replace “The I functions” with “The modified Bessel functions of the first kind”.
- p. A.9. Replace “Bessel functions J ” with “Bessel functions J_ν ”.
- p. C.{2,4,5,6,7,8}. The author gives formulas for $x > \{15.9, 15.9, \{15.9, 10\}, 10, 10, 10\}$ in Appendix C, when entries are not given in this range of x . There is no way to compare these formulae with numerical values. Furthermore, there is no discussion on their numerical comparison. No motivation is explained as to why these formulas are inserted or where they originated from. Since these are approximate formulas, the reader does not understand why $>$ is sometimes used and \geq used other times.
- p. C.5, Table C.9, Entry ($x = 0, Y_1(x)$). Remove typographical error “box”.

Replacements

Location	For	Read
p. 2.3, Fig. 2.1.	J_0, J_1, J_2, J_3	J_1, J_2, J_3, J_4
p. 2.5, Table 2.1.	$J_0 x$	$J_0(x)$
p. 3.7, (3.7).	$J_0 \left[\right.$	$J_0 - \left[\right.$
p. 3.12, Section 3.4.	$H_0^{(1)}, H_0^{(2)}$	$H_0^{(1)}(x), H_0^{(2)}(x)$
p. 4.2, line 4.	$\Gamma(x)$	$x\Gamma(x)$
p. 4.6, line 2.	$\frac{\sqrt{2}}{\sqrt{2}\Gamma(\frac{1}{2})}$	$\frac{\sqrt{x}}{\sqrt{2}\Gamma(\frac{1}{2})}$
p. 4.3–4.4, (4.4).	$(n - m + 1)!$	$(n - m - 1)!$
p. 4.3–4.4, (4.4).	$(-1)^m$	$\frac{1}{\pi}(-1)^m$
p. 4.6, line 3.	$\frac{\sqrt{2}}{\sqrt{2}\cdot\sqrt{\pi}}$	$\frac{\sqrt{x}}{\sqrt{2}\sqrt{\pi}}$
p. 5.2, eq. 2.	$e_i \cdot e_j$	$e_i \cdot e_j$
p. 5.2, eq. 3.	e_i, e_j	$\langle e_i, e_j \rangle$ or $e_i \cdot e_j$
p. 5.3.	$i = 0$	$i = 1$
p. 5.3, line -1.	$\neq 0$ if $m = n$.	$= 0$ if $m = n$.
p. 5.4, Section 5.1.4.	$\phi_1, \phi_2, \phi_3, \dots$	$\phi_0, \phi_1, \phi_2, \dots$
p. 5.4, Section 5.1.4.	$\langle \phi_m(x) \phi_n(x) \rangle$	$\langle \phi_m(x), \phi_n(x) \rangle$
p. 5.4, Section 5.1.4.	if $m \neq n \neq 0$ if $m = n$.	if $m \neq n$ and $\neq 0$ if $m = n$.
p. 5.12.	$R(\nu) > -1$	$\text{Re}(\nu) > -1$
p. 5.17, Line -4.	cm	c_m
p. 5.21, Line 2.	xi	ξ
p. 6.3, Fig. 6.3.	$I_2(x)$ in lower right-hand side	$I_3(x)$
p. 6.4, (6.6).	$\left(\frac{\pi}{2}\right)^{n+m}$	$\left(\frac{x}{2}\right)^{n+2m}$
p. 6.6, (6.8).	–	+
p. 6.6, Section 6.1.4, eq. 1.	–	+
p. 6.6, (6.9)–(6.11).	–	+
p. 6.8, line 2.	$m = 0$	$m = 1$

p. 6.8.	$\frac{d}{dr} [r^{-\nu} I_{\nu}(r)] = -r^{-\nu} I_{\nu+1}(r)$	$\frac{d}{dr} [r^{-\nu} I_{\nu}(r)] = r^{-\nu} I_{\nu+1}(r)$
p. 6.8, line -1.	$\frac{e^{-x}}{\sqrt{2\pi x}}$	$\sqrt{\frac{\pi}{2x}} e^{-x}$
p. 6.9, eq. 2.	$(n = 0, 1, 2, \dots)$	$(n = 1, 2, 3, \dots)$
p. 6.14, (6.17a).	$I_0(\alpha.1)$	$I_0(\alpha\alpha)$
p. 7.1, (7.2).	I_0	I_0
p. 7.5, between (7.13) & (7.14).	$(-1)^{1/2}$ or $\sqrt{-i}$	$(-i)^{1/2}$ or $\sqrt{-i}$
p. 7.5, between (7.14) & (7.15).	$(-i)^{1/2}$ or \sqrt{i}	$(i)^{1/2}$ or \sqrt{i}
p. 7.6.	$i^{3m} = -1$ for $m = 1, 5, 9, \dots$	$i^{3m} = -i$ for $m = 1, 5, 9, \dots$
p. 7.6.	$i^{3m} = -i$ for $m = 2, 6, 10, \dots$	$i^{3m} = -1$ for $m = 2, 6, 10, \dots$
p. 7.6, eq. 4.	$2l + i$	$2l + 1$
p. 8.19.	$d \left[\left(\frac{\omega\mu}{\rho} \right)^{-1/2} \right]$	$d = \left[\frac{\omega\mu}{\rho} \right]^{-1/2}$ or $d \left(\left[\frac{\omega\mu}{\rho} \right]^{-1/2} \right)$
p. 8.19, Fig. 8.3.	Jz	Jz
p. 9.12, line 7.	Bessel	BesselJ
p. 9.12, line -7.	Beselj	BesselJ
p. 9.18, Fig. 9.7.	$x^2 e^x$	$x^2 e^x$
p. 10.5.	$y = y(x) = t^2$	$y = y(t) = t^2$
p. 10.5.	$\frac{y[x(t)]}{dt} = \frac{dy_1}{dt}$	$\frac{dy[x(t)]}{dt} = \frac{dy_1}{dt}$
p. 10.14, Section 10.3.4, line 2.	eigenvalues are	eigenvalues are
p. 10.17, eq. 1.	$K_n(x) \approx \frac{e^{-x}}{\sqrt{2\pi x}}$	$K_n(x) \approx \sqrt{\frac{\pi}{2x}} e^{-x}$
p. 10.24, eq. -1.	$a_n P_n$	$a_n P_n(x)$
p. 10.27, eq. -1.	$a_n H_n$	$a_n H_n(x)$
p. 10.28, Orthogonality.	T_n	$T_n(x)$
p. 10.28, eq. -2.	$a_n T_n$	$a_n T_n(x)$
p. 10.28, eq. -1, $a_n, n \geq 1$.	$f(x)$	$f(x) T_n(x)$
p. 10.29, eq. -2.	$a_n U_n$	$a_n U_n(x)$
p. 10.29, eq. -1, a_n .	$f(x)$	$f(x) U_n(x)$
p. A.1, (ii).	$J(x)$	$J_{\nu}(x)$
p. A.1, (ii).	arguments x	arguments $x > 0$
p. A.2, (vi).	$J(0)$	$J_{\nu}(0)$
p. A.2, (x).	J_{-n}	$J_{-n}(x)$
p. A.3, Fig. A.1.	J_0, J_1, J_2, J_3	J_1, J_2, J_3, J_4
p. A.4, (A.4).	$(-1)^m$	$(-1)^m \frac{(x^2/4)^m}{(m!)^2}$
p. A.4, eq. 2.	$n \rightarrow \infty$	$m \rightarrow \infty$
p. A.4, eq. -2.	$\sin \nu x$	$\sin(\nu\pi)$
p. A.5.	$Y(x)$	$Y_n(x)$
p. A.5, (iii).	arguments x	arguments $x > 0$
p. A.5.	$H_0^2(x)$	$H_0^{(2)}(x)$
p. A.5.	$H_{\nu}^{(1)}(x) = J_{\nu}(x) + iY_{\nu}(x)$	$H_{\nu}^{(1)}(x) = J_{\nu}(x) + iY_{\nu}(x)$
p. A.8, eq. 3.	$\left(\frac{x}{2} \right)^{n+m}$	$\left(\frac{x}{2} \right)^{n+2m}$
p. A.8, eq. 5.	$K_n(x)$	$K_{\nu}(x)$

p. A.8, line -3.	$\frac{e^x}{\sqrt{x}}$	$\frac{e^x}{\sqrt{2\pi x}}$
p. A.8, line -3.	$\frac{e^{-x}}{\sqrt{x}}$	$\sqrt{\frac{\pi}{2x}}e^{-x}$.
p. A.8, line -2.	I_n	$I_n(x)$
p. A.13, line 5.	$\Gamma(\nu + m + 1)!$	$\Gamma(\nu + m + 1)$
p. A.13, line 8.	$m = 0$	$m = 1$
p. A.13, line 9.	K'_0	$K'_0(x)$
p. A.13, line 11.	$K_\nu(x) \approx \frac{e^{-x}}{2\pi x}$	$K_\nu(x) \approx \sqrt{\frac{\pi}{2x}}e^{-x}$
p. A.13, line -7.	$\frac{d}{dr} [r^{-\nu} I_\nu(r)] = -r^{-\nu} I_{\nu+1}(r)$	$\frac{d}{dr} [r^{-\nu} I_\nu(r)] = r^{-\nu} I_{\nu+1}(r)$
p. A.13, line -7.	$\frac{d}{dx} [x^{-\nu} I_\nu(x)] = -x^{-\nu} I_{\nu+1}(x)$	$\frac{d}{dx} [x^{-\nu} I_\nu(x)] = x^{-\nu} I_{\nu+1}(x)$
p. A.14, line 2.	$\frac{d}{dr} [r^{-\nu} I_\nu(r)] = -r^{-\nu} I_{\nu+1}(r)$	$\frac{d}{dr} [r^{-\nu} I_\nu(r)] = r^{-\nu} I_{\nu+1}(r)$
p. A.14, line 2.	$\frac{d}{dx} [x^{-\nu} I_\nu(x)] = -x^{-\nu} I_{\nu+1}(x)$	$\frac{d}{dx} [x^{-\nu} I_\nu(x)] = x^{-\nu} I_{\nu+1}(x)$
p. C.1, Title.	Numerical	Numerical

Topic #5 ——— OP – SF Net 22.3 ——— May 15, 2015

From: Pavel Winternitz (wintern@CRM.UMontreal.CA) and Decio Levi (levi@roma3.infn.it)
 Subject: SIDE 12 and preceding Abecederian of SIDE (ASIDE) Summer School

Centre de Recherches Mathématiques (CRM), Université de Montréal, will hold SIDE 12, the twelfth in a series of biennial conferences devoted to Symmetries and Integrability of Difference Equations and related topics such as ordinary and partial difference equations, analytic difference equations, orthogonal polynomials and special functions, symmetries and reductions, difference geometry, integrable discrete systems on graphs, integrable dynamical mappings, discrete Painlevé equations, singularity confinement, algebraic entropy, complexity and growth of multivalued mapping, representations of affine Weyl groups, quantum mappings and quantum field theory on the space-time lattice and physical applications. SIDE 12 will take place at Hôtel Le Chanteclerc, Saint Adèle, Québec, Canada from July 3 through July 9, 2016.

SIDE 12 will be preceded by the Abecederian of SIDE (ASIDE) summer school for young participants (graduate students and postdoctoral fellows) which will be held at Centre de Recherches Mathématiques, Université de Montréal from June 27 through July 1, 2016. The goal of ASIDE is to gather early career researchers, post-doctoral fellows and PhD students for a week-long series of mini-courses taught by their peers. These introductory mini-courses will cover the topics represented at the SIDE meeting. The Organizing Committee for ASIDE 12 consists of:

- Vincent Genest, Chair (CRM)
- Ferenc Balogh (Concordia)
- Rapahël Rebelo (CRM)
- Bart Vlaar (Nottingham)

The website for SIDE 12 is <http://www.crm.umontreal.ca/2016/SIDE12/index.php>.

From: Bruce C. Berndt (berndt@math.uiuc.edu), Wen-Ching Winnie Li (wli@math.psu.edu), and J. R. Smart (smart@math.wisc.edu)
Subject: Joseph Lehner Obituary

Joseph Lehner was born in New York City on October 29, 1912 to Louis and Rachel (Rosenblum), and died in Haverford, PA on August 5, 2013. Lehner's scientific work was primarily concerned with automorphic forms for discontinuous groups and related number theory.

In 1964, Lehner published one of the first treatises in English on automorphic forms [3], which included both the historical beginnings of the subject as well as later developments, due especially to E. Hecke, H. Petersson, H. Rademacher, and C. L. Siegel. This book has become a classic, and has served as an introductory book to automorphic forms for many generations.

Lehner's seminal joint paper with Oliver Atkin [1] introduced a method to study the arithmetic of modular forms which fundamentally changed the field. The idea was to collect together forms for congruence subgroups $\Gamma_0(m)$ for all $m \geq 1$. For forms of $\Gamma_0(m)$, they distinguished "oldforms," which arise naturally from cusp forms of $\Gamma_0(n)$ with n dividing m , and "newforms," which are forms which genuinely live in $\Gamma_0(m)$. They pioneered the "theory of newforms," which reduces the study of the arithmetic of modular forms for $\Gamma_0(m)$ to that of newforms, and they studied the arithmetic of newforms in detail. The Atkin-Lehner theory of newforms was extended by T. Miyake [5] to forms for congruence subgroups $\Gamma_1(n)$ and completed by the second author [4] for forms for all congruence subgroups of $SL_2(\mathbb{Z})$. The newform theory naturally led to automorphic forms and automorphic representations for GL_2 in the adelic setting, and it therefore has had a far-reaching impact in the theory of automorphic forms. To date, researchers have been extending their ideas to study newforms for orthogonal groups. The Atkin-Lehner operators introduced in their paper [1] also play a fundamental role in the study of elliptic and Shimura modular forms.

Lehner earned his BS at New York University (1938) and his MA (1939) and PhD (1941) at the University of Pennsylvania under the supervision of Hans Rademacher, whom he was later to describe as his teacher and friend. Lehner was the 6th of Rademacher's 21 doctoral students, and the third author of this obituary is the second of Lehner's four doctoral students.

Lehner began his academic career as an instructor at Cornell in 1941–1943. In 1943, he joined Kellex Corporation, which was involved with the design and initial operation of the K25 diffusion plant at Oak Ridge (Manhattan Project). His group, which included Manson Benedict and Elliott Montroll, developed a highly advanced mathematical theory of how a cascade of approximately four thousand stages would respond to time-dependent disturbances in its performance – whether a cascade would magnify the disturbance, leading ultimately to a disruptive or destructive surge in pressure, or whether the cascade would damp out these fluctuations as it went through successive stages. Lehner and Montroll were the principal mathematicians who consequently solved systems of time and space dependent partial differential equations, which led to an efficient method for producing U-235 at a newly constructed plant at Oak Ridge. For further information about contributions made by Lehner, nuclear engineer Manson Benedict (recipient of the National Medal of Science in 1975), and their group, consult an informative interview with Bene-

dict [2]. After three years, Lehner left Kellex to become head of the mathematics group at Hydrocarbon Research Inc.

Lehner returned briefly to academia as an Associate Professor at the University of Pennsylvania in 1949–1952, before joining Los Alamos Scientific Laboratory for five years, followed by two additional years as a consultant. At Los Alamos, he further worked on the Manhattan Project, developing theories of diffusion cascades and neutron transport. In 1957, he returned to academic pursuits to become Professor at Michigan State University for six years, which were followed by nine years at the University of Maryland. His last position was as Mellon Professor at the University of Pittsburgh from 1972 to 1980.

Lehner published two books and, according to MathSciNet, a total of 67 research papers with five arising from his work in defense. Furthermore, he published 16 problems and/or solutions in the *American Mathematical Monthly*. Including Atkin, Lehner had 11 coauthors, with Marvin Knopp and Morris Newman being his most frequent collaborators.

Before meeting Joseph's father, Lehner's mother Rachel had been married to Samuel Rapport (abbreviated from the original Rappaport), giving birth to five children before her husband died. She then married Joe's father, who had had five children by his previous marriage. The family then moved to Massachusetts. Their first child was born at home and soon died. With her second pregnancy, her sister forced her to return to New York City to have her baby at a hospital; this baby was Joe Lehner. For her third and last pregnancy, Joe's mother again chose to have the birth at home, and the baby died. When Joe was three years old, his parents divorced, and he and his family moved back to New York. As a teenager in New York City, he built a radio, participated on the debate team in high school, and helped one of his half-brothers in his law office, sometimes appearing in court in place of his half-brother. Lehner graduated from high school in 1928 at the age of 15. After a short stay at an experimental college in Wisconsin, he returned home to take courses in mathematics and science at New York University (NYU). While doing so, he obtained a position keeping records at Macy's which was to last about seven years. Due to the Depression, he lost his job at Macy's but eventually landed a position on a WPA project, writing his first published paper, "Production, Employment, and Productivity in 57 Selected Industries." This project took him to Philadelphia in 1936, where he took courses at the University of Pennsylvania that could be transferred to NYU, from which he earned a BS in 1938. Lehner met his future wife, Mary Beluch, in the University of Pennsylvania Library, where she worked as a WPA proofreader. They married on August 17, 1938 and had 65 years together. Mary's editorial work continued in helping Joe prepare his two books. They had one daughter, Zheindl, nee Janet.

References

- [1] A. O. L. Atkin and J. Lehner, *Hecke operators on $\Gamma_0(m)$* , Math. Ann. 185 (1970), 134–160.
- [2] M. Benedict, *Interview with Center for Oral History*, January 24, 1991, Chemical Heritage Foundation, Philadelphia.
- [3] J. Lehner, *Discontinuous Groups and Automorphic Functions*, Mathematical Surveys, No. VIII, American Mathematical Society, Providence, RI, 1964.
- [4] W.-C. W. Li, *Newforms and functional equations*, Math. Ann. 212 (1975), 285–315.
- [5] T. Miyake, *On automorphic forms on GL_2 and Hecke operators*, Ann. Math. 94 (1971), 174–189.

From: OP–SF NET Editors
Subject: Preprints in arXiv.org

The following preprints related to the fields of orthogonal polynomials and special functions were posted or cross-listed to one of the subcategories of arXiv.org, mostly during March and April 2015.

<http://arxiv.org/abs/1501.00138>

Generalization of Lambert W–function, Bessel polynomials and transcendental equations
Giorgio Mugnaini

<http://arxiv.org/abs/1503.02147>

Padé interpolation and hypergeometric series
Masatoshi Noumi

<http://arxiv.org/abs/1503.02226>

Fourier–Bessel heat kernel estimates
Jacek Malecki, Grzegorz Serafin, Tomasz Zorawik

<http://arxiv.org/abs/1503.03020>

Some inequalities for the trigamma function in terms of the digamma function
Feng Qi, Cristinel Mortici

<http://arxiv.org/abs/1503.04016>

The asymptotics of a generalised Beta function
R. B. Paris

<http://arxiv.org/abs/1503.04786>

Darboux transformations for multivariate orthogonal polynomials
Gerardo Ariznabarreta, Manuel Mañas

<http://arxiv.org/abs/1503.04972>

Some new inequalities for the gamma function
Xiaodong Cao

<http://arxiv.org/abs/1503.05003>

Darboux transformations for CMV matrices
M.J. Cantero, F. Marcellán, L. Moral, L. Velázquez

<http://arxiv.org/abs/1503.05387>

Asymptotic properties of biorthogonal polynomials systems related to Hermite and Laguerre polynomials
Yan Xu

<http://arxiv.org/abs/1503.05391>

Generalizations of Some Inequalities for the p –Gamma, q –Gamma and k –Gamma Functions
Kwara Nantomah, Edward Prempeh

<http://arxiv.org/abs/1503.05399>

The monic Laguerre polynomials preserve real-rootedness
Praveen S. Venkataramana

<http://arxiv.org/abs/1503.05402>

Orthogonal polynomials on the disk in the absence of finite moments
Peter C. Gibson

<http://arxiv.org/abs/1503.06290>

Nicholson-type integrals for products of parabolic cylinder functions involving Laplace transform of confluent hypergeometric functions
Ridha Nasri

<http://arxiv.org/abs/1503.07329>

The asymptotic expansion of a generalisation of the Euler-Jacobi series
R.B. Paris

<http://arxiv.org/abs/1503.07404>

On (p,q) -analogue of Bernstein Operators
M. Mursaleen, Khursheed J. Ansari, Asif Khan

<http://arxiv.org/abs/1503.07897>

Characterization of the Generalized Chebyshev Polynomials of First Kind
Mohammad A. AlQudah

<http://arxiv.org/abs/1503.00971>

Selberg integrals, Askey-Wilson polynomials and lozenge tilings of a hexagon with a triangular hole
Hjalmar Rosengren

<http://arxiv.org/abs/1503.02833>

Special polynomials related to the supersymmetric eight-vertex model: A summary
Hjalmar Rosengren

<http://arxiv.org/abs/1503.07393>

Generalized Chebyshev I-Bernstein bases transformation
Mohammad A. AlQudah

<http://arxiv.org/abs/1504.00883>

Asymptotic expansions of zeros of a partial theta function
Vladimir Petrov Kostov

<http://arxiv.org/abs/1504.00885>

On the spectrum of a partial theta function
Vladimir Petrov Kostov

<http://arxiv.org/abs/1504.01141>

Generalized Tschebyscheff polynomials of the second kind
Mohammad A. AlQudah

<http://arxiv.org/abs/1504.01238>

On convergence of basic hypergeometric series

Toshio Oshima

<http://arxiv.org/abs/1504.01524>

A property of a partial theta function

Vladimir Petrov Kostov

<http://arxiv.org/abs/1504.01658>

Tschebyscheff of the second kind and Bernstein polynomials change of bases

Mohammad A. AlQudah

<http://arxiv.org/abs/1504.01769>

Zeros of random functions generated with de Branges kernels

Jorge Antezana, Jordi Marzo, Jan-Fredrik Olsen

<http://arxiv.org/abs/1504.01946>

Orthogonal Polynomials appearing in SIE grid representations

Stefan Hilger

<http://arxiv.org/abs/1504.03140>

On Some Hypergeometric Summations II. Duality and Reciprocity

Katsunori Iwasaki

<http://arxiv.org/abs/1504.03297>

Differential orthogonality: Laguerre and Hermite cases with applications

J. Borrego-Morell, H. Pijeira-Cabrera

<http://arxiv.org/abs/1504.03825>

On an Orbifold Hamiltonian Structure for the First Painlevé Equation

Katsunori Iwasaki, Shu Okada

<http://arxiv.org/abs/1504.03968>

Christoffel functions with power type weights

Tivadar Danka, Vilmos Totik

<http://arxiv.org/abs/1504.05786>

On the double zeros of a partial theta function

Vladimir Petrov Kostov

<http://arxiv.org/abs/1504.05976>

Asymptotics of orthogonal polynomials generated by a Geronimus perturbation of the Laguerre measure

Alfredo Deaño, Edmundo J. Huertas, Pablo Román

<http://arxiv.org/abs/1504.06072>

A Lagrangian Method for Deriving New Indefinite Integrals of Special Functions

John T. Conway

<http://arxiv.org/abs/1504.06245>

Christoffel functions on Jordan curves with respect to measures with jump singularity
Tivadar Danka

<http://arxiv.org/abs/1504.06448>

Turán type inequalities for regular Coulomb wave functions
Árpád Baricz

<http://arxiv.org/abs/1504.07297>

The kissing polynomials and their Hankel determinants
Alfredo Deaño, Daan Huybrechs, Arieh Iserles

<http://arxiv.org/abs/1504.08144>

Fractional integral and generalized Stieltjes transforms for hypergeometric functions as transmutation operators
Tom H. Koornwinder

<http://arxiv.org/abs/1504.08345>

A Method of Proving a Class of Trigonometric Inequalities Based on Approximations of the Sine and Cosine Functions by Maclaurin Polynomials
Branko Malešević, Milica Makragić

<http://arxiv.org/abs/1504.00440>

Hankel Determinant Approach to Generalized Vorob'ev–Yablonski Polynomials and their Roots
Ferenc Balogh, Marco Bertola, Thomas Bothner

<http://arxiv.org/abs/1504.01748>

Properties of the zeros of generalized basic hypergeometric polynomials
Oksana Bihun, Francesco Calogero

<http://arxiv.org/abs/1504.04539>

A Riemann–Hilbert problem for equations of Painlevé type in the one matrix model with semi-classical potential
Max R. Atkin

<http://arxiv.org/abs/1504.07696>

On a family of polynomials related to $(2, 1) = (3)$
Wadim Zudilin

<http://arxiv.org/abs/1504.04555>

Hypergeometric/Difference–Equation–Based Separability Probability Formulas and Their Asymptotics for Generalized Two–Qubit States Endowed with Random Induced Measure
Paul B. Slater

<http://arxiv.org/abs/1504.03408>

Weighted Hurwitz numbers and hypergeometric ${}_2F_1$ -functions: an overview
J. Harnad

<http://arxiv.org/abs/1503.07962>

CM periods, CM regulators and hypergeometric functions, I
Masanori Asakura, Noriyuki Otsubo

<http://arxiv.org/abs/1503.08894>

CM periods, CM regulators and hypergeometric functions, II
Masanori Asakura, Noriyuki Otsubo

<http://arxiv.org/abs/1503.05519>

Hypergeometric series, modular linear differential equations, and vector-valued modular forms
Cameron Franc, Geoffrey Mason

<http://arxiv.org/abs/1504.03438>

A complete Riemann zeta distribution and the Riemann hypothesis
Takashi Nakamura

<http://arxiv.org/abs/1504.08058>

On Littlewood Polynomials
Raphael Renya, Steven Damelin

<http://arxiv.org/abs/1504.07944>

Jacobi polynomials and congruences involving some higher-order Catalan numbers and binomial coefficients
Khodabakhsh Hessami Pilehrood, Tatiana Hessami Pilehrood

<http://arxiv.org/abs/1504.05801>

Some identities of symmetry for q -Euler polynomials under the symmetric group of degree n arising from fermionic p -adic q -integrals on \mathbb{Z}_p
Dmitry V. Dolgy, Dae San Kim, Taekyun Kim

<http://arxiv.org/abs/1504.05499>

Some identities of symmetry for q -Bernoulli polynomials under symmetric group of degree n
Dae san Kim, Taekyun Kim

<http://arxiv.org/abs/1504.05355>

CLT for the zeros of Kostlan Shub Smale random polynomials
Federico Dalmao

<http://arxiv.org/abs/1504.03856>

Sparse multivariate polynomial interpolation in the basis of Schubert polynomials
Priyanka Mukhopadhyay, Youming Qiao

<http://arxiv.org/abs/1504.03705>

Racah Polynomials and Recoupling Schemes of $\mathfrak{su}(1, 1)$
S. Post

<http://arxiv.org/abs/1504.03311>

Quantum Hurwitz numbers and Macdonald polynomials
J. Harnad

<http://arxiv.org/abs/1504.01227>

Chebyshev polynomials, moment matching, and optimal estimation of the unseen
Yihong Wu, Pengkun Yang

<http://arxiv.org/abs/1503.07076>

h-Transforms and Orthogonal Polynomials
Dominique Bakry (IMT), Olfa Zribi (IMT)

<http://arxiv.org/abs/1503.06376>

Expected number of real zeros for random linear combinations of orthogonal polynomials
D. S. Lubinsky, I. E. Pritsker, X. Xie

<http://arxiv.org/abs/1503.04787>

Pre-sequences of matrix orthogonal polynomials
Juan Tirao

<http://arxiv.org/abs/1503.03497>

Pseudo prolate spheroidal functions
Luís Daniel Abreu, João M. Pereira

<http://arxiv.org/abs/1504.07971>

Longitudinal electric current in classical collisional Maxwellian plasma, inducted by transversal electromagnetic wave
A. V. Latyshev, A. A. Yushkanov

Topic #8 ——— OP – SF Net 22.3 ——— May 15, 2015

From: OP–SF NET Editors

Subject: About the Activity Group

The SIAM Activity Group on Orthogonal Polynomials and Special Functions consists of a broad set of mathematicians, both pure and applied. The Group also includes engineers and scientists, students as well as experts. We have around 115 members scattered about in more than 20 countries. Whatever your specialty might be, we welcome your participation in this classical, and yet modern, topic. Our WWW home page is:

<http://math.nist.gov/opsf>

This is a convenient point of entry to all the services provided by the Group. Our Webmaster is Bonita Saunders (bonita.saunders@nist.gov).

The Activity Group sponsors OP–SF NET, an electronic newsletter, and SIAM–OPSF (OP–SF Talk), a listserv, as a free public service; membership in SIAM is not required. OP–SF NET is transmitted periodically through a post to OP–SF Talk. The OP–SF NET Editors are Howard Cohl (howard.cohl@nist.gov) and Kerstin Jordaan (kerstin.jordaan@up.ac.za).

Back issues of OP–SF NET can be obtained at the websites:

<https://staff.fnwi.uva.nl/t.h.koornwinder/opsfnet>

<http://math.nist.gov/~DLozier/OPSFnet>

SIAM-OPSF (OP-SF Talk), which was recently moved to a SIAM server, facilitates communication among members and friends of the Activity Group. To subscribe or to see a link the archive of all messages, go to <http://lists.siam.org/mailman/listinfo/siam-OPSF> and follow the instructions under the sub-heading "Subscribing to SIAM-OPSF". To contribute an item to the discussion, send e-mail to siam-opsf@siam.org. The moderators are Bonita Saunders (bonita.saunders@nist.gov) and Diego Dominici (dominicd@newpaltz.edu).

SIAM has several categories of membership, including low-cost categories for students and residents of developing countries. In addition, there is the possibility of reduced rate membership for the members of several societies with which SIAM has a reciprocity agreement; see <http://www.siam.org/membership/individual/reciprocal.php> For current information on SIAM and Activity Group membership, contact:

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3600 University City Science Center
Philadelphia, PA 19104-2688 USA
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e-mail: service@siam.org
WWW : <http://www.siam.org>

Topic #9 ——— OP – SF Net 22.3 ——— May 15, 2015

From: OP-SF NET Editors

Subject: Submitting contributions to OP-SF NET and SIAM-OPSF (OP-SF Talk)

To contribute a news item to OP-SF NET, send e-mail to one of the OP-SF Editors howard.cohl@nist.gov or kerstin.jordaan@up.ac.za.

Contributions to OP-SF NET 22.4 should be sent by July 1, 2015.

OP-SF NET is an electronic newsletter of the SIAM Activity Group on Special Functions and Orthogonal Polynomials. We disseminate your contributions on anything of interest to the special functions and orthogonal polynomials community. This includes announcements of conferences, forthcoming books, new software, electronic archives, research questions, and job openings as well as news about new appointments, promotions, research visitors, awards and prizes. OP-SF Net is transmitted periodically through a post to SIAM-OPSF (OP-SF Talk).

SIAM-OPSF (OP-SF Talk) is a listserv of the SIAM Activity Group on Special Functions and Orthogonal Polynomials, which facilitates communication among members, and friends of the Activity Group. See the previous Topic. To post an item to the listserv, send e-mail to siam-opsf@siam.org.

WWW home page of this Activity Group:

<http://math.nist.gov/opsf>

Information on joining SIAM and this activity group: service@siam.org

The elected Officers of the Activity Group (2014-2016) are:

Walter Van Assche, Chair
Jeff Geronimo, Vice Chair
Diego Dominici, Program Director
Yuan Xu, Secretary

The appointed officers are:

Howard Cohl, OP-SF NET co-editor

Kerstin Jordaan, OP-SF NET co-editor

Diego Dominici, OP-SF Talk moderator

Bonita Saunders, Webmaster and OP-SF Talk moderator

Thought of the month

“A ten minute nap in a colloquium is equivalent to one hour of a good night’s sleep.”

by Richard Askey