Quantum Algorithms for Quantum Field Theories

Stephen Jordan

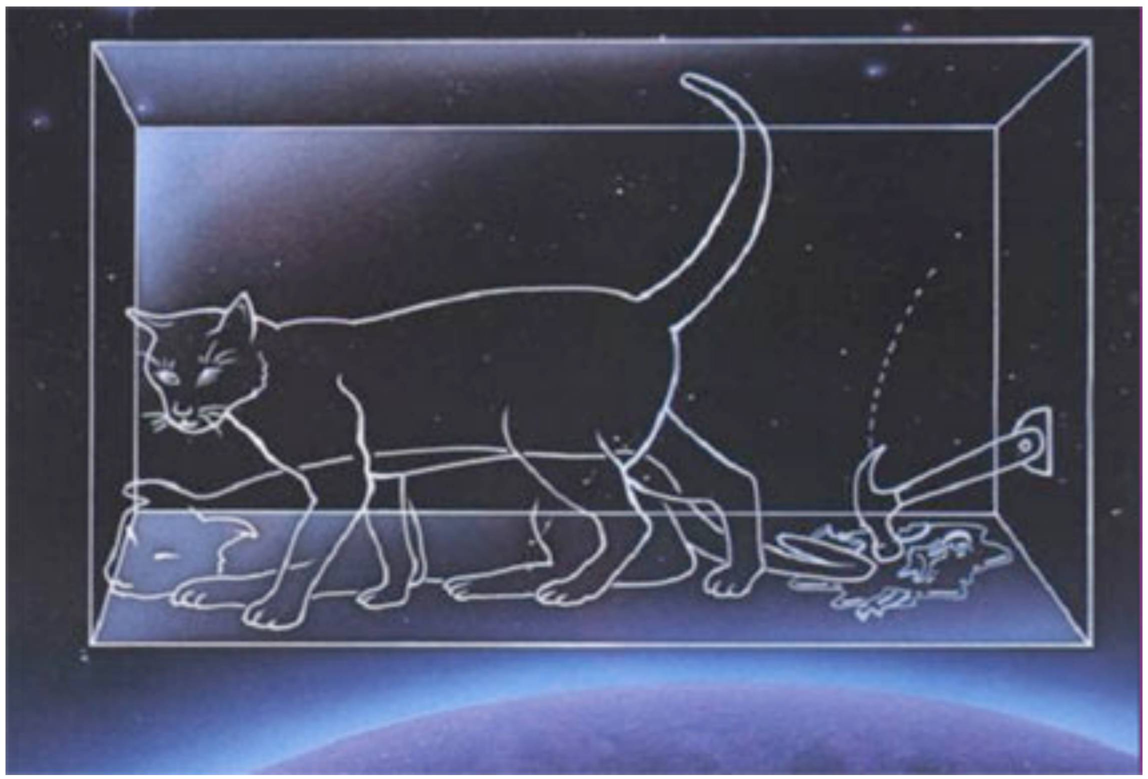
Joint work with
Keith Lee
John Preskill

[arXiv:1111.3633 and 1112.4833]

NIST

Feb 21, 2012

Quantum Mechanics



Each state of the system is a basis vector.

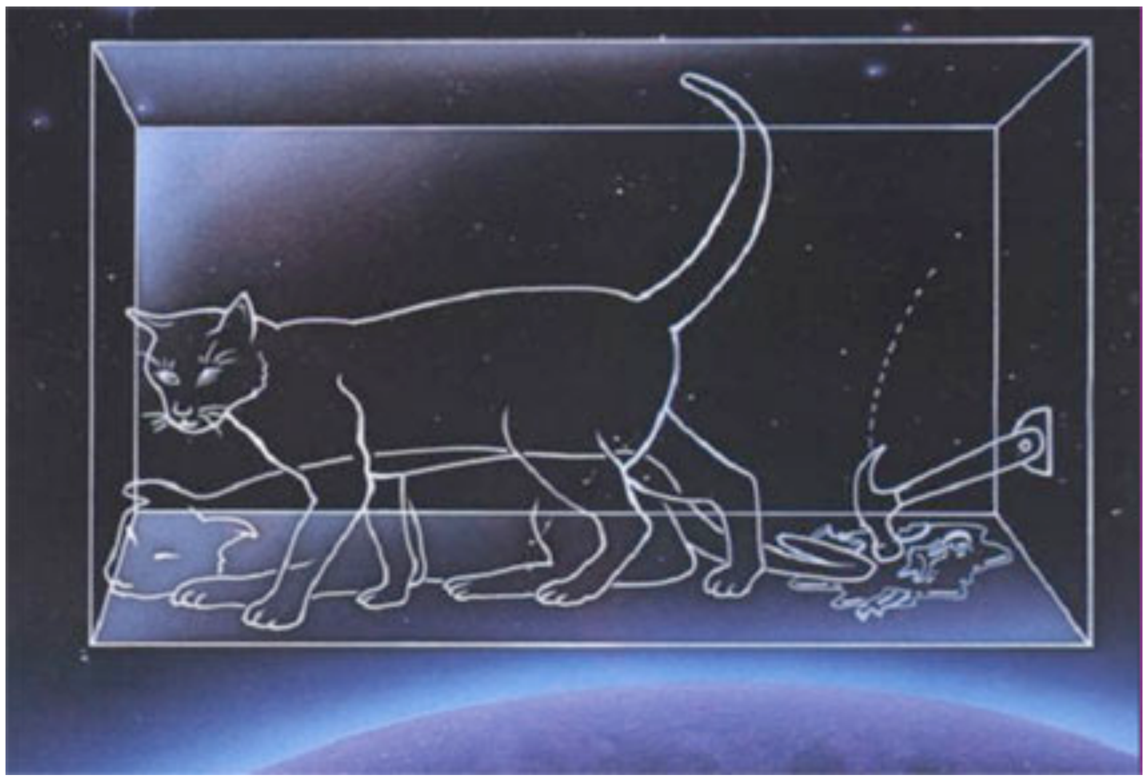
$$|\mathrm{dead}\rangle$$

$$|\mathrm{alive}\rangle$$

A general state is a linear combination of this basis:

$$\alpha|\mathrm{dead}\rangle + \beta|\mathrm{alive}\rangle \qquad \alpha, \beta \in \mathbb{C}$$

Quantum Mechanics



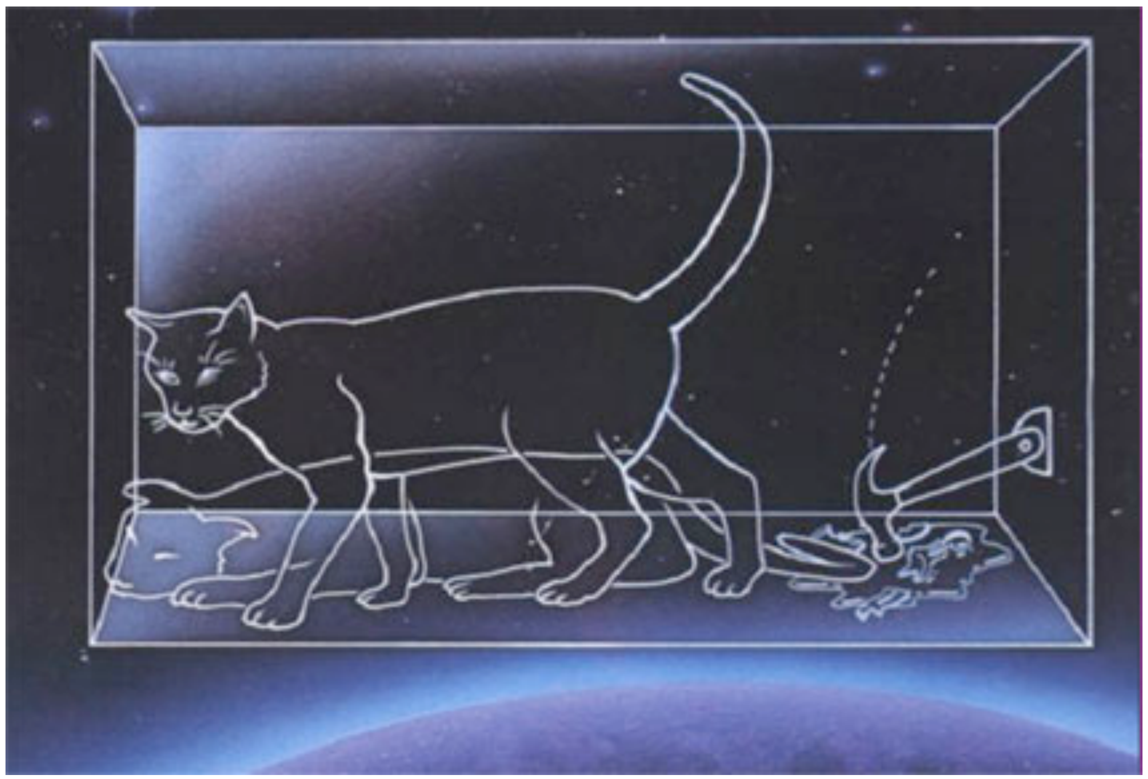
$$\alpha|\text{dead}\rangle + \beta|\text{alive}\rangle$$

If we look inside the box we see:

A dead cat with probability $|\alpha|^2$

A living cat with probability $|\beta|^2$

The Classical World



In most macroscopic systems, noise from the environment randomizes the phases.

The linear combination of states then acts like an ordinary probability distribution.

$$(p_{\text{dead}}, p_{\text{alive}}) \in \mathbb{R}^2$$

Qubits

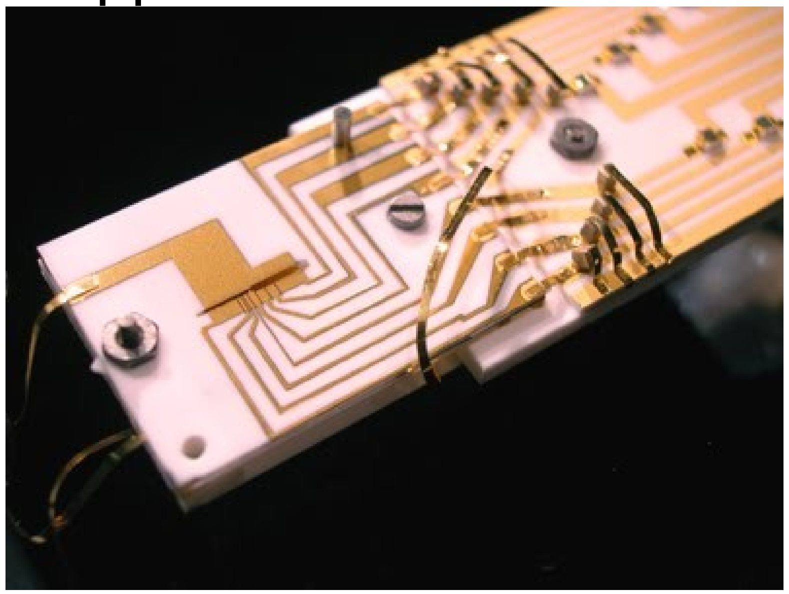
To exhibit quantum-mechanical effects we want a system that is simple and well isolated from its environment.

One qubit: $\alpha|0\rangle + \beta|1\rangle$

n qubits: $$\sum_{x\in\{0,1\}^n} \alpha(x)|x\rangle$$

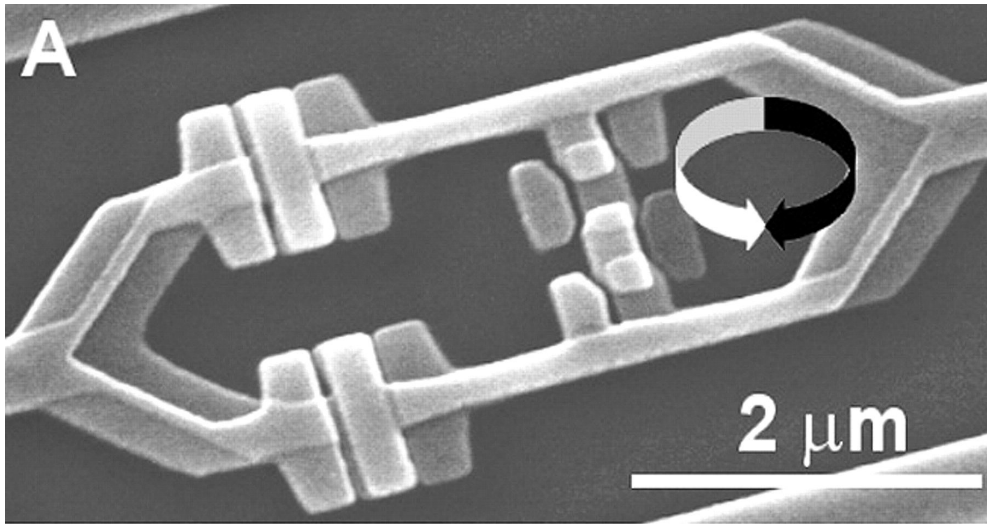
Qubits

Trapped Ions



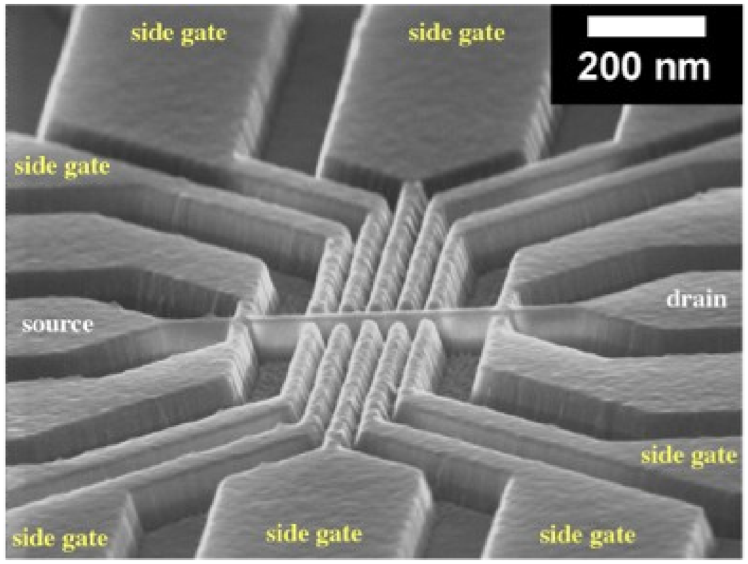
[Wineland group, NIST]

Superconducting Circuits



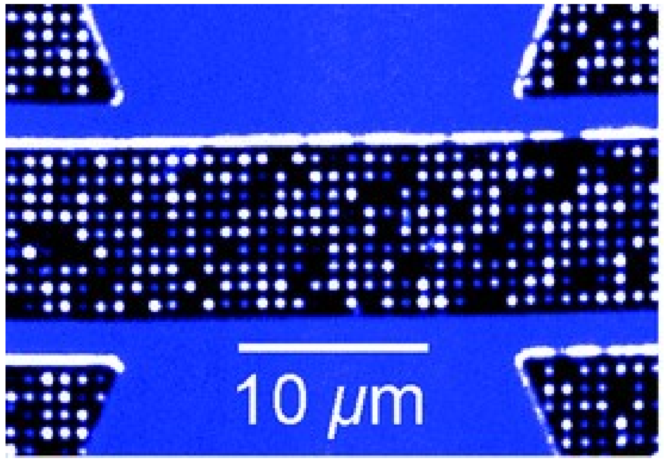
[Mooij group, TU Delft]

Quantum Dots



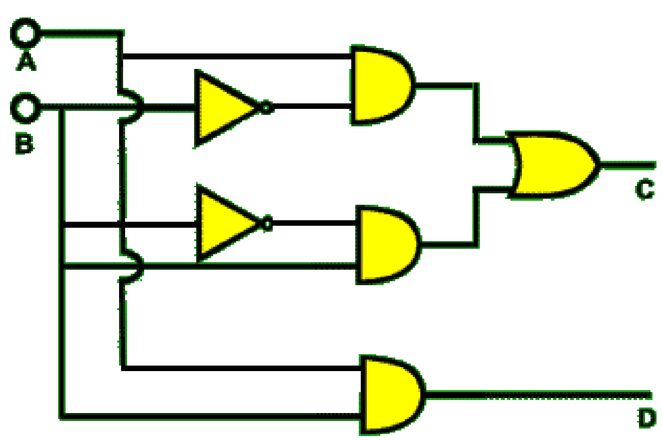
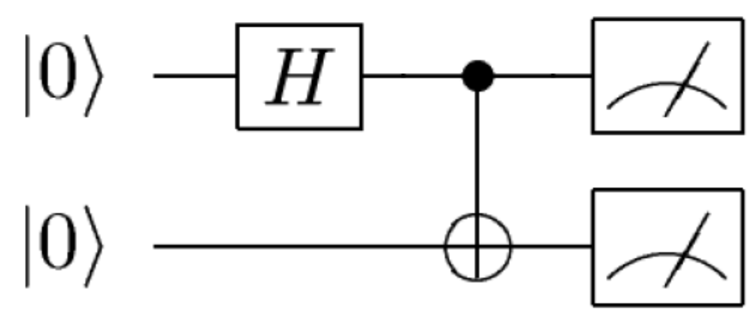
[Paul group, U. Glasgow]

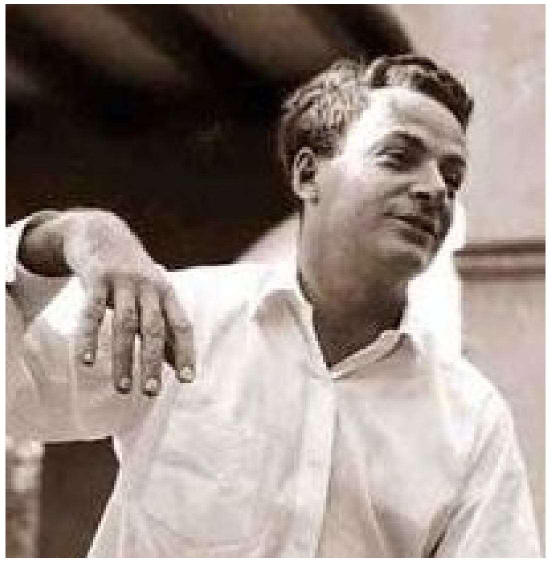
NV Centers in Diamond



[Awshalom group, UCSB]

Quantum Circuits

Classical	Quantum
0101101	$$\|\psi\rangle = \sum_{x \in \{0,1\}^n} \alpha(x)\|x\rangle$$
	 $$\boxed{H} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

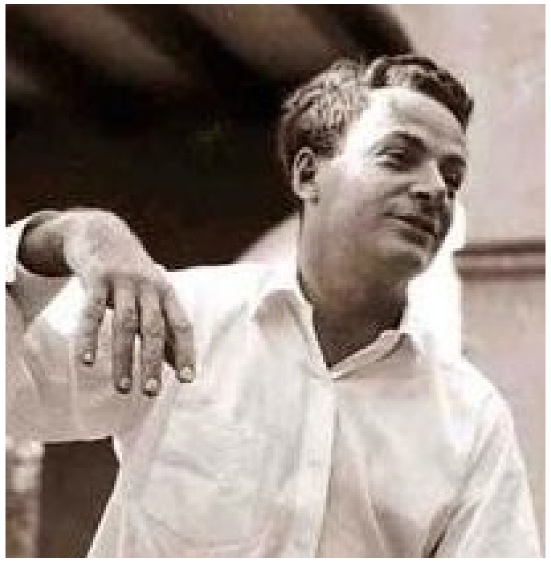


The full description of quantum mechanics for a large system with R particles has too many variables. It cannot be simulated with a normal computer with a number of elements proportional to R.

-Richard Feynman, 1982

An n-bit integer can be factored on a quantum computer in $O(n^2)$ time.

-Peter Shor, 1994



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Are there any systems that remain hard to simulate even with quantum computers?

Quantum Simulation

Condensed-matter lattice models:

[Lloyd, 1996]

[Abrams, Lloyd, 1997]

[Berry, Childs, 2012]

Many-particle Schrödinger and Dirac Equations:

[Meyer, 1996]

[Zalka, 1998]

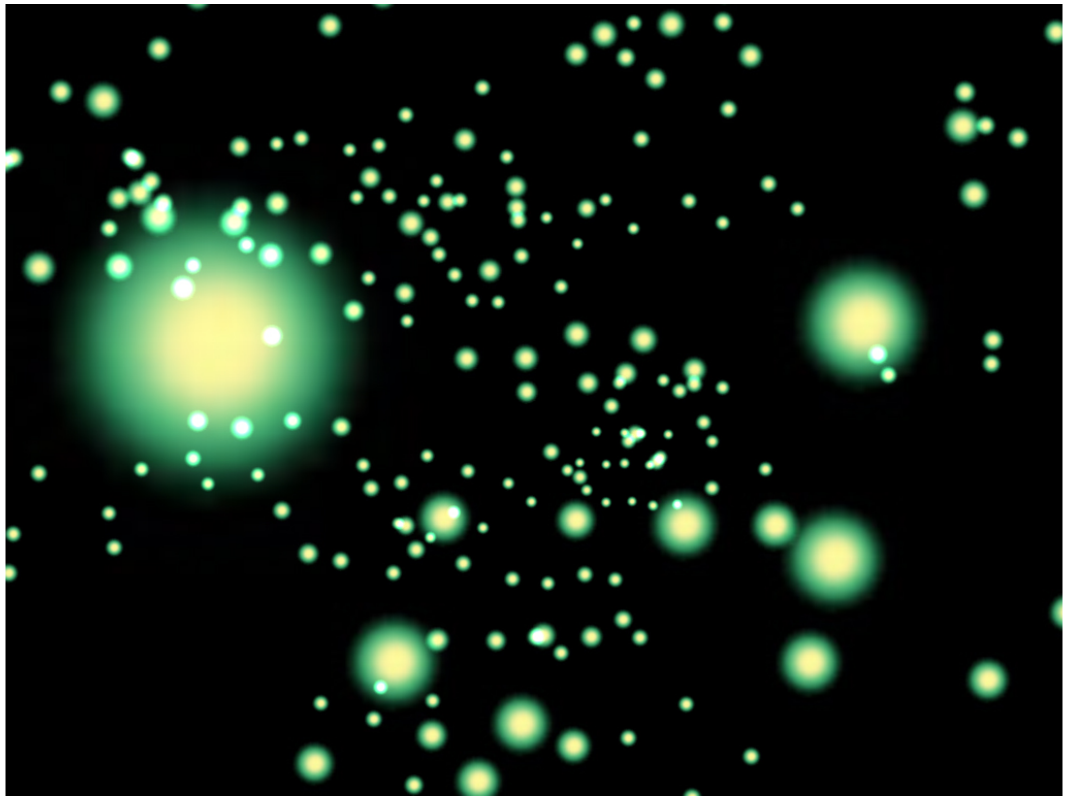
[Taylor, Boghosian, 1998]

[Kassal, S.J., Love, Mohseni, Aspuru-Guzik, 2008]

Quantum Field Theory

- Much is known about using quantum computers to simulate quantum systems.
- Why might QFT be different?
    - Field has infinitely many degrees of freedom
    - Relativistic
    - Particle number not conserved
    - Formalism looks different

Quantum Particles



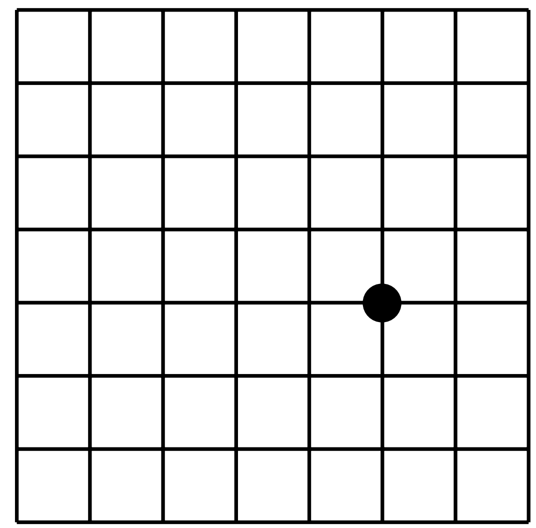
A classical particle is described by its location coordinates.

$$\vec{r} = (x, y, z)$$

The state of a quantum particle is linear combination of positions.

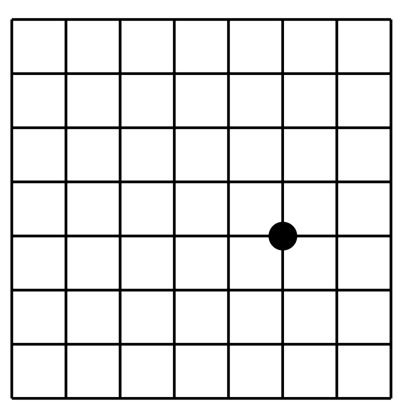
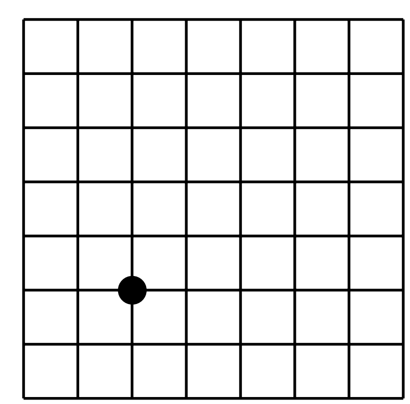
$$|\psi\rangle = \int d^3r \; \psi(r) \; |r\rangle$$

A configuration is a list of particle coordinates.

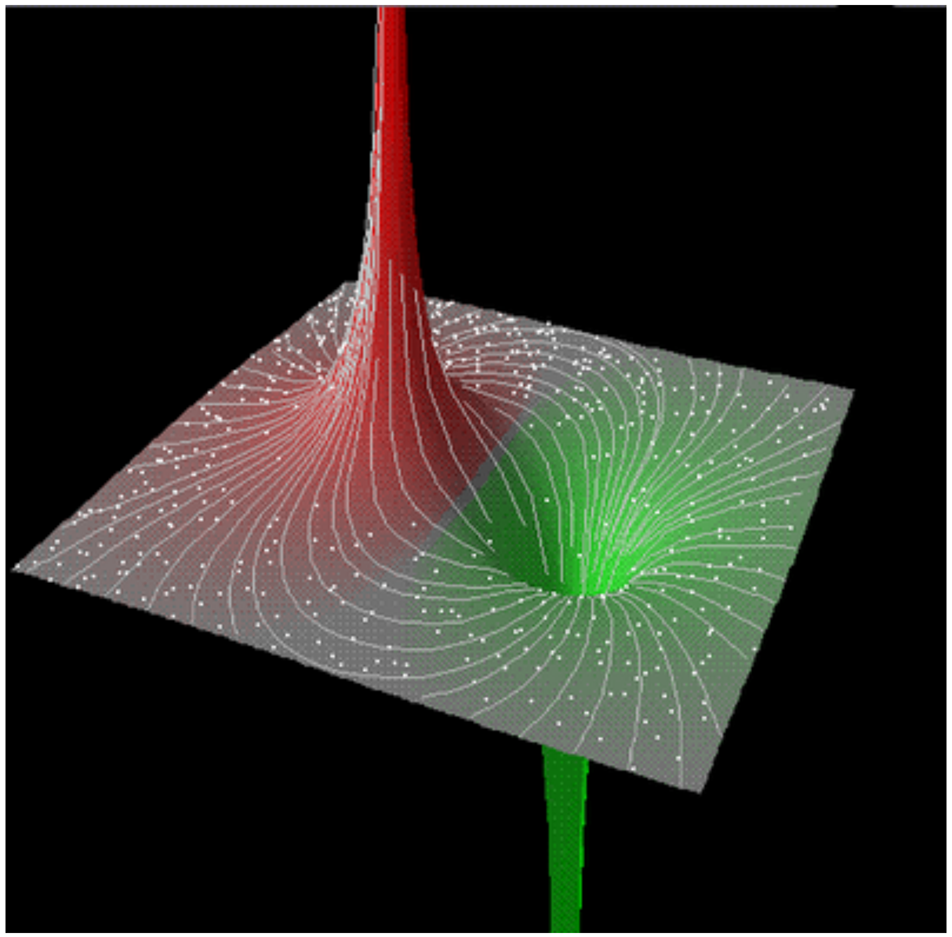


(5,3)

A quantum particle can be in a superposition of locations.

$\frac{1}{\sqrt{2}} \Bigg|$  (5,3) $\Bigg\rangle - \frac{i}{\sqrt{2}} \Bigg|$  (2,2) $\Bigg\rangle$

Quantum Fields



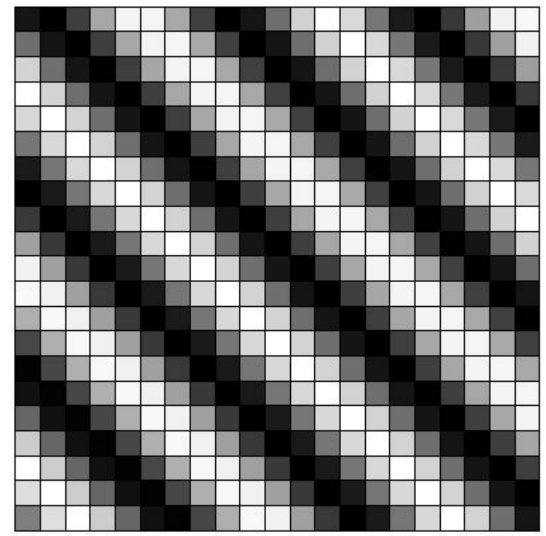
A classical field is described by its value at every point in space.

$$E(r) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

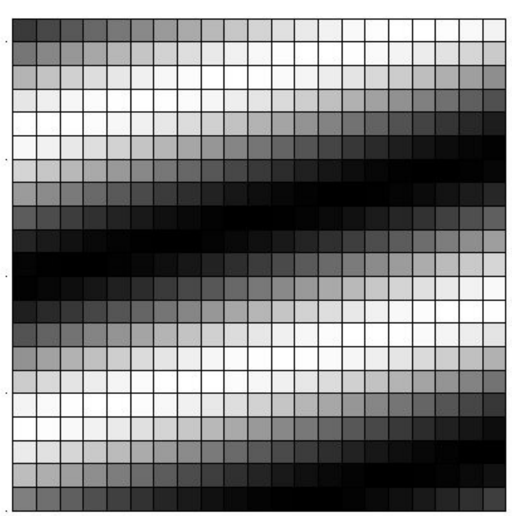
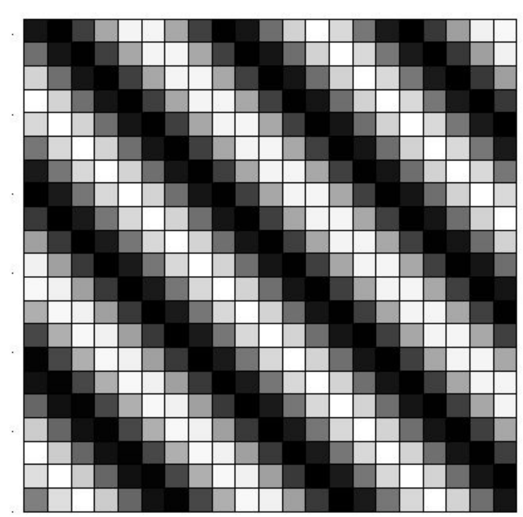
A quantum field is a linear combination of classical field configurations.

$$|\Psi\rangle = \int \mathcal{D}[E]\Psi[E]\ |E\rangle$$

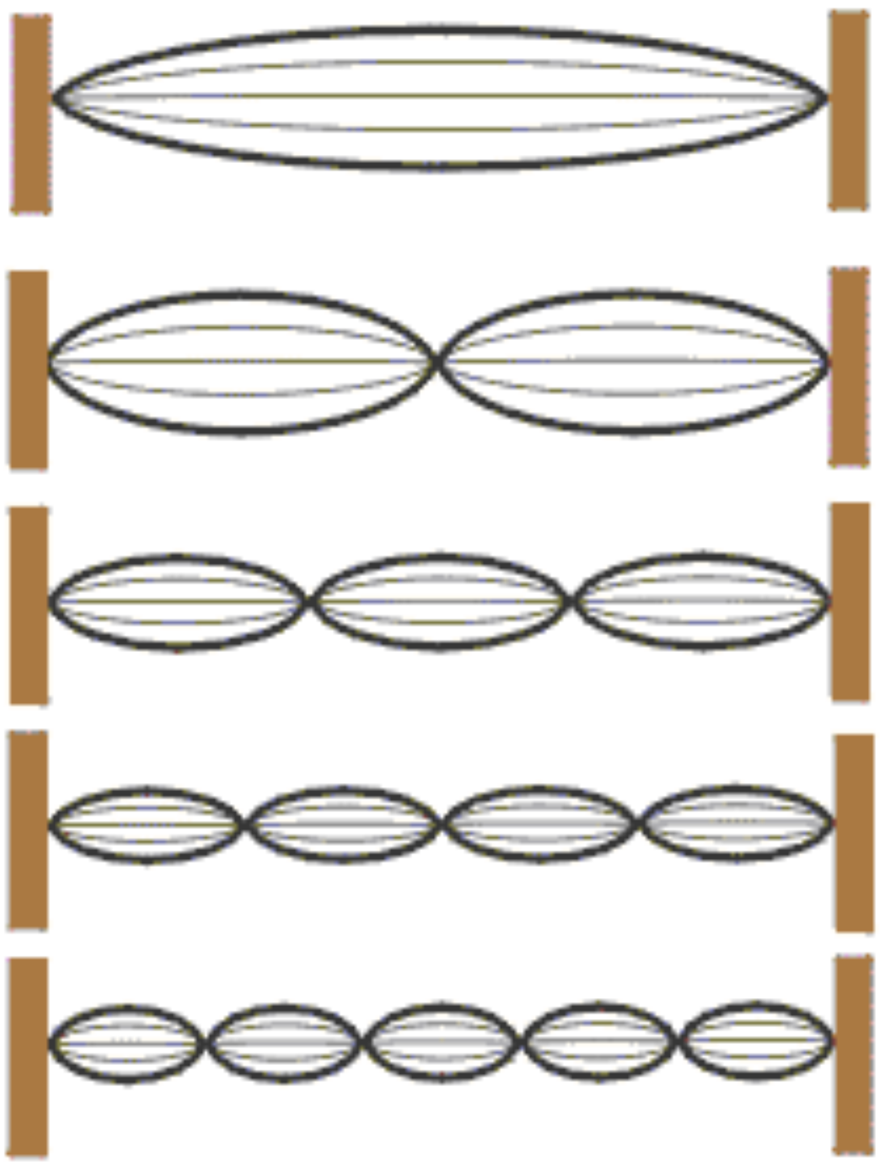
A configuration of the field is a list of field values, one for each lattice site.



A quantum field can be in a superposition of different field configurations.

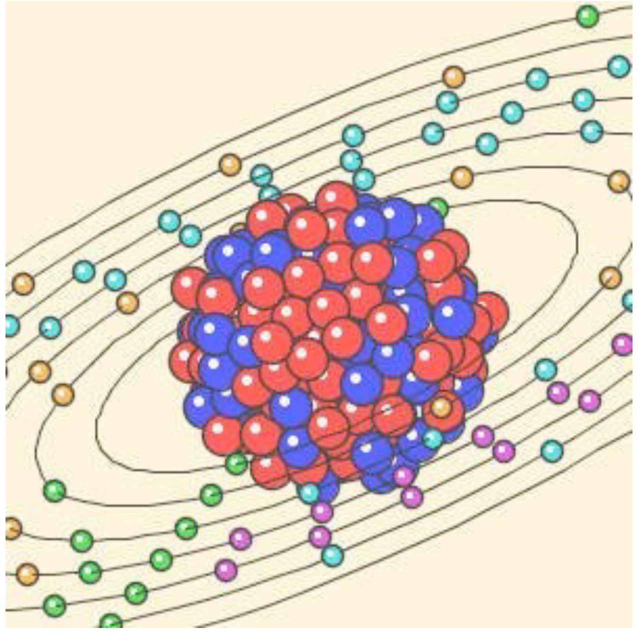
$\frac{1}{\sqrt{2}}\Big|$  $\Big\rangle - \frac{i}{\sqrt{2}}\Big|$  $\Big\rangle$

Particles Emerge from Fields

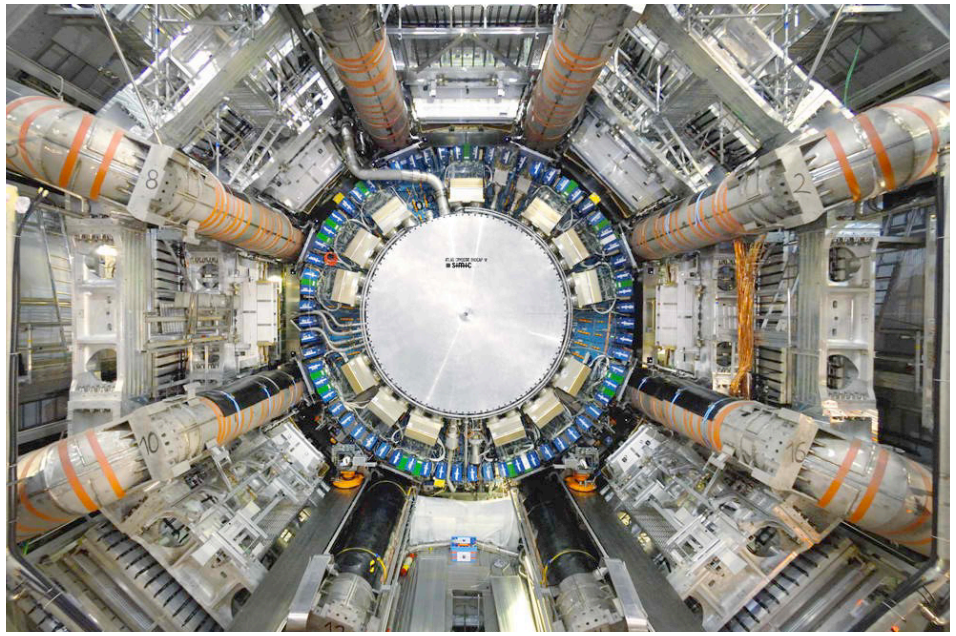


Particles of different energy are different resonant excitations of the field.

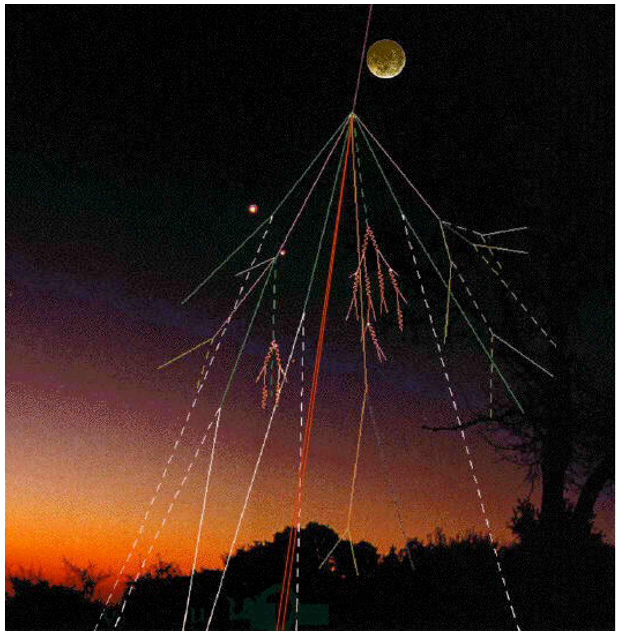
When do we need QFT?



Nuclear Physics



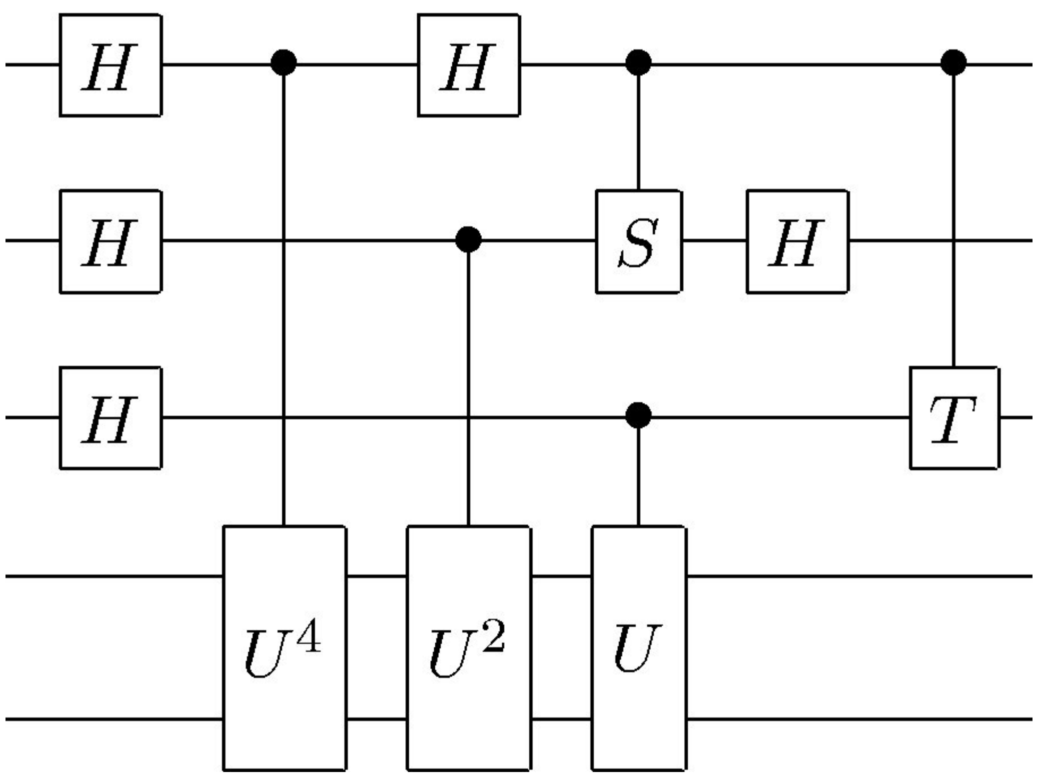
Accelerator Experiments



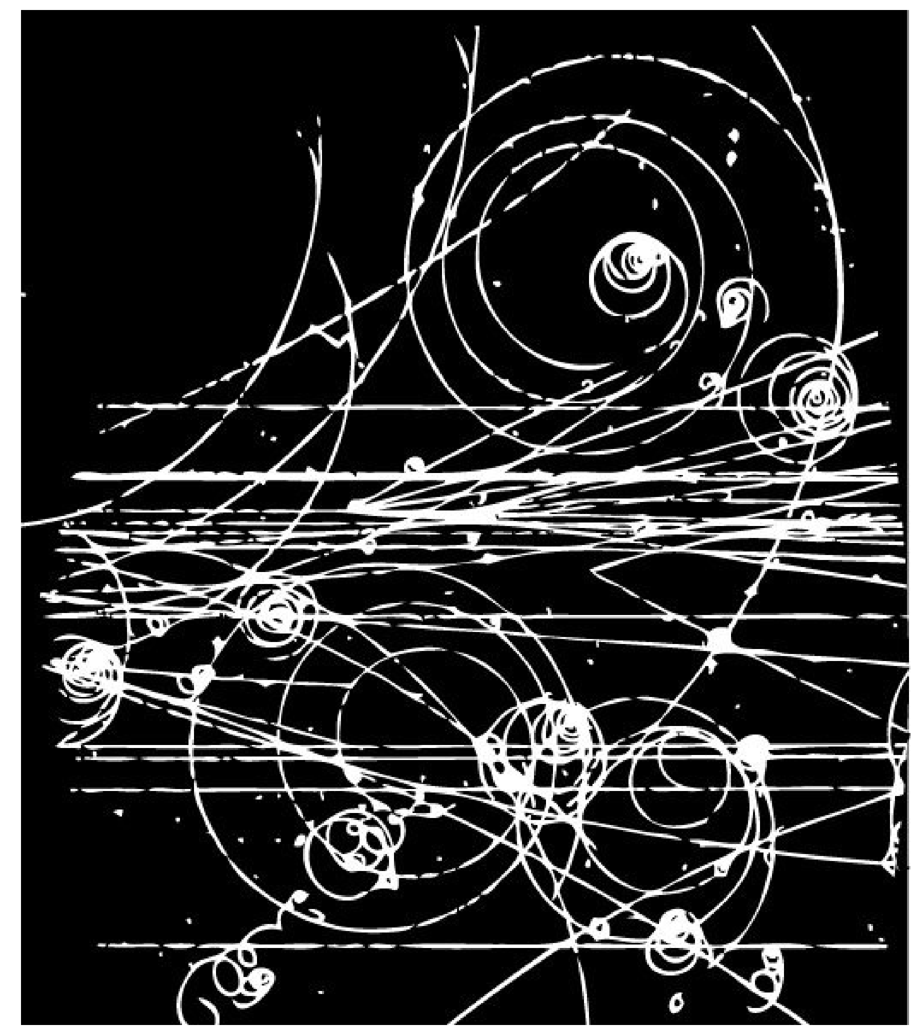
Cosmic Rays

→Whenever quantum mechanical and relativistic effects are both significant.

What is the computational power of our universe?

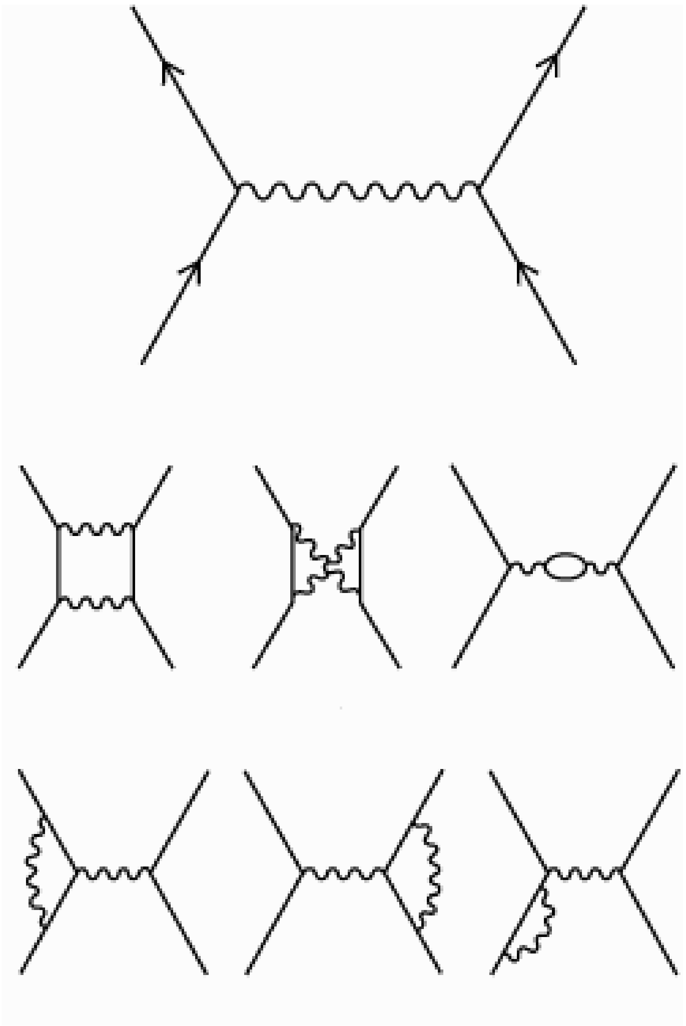


simulate



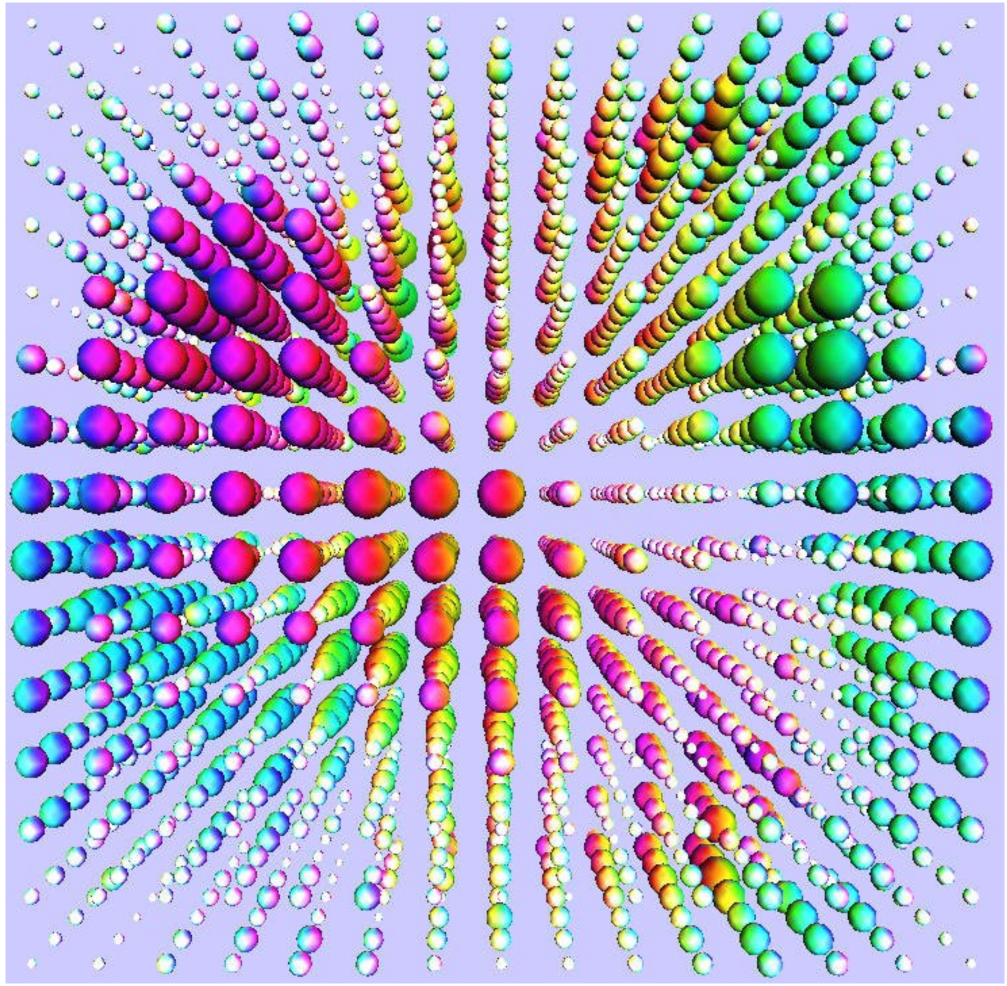
Classical Algorithms

Feynman diagrams



Break down at strong
coupling or high precision

Lattice methods

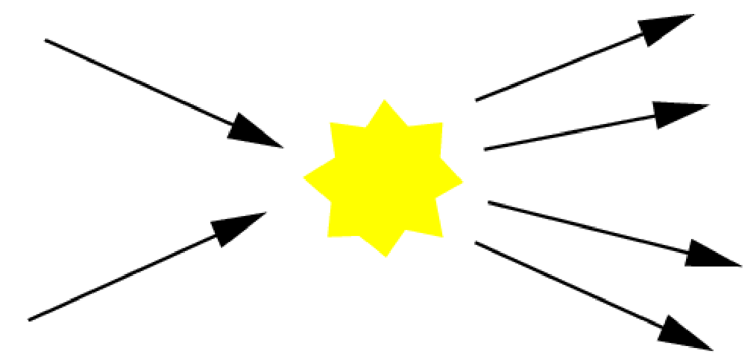


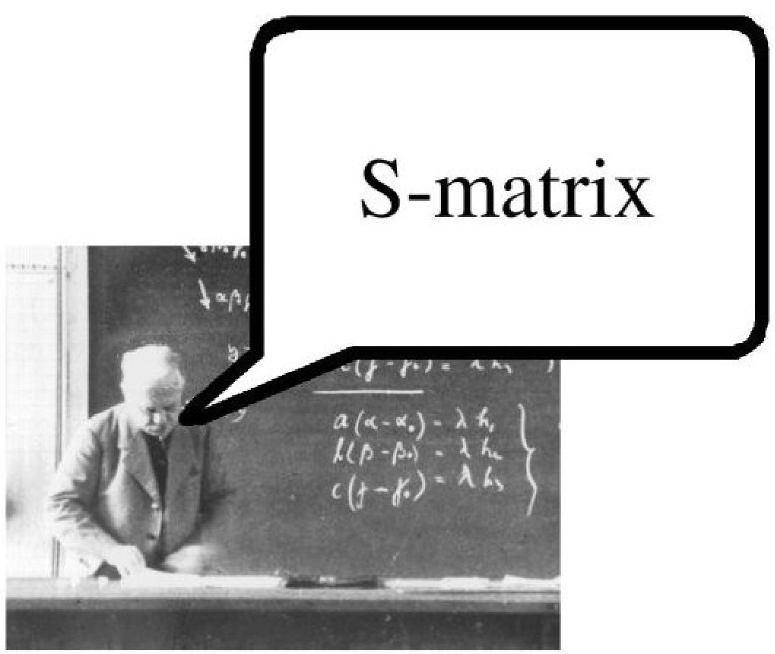
Cannot calculate
scattering amplitudes

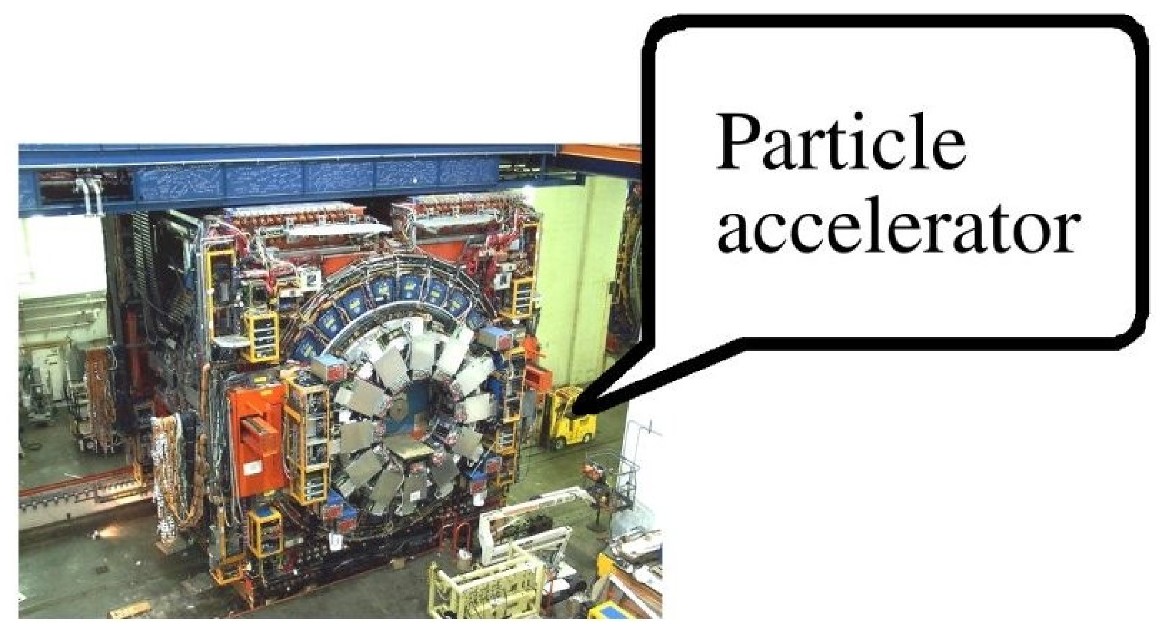
A QFT Computational Problem

Input: a list of momenta of incoming particles

Output: a list of momenta of outgoing particles







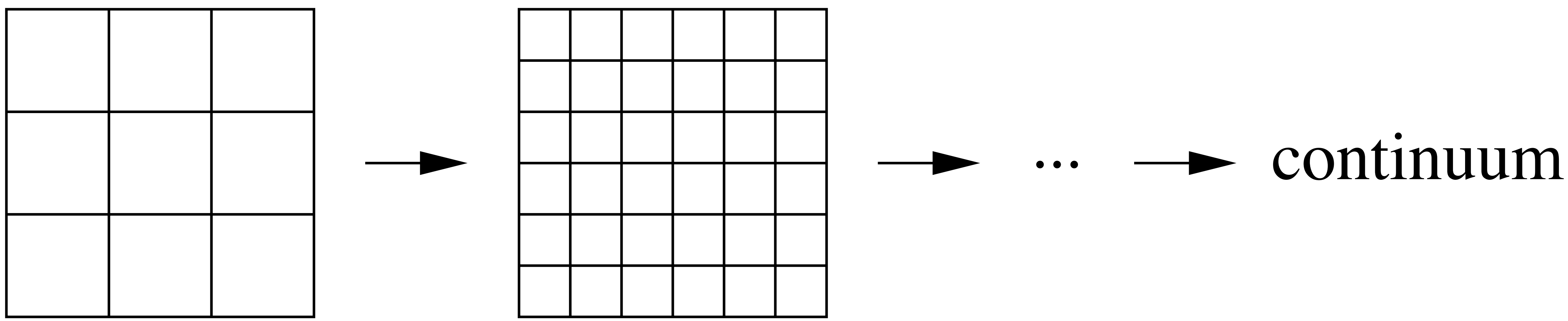
I will present a polynomial-time quantum algorithm to compute scattering probabilities in ϕ^4-theory with nonzero mass

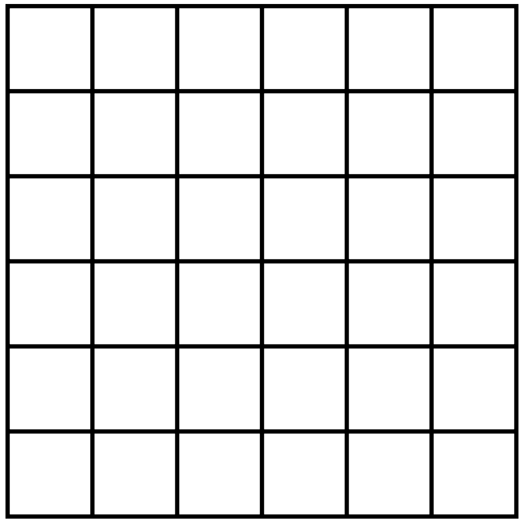
ϕ^4-theory is a simple model that illustrates some of the main difficulties in simulating a QFT:

- Discretizing spacetime
- Preparing initial states
- Measuring observables

Lattice cutoff

Continuum QFT = limit of a sequence of theories on successively finer lattices

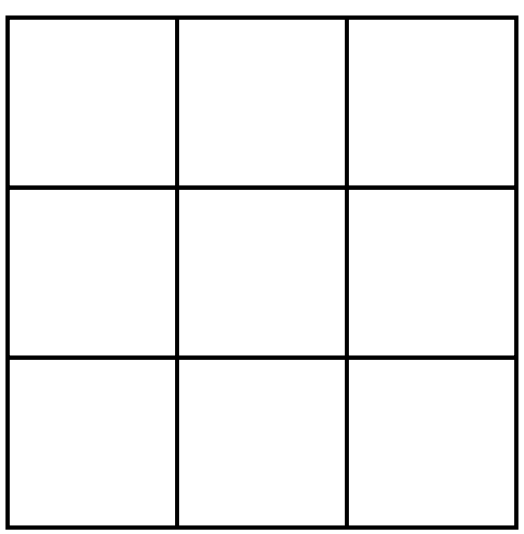




Mass: m

Interaction strength: λ

Coarse grain

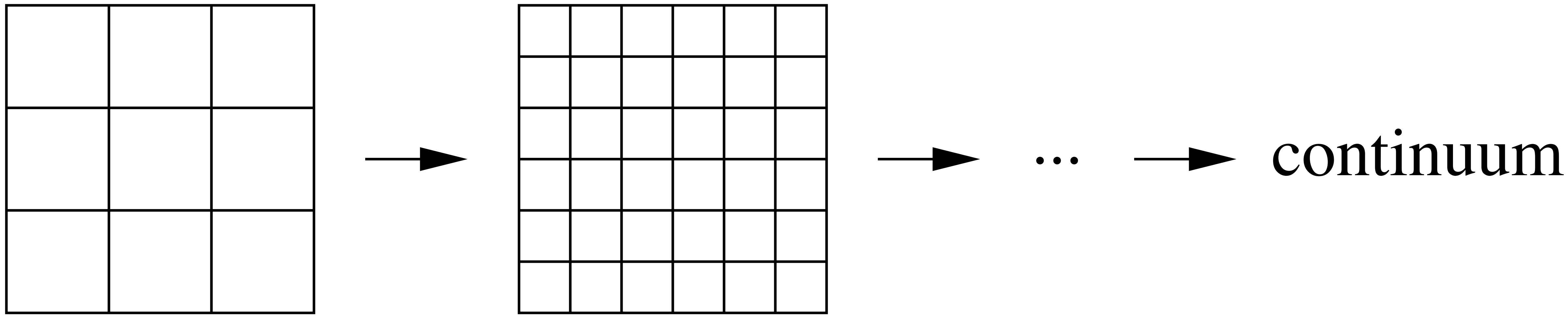


Mass: m'

Interaction strength: λ'

Lattice cutoff

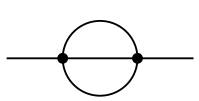
Continuum QFT = limit of a sequence of theories on successively finer lattices



m and λ are functions of lattice spacing!

Discretization Errors

- Renormalization of m and make discretization tricky to analyze
- In ϕ^4-theory, in d=1,2,3, discretization errors scale as a^2



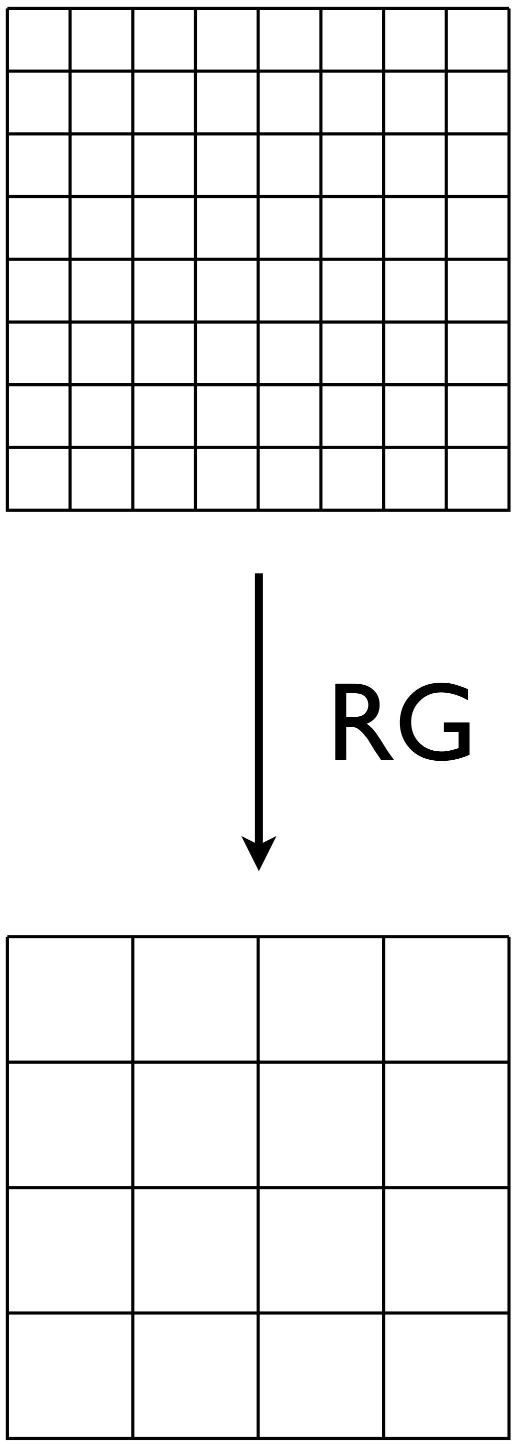
$$= \frac{(-i\lambda_0)^2}{6}\iint \frac{d^Dk}{(2\pi)^D}\frac{d^Dq}{(2\pi)^D}\frac{i}{(k^0)^2-\sum_i \frac{4}{a^2}\sin^2\left(\frac{ak^i}{2}\right)-m^2}\;\frac{i}{(q^0)^2-\sum_i \frac{4}{a^2}\sin^2\left(\frac{aq^i}{2}\right)-m^2}$$

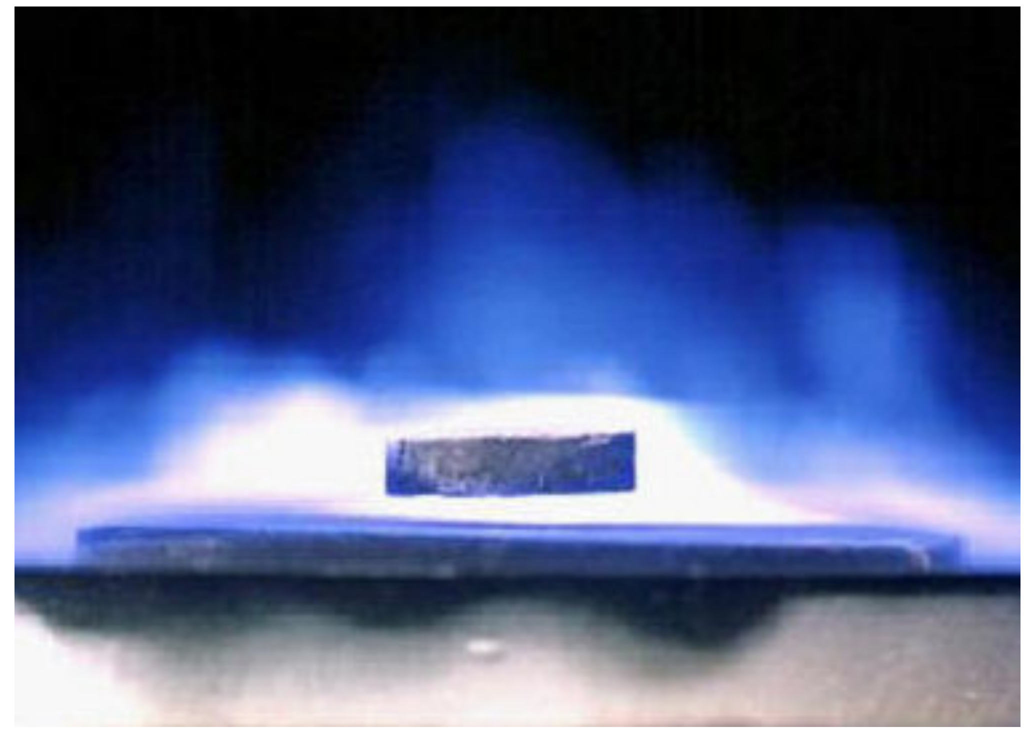
$$\times\frac{i}{(p^0+k^0+q^0)^2-\sum_i \frac{4}{a^2}\sin^2\left(\frac{a(p^i+k^i+q^i)}{2}\right)-m^2} \qquad (207)$$

$$= \frac{i\lambda_0^2}{3}\int_0^1\!\!\int_0^1\!\!\int_0^1 dx\,dy\,dz\,\delta(x+y+z-1)\iint\frac{d^Dk}{(2\pi)^D}\frac{d^Dq}{(2\pi)^D}\frac{1}{\mathsf{D}^3}\,, \qquad (208)$$

...its complicated

Condensed Matter





There is a fundamental lattice spacing.

But:

We may save qubits by simulating a coarse-grained theory.

After imposing a spatial lattice we have a many-body quantum system with a local Hamiltonian

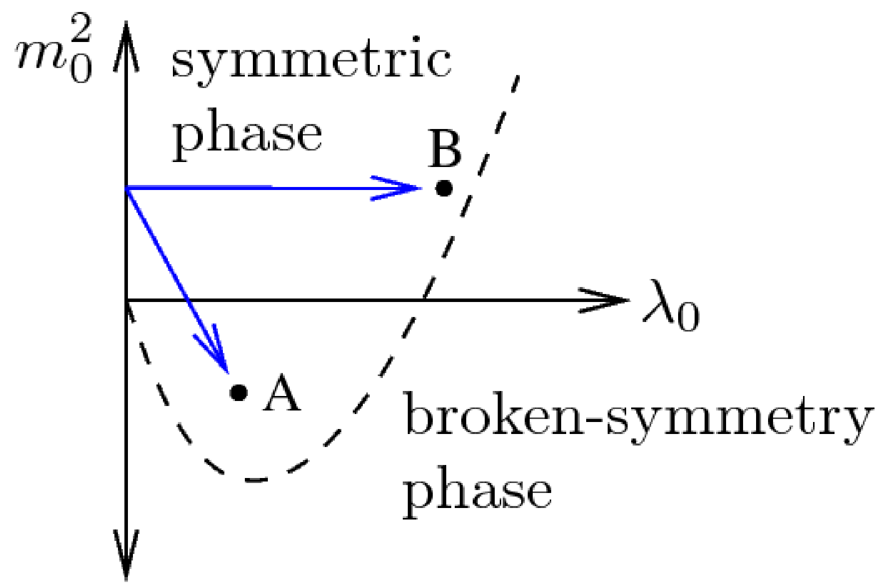
Simulating the time evolution in polynomial time is a solved problem

Standard methods scale as N^2. We can do N.

- Convergence as $a \to 0$
- Preparing wavepackets
- Measuring particle momenta

Strong Coupling

ϕ^4-theory in 1+1 and 2+1 dimensions has a quantum phase transition in which the $\phi \to -\phi$ symmetry is spontaneously broken



Near the phase transition perturbation theory fails and the gap vanishes.

$$m_{\text{phys}} \sim (\lambda_c - \lambda_0)^\nu \qquad \nu = \begin{cases} 1 & d = 1 \\ 0.63\ldots & d = 2 \end{cases}$$

Complexity

Weak Coupling:

$d = 1$	$(1/\epsilon)^{1.5}$
$d = 2$	$(1/\epsilon)^{2.376}$
$d = 3$	$(1/\epsilon)^{5.5}$

Strong Coupling:

	$\lambda_c - \lambda_0$	p	n_{out}
$d = 1$	$\left(\frac{1}{\lambda_c - \lambda_0}\right)^8$	p^4	n_{out}^5
$d = 2$	$\left(\frac{1}{\lambda_c - \lambda_0}\right)^{5.04}$	p^6	$n_{\text{out}}^{7.128}$

Eventual goal:

Simulate the standard model in BQP

Solved problems:

ϕ^4-theory [arXiv:1111.3633 and 1112.4833]

Gross-Neveu [S.J., Lee, Preskill, *in preparation*]

Open problems:

Gauge symmetries, massless particles

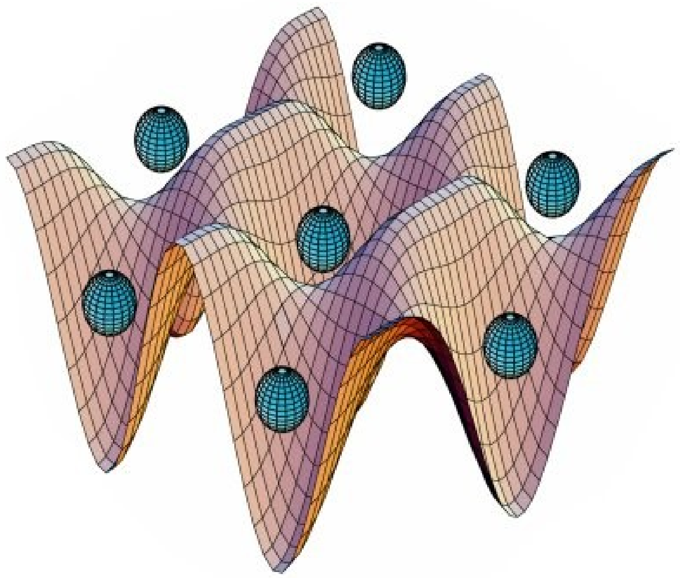
Spontaneous symmetry breaking

Bound states, confinement

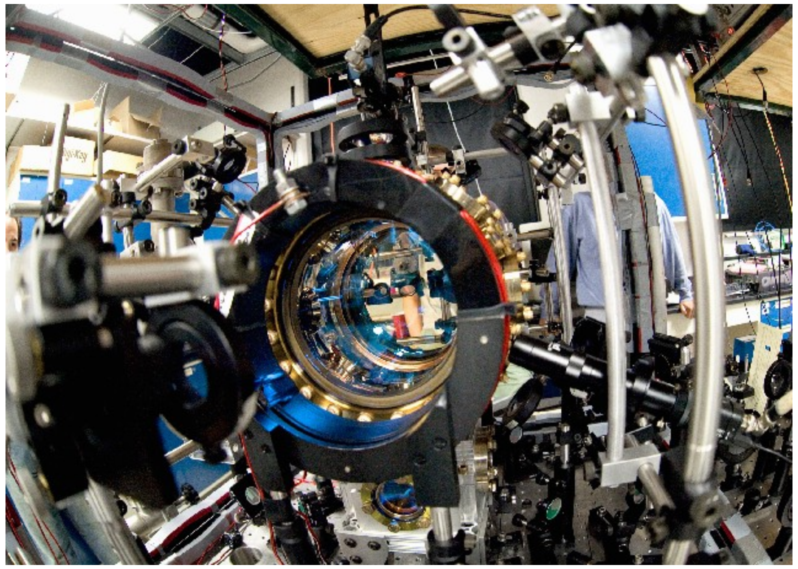
Chiral Fermions

Analog Simulation

- No gates: just implement a Hamiltonian and let it time-evolve



- Current experiments do this!

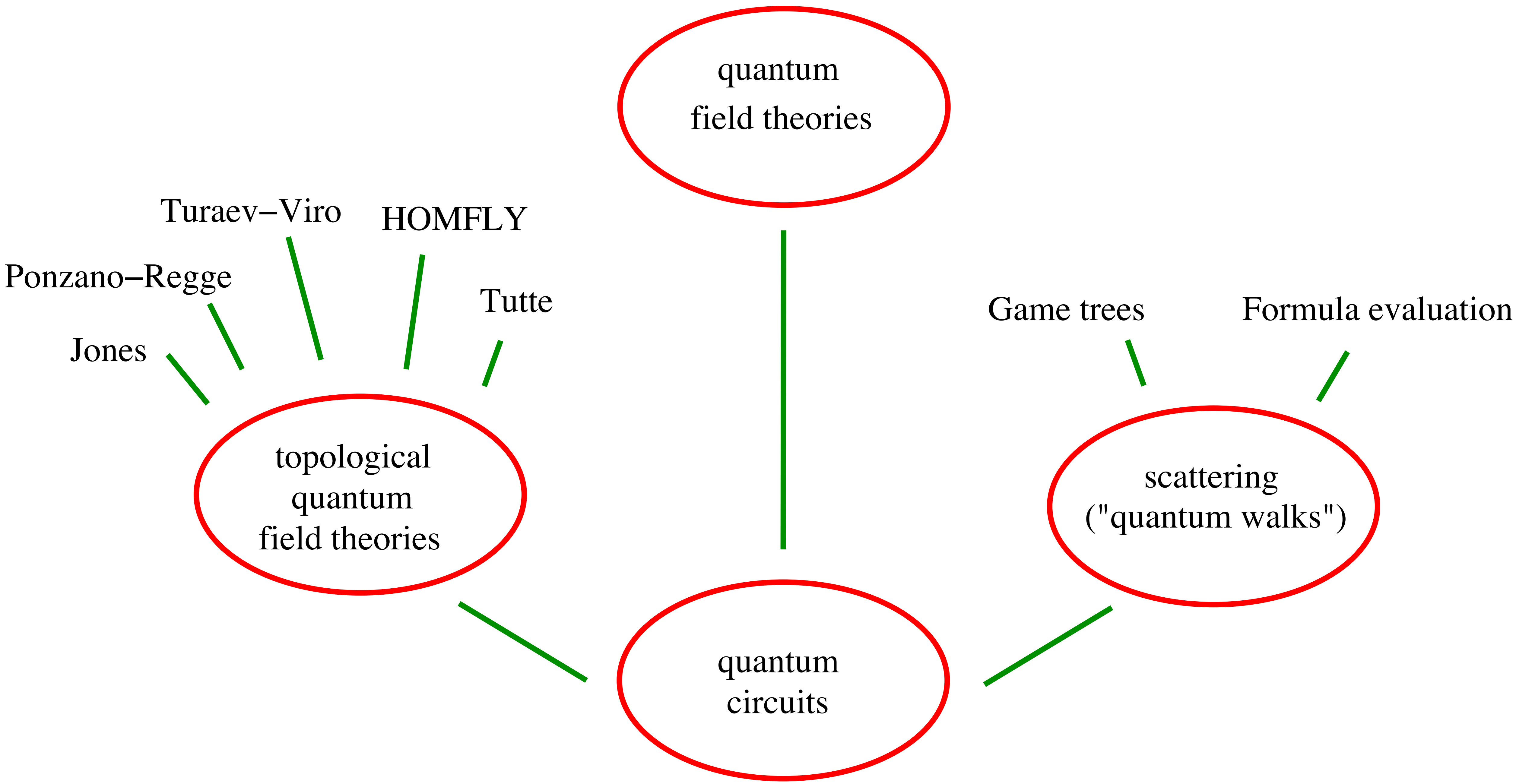


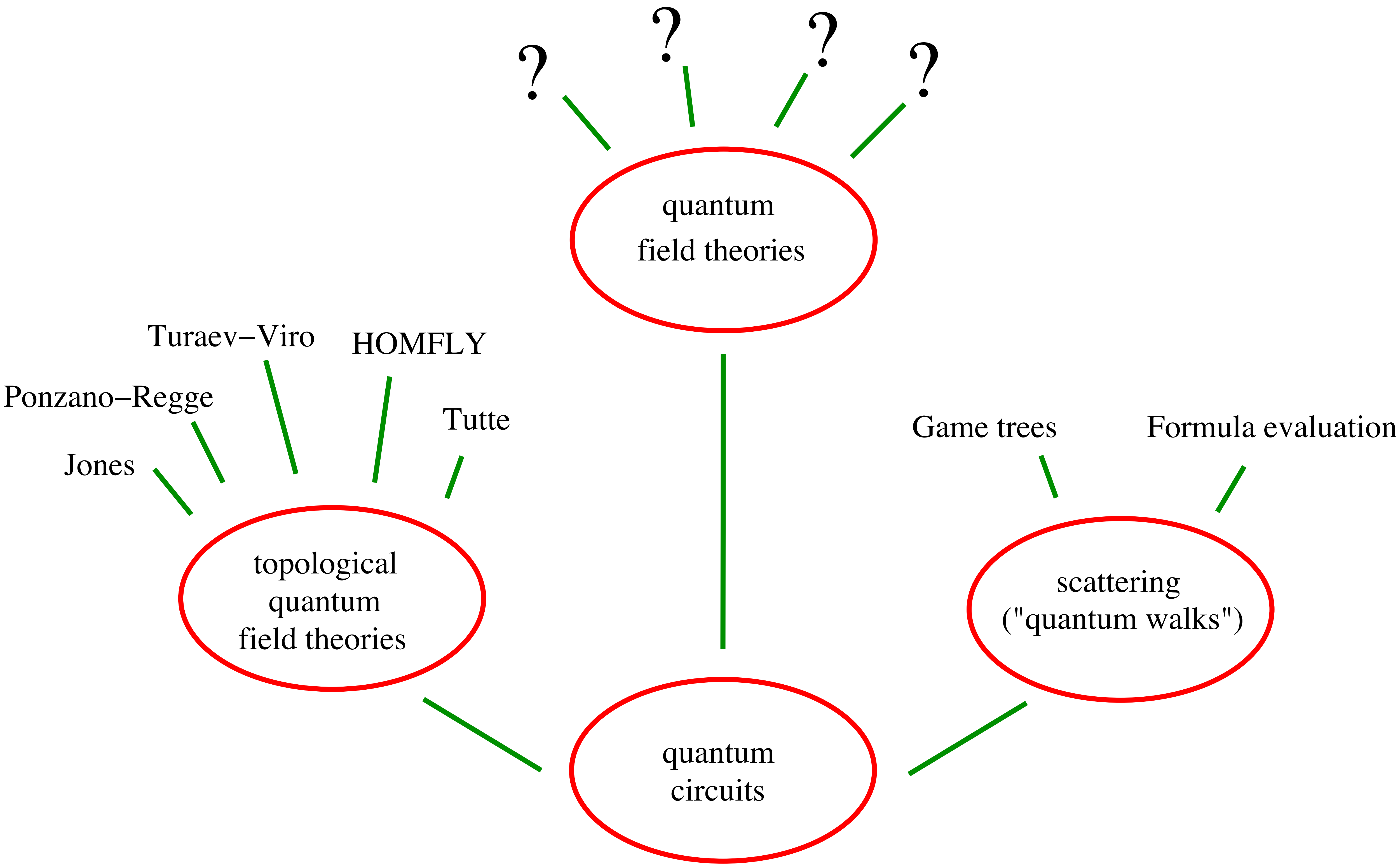
Analog Simulation

- Experiments so far have concentrated on mapping out phase diagrams
- We are developing a proposal to simulate ϕ^4 scattering processes using Rydberg atoms trapped in optical lattices

[Gorshkov, S.J., Preskill, Lee, *In Preparation*]

Broader Context

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quantum
field theories
Turaev−Viro
HOMFLY
Ponzano−Regge
Tutte
Jones
topological
quantum
field theories
Game trees
Formula evaluation
scattering
("quantum walks")
quantum
circuits

What I'm trying to do is get you people who think about computer simulation to see if you can't invent a different point of view than the physicists have.

-Richard Feynman, 1981

In thinking and trying out ideas about "what is a field theory" I found it very helpful to demand that a correctly formulated field theory should be soluble by computer... It was clear, in the '60s, that no such computing power was available in practice.

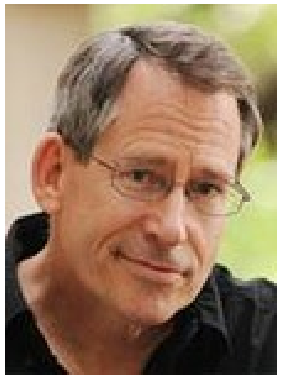
-Kenneth Wilson, 1982

Conclusion

Quantum computers can simulate scattering in ϕ^4-theory.

There are many exciting prospects for quantum computation and quantum field theory to contribute to each other's progress.

I thank my collaborators:




Thank you for your attention.