Secondary Spectrum Trading Market – Auction-Based Approach to Spectrum Allocation and Profit Sharing

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Outline

- Background
- Motivation
- Problem formulation
- Efficient vs. optimal mechanism
  - Generalized Branco's mechanism
- Incentive for cooperation among sellers
- Equitable profit sharing among sellers
  - Existence of nonempty core of cooperative game
  - Existence of equitable profit sharing scheme
- Conclusion

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Background (1)

- Inefficient spectrum allocation today
  - Conventional way
    - Static allocation by a government agency (e.g., Federal Communications Commission (FCC) in the U.S.)
  - Drawbacks
    - Hampers the entrance of a new service provider
      - Reduced competition
    - Under-utilized in many places

Background (2)

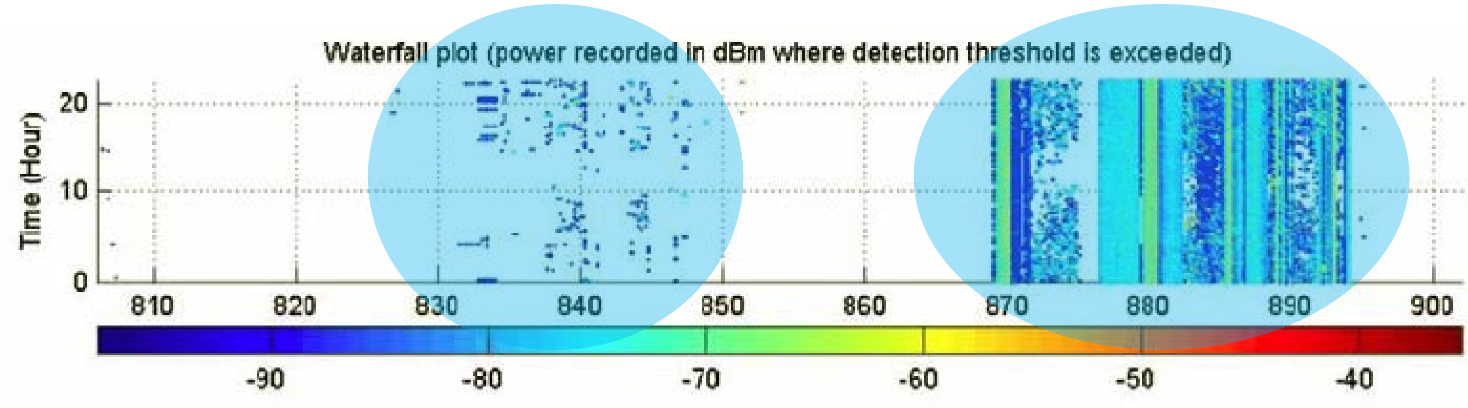
- Example of spectrum allocation (in the U.S.)
  - 614 ~ 806 MHz : Broadcasting (TV, channels 38-69)
  - 806 ~ 824 MHz : Pagers and public safety (uplink (e.g., T-GSM 810))
  - 824 ~ 849 MHz : Mobile phone (wireless comm. uplink)
  - 849 ~ 869 MHz : Pagers and public safety (downlink)
  - 869 ~ 894 MHz : Base station (wireless comm. downlink)

Current / Planned Technologies	Previous Technologies	Band	Frequency (MHz)
SMR iDEN, ESMR CDMA (future), ESMR LTE (future)		800	806-824 and 851-869
GSM, IS-95 (CDMA), 3G	AMPS, IS-136 (D-AMPS)	Cellular	824-849 and 869-894
GSM, IS-95 (CDMA), 3G, 4G	IS-136 (D-AMPS)	PCS	1,850–1,910 and 1,930–1,990
3G, 4G, MediaFlo, DVB-H		700 MHz	698-806
Unknown		1.4 GHz	1,392–1,395 and 1,432–1,435
3G, 4G		AWS	1,710–1,755 and 2,110–2,155
4G		BRS/EBS	2,496–2,690

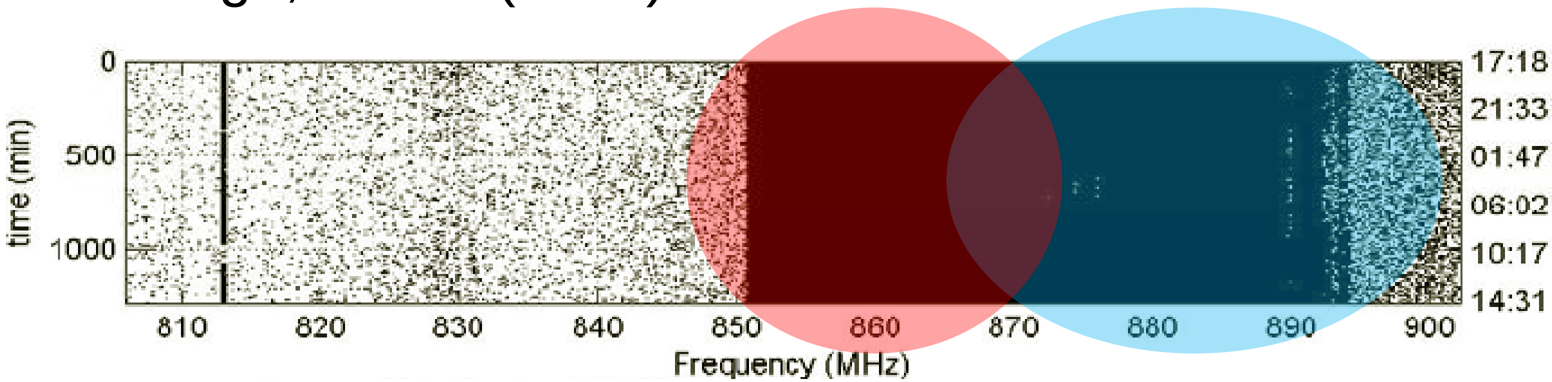
Source: http://en.wikipedia.org/wiki/Cellular_frequencies

Background (3)

- Limestone, Maine (2007)

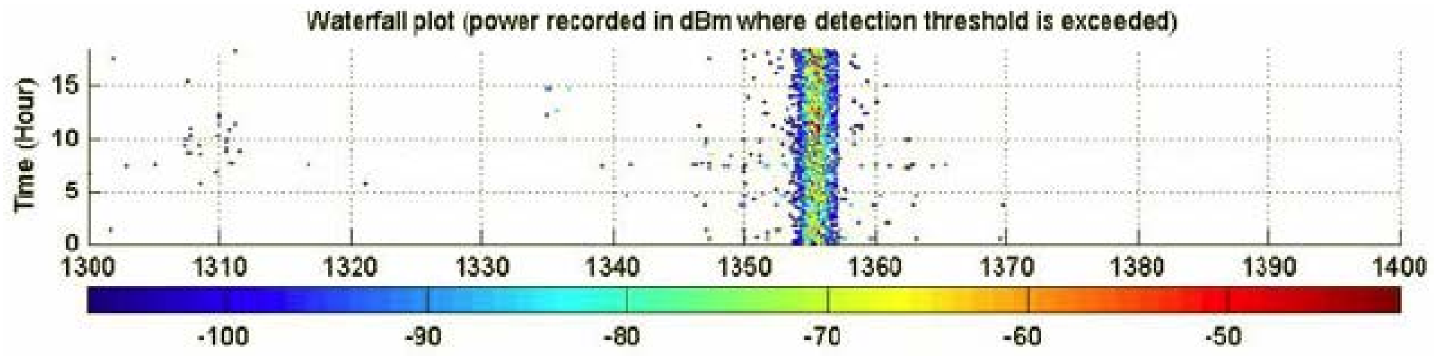


- Chicago, Illinois (2005)

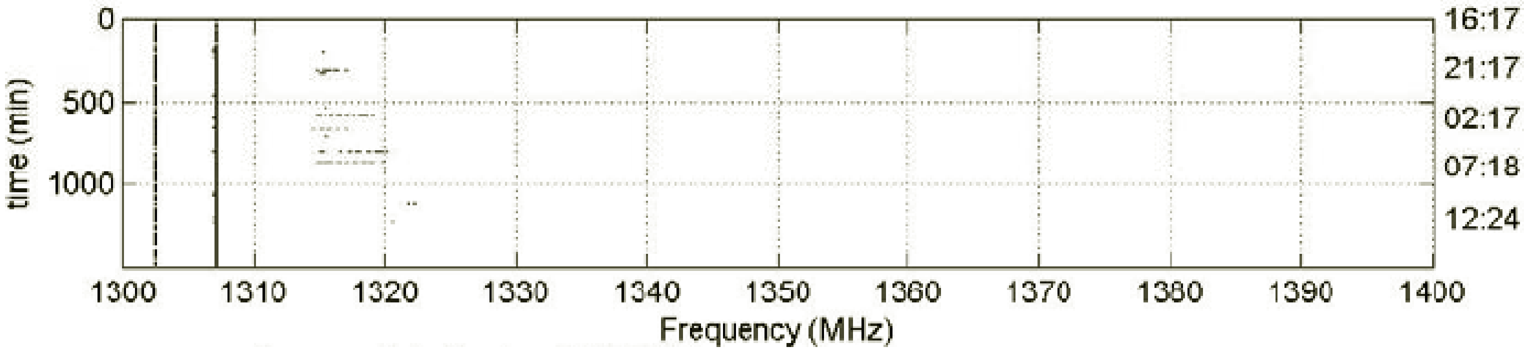


Background (4)

- Limestone, Maine (2007)



- Chicago, Illinois (2005)



Background (5)

- Lessons from the measurements
  - While spectrum is considered scarce (and expensive), allocated frequency bands are often under-utilized
- Natural Question – In light of rapidly increasing demand for spectrum
  - How can we increase frequency usage efficiency?
  - Is there any way to allow other users (who need the frequency) to use under-utilized frequency bands?

Background (6)

- Proposed approaches
  - Pack more users in frequency spectrum
    - Mobile Virtual Network Operators (MVNOs), e.g., Virgin Mobile USA, 7-Eleven Speak Out Wireless, AirLink mobile, Credo Mobile
      - Share spectrum or infrastructure with Mobile Network Operators (MNOs), e.g., AT&T, Sprint, Verizon, T-Mobile
  - Allow dynamic frequency access to unlicensed users (secondary users)
    - e.g., Cognitive Radio (CR)

Background (7)

- Mobile Virtual Network Operator (MVNO)
  - Business agreement to use the spectrum and infrastructure of licensed Mobile Network Operators (MNOs)
    - Examples
      - Virgin Mobile USA (MVNO) with Sprint Nextel (MNO)
      - Credo Mobile (MVNO) with Spring Nextel (MNO)
      - Firefly Mobile (MVNO) with AT&T (MNO)
  - Runs own cellular mobile service business with its own brand, pricing scheme, numbering resources, and featured services

Background (8)

- Cognitive Radio (CR):
  - Underlying technology : Software-Defined Radio (SDR)
  - CR users (CRUs) can
    - switch its radio access technology based on the availability and/or performance of available networks
    - use any available frequency band
  - CRUs often called unlicensed users

- **Key constraint:**
  - Licensed users shall not be affected by CRUs’ use of frequency band

Background (9)

- Proposed methods for honoring the constraint include
  - Frequency rental protocol
    - Primary provider (i.e., licensed user) broadcasts available frequency bands
    - CRUs request (and use those bands granted for use)
    - When a licensed user needs the frequency bands, it sends a signal to stop CRUs
  - Frequency sensing
    - CRUs continuously monitor the usage on frequency bands
    - If no activity is detected, use the bands
    - When activity is detected, stop using the bands
  - Interference temperature model
    - Use frequency bands while total interference level at licensed user receivers remains below a predefined threshold

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Motivation (1)

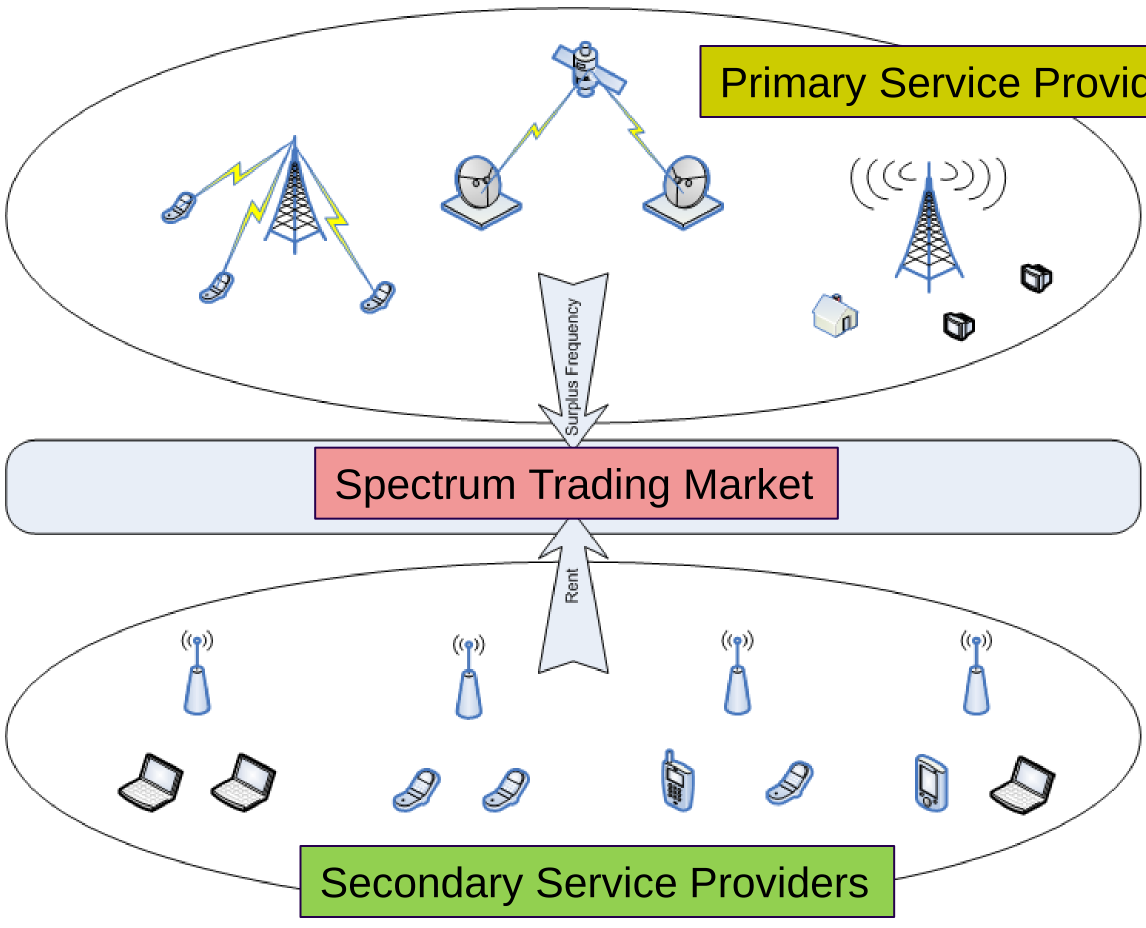
- Drawbacks of MVNOs
  - Low flexibility for under-utilized frequency
    - Constrained to use the same radio technologies employed by MNOs
    - Can provide only (almost) the same set of services as MNOs
- Research on CR
  - Most of previous studies focus on resource allocation among CRUs
  - Often assume CRUs can use the spectrum free of charge
    - Private primary service providers may not be so generous
      - Likely to demand a payment
  - Individual CRUs responsible for finding and using under-utilized frequency spectrum (especially under frequency sensing and interference temperature model)
    - **Uncoordinated** access/use of under-utilized spectrum

Motivation (2)

- Secondary trading market for spectrum trading (to marry the previous two)
  - What if secondary service providers (acting as middle men)
    - Have own infrastructure with dynamic frequency access capability at both access point and user equipment (UE)
    - Lease the spectrum from primary service providers (licensees)
    - Collect the service/usage fee from their customers (CRUs)
  - Can use under-utilized spectrum in a more efficient and organized manner
  - Can provide more services
    - Not tied to the same radio technologies as MNOs

Motivation (3)

- Model:



Motivation (4)

- Need to design a spectrum sharing and pricing scheme between the primary service providers (PSPs) and secondary service providers (SSPs)

Motivation (5)

- Propose an auction-based framework for secondary spectrum trading market
  - Offers a natural tool for spectrum trading
    - Strategies of buyers
    - Methods for exchange of information
    - Allocation and payment schemes

  - Well designed auction mechanisms have desirable properties
    - Efficiency and/or optimality
    - Incentive compatibility
    - Individual rationality

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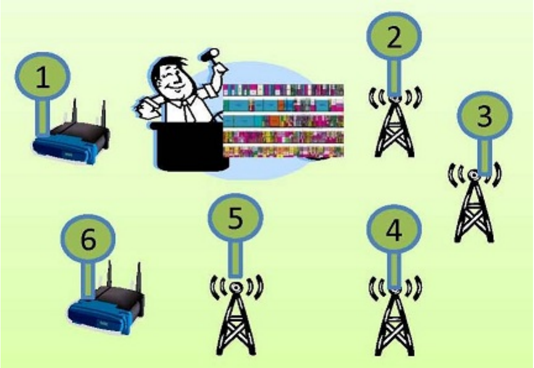
Problem formulation (1)

- In spectrum auction

Goods/Items:	Available frequency bands
Sellers:	Primary service providers
Buyers/Bidders:	Secondary service providers



- Frequency spectrum traded in a fixed unit
  - e.g., unit of 100 kHz
    Total available spectrum from a primary service provider: 1 MHz
    Primary service provider has 10 units of homogeneous good

- Frequency trading performed periodically or whenever needed

Problem formulation (2)

- Sellers – primary service providers
  - Each seller interested in lending (a portion of) under-utilized spectrum it owns in different regions (i.e., operating markets)
  - Available spectrum divided according to a fixed unit (e.g., 100 kHz)
  - Sellers free to cooperate among themselves and form *coalitions* to sell their spectrum together
  - Each seller has a value associated with each unit of frequency band it wishes to lend
    - Determines its reserve price
  - *Risk neutral* – wish to maximize expected profit (i.e., revenue minus its values for sold frequency bands)

Problem formulation (3)

- Buyers – secondary service providers
  - Interested in purchasing frequency bands in different regions/markets
  - Have private information – **type** of buyer *j* denoted by T_j
    - Has distribution $\mathcal{G}_j$ with density g_j
    - Value of the *k*-th frequency band won by buyer *j* given by $V_k^j(T_j)$
    - Independent and identically distributed (i.i.d.)
  - Interested in maximizing own expected payoffs
    - Payoff = total value from items won – price paid for the items

Problem formulation (4)

- Setup
  - Consider only a single market
  - $\mathcal{S}$ = set of primary service providers (sellers)
  - $\mathcal{B}$ = set of secondary service providers (buyers)
  - For each $s \in \mathcal{S}$, f_s denotes the number of frequency bands available for lease from seller s

Problem formulation (5)

- Seller:
  - Announces the list of frequency bands it wishes to lend and its reserve prices
  - May join other sellers to form a *coalition*
    - $\mathcal{P}$ - set of all possible partition of $\mathcal{S} = \{1, 2, \ldots, S\}$
    - Each coalition of sellers holds a *separate* auction, sharing information among coalition members

Problem formulation (6)

- Buyer:
  - Each buyer first chooses one seller and participates in the auction of a coalition to which the chosen seller belongs
    - Assume that the selection of a seller by a buyer does **not** depend on its type
  - Places a bid with the selected seller based on its private information

Problem formulation (7)

- Trading system: For each auction,
  - Identifies winning bids and allocates goods (allocation scheme)
  - Computes the prices to charge winning bids (pricing scheme)
  - Distributes the revenue from the auction to the sellers according to a fixed and *known* revenue sharing scheme (revenue sharing scheme)

Problem formulation (8)

- Goal: Design a secondary spectrum trading market that will encourage and support trading between potential sellers and buyers
  - Should provide potential sellers with proper incentives to make their under-utilized frequency bands available to prospective buyers
    - Sellers likely to feel more compelled to put their under-utilized frequency bands up for sale when they anticipate higher revenue
- Questions of interest
  - How can the sellers maximize their revenue from auction?
  - Could they increase their revenue by cooperating with each other?
    - Cooperation would be “possible” only if (i) sellers feel that they can benefit from it and (ii) the revenue is shared fairly in sellers’ views
  - Is it possible to sustain cooperation among sellers?
  - If so, how should the revenue be shared among them to maintain such cooperation?

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Efficient vs. optimal mechanisms (1)

- Efficient mechanism
  - Maximizes social welfare
    - Assigns the item(s) to the buyer(s) who value the item(s) most
  - Suitable for auction of the public asset
  - Well studied - buyers' strategies, allocation and payment rule
    - Well-known single item auctions
      - Dutch auction, English auction, first-price auction, second-price auction (Vickrey auction)
    - Well-known multiple item auctions
      - Discriminatory auction, uniform price auction, VCG mechanism
  - Designed for a single seller

Efficient vs. optimal mechanisms (1)

- Optimal mechanism
  - Maximizes seller's *expected* revenue
  - Suitable for auction of a private asset
  - Much studied - buyers' strategies, allocation, payment
    - Single item auction : Myerson's mechanism
    - Multiple item auction : Branco's mechanism
    - Mechanism given by a pair of functions (*p, c*)
      - e.g., in Branco's mechanism with *m* units of item

$$p : T \rightarrow \mathbb{R}^{(N \times m)}, \; c : T \rightarrow \mathbb{R}^N$$

$p_{jk}(t)$: probability that bidder *j* will receives at least *k* units

$c_j(t_j)$: bidder *j*'s expected payment

  - Designed for a single seller

Generalized Branco's mechanism (GBM) (1)

- M buyers
  - $T_j \in \mathcal{T}_j$ - type of buyer j (private information)
  - Each buyer reports its type to seller(s) - $\mathbf{t}^* = (t_j^*; j = 1, 2, \ldots, M)$
    - Not necessarily its true type

- Seller(s)
  - Have values for items for sale – $0 \leq V_0^{(1)} \leq V_0^{(2)} \leq \cdots \leq V_0^{(m)}$
  - Compute contributions: For each $j = 1, \ldots, M, k = 1, \ldots, m$

$$\pi_{j,k}(t_j^*) = V_{j,k}(t_j^*) - \left.\frac{\partial V_{j,k}(t_j)}{\partial t_j}\right|_{t_j = t_j^*} \frac{1 - \mathcal{G}_j(t_j^*)}{g_j(t_j^*)}$$

  - Order the contributions by decreasing value
    - $\pi_{(\ell)}(\mathbf{t}^*)$ - ℓ-th largest contribution among ***all*** buyers

Generalized Branco's Mechanism (GBM) (2)

- Regularity assumptions
  - $(t_j - \tilde{t}_j)(\pi_{j,k}(t_j) - \pi_{j,k}(\tilde{t}_j)) \geq 0$ for all $t_j, \tilde{t}_j \in \mathcal{T}_j$
  - if $\pi_{j,k+1}(t_j) \geq 0$, then $\pi_{j,k}(t_j) \geq \pi_{j,k+1}(t_j)$ for all $t_j \in \mathcal{T}_j$

Generalized Branco's Mechanism (GBM) (3)

- In a nutshell,
  - $m^\star(\mathbf{t}^*)$ items are awarded to the buyers with the $m^\star(\mathbf{t}^*)$ highest *contributions*, where

$$m^\star(\mathbf{t}^*) := \max\{\ell \in \{1,2,\ldots,m\} \mid \pi_{(\ell)}(\mathbf{t}^*) > V_0^{(\ell)}\}.$$

  - Buyer *j* pays $V_{j,k}(\varsigma_{j,k}(\mathbf{t}^*_{-j}))$ for the *k*-th item it wins, where

$$\varsigma_{j,k}(\mathbf{t}^*_{-j}) := \inf\{\hat{t}_j \in \mathcal{T}_j \mid \pi_{j,k}(\hat{t}_j) \geq \min\{\eta_\ell(\hat{t}_j, \mathbf{t}^*_{-j});\ \ell = 1,2,\ldots,m\}\}$$

and $\eta_\ell(\mathbf{t}^*) = \max\{\pi_{(\ell+1)}(\mathbf{t}^*), V_0^{(\ell)}\}$

- Smallest value for the *k*-th item that would win the item

Properties of GBM (1)

Theorem: The GBM satisfies following properties:

- Incentive compatible
  - Reporting true type is an optimal strategy for bidders
  - We will assume buyers report their true types when GBM is employed by coalitions of sellers in our framework
- Individually rational
  - No buyer will be worse off by participating in the auction
- Optimal mechanism
  - Maximizes the expected profit of the seller(s)
    - Profit = total revenue – total value of sold items

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Selfish buyers and non-cooperative game (1)

- Buyers assumed selfish
  - Interested in maximizing own expected payoffs
  - Interaction among selfish buyers modeled using a non-cooperative game
  - Only action is to select a seller

- Seller selection probability vectors:

$$\mathbf{p} = (p_b;\ \ b \in \mathcal{B})$$

  - $p_b = (p_{b,s}; s \in \mathcal{S})$, where $p_{b,s}$ is the probability that buyer b selects seller s

Selfish buyers and non-cooperative game (2)

- Non-cooperative game among buyers $(\mathcal{B} = \{1, 2, \ldots, B\})$
  - Payoff of buyer b given by $p_b(A, T; P)$
    - $A = (a_1, a_2, \ldots, a_B)$ - sellers selected by buyers $(a_b \in \mathcal{S})$
    - $T = (T_b; b \in \mathcal{B})$ - vector of buyers' (reported) types
    - P - partition of sellers, i.e., set of coalitions that emerge
      - $P(s)$ - coalition to which seller $s \in \mathcal{S}$ belongs
      - **Each** coalition $C \in P$ holds a separate auction employing the generalized Branco's mechanism (GBM)
    - $p_b(A, T; P)$ = total value from items won – total price paid for the items won (according to the GBM)

Incentive for cooperation among sellers (1)

- Assume that buyers fix their seller selection probabilities
  - *Any arbitrary* probability vectors (mixed-strategy profile)

- For every $C \subset \mathcal{S}$, let $v(C)$ denote the expected profit of coalition C under the GBM

- Theorem: For every $C_1, C_2 \subset \mathcal{S}$ such that $C_1 \cap C_2 = \emptyset$

$$v(C_1) + v(C_2) \leq v(C_1 \cup C_2)$$

⇨ Sellers are better off cooperating among themselves to maximize their expected profit

Source of difficulty (1)

- Calculation of prices to charge, hence total revenue from auction, difficult
- **Lack of monotonicity**
  - Profit/revenue does **NOT** necessarily increase with the set of items to be sold
    - Can easily find examples where introducing additional items to sell reduces the *total* revenue

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Cooperative game (1)

- How should sellers share the (expected) profit among themselves to promote and sustain cooperation?
  - Model the interaction as a cooperative game
  - Characteristic function v defined through expected profit for different possible coalitions
    - $v(C)$ - Expected payoff (i.e., expected profit) sellers in coalition C can guarantee themselves

Definition: An imputation is an expected payoff vector

$\mathbf{x} = (x_1, \ldots, x_S)$ satisfying

- $\sum_{s \in \mathcal{S}} x_s = v(\mathcal{S})$
- $x_s \geq v(\{s\})$ for all $s \in \mathcal{S}$

Cooperative game (2)

Definition: Let $\mathbf{x}$ and $\mathbf{y}$ be two imputations. We say $\mathbf{x}$ dominates $\mathbf{y}$ through $C \subset \mathcal{S}$ if

- $x_s > y_s$ for all $s \in C$
- $\sum_{s \in C} x_s \leq v(C)$

Definition: We say $\mathbf{x}$ dominates $\mathbf{y}$ if there is any coalition $C \subset \mathcal{S}$ such that $\mathbf{x}$ dominates $\mathbf{y}$ through C

Definition: The set of all undominated imputations is called the core.

- ***Not*** guaranteed to exist (i.e., non-empty)

Existence of non-empty core (1)

- <u>Theorem:</u> The core of the cooperative game among the sellers is always non-empty
  - Implication – There always <u>*exists*</u> a way for sellers to share profit so that no subset of sellers will have an incentive or power to deviate from cooperation and increase their expected payoffs

Revenue sharing (1)

- Equitable sharing of revenue is possible
  - But, only in “expected” sense
  - Does not tell us how to share the revenue for each realization so as to achieve expected payoffs in the core
- Given an expected payoff vector $\mathbf{x}^{\star}$ in the core of cooperative game, how should the sellers distribute the profit for each realization so that their expected payoffs equal $\mathbf{x}^{\star}$?
- We would like to impose some additional natural constraints on the revenue sharing scheme we wish to design

Revenue sharing (2)

Revenue allocation scheme: $\Theta : \Omega \to [0, \infty)^S$ with $\sum_{s=1}^{S} \Theta_s(\omega) = 1$

C1. A seller that does not contribute anything to the auction (i.e., it brings neither winning contribution(s) nor allocated item(s)), called a non-contributing seller, receives nothing

- Only contributing sellers receive positive payments

C2. Sellers shall have a nonnegative profit for *every* realization

- Seller always receives a payment that is at least its total value of its items sold to the buyers

C3. $\Theta(\omega)$ depends only on the set of contributing sellers

- Can maintain the revenue allocation vectors in a finite table

Revenue sharing (2)

- Question: Is there a revenue allocation scheme, i.e., a mapping $\Theta^{\star}$, that satisfies C1 through C3?

Theorem: Given any expected payoff vector in the core of cooperative game, there always exists a revenue allocation scheme that satisfies C1 through C3

- Recursive method for finding a mapping $\Theta^{\star}$

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Conclusion

- Proposed an auction-based framework for designing a secondary spectrum trading market
  - Proposed an optimal auction mechanism (GBM) for allocating and pricing frequency bands
  - Showed the existence of an incentive for risk neutral sellers to cooperate in order to maximize their profits
  - By modeling the interaction among the sellers as a cooperative game, proved the existence of non-empty core of cooperative game
  - Designed a revenue sharing scheme that allows sellers to achieve *any* expected payoff vector in the non-empty core

Thank you….

Any questions?