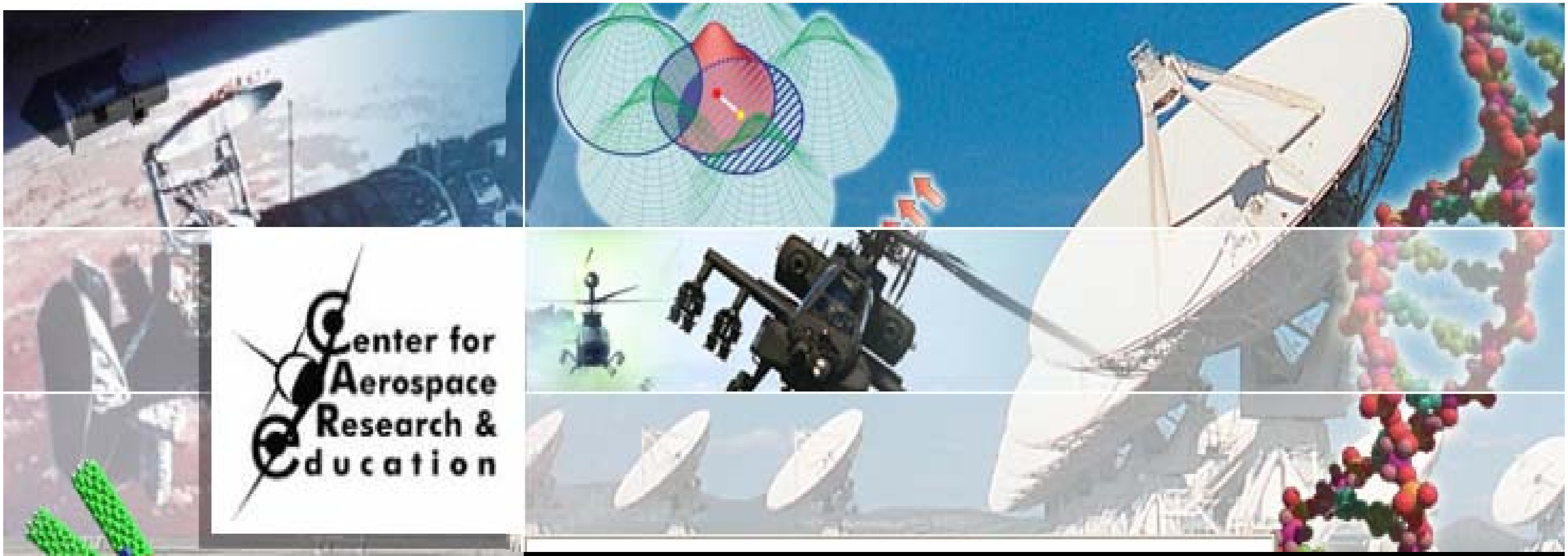
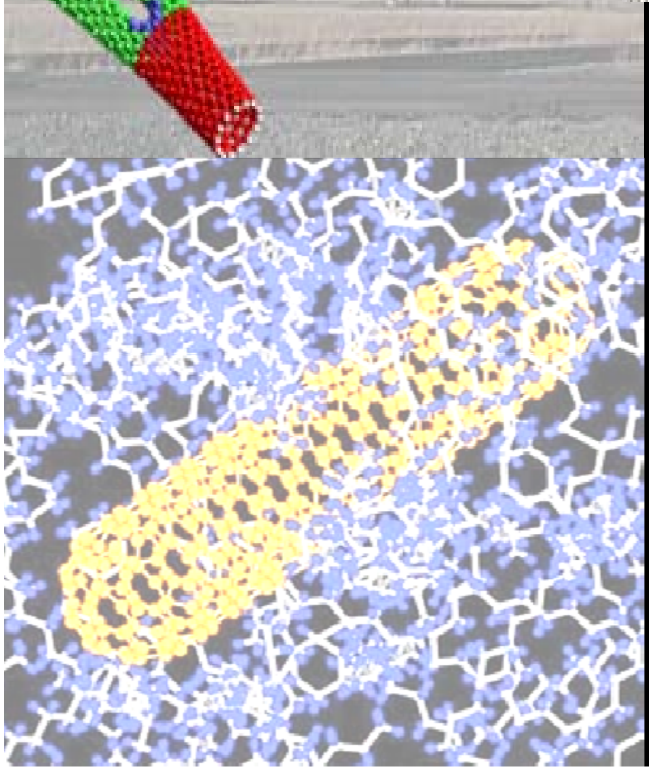




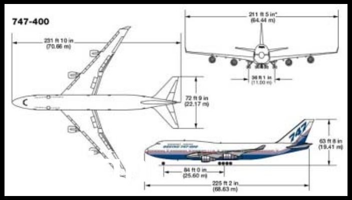
Multi-Scale Analysis of Aircraft Structural Longevity

(Research Conducted in the early 1990s)

Satya N. Atluri, UCI

Life Cycle of an Aircraft

Market Requirements



Design
Prototype
Certification

Production

AGILE

FEM, BEM
Meshless
MDO, IPPD,
Inverse
Problem

AGILE

St
Integ
Durabilit

Scheduling,
Air Traffic
Control and
Navigation

Operation

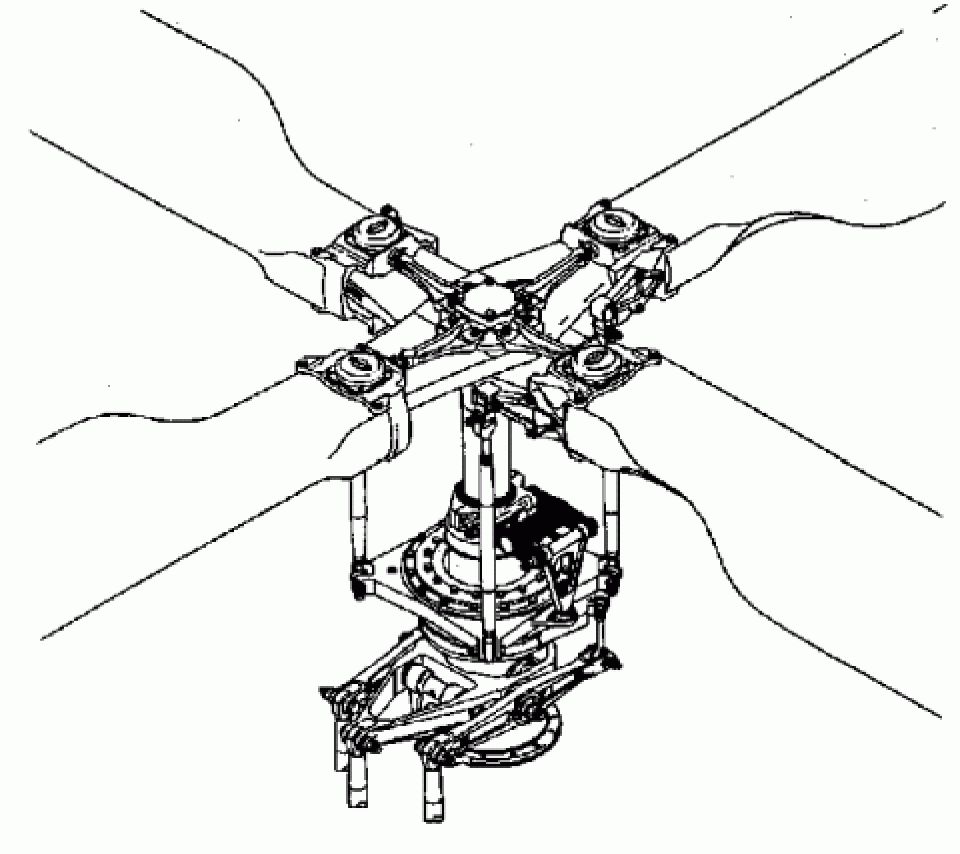
Damage
Tolerance &
Life
Enhancement

tenance

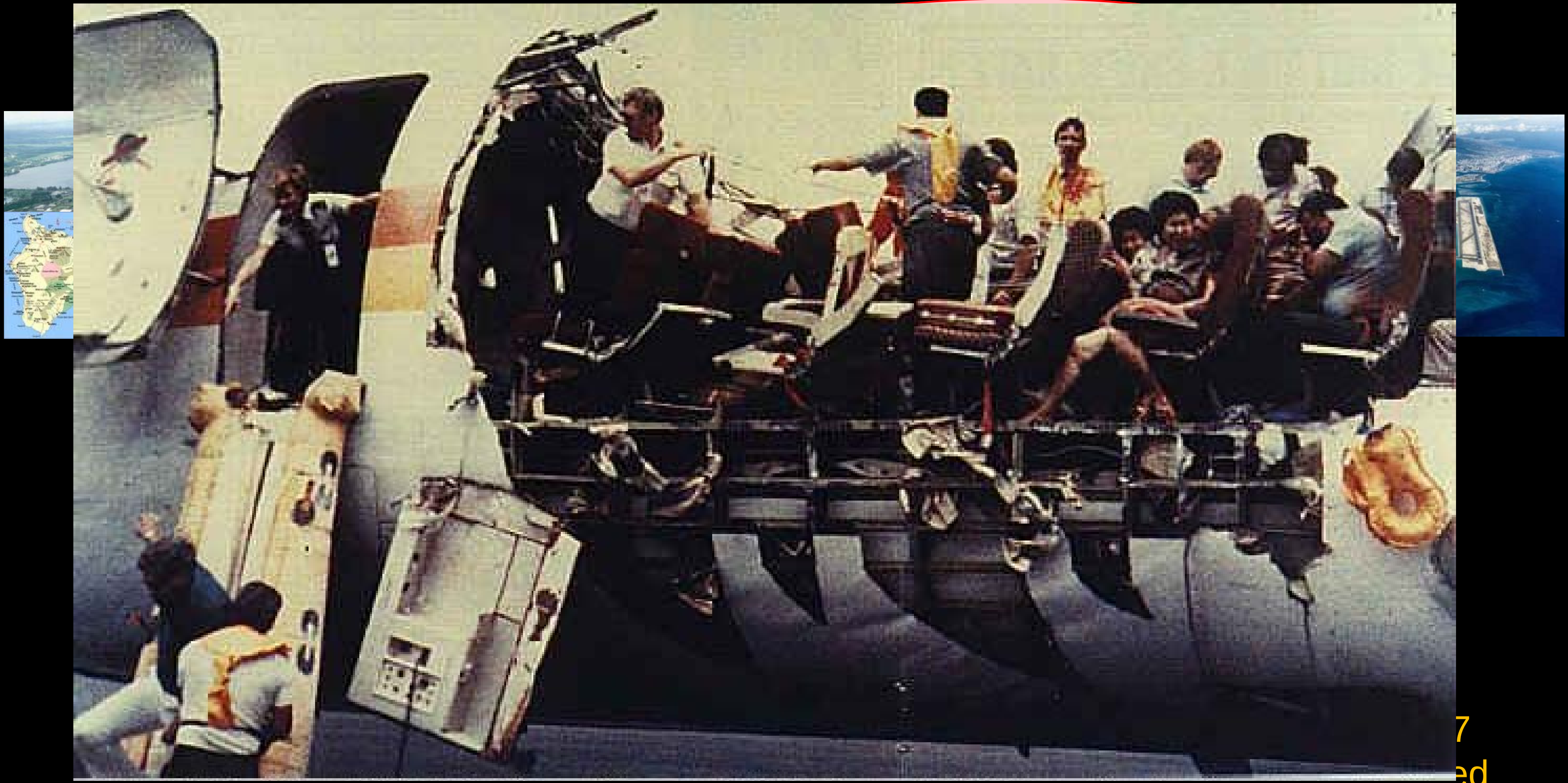
Overhauls

Retirement

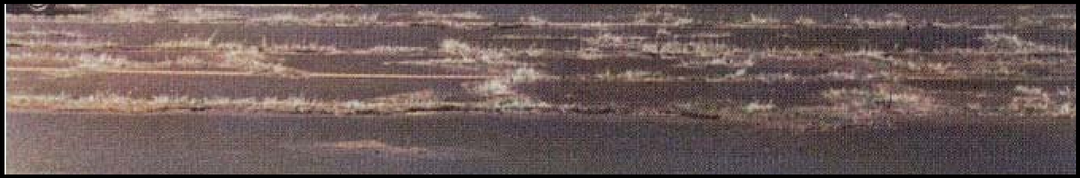
Structural Integrity of Rotorcraft Components (DTA?)



Aircraft Fatigue Failure: Loss of Integrity

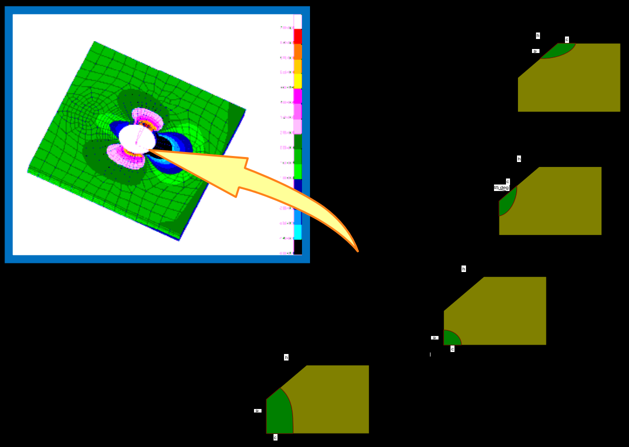


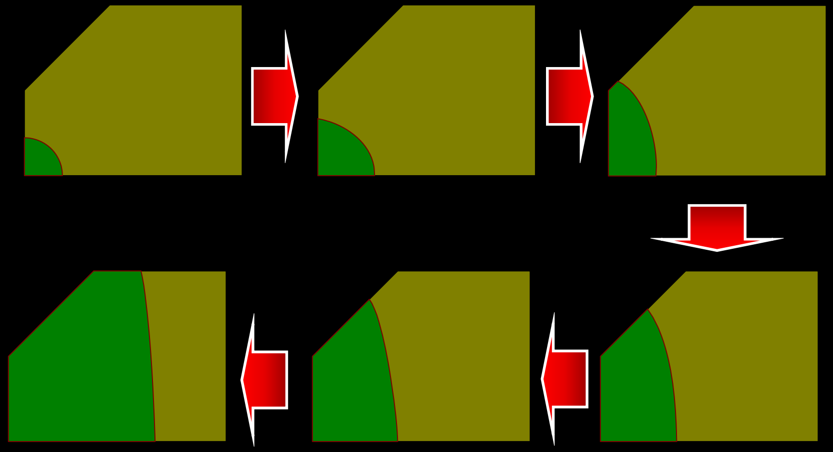
4-28-1988 After 89,090 flight cycles on a 737-200, metal fatigue lets the top go in flight

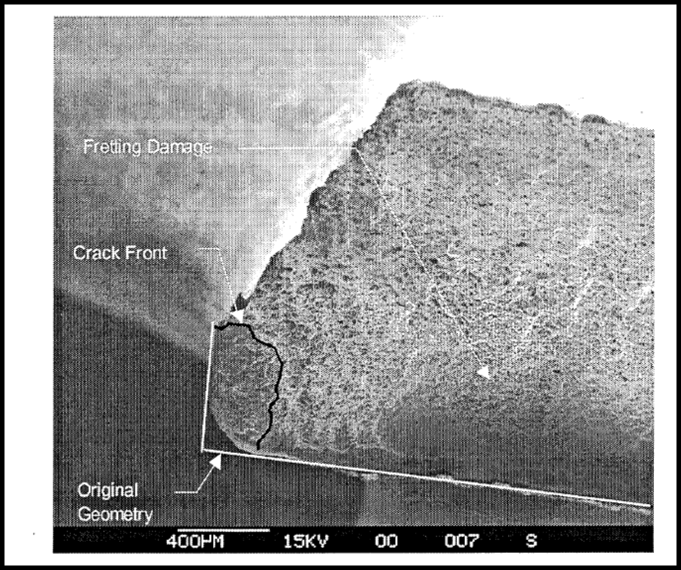


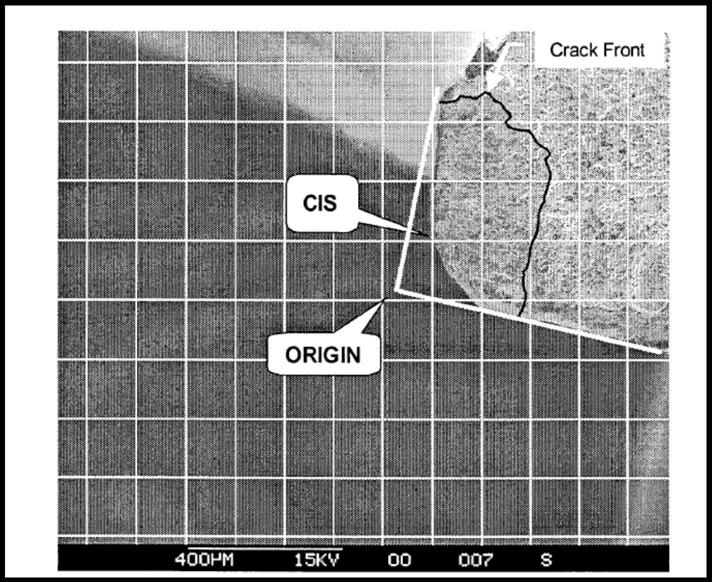
7
ed
an
explosive decompression in flight, but was able to land safely.

Micro Crack Level: 10^{-5} m
DTALE: MLPG-SGBNM Alternating

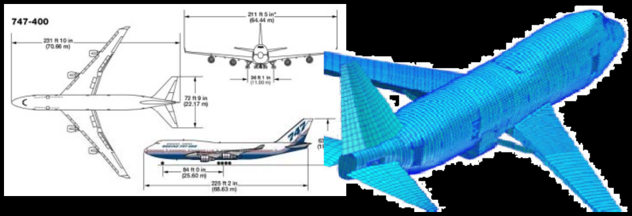


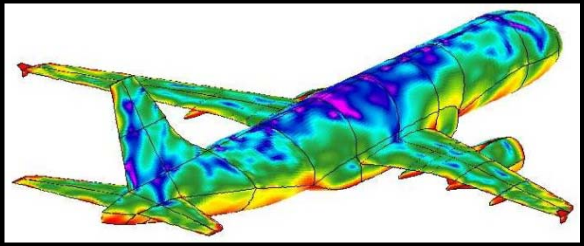


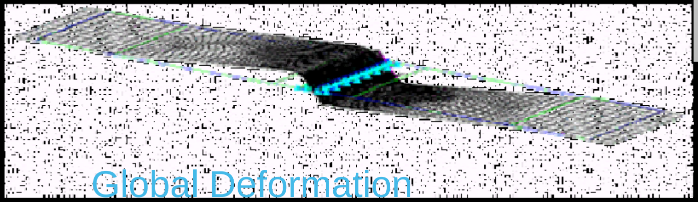




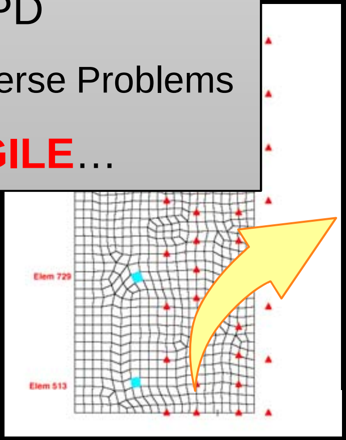
Mega- to Micro-Level Multiple-Scale Analyses

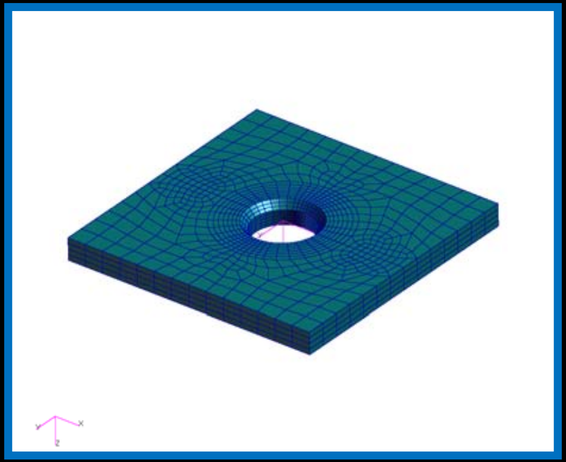






- Finite volume
- Finite Element
- Panel Methods
- Meshless Methods
- BEM
- MDO
- IPPD
- Inverse Problems
- **AGILE**…



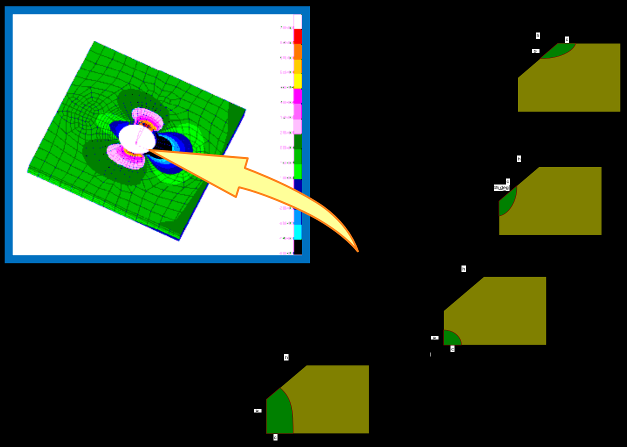


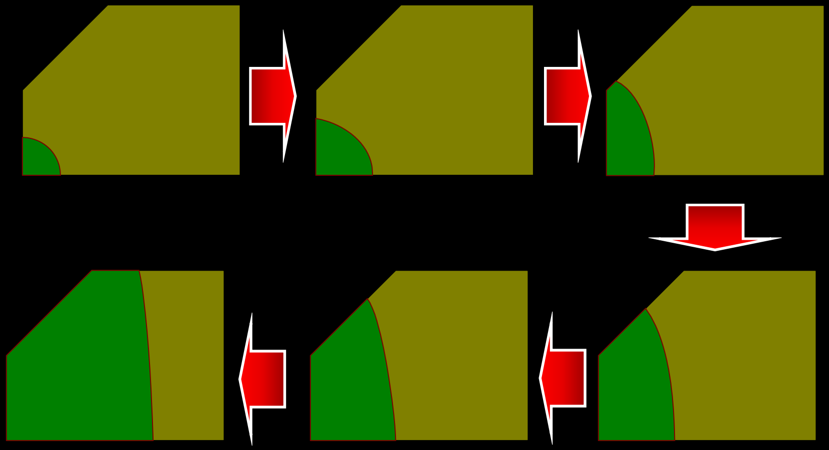
System Level:
10^2m

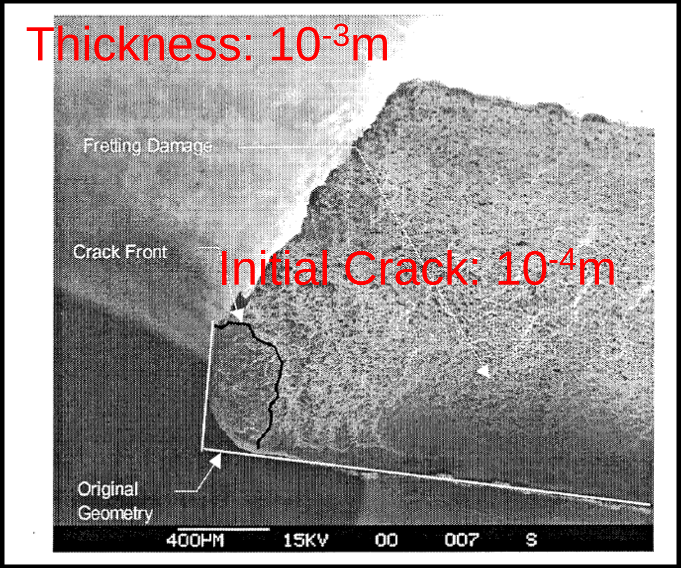
Component Level:
1~ 10^{-2} m

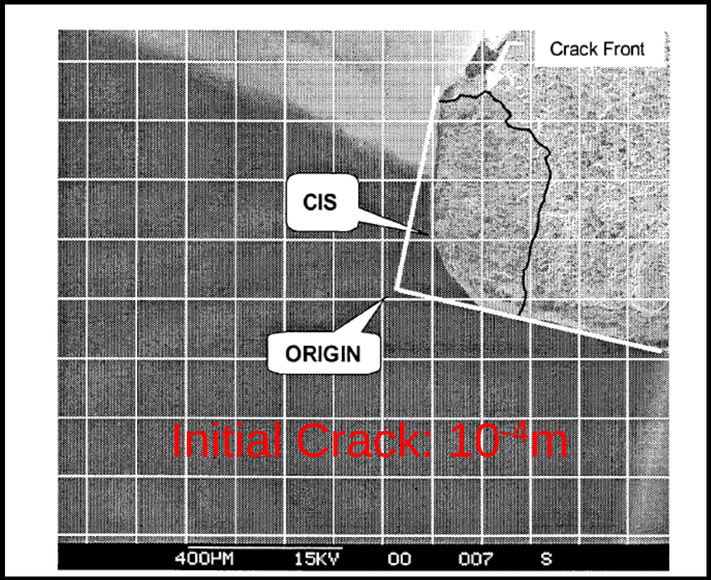
Micro Crack Level:
10^{-4} ~ 10^{-6} m

Initial Detected Crack Level: 10^{-4} m
AGILE Alternating Techniques



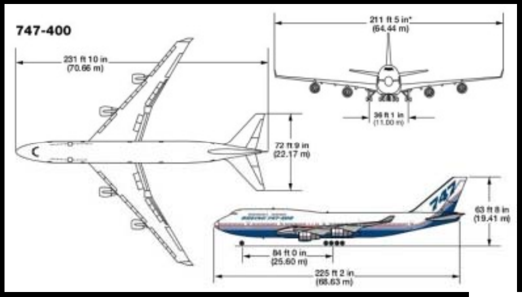


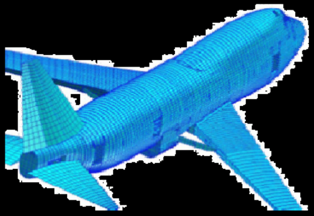




Multi-Scale Damage Tolerance for Initially Detectable Cracks

10^2





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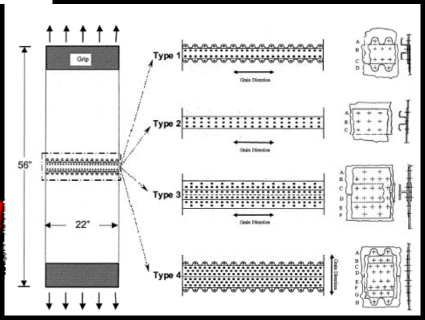
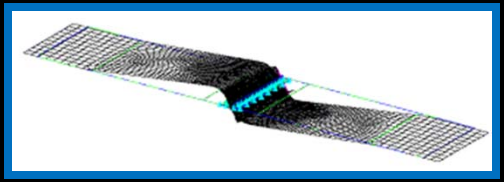
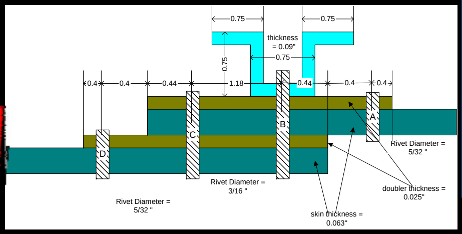
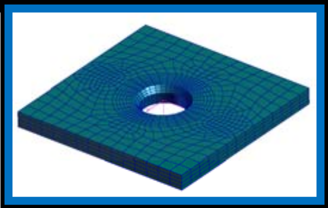


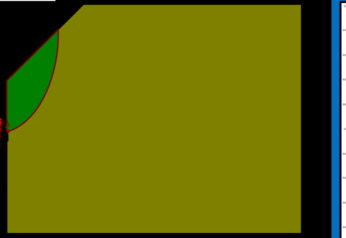


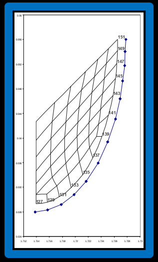
10^{-2}





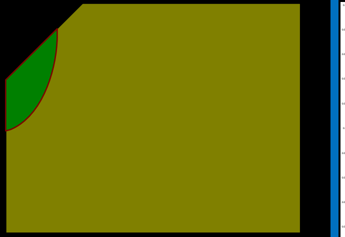
10^{-4}

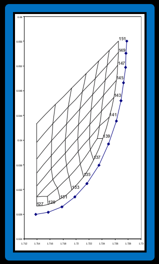




Micro-Crack Initiation?
Simply using continum-stress mechanics

10^{-4}





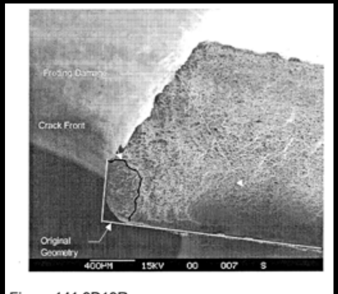
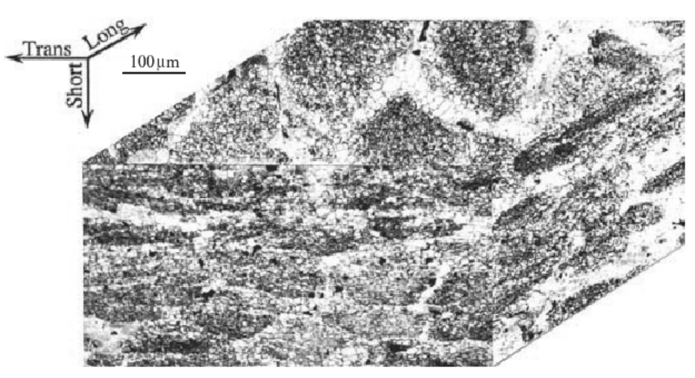
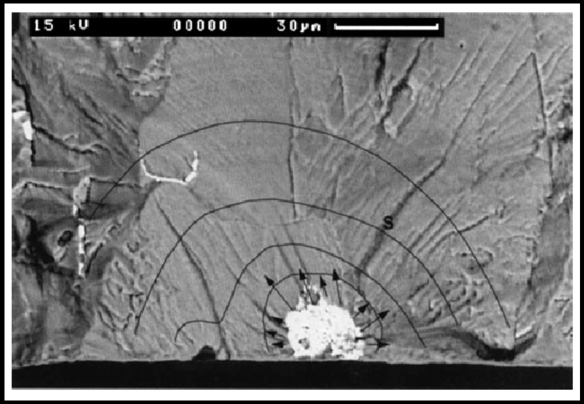


Figure 141 3D13R

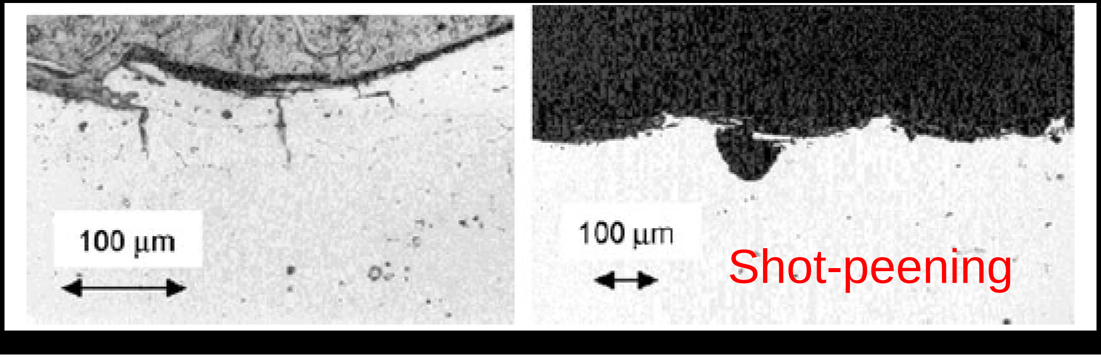
10^{-6}



Micro-Structure

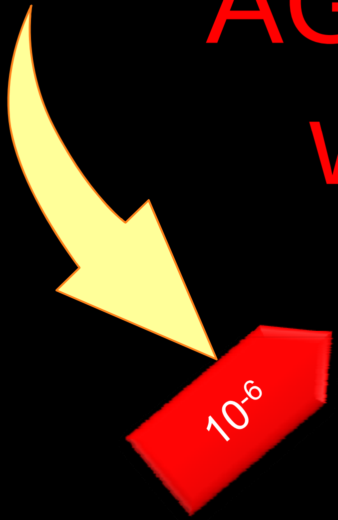


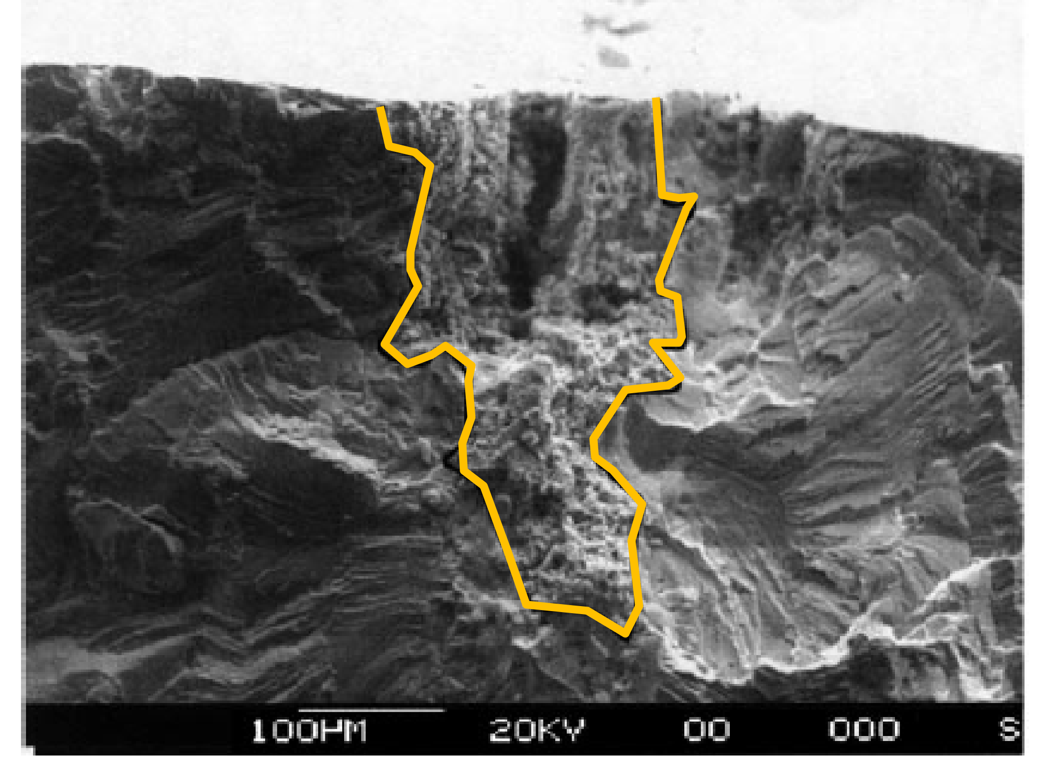
Inclusion



Shot-peening

AGILE: Model at 10^{-6} Level with Continuum Details





AGILE: Boundary surface mesh only, without refining FEM mesh. Higher order boundary-elements fit curved surfaces much better!

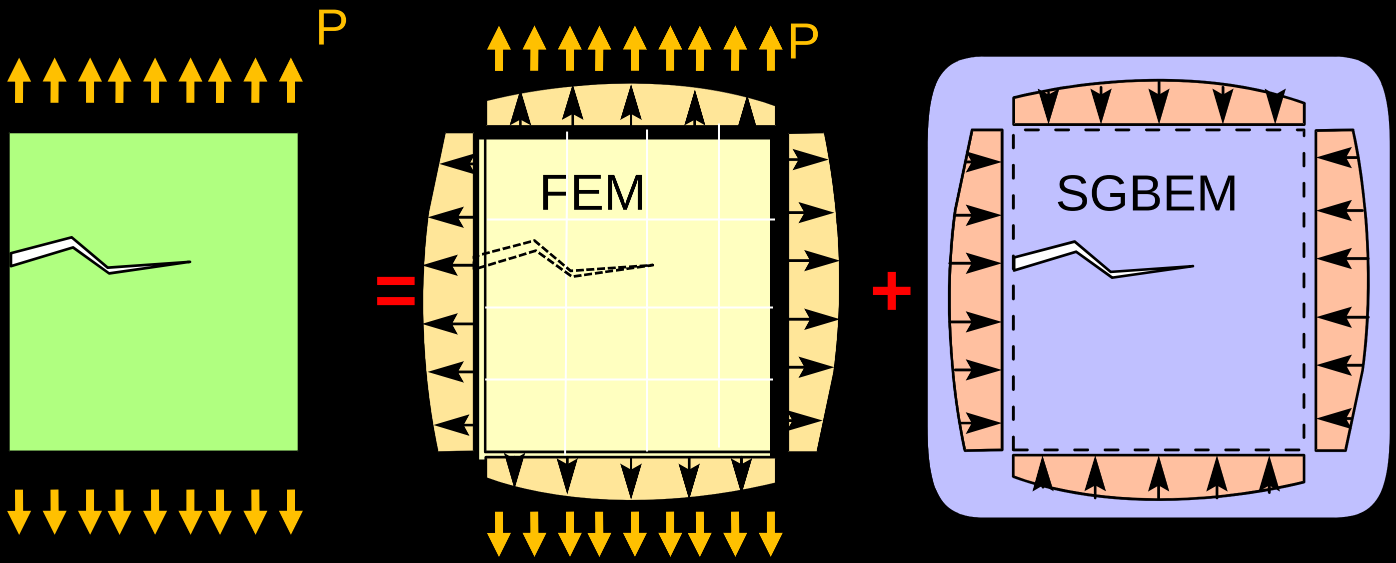
AGILE

- Continum Damage Mechanics
- Anisotropic Damage Mechanics
- Grain Boundary Fracture Mechanics
- Gradient Theories of Material Behavior
- ________________? Far in the Future
- Ab Initio……Dislocation Dynamics
- MD
- Statistical Mechanics
- DFT……..

AGILE (LOCAL): SGBEM-FEM Alternating

(Symmetric Galerkin Boundary Element – FEM Alternating Method)

(Overall Accuracies of KI, KII,KIII, Jk are the best of any available method)



Loaded Finite body
with a crack

Loaded Finite body
without a crack

Infinite body
with a crack

FEM Stiffness matrix inverted only ONCE, Faster!

Why AGILE?

- Accuracy is the best:
  - State-of-the-art advanced theories & analytical developments are used, in conjunction with the most efficient computational algorithms.
  - Most advanced closed-form mathematics, and only minimal numerics

Advanced Theories

- Solvers are developed, based on both FEM(for uncracked structure) and SGBEM(for a subdomain w/2-D or 3-D crack).
- SGBEM is developed, using the newly developed weakly-singular BIEs:
  - Support higher-order elements for curved surfaces
  - higher performance and accuracy
  - Preserve the symmetry of the matrices
- FEM & SGBEM are coupled through the Schwartz alternating method:
  - FE mesh, and the SG-BEM crack-model are totally uncoupled
  - Ease of mesh creation
  - Very Fast algorithm for automated crack growth, FE model is factorized and solved only once.

AGILE: Faster and more accurate than traditional BIE

- Weakly-singular integrals are numerically tractable, with
  Gaussian quadrature algorithms using lower order integrations
- Higher-order elements with curved sides can be used,
  because of its requirement of only C_0 continuity, which is especially useful for modeling 3D non-planar cracks with less elements.

AGILE: More applicable than pure BIE

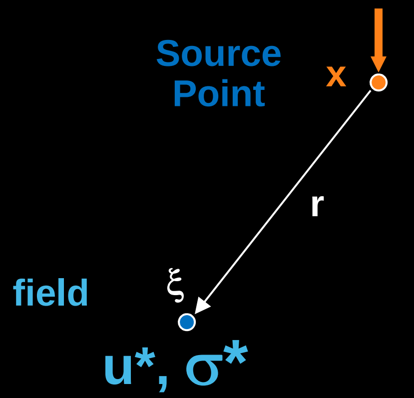
- Built-in FE solver handles more complicated geometries, including structural elements, such as beams, plates, shells, and MPCs.
- More efficient for problems with high volume/surface ratios, for example, thin-walled structures, manifold domains, and bi-material parts.
- 2-D, 2-D/3-D transition, & 3-D modeling of structures w/ mixed-mode crack-growth

SGBEM: Fundamental Solutions

3D Problems

$$u_i^{*p}(\mathbf{x},\xi)=\frac{1}{16\pi\mu(1-\upsilon)r}[(3-4\upsilon)\delta_{ip}+r_{,i}r_{,p}]$$

$$\sigma_{ij}^{*p}(\mathbf{x},\xi)=\frac{1}{8\pi(1-\upsilon)r^2}[(1-2\upsilon)(\delta_{ij}r_{,p}-\delta_{ip}r_{,j}-\delta_{jp}r_{,i})-3r_{,i}r_{,j}r_{,p}]$$



2D Problems

$$u_i^{*p}(\mathbf{x},\xi)=\frac{1}{8\pi\mu(1-\upsilon)}[-(3-4\upsilon)\ln r\,\delta_{ip}+r_{,i}r_{,p}]$$

$$\sigma_{ij}^{*p}(\mathbf{x},\xi)=\frac{1}{4\pi(1-\upsilon)r}[(1-2\upsilon)(\delta_{ij}r_{,p}-\delta_{ip}r_{,j}-\delta_{jp}r_{,i})-2r_{,i}r_{,j}r_{,p}]$$

where $\mathbf{r}=\xi-\mathbf{x}$

Displacement BIE

Using the fundamental solution **u*** as the test function , we obtain:

DBIE:

$$u_p(\mathbf{x}) = \int_{\partial\Omega} t_j(\xi) u_j^{*p}(\mathbf{x},\xi)\, dS - \int_{\partial\Omega} u_m(\xi) t_m^{*p}(\mathbf{x},\xi)\, dS$$

in which, displacements **u** are determined from

- the boundary displacements and
- the boundary tractions

Singularity *O(1/r^2)*

when differentiated directly, this leads to a Traction BIE, which is, unfortunately, hyper-singular: ***O(1/r ^{3})***

New Non-hyper Singular *O(1/r²)* Traction BIE

Using the test function, the global weak form of solid mechanics becomes

$$\int_{\partial\Omega} n_i E_{ijmn} u_{m,n} \bar{u}_{j,k} \, dS - \int_{\partial\Omega} n_k E_{ijmn} u_{m,n} \bar{u}_{j,i} \, dS + \int_{\partial\Omega} n_n E_{ijmn} u_{m,k} \bar{u}_{j,i} \, dS - \int_{\Omega} u_{m,k} (E_{ijmn} \bar{u}_{j,i})_{,n} \, d\Omega = 0$$

Replacing the test function with the gradients of fundamental solution, we obtain:

TBIE:

$$-\sigma_{ab}(\mathbf{x}) = \int_{\partial\Omega} t_q(\xi) \sigma^{*q}_{ab}(\mathbf{x},\xi) \, dS + \int_{\partial\Omega} D_p u_q(\xi) \Sigma^{*}_{abpq}(\mathbf{x},\xi) \, dS$$

in which, stresses are determined from

- the boundary displacements and
- the boundary tractions

Singularity *O(1/r²)*

De-sigularization
of Symmetric Galerkin Form

Applying Stoke's Theorem to Symmetric Galerkin form

$$\begin{aligned}\frac{1}{2}\int_{\partial\Omega}\hat{t}_p(\mathbf{x})u_p(\mathbf{x})dS_x &= \int_{\partial\Omega}\hat{t}_p(\mathbf{x})dS_x\int_{\partial\Omega}t_j(\boldsymbol{\xi})u_j^{*p}(\mathbf{x},\boldsymbol{\xi})\,dS_\xi \\ &+ \int_{\partial\Omega}\hat{t}_p(\mathbf{x})dS_x\int_{\partial\Omega}D_i(\boldsymbol{\xi})u_j(\boldsymbol{\xi})G_{ij}^{*p}(\mathbf{x},\boldsymbol{\xi})\,dS_\xi \\ &+ \int_{\partial\Omega}\hat{t}_p(\mathbf{x})dS_x\int_{\partial\Omega}^{CPV} n_i(\boldsymbol{\xi})u_j(\boldsymbol{\xi})\phi_{ij}^{*p}(\mathbf{x},\boldsymbol{\xi})\,dS_\xi\end{aligned}$$

$$\begin{aligned}-\frac{1}{2}\int_{\partial\Omega}t_b(\mathbf{x})\hat{u}_b(\mathbf{x})dS_x &= \int_{\partial\Omega}D_a\hat{u}_b(\mathbf{x})dS_x\int_{\partial\Omega}t_q(\boldsymbol{\xi})G_{ab}^{*q}(\mathbf{x},\boldsymbol{\xi})\,dS_\xi \\ &- \int_{\partial\Omega}t_q(\boldsymbol{\xi})\,dS_\xi\int_{\partial\Omega}^{CPV} n_a(\mathbf{x})\hat{u}_b(\mathbf{x})\phi_{ab}^{*q}(\mathbf{x},\boldsymbol{\xi})dS_x \\ &+ \int_{\partial\Omega}D_a\hat{u}_b(\mathbf{x})dS_x\int_{\partial\Omega}D_p u_q(\boldsymbol{\xi})H_{abpq}^{*}(\mathbf{x},\boldsymbol{\xi})\,dS_\xi\end{aligned}$$

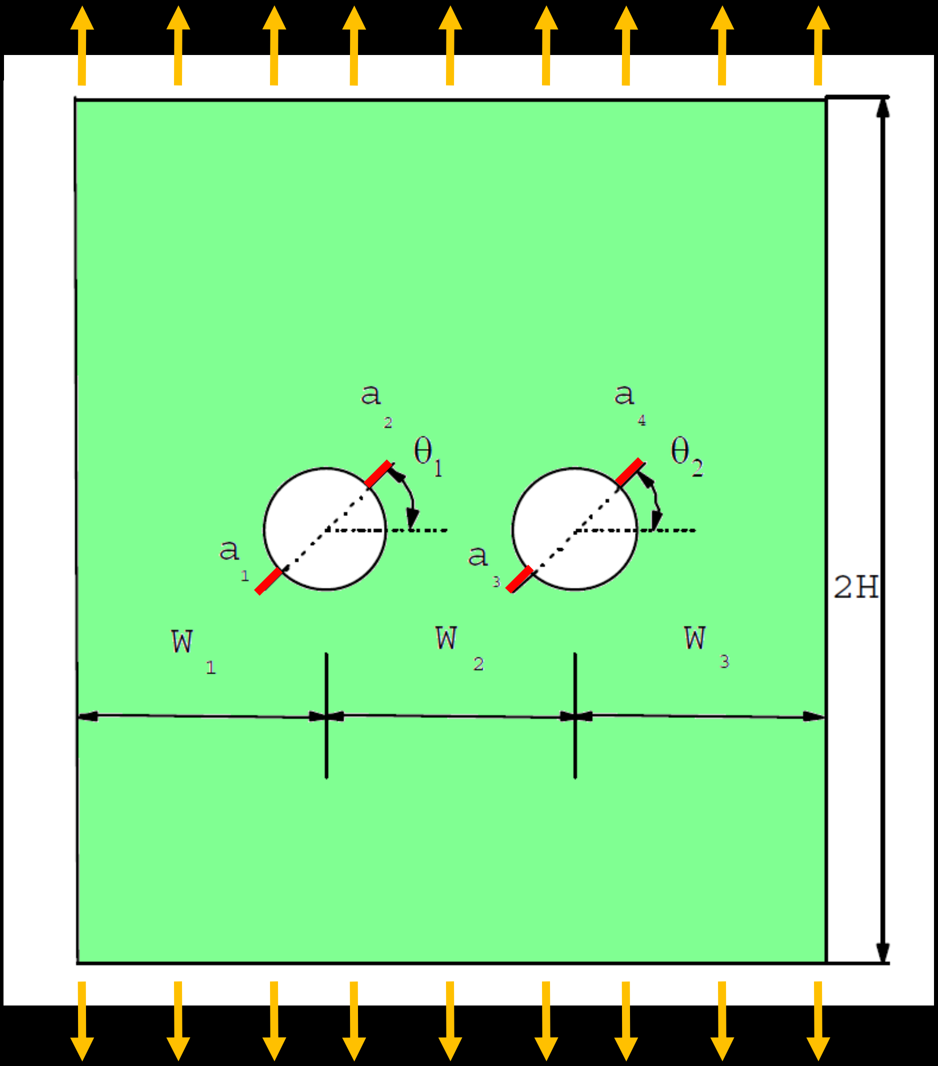
Singularity *O(1/r)*

Han. Z. D.; Atluri, S. N. (2003): On Simple Formulations of Weakly-Singular Traction & Displacement BIE, and Their Solutions through Petrov-Galerkin Approaches, CMES: *Computer Modeling in Engineering & Sciences*, vol. 4 no. 1, pp. 5-20.

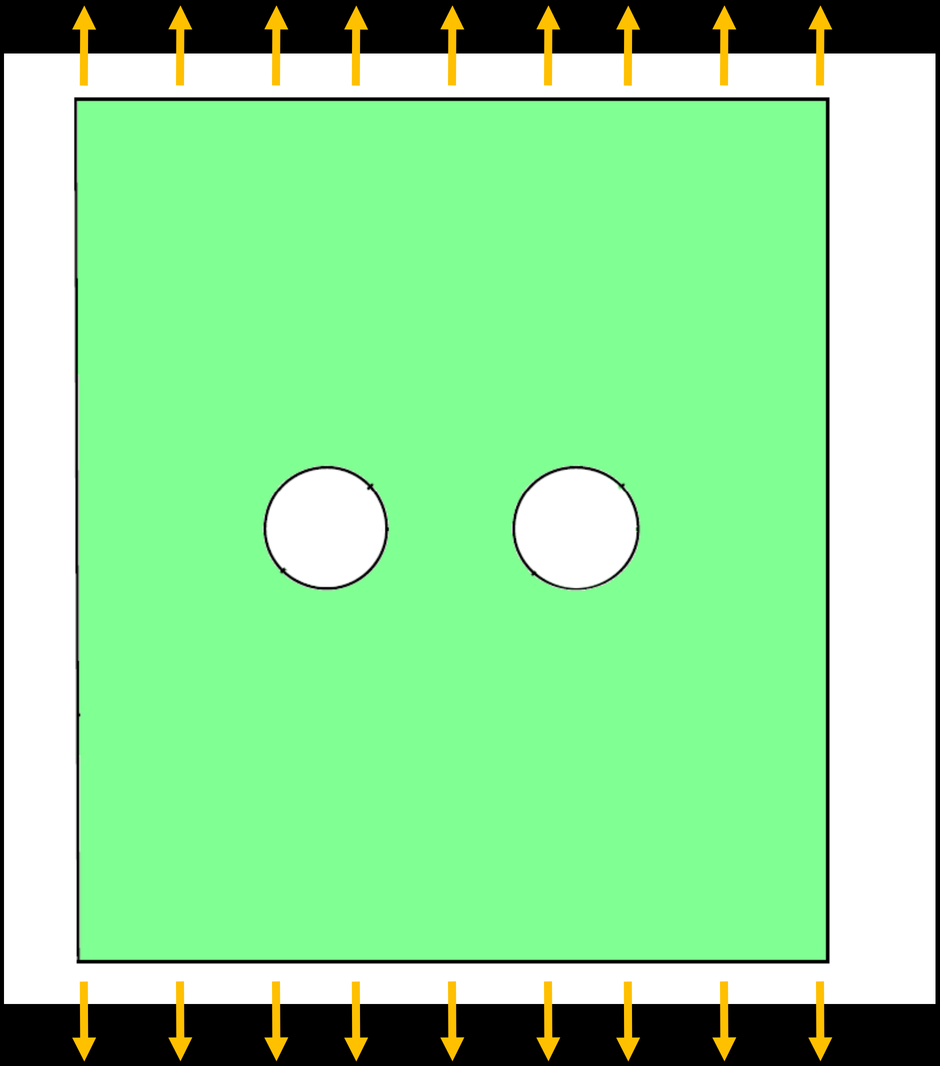
Intrinsic Features of the SGBEM

- weak singularity of the kernel: O(1/r)
- symmetric structure of the global "stiffness" matrix
- the possibility of using higher-order elements with curved sides

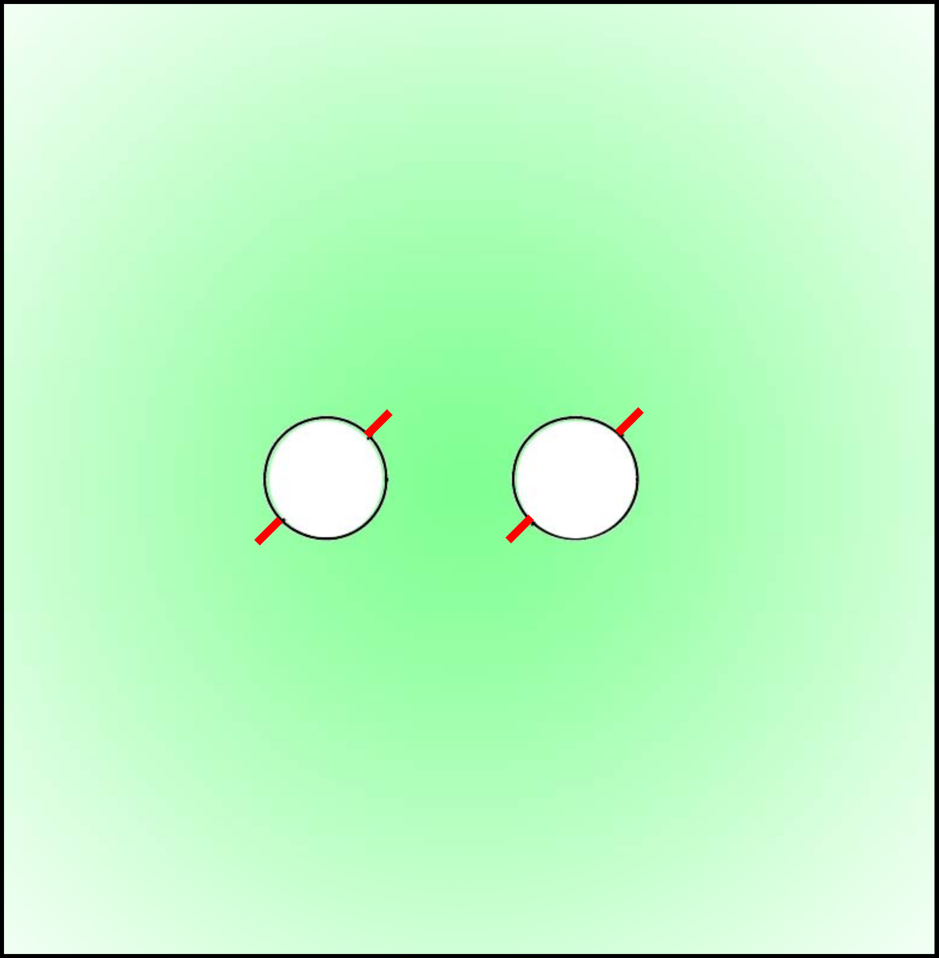
AGILE-2D: Cracks Emanating from Fastener Holes in a Fuselage Lap-Joint



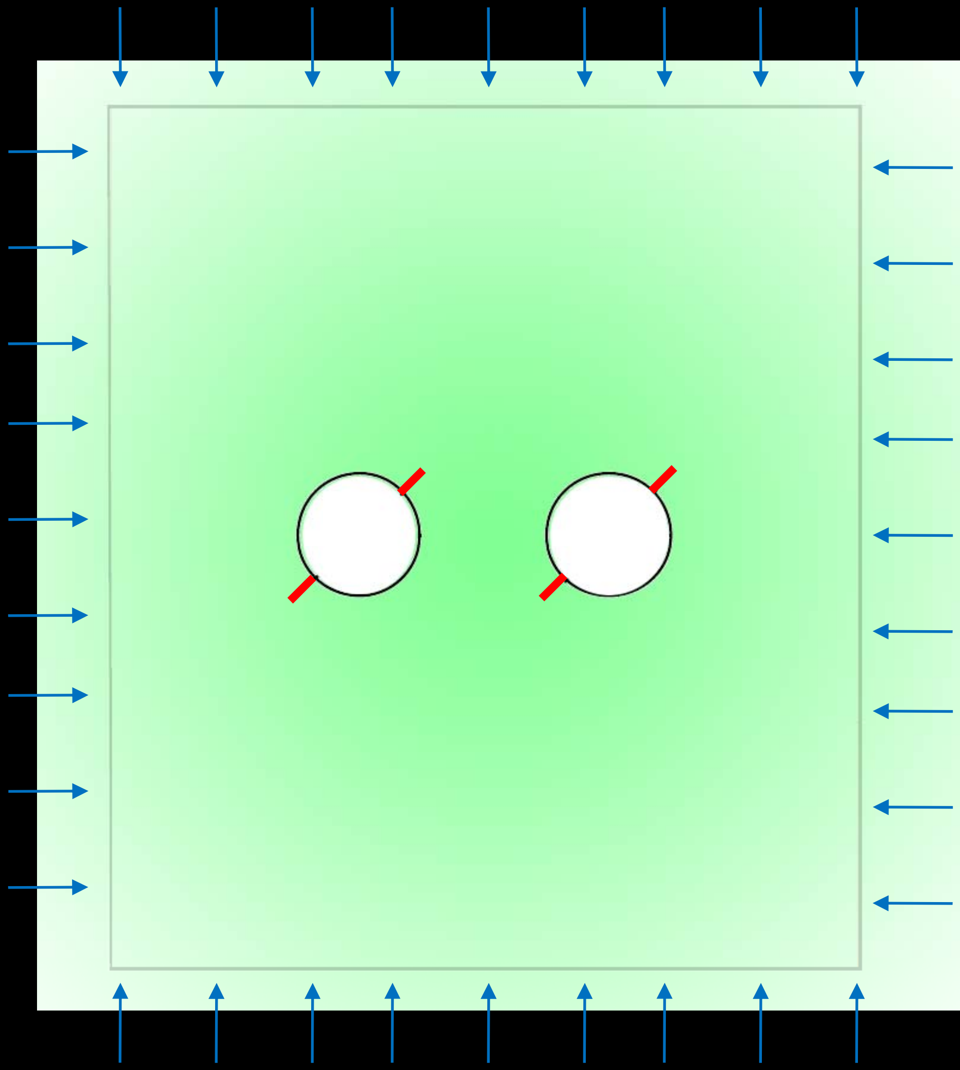
FEM Model with Boundary and Load Conditions but NO Crack



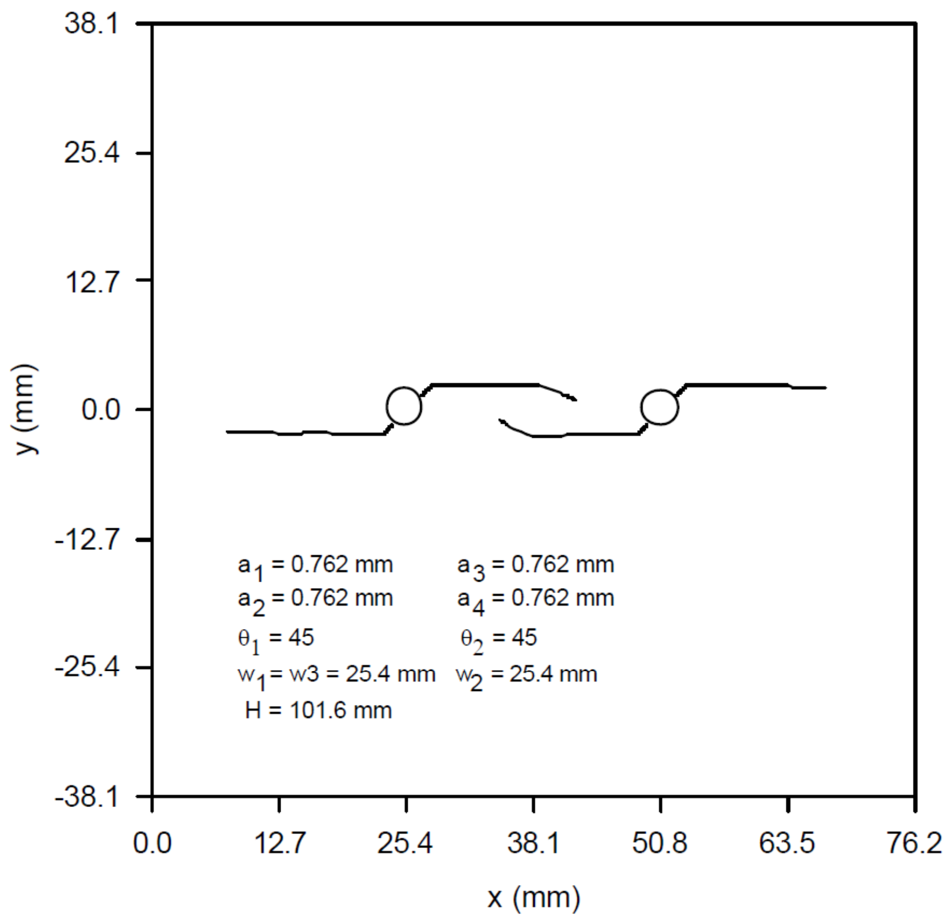
2-D Infinite body
with loaded arbitrarily-shaped line cracks
ONLY: Singular Integral equations



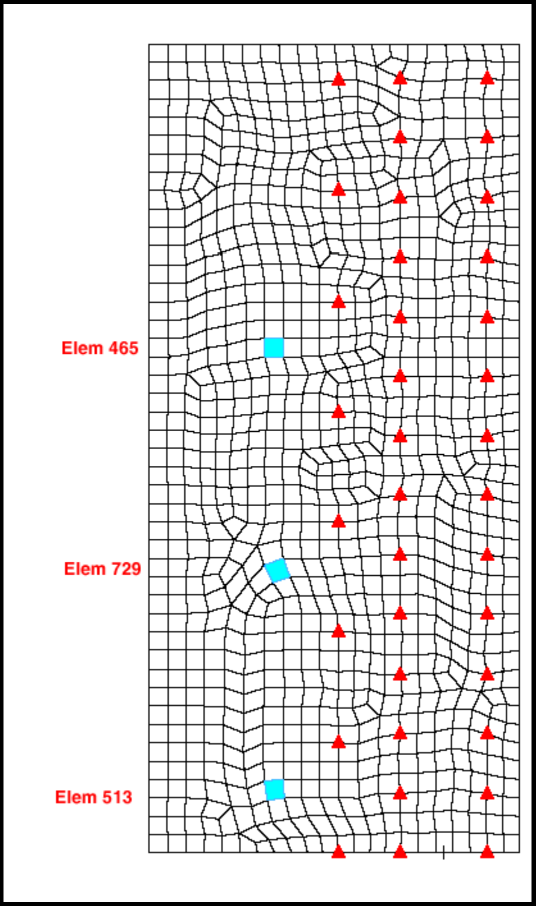
Alternating Procedure: Apply the residual tractions back on to the FEM



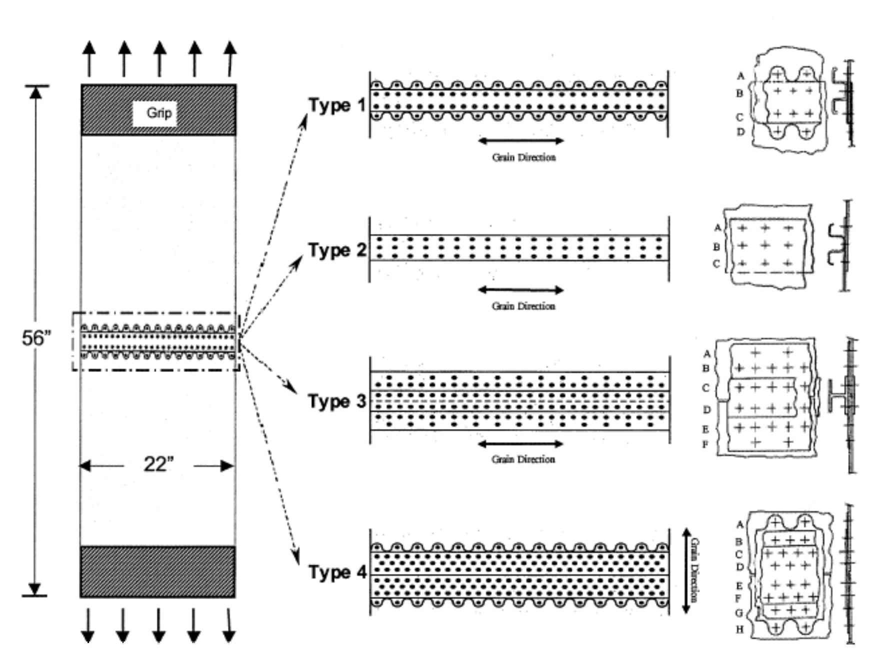
AGILE-2D Mixed Mode Crack Growth

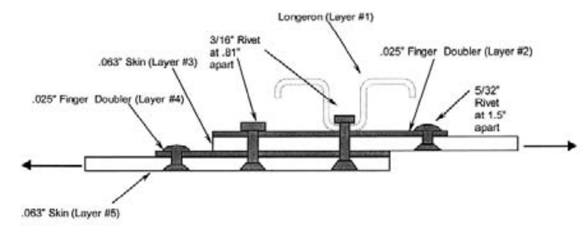


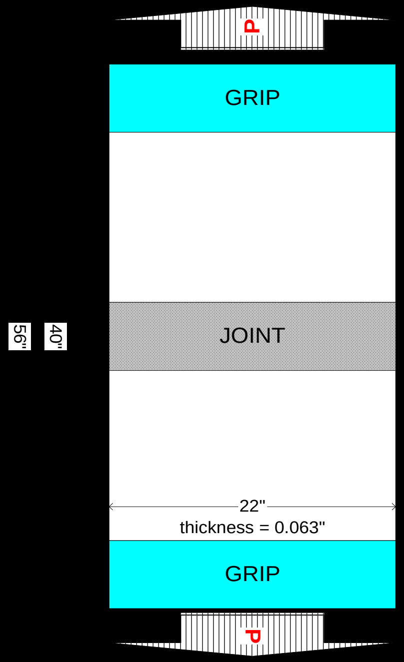
AGILE-2D: Multiple Holes



2D/3D Mixed Analyses with Parametric Crack Study

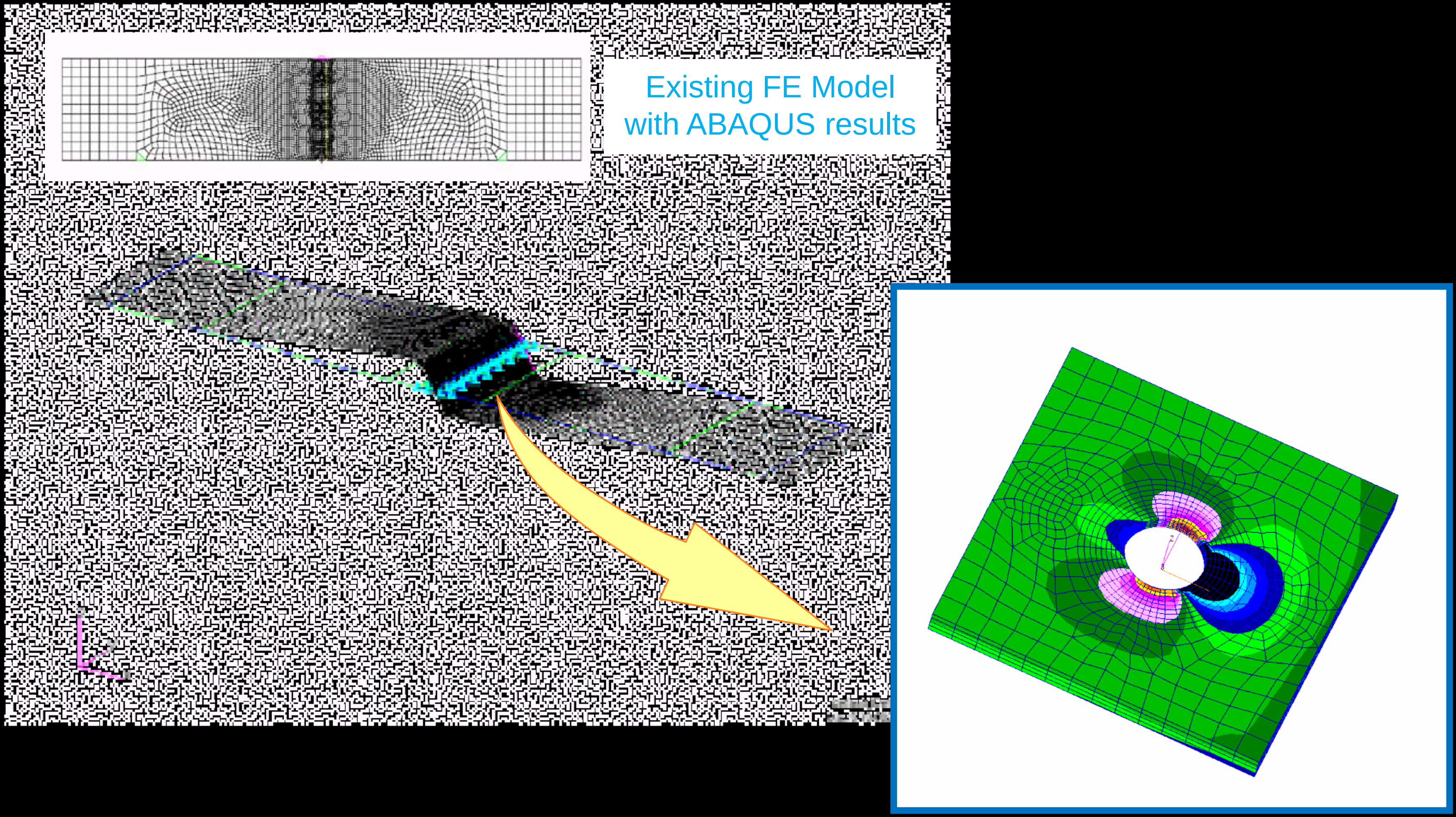






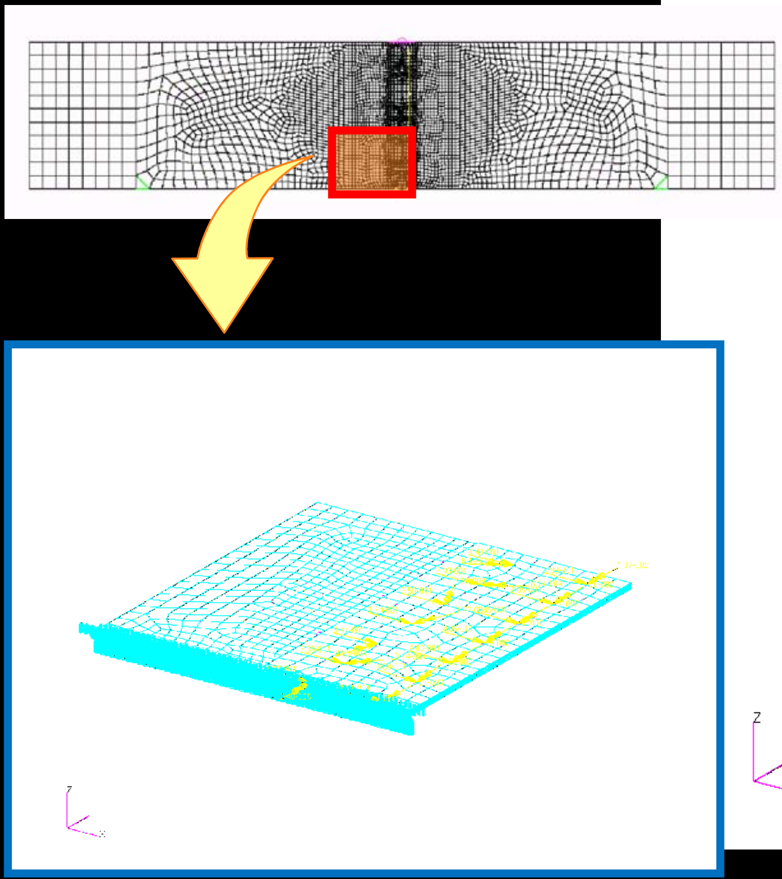
Skin Thickness = 0.063"

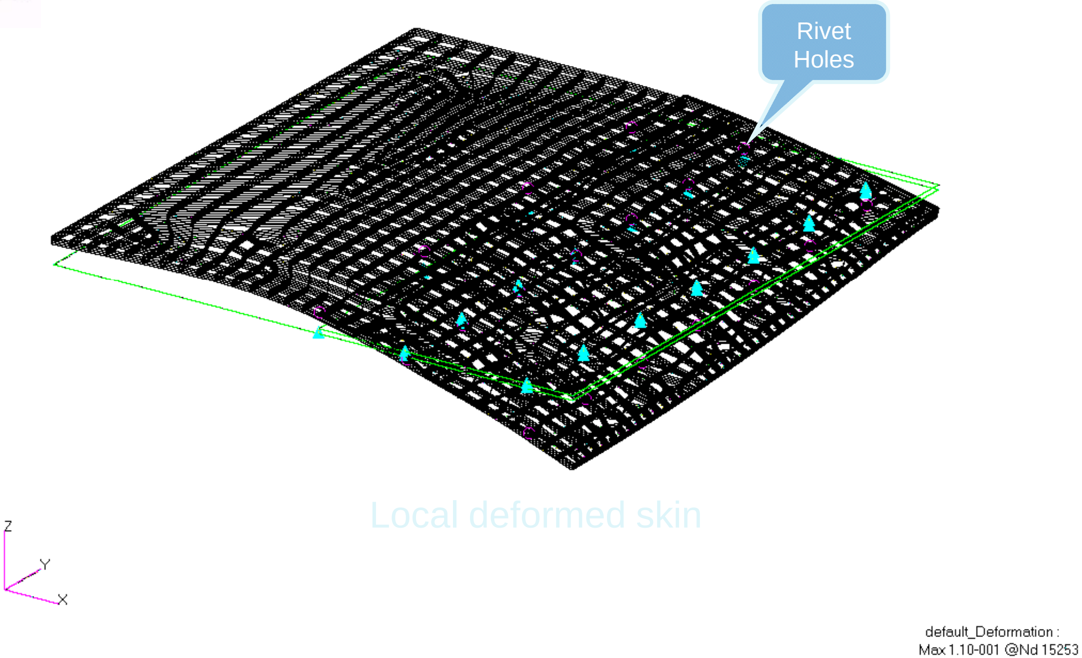
AGILE: Mixed 2D/3D Crack Parametric Analysis



Existing FE Model with ABAQUS results

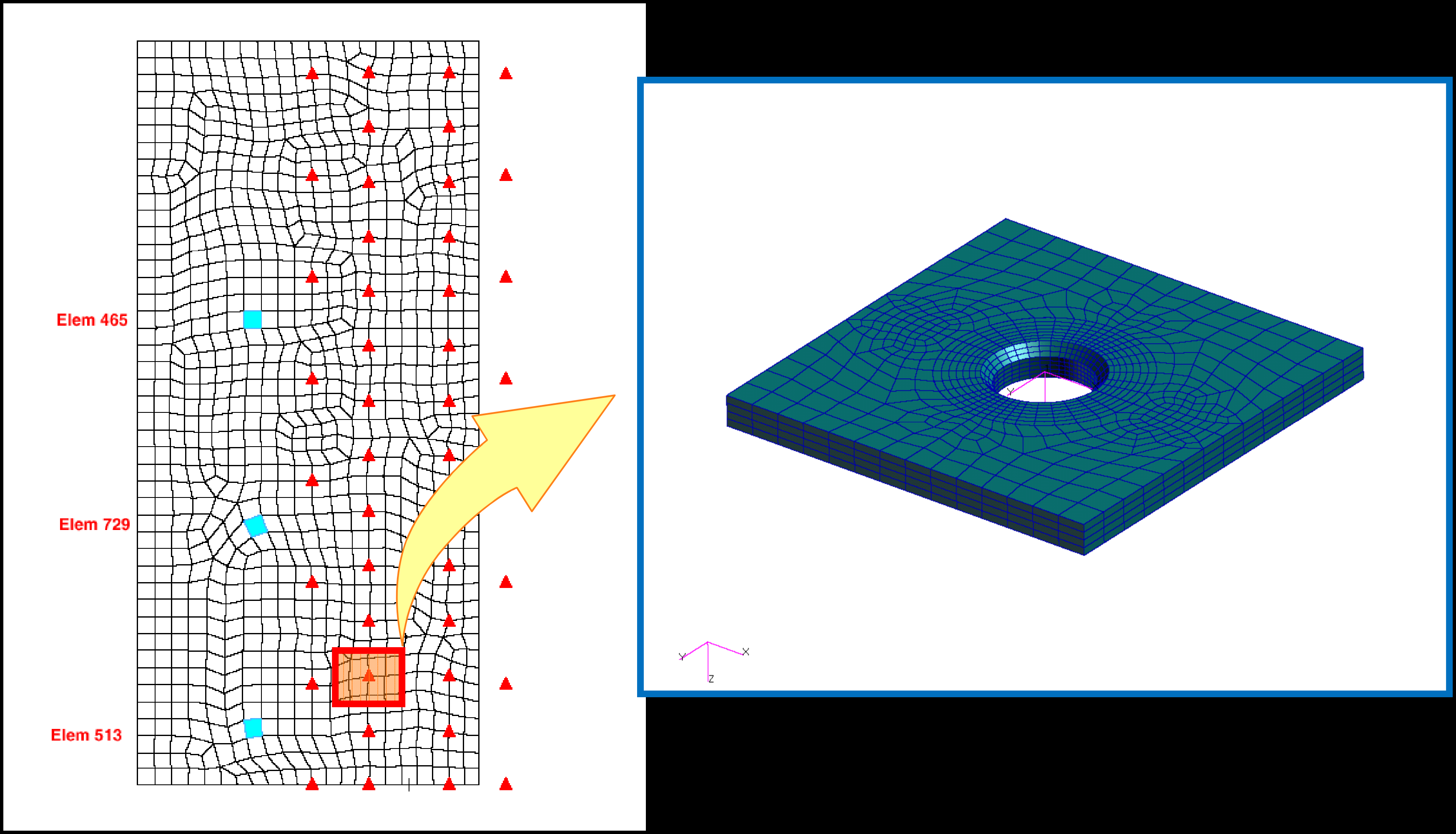
Intermediate FE Model (Joint)



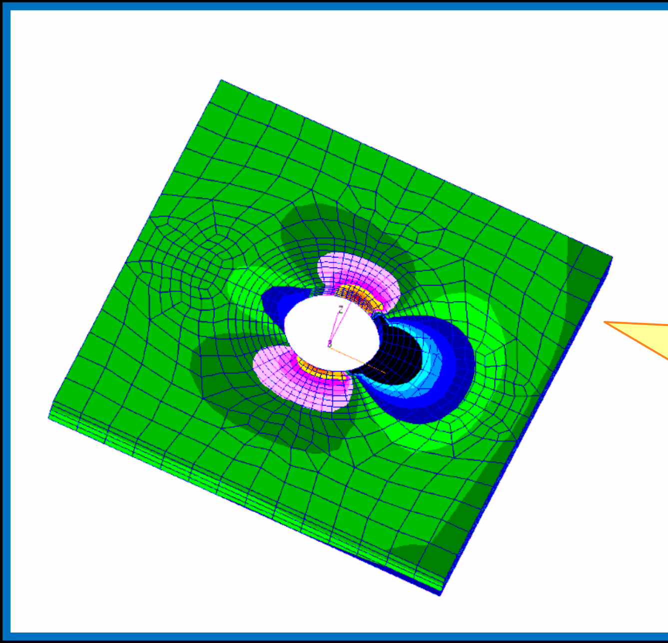


3D FE model with LBCs transferred from the global shell analysis by using AGILE GUI

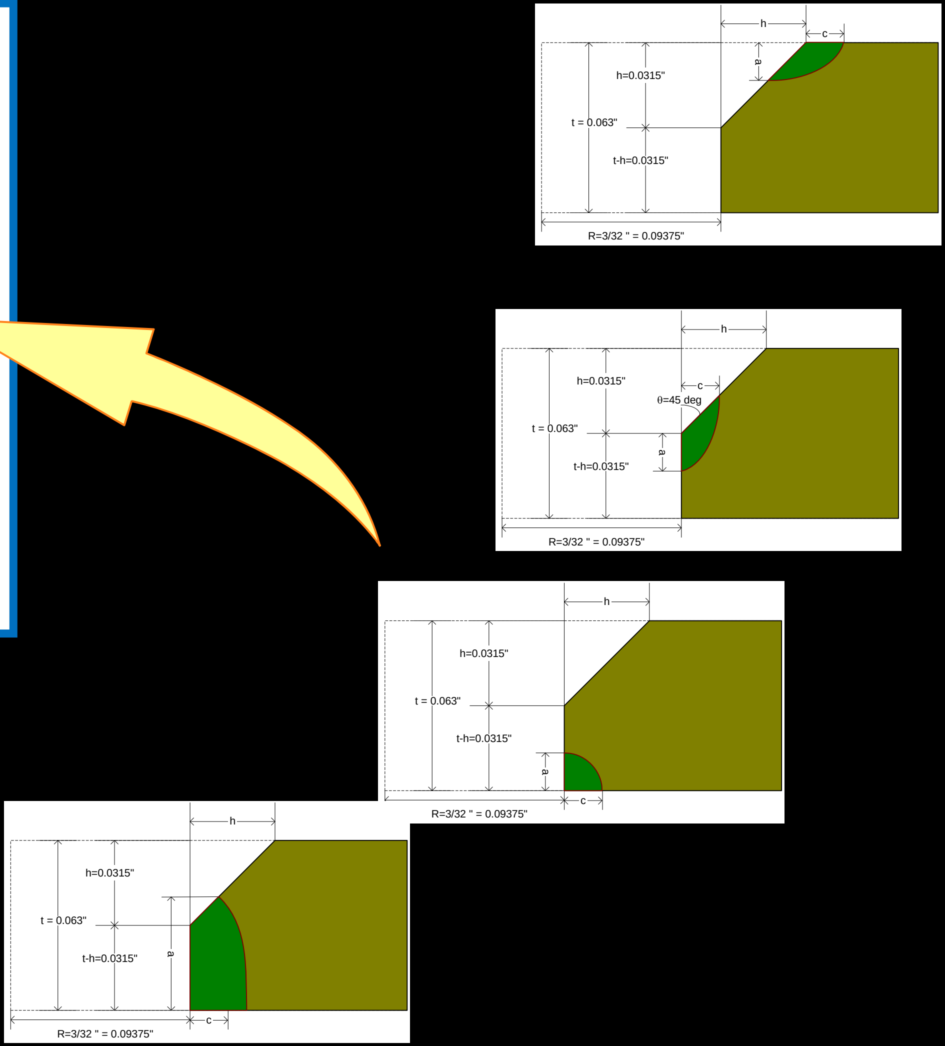
Local FE Model of Rivet Hole



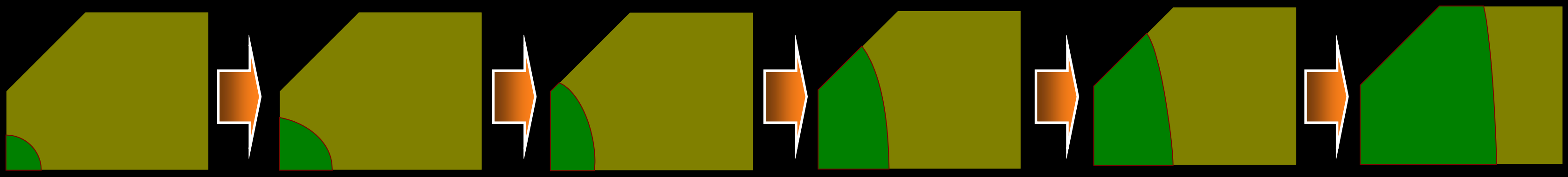
Multiple Crack Location study



AGILE FE model



Possible Crack Development



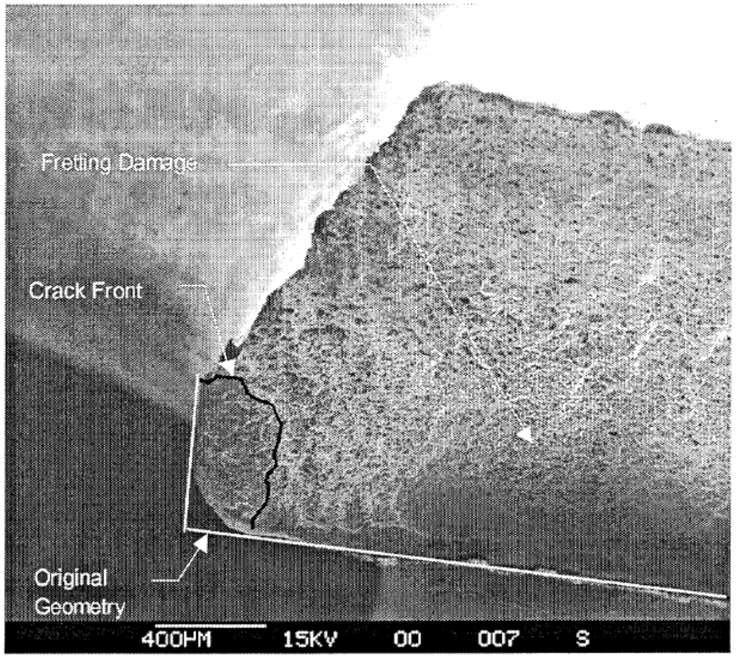


Figure 141 3D13R

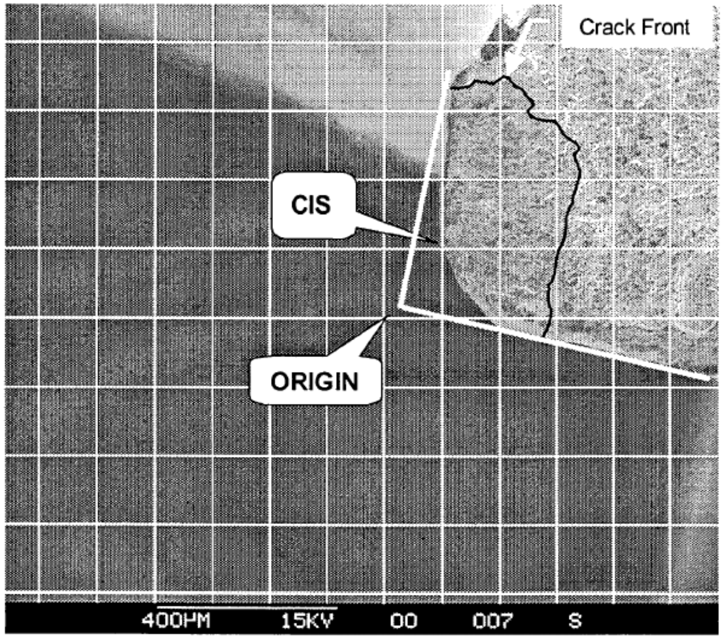
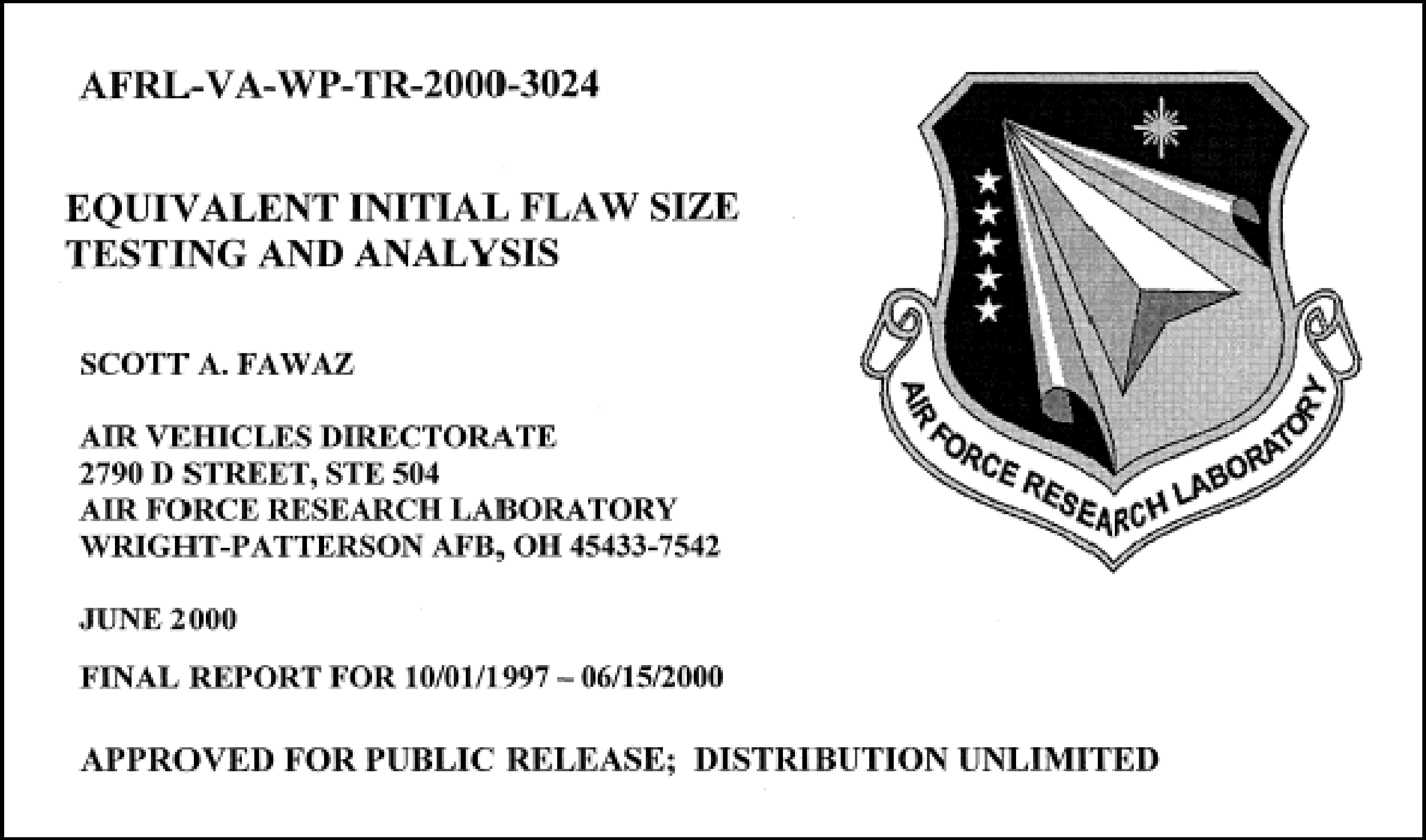


Figure 142 3D13R

Experiment Report by Air Force



AFRL-VA-WP-TR-2000-3024

EQUIVALENT INITIAL FLAW SIZE TESTING AND ANALYSIS

SCOTT A. FAWAZ

AIR VEHICLES DIRECTORATE
2790 D STREET, STE 504
AIR FORCE RESEARCH LABORATORY
WRIGHT-PATTERSON AFB, OH 45433-7542

JUNE 2000

FINAL REPORT FOR 10/01/1997 – 06/15/2000

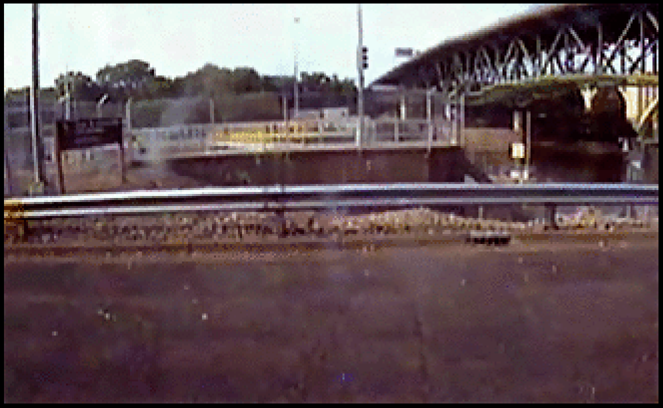
APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED

CPU Time

- Global Analysis
  3 Minutes
- Intermediate Analysis (Joint)
  21.5 Minutes
- Local Analysis (Rivet Hole)
  4.5 Minutes
- Crack Analysis (AGILE)
  100 Minutes for **31** cases

Total CPU Time ≈ 2 Hours in a normal lap-top! (in 2003!)

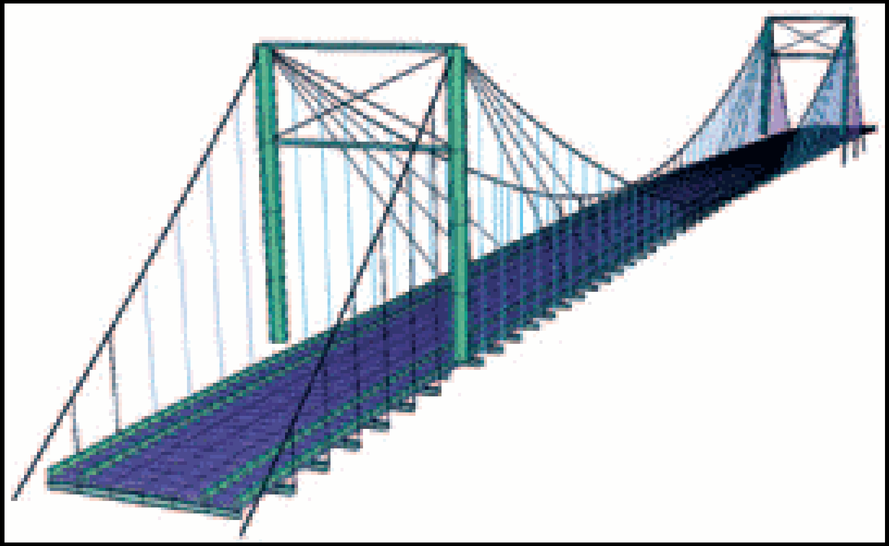
Bridge Collapse: Catastrophic Failure



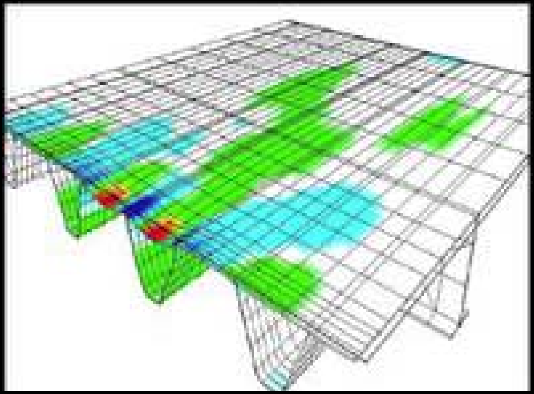
In 2007, a highway bridge over the Mississippi River in Minneapolis collapsed into the river and onto the riverbanks beneath during evening rush hour.

Application of AGILE-3D in the Fatigue Crack-Growth Analyses of Orthotropic Deck Bridges

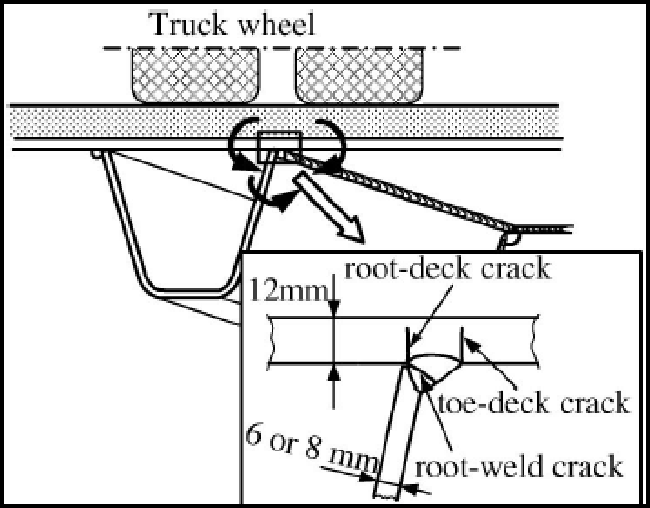
Orthotropic Deck Bridges



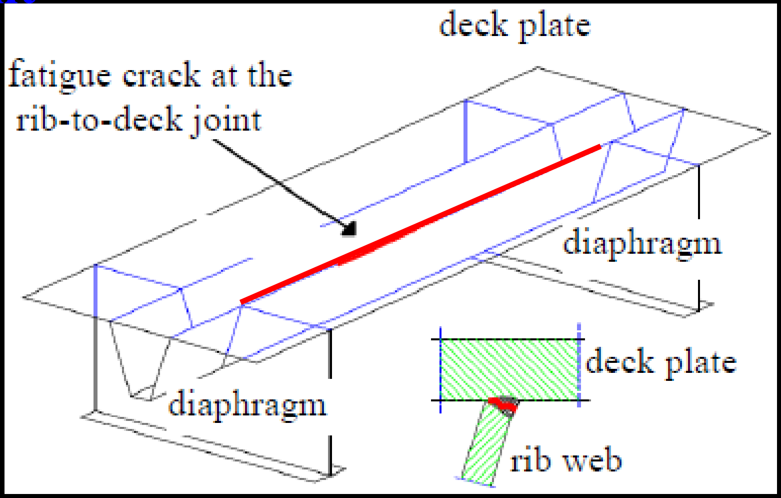
Fatigue crack at the rib-deck welded joint



dynamic load at the U-rib joint

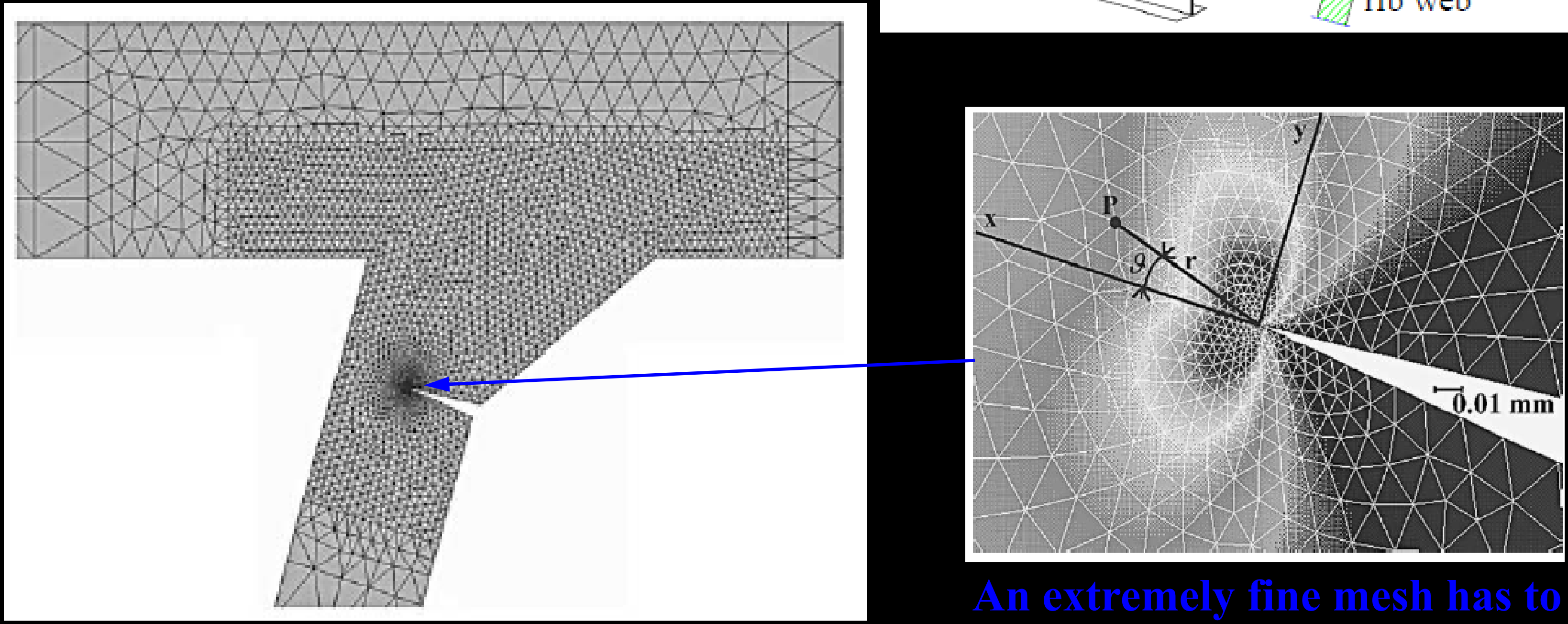


The Computational Model (XFEM) used for the Fatigue Crack Analysis of the Rib-Deck Welded Joint



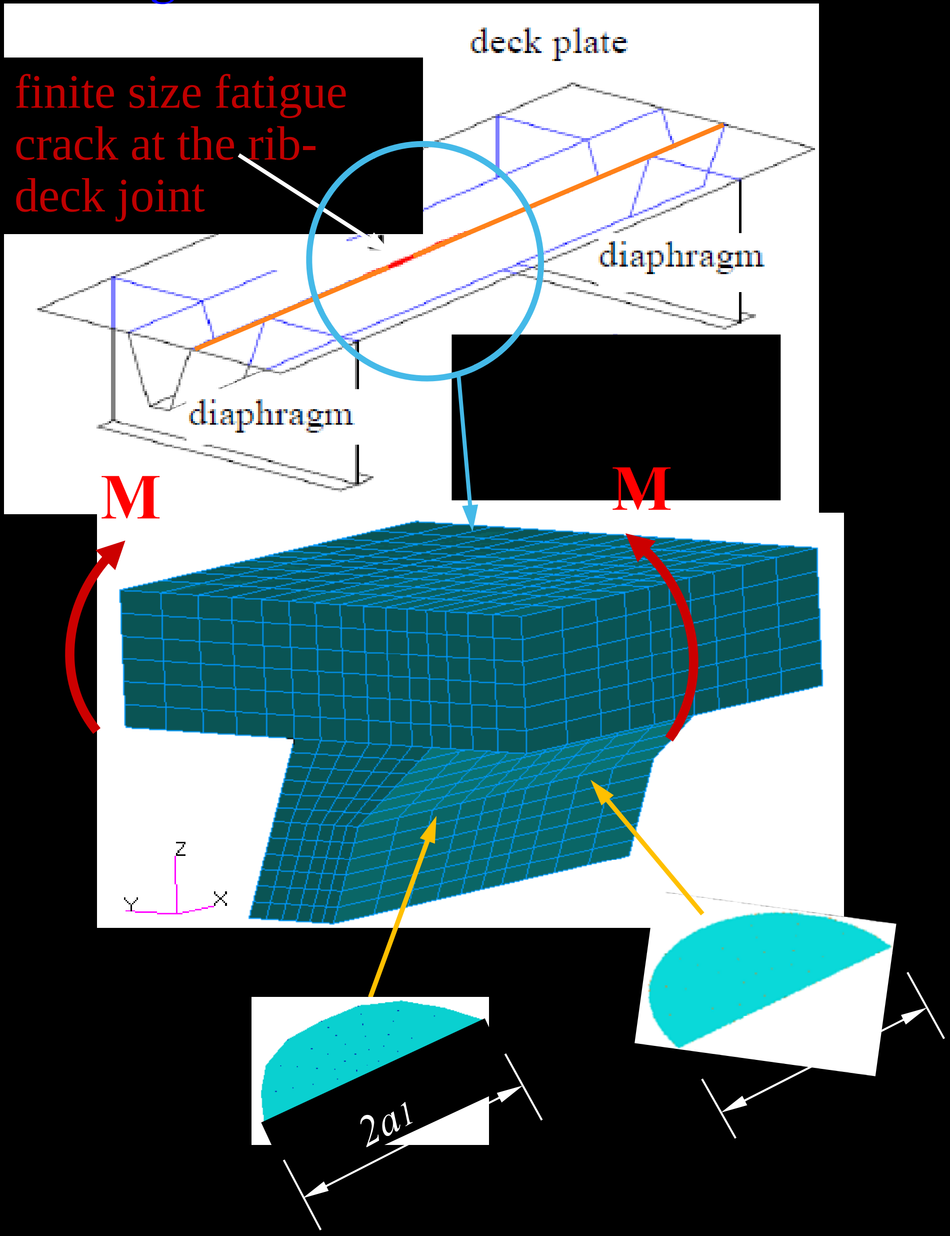
2-D Plane Strain Model

which implies that the crack at the rib-deck is "infinitely" long, across the whole span of two horizontal floor beams / stiffeners



An extremely fine mesh has to be used at the crack tip

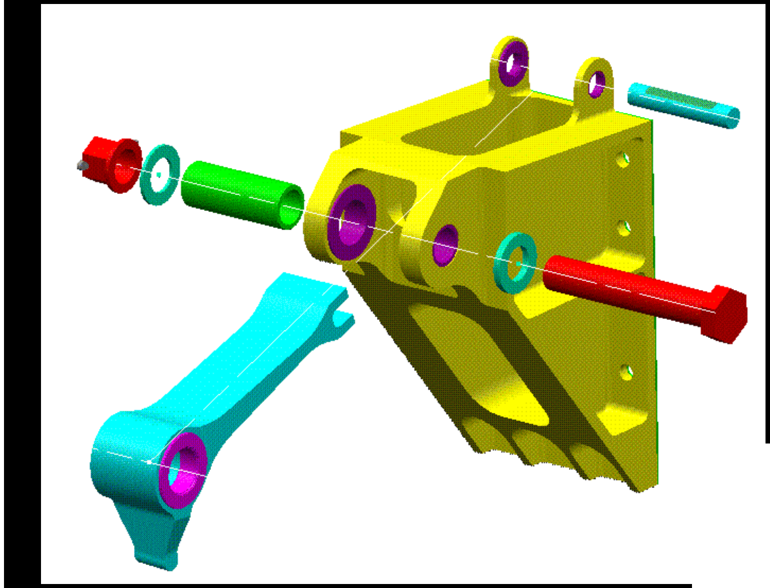
Using AGILE-3D for the Prediction of Fatigue Life of Orthotropic Deck Bridges

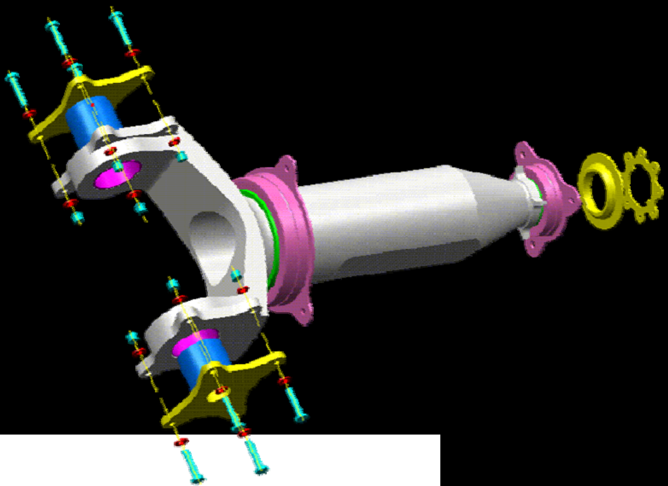


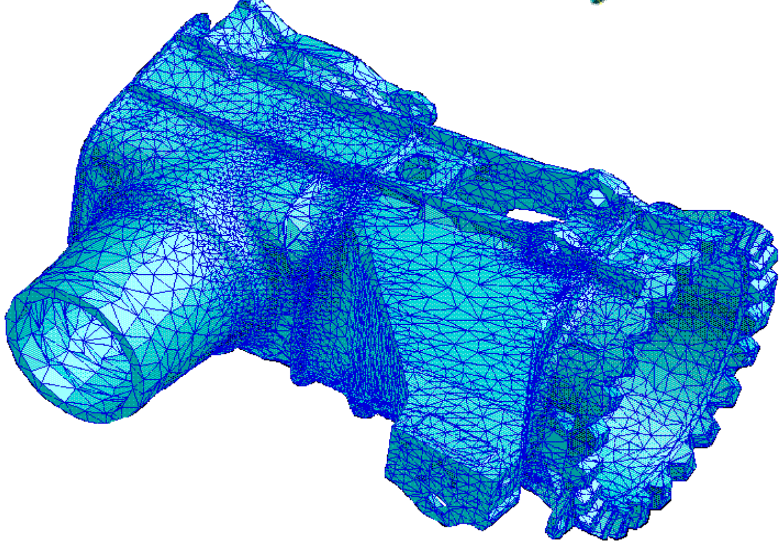
The advantages of using AGILE-3D for the fatigue crack analysis of orthotropic deck bridges:

1) **3-D model can be used to account for the different sizes and geometries of cracks;**
2) **Computationally efficient as a coarse mesh is able to give accurate results.**

Typical structural components

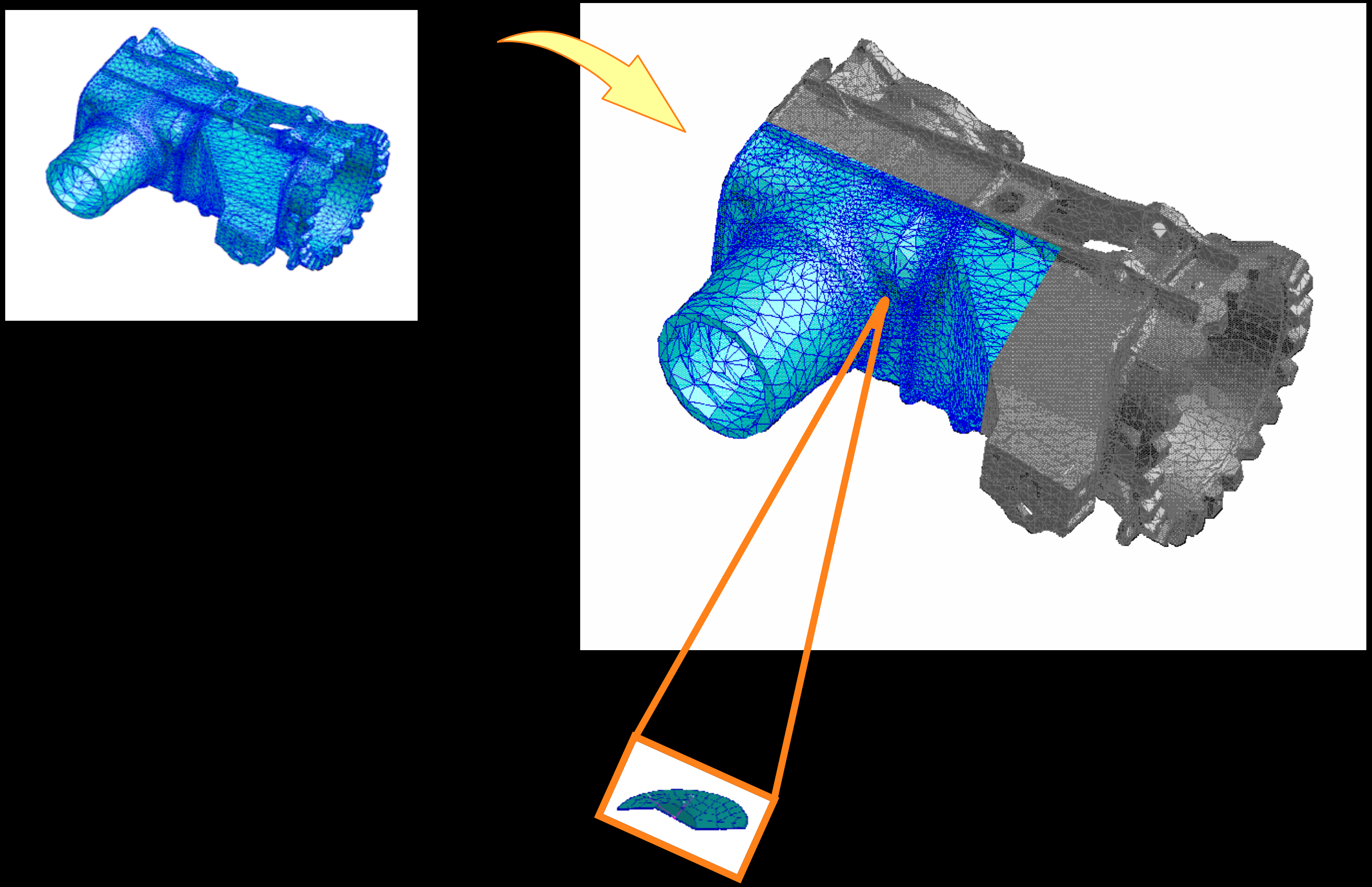




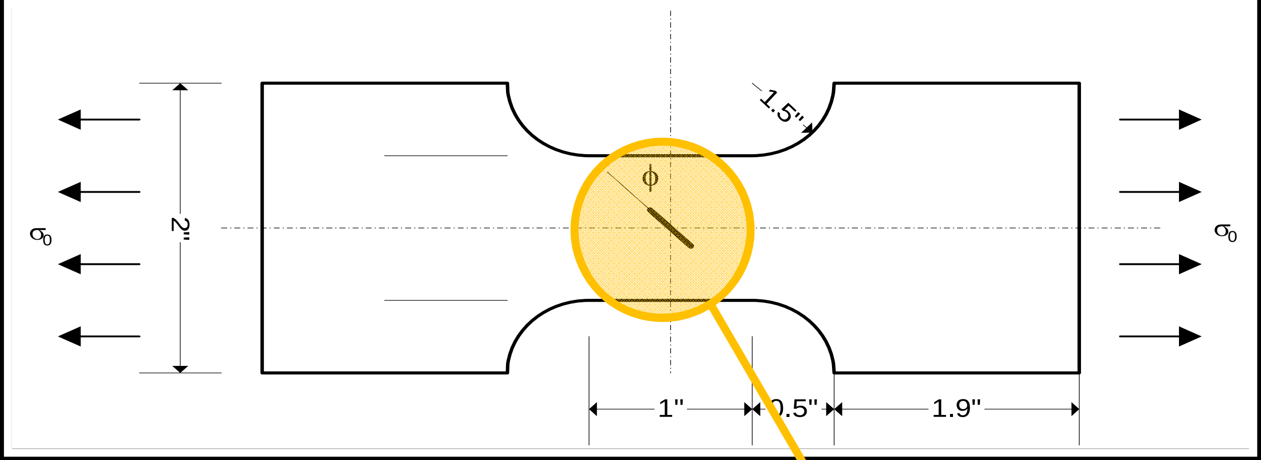


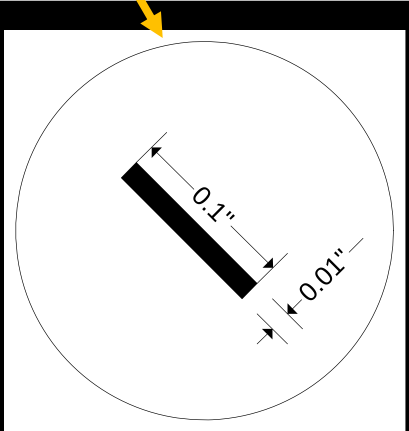
High Surface/Volume ratio

Multiple Level Analyses



AGILE:
Non-planar 3D fatigue growth

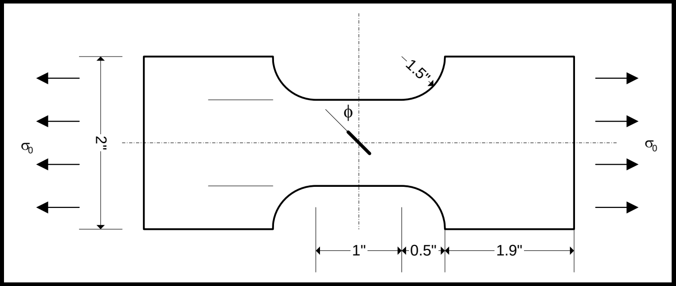




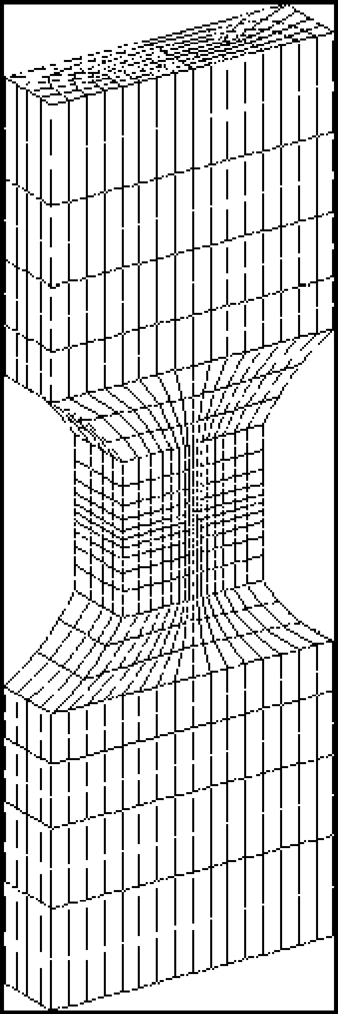
Non-planar 3D fatigue
growth of an inclined
semi-circular surface crack

Nonplanar fatigue growth of
an inclined semi-circular surface crack

- ASTM E740 specimen
- Mixed-mode fatigue growth

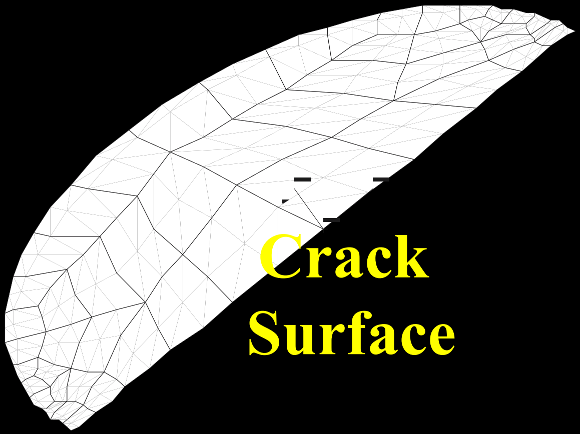


AGILE Models



Finite Body
w/o Crack

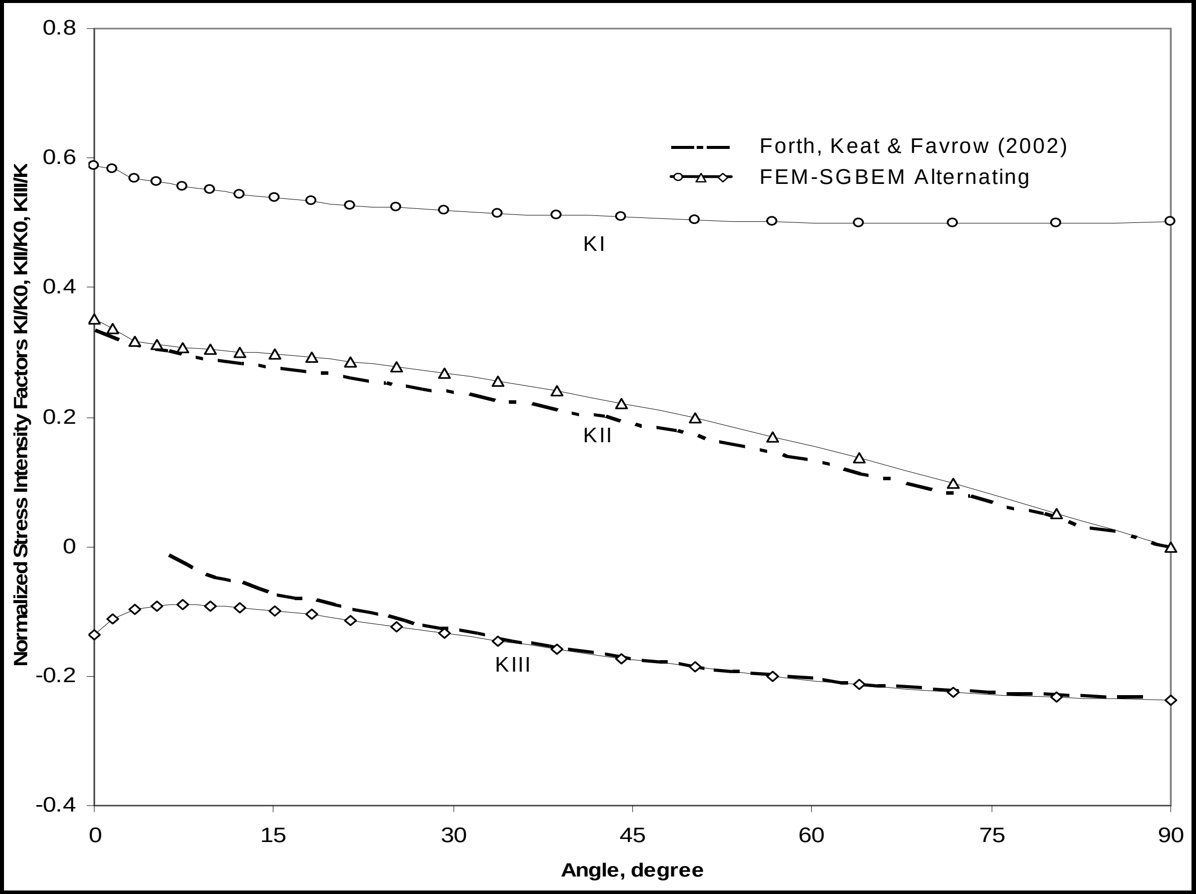
2304 Elements
(Hexa 20)



Crack
Surface

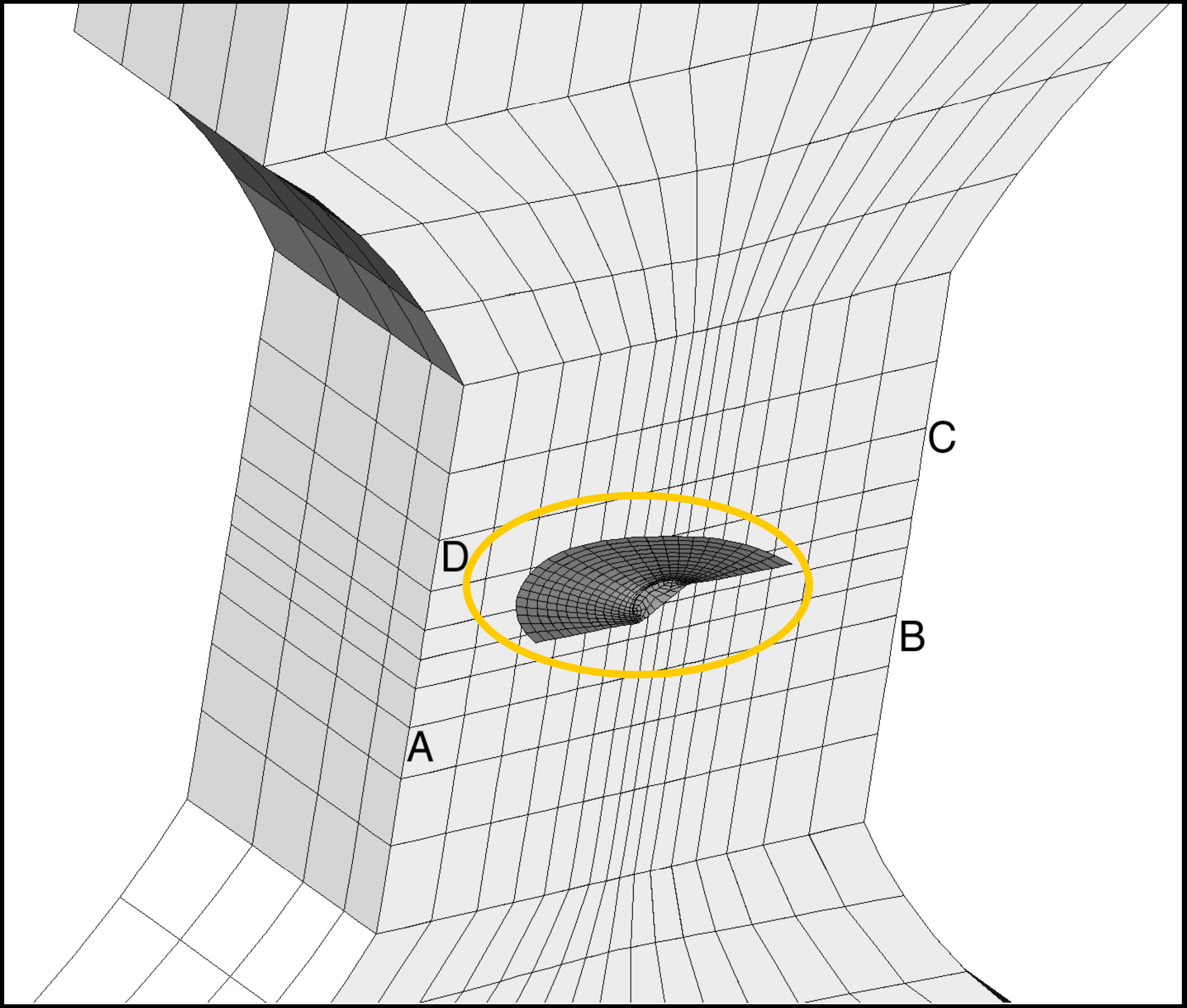
24 Elements
along crack front
(Quad 8)

Stress Intensity Factors
:Initial Crack

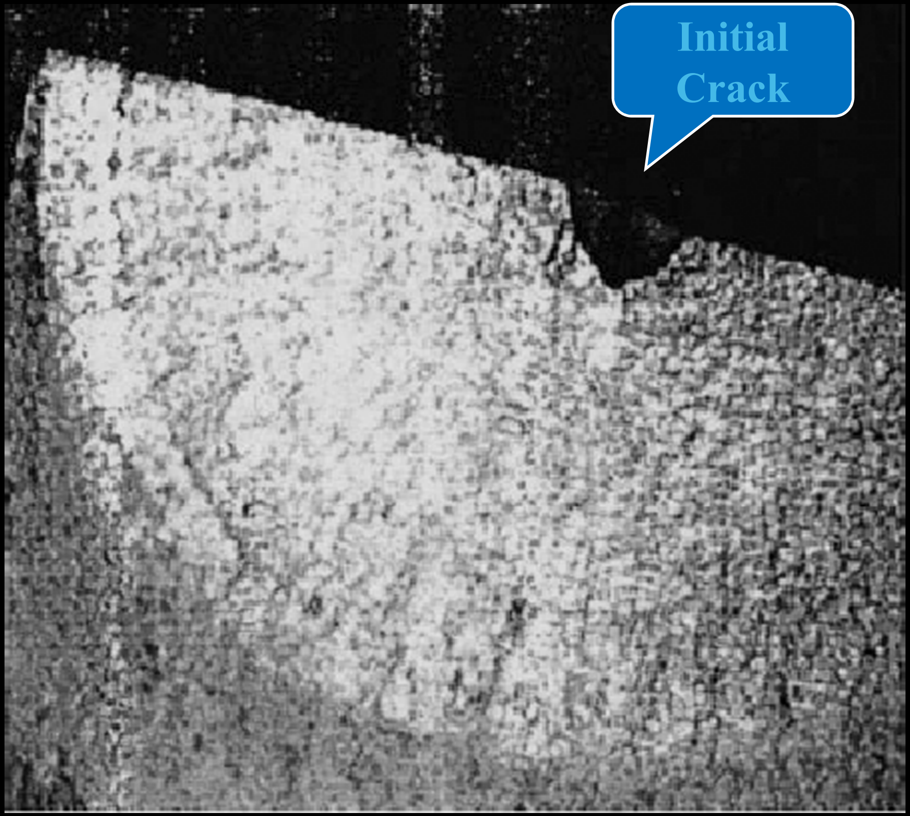


Han. Z. D.; Atluri, S. N. (2002): SGBEM (for Cracked Local Subdomain) – FEM(for uncracked global Structure) AlternatingMethod for Analyzing 3D Surface Cracks and Their Fatigue-Growth, CMES: *Computer Modeling in Engineering & Sciences*, vol. 3 no. 6, pp. 699-716.

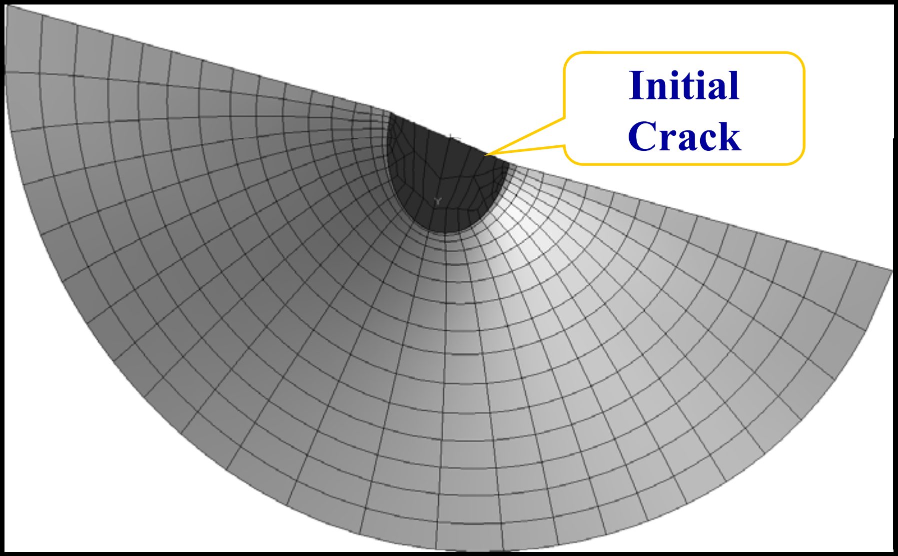
Crack in the specimen



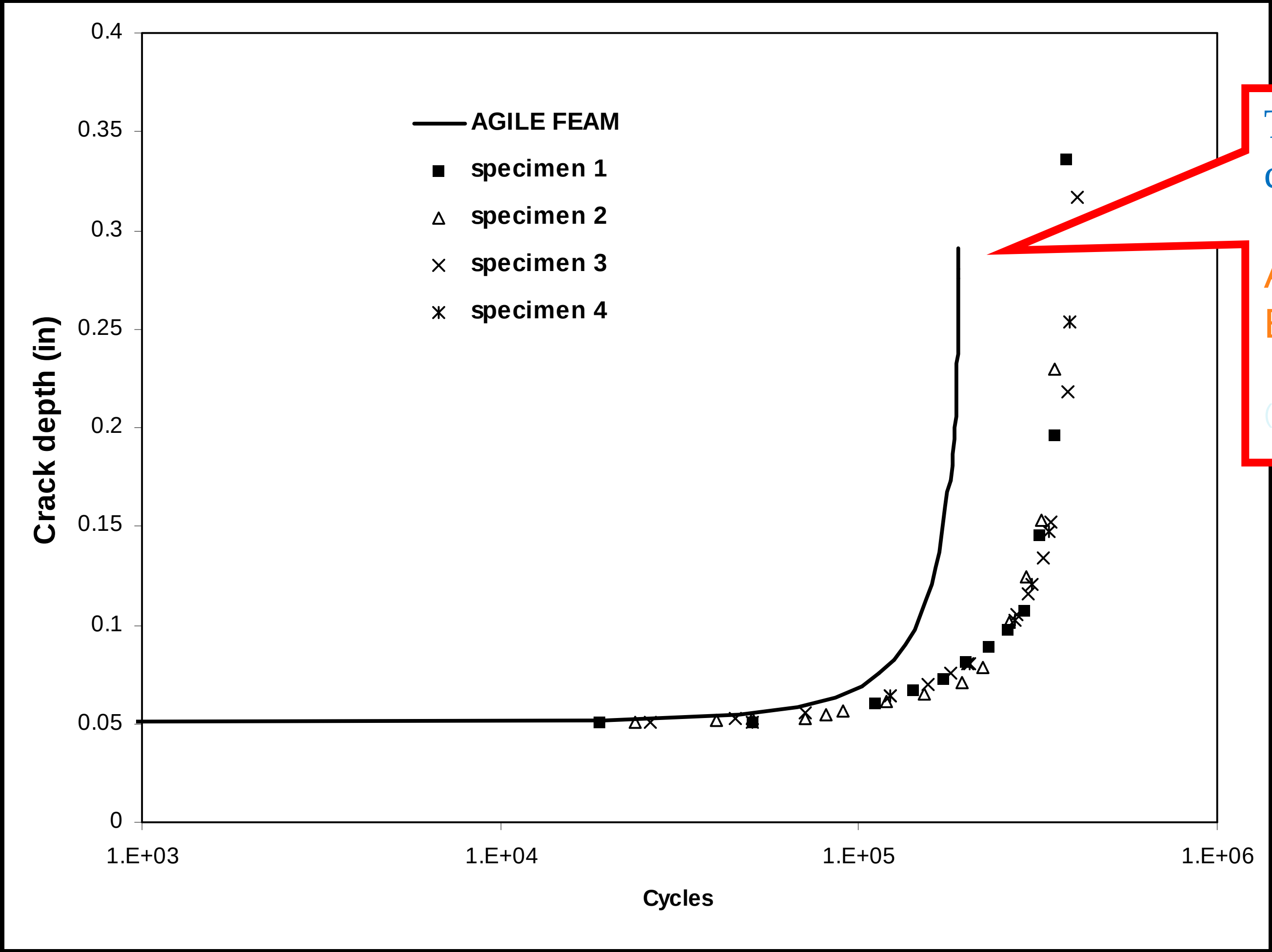
Final Crack


Initial Crack

Final Crack Predicted by using AGILE


Initial Crack

Fatigue Loading Cycles

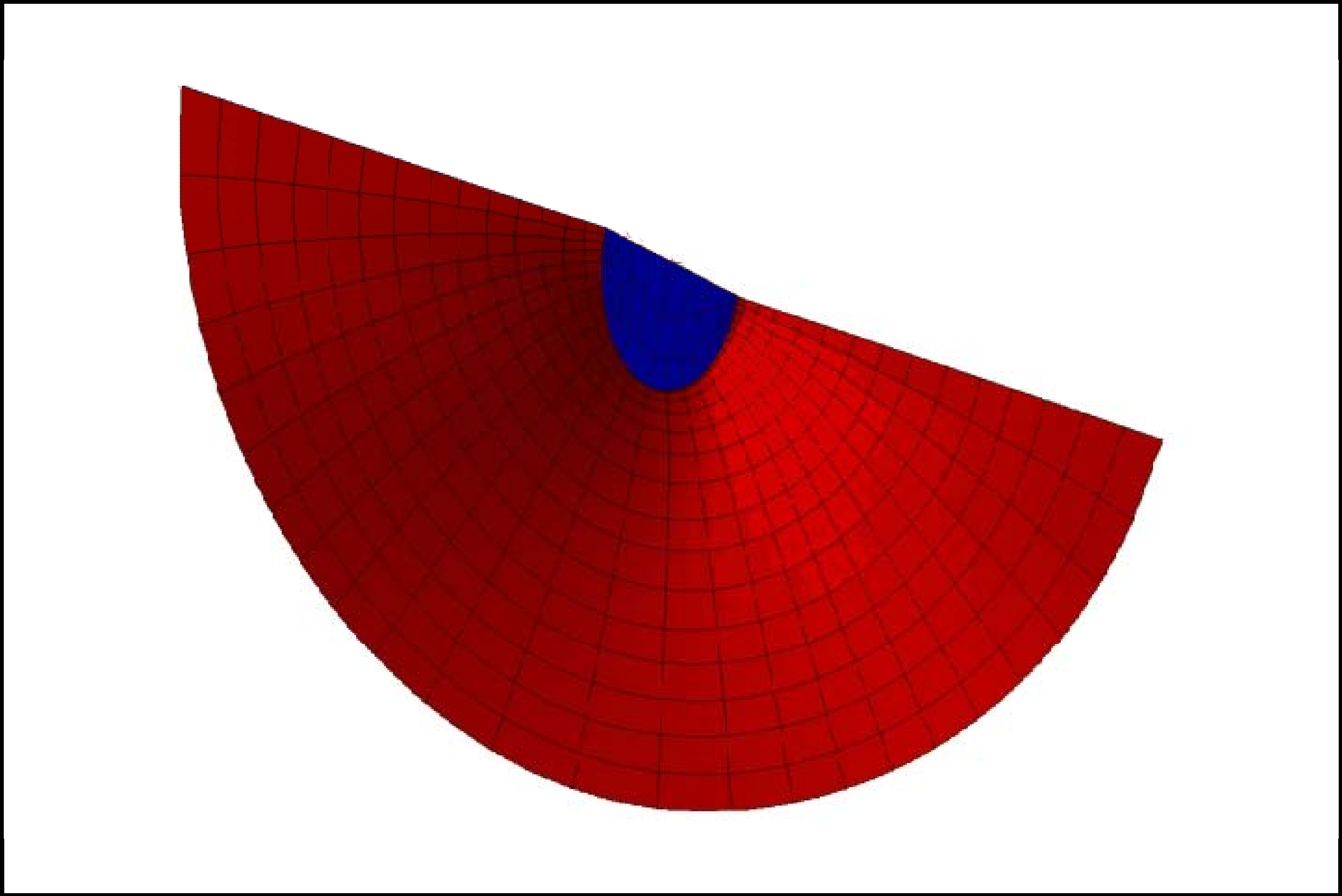


The critical depth of the crack

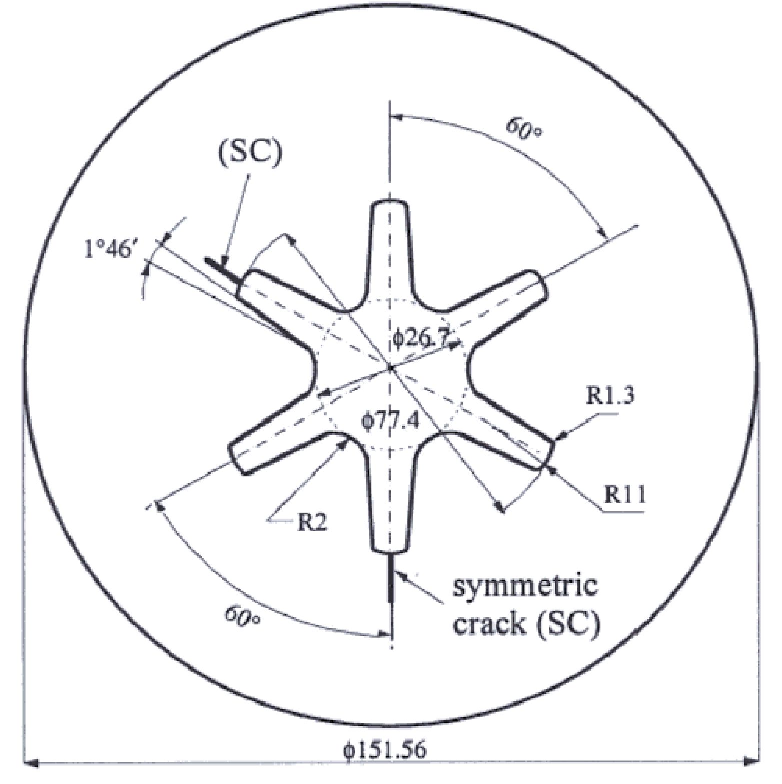
AGILE 0.29”
Exp. Ave. 0.284”

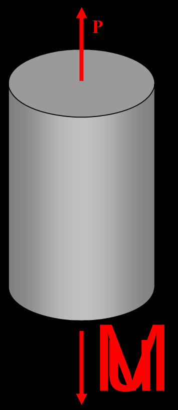
(0.34”, 0.23”, 0.32”, and 0.25”)

The Non-planarly Growing Crack...



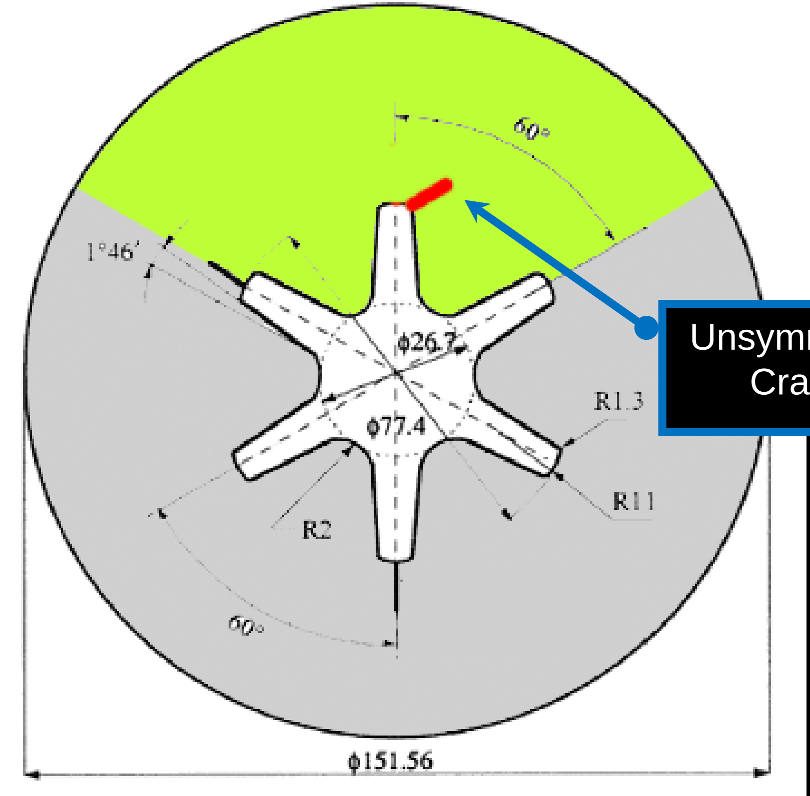
Analysis of Cracks in Solid Propellant Rocket Grains



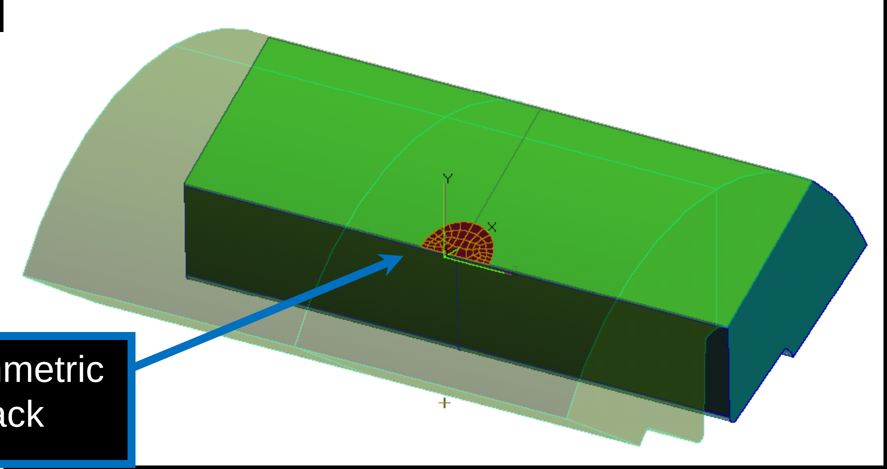


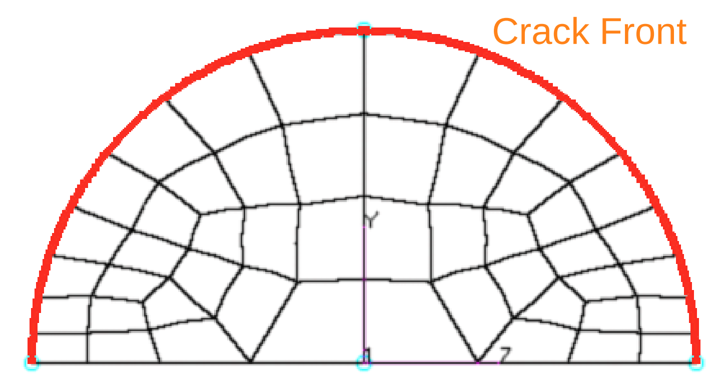
Solid Propellant Rocket Grain under tension and inner pressure

Unsymmetric BE Crack Model



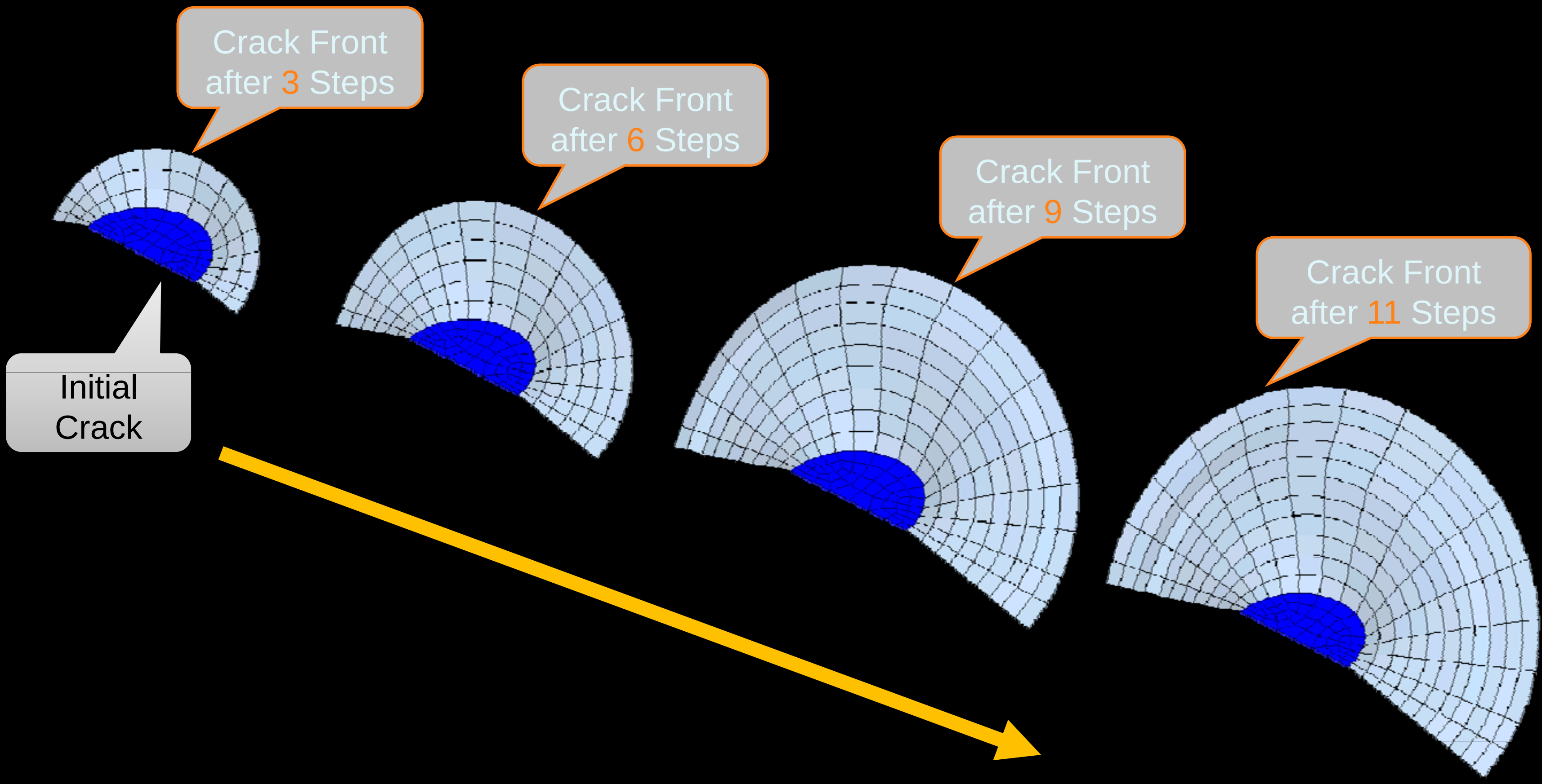
Unsymmetric Crack





Semi-Circular Crack

Crack Front Advancements



Initial Crack

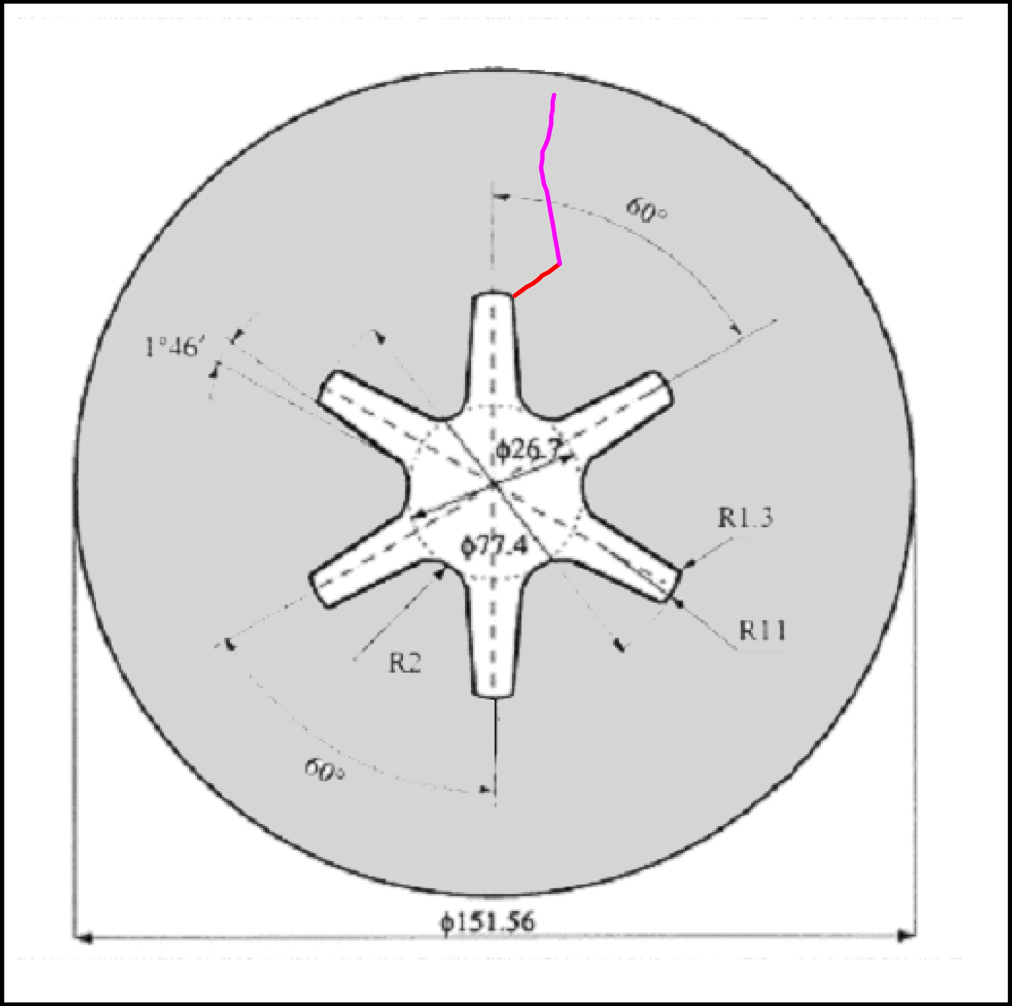
Crack Front after 3 Steps

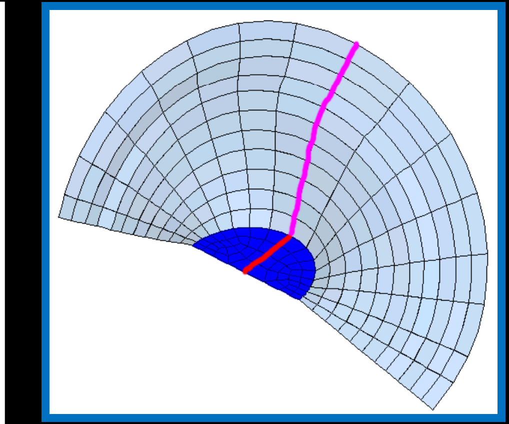
Crack Front after 6 Steps

Crack Front after 9 Steps

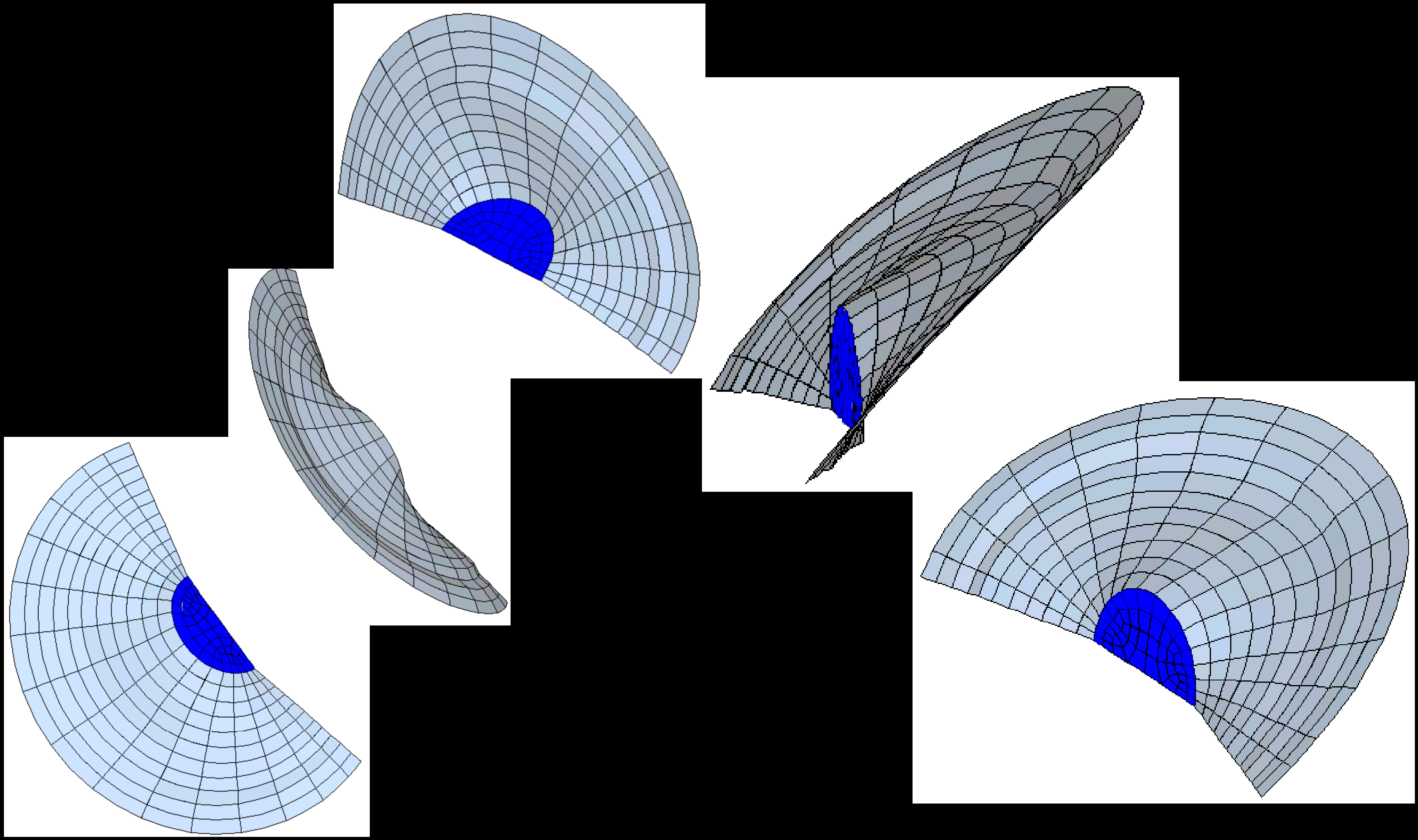
Crack Front after 11 Steps

Center Line of Growing Crack

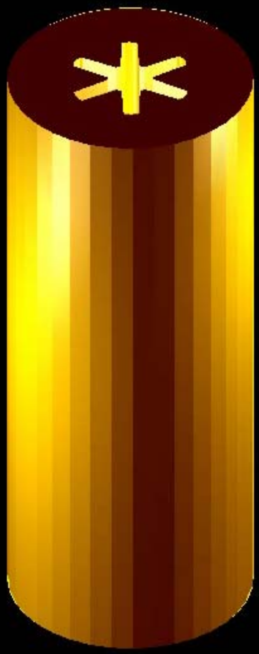




Final Crack Surface



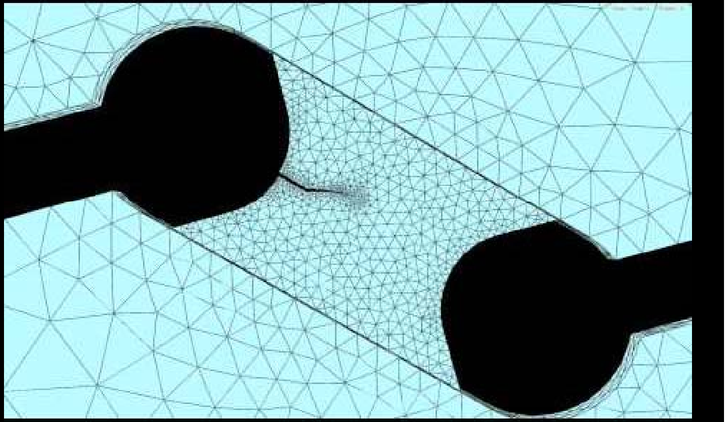
Simulation: Growth of the Crack



Some Other Fracture Codes

- Codes based on analytical/handbook solutions
  - NASGRO, FASTRAN
- Full BEM codes
  - BEASY, FRANC3D
- Full FEM codes with specific elements
  - ABAQUS, MARC, ZenCrack, XFEM
- FEM-SGBEM Alternating Code
  - ***AGILE (Most Efficient & Most Accurate)***

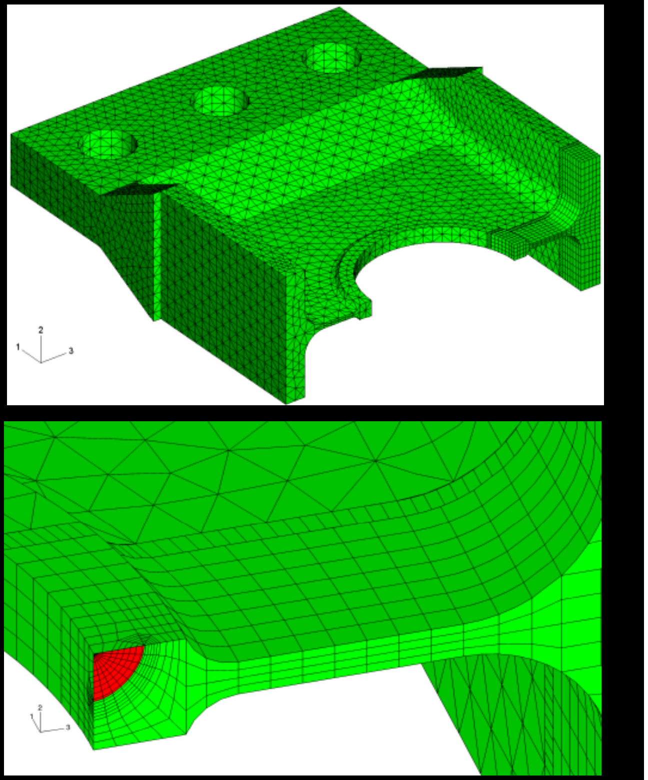
From FEM, ZenCrack to XFEM



- FEM: Enriched Singular Elements (developed in 1970’s, pioneered by Atluri and his colleagues, implemented in ABAQUS, MARC, etc.)
  - Confirming & adaptive Meshes.
  - Accuracy dependent on the mesh quality.
  - Costly labor of Meshing & Re-Meshing
  - No automated crack growth.

Enrichment Elements are the KEY!

From FEM, ZenCrack to XFEM

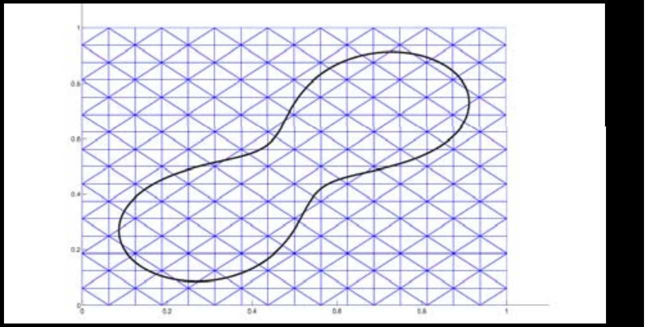


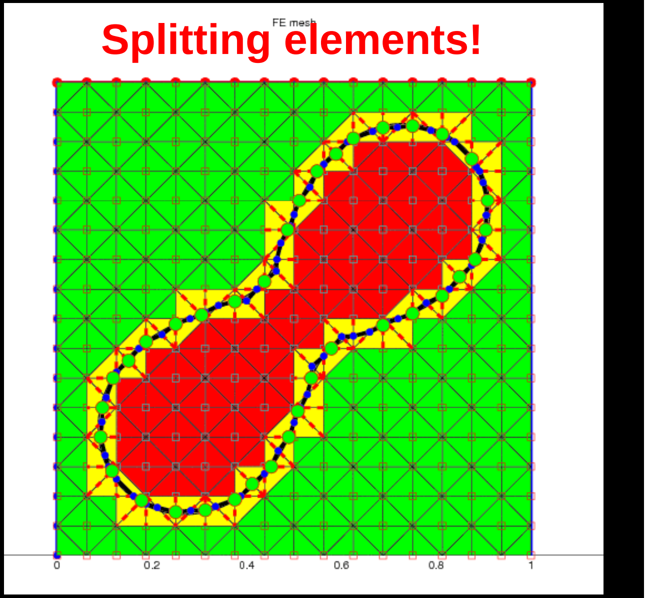
- Zen Crack: a crack mesh generator
  - – Insert a crack into a non-cracked FEM Mesh
  - – Create the meshes outside involving FEM Solvers.
  - – Reduce labor work in creating the conforming and adaptive meshes
  - – Algorithm is unstable.

Enriched Elements still play the KEY role!

From FEM, ZenCrack to XFEM

- XFEM: Split elements to match the cracks
  - – Integrate the element manipulation into the FEM Solvers, and HIDE it from the users.
  - – No adaptive meshes
  - – Splitted elements without quality.
  - – No accuracy control.





Only 2D Enriched Elements can be used.

What about XFEM 3D?

(up to 2010)

- Only Tet Mesh but No Hexa Mesh.
- No 3D enrichment element for non-planar cracks.
- The accuracy is heavily dependent on the initial FEM Mesh.

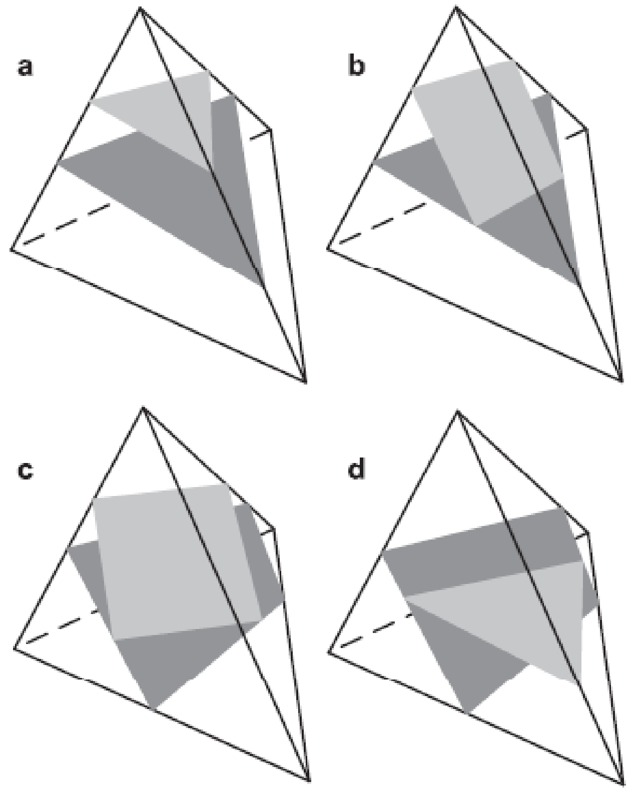


Fig. 6. Crack junction.

FEM without Enrichment Elements!

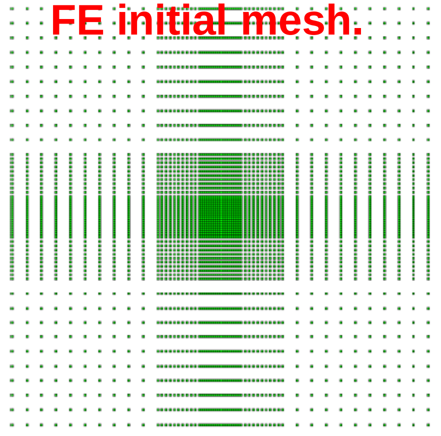
What about XFEM 3D?

(Rabczuk, Bordas, Zi (2010): Computers and Structures 88, pp. 1391–1411)

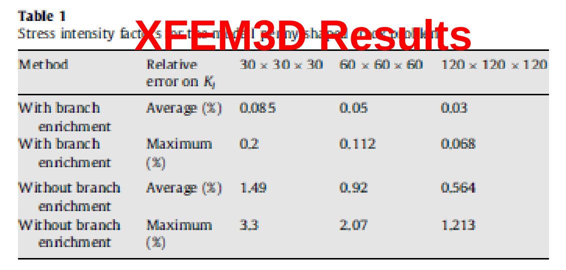
Penny-shaped embedded crack in a tension bar

- 30x30x30=27,000 elements: Error = 3.3%
- 60x60x60=216,000 elements: Error = 2.07%
- 120x120x120=1,728,000 elements: Error = 1.21%
- AGILE: 20 elements Error = 0.3%

XFEM-3D is NOT suitable for fatigue & fracture analyses



FE initial mesh.



XFEM3D Results

Table 1
Stress intensity factors for the mode I penny-shaped crack problem.

Method	Relative error on K_I	30 × 30 × 30	60 × 60 × 60	120 × 120 × 120
With branch enrichment	Average (%)	0.085	0.05	0.03
With branch enrichment	Maximum (%)	0.2	0.112	0.068
Without branch enrichment	Average (%)	1.49	0.92	0.564
Without branch enrichment	Maximum (%)	3.3	2.07	1.213

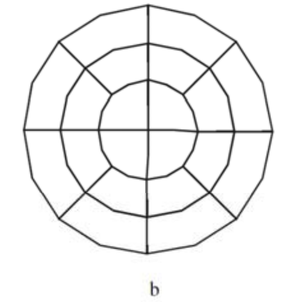
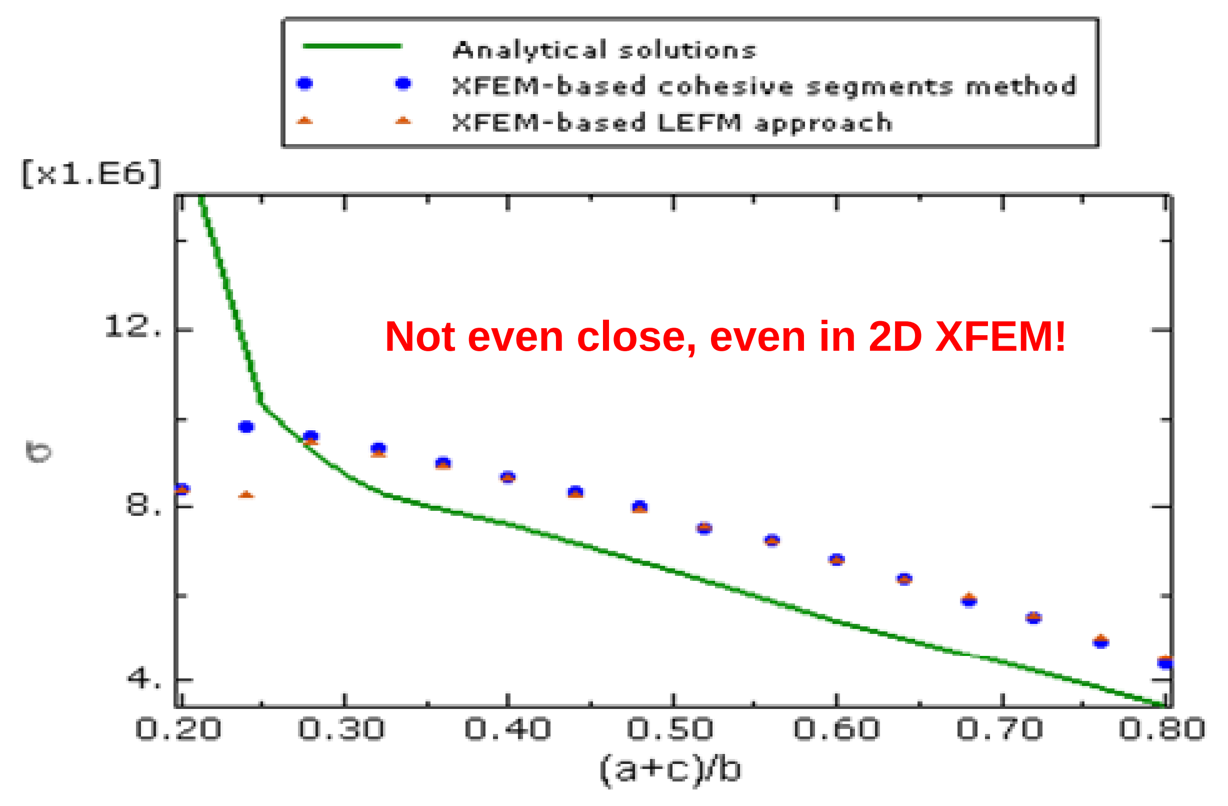


Figure 9 : Meshes for a penny-shaped crack: (a) 12 quadratic elements, (b) 20 quadratic elements.

AGILE mesh.

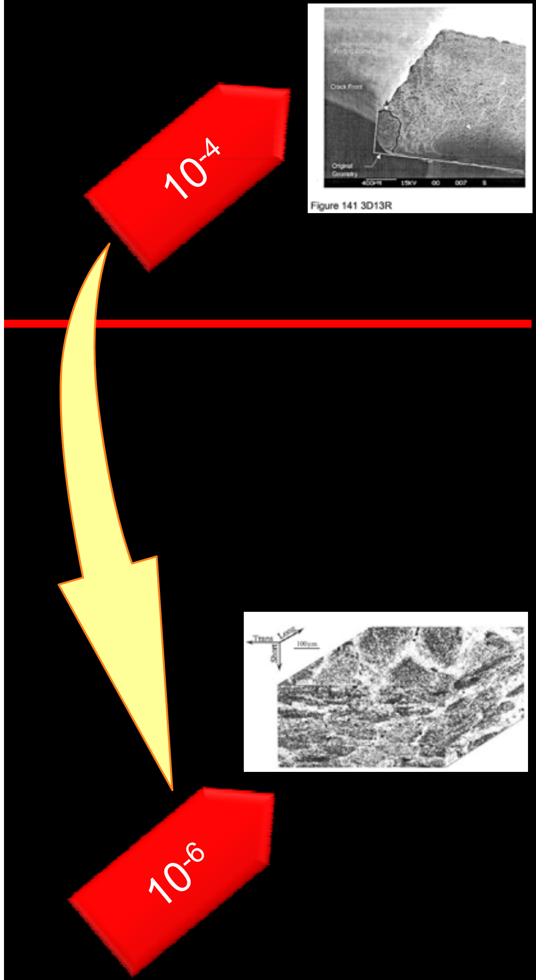
What about XFEM 3D in Commercial Codes?



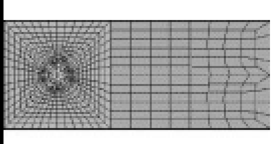
Not even close, even in 2D XFEM!

XFEM3D, without singularity enrichment, is NOT suitable for fracture analysis!

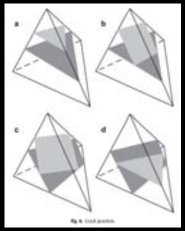
How to Reach 10^{-6} Level even using continuum mechanics?



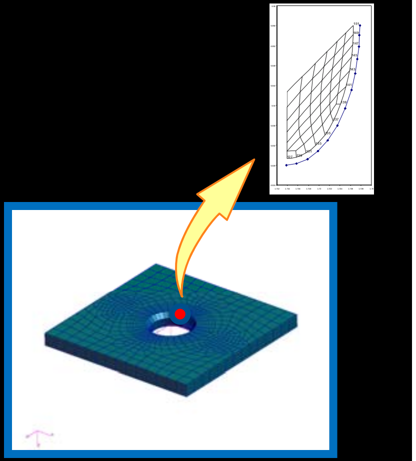
- **FEM:** Zoom-in refined localized mesh, => 10^{-5}



- **XFEM:** Splitting Elements without mesh quality control, => 10^{-5}



- **AGILE:** Completely de-coupled FEM-SGBEM LOCAL model, Cracks can be two orders lower, => 10^{-6}



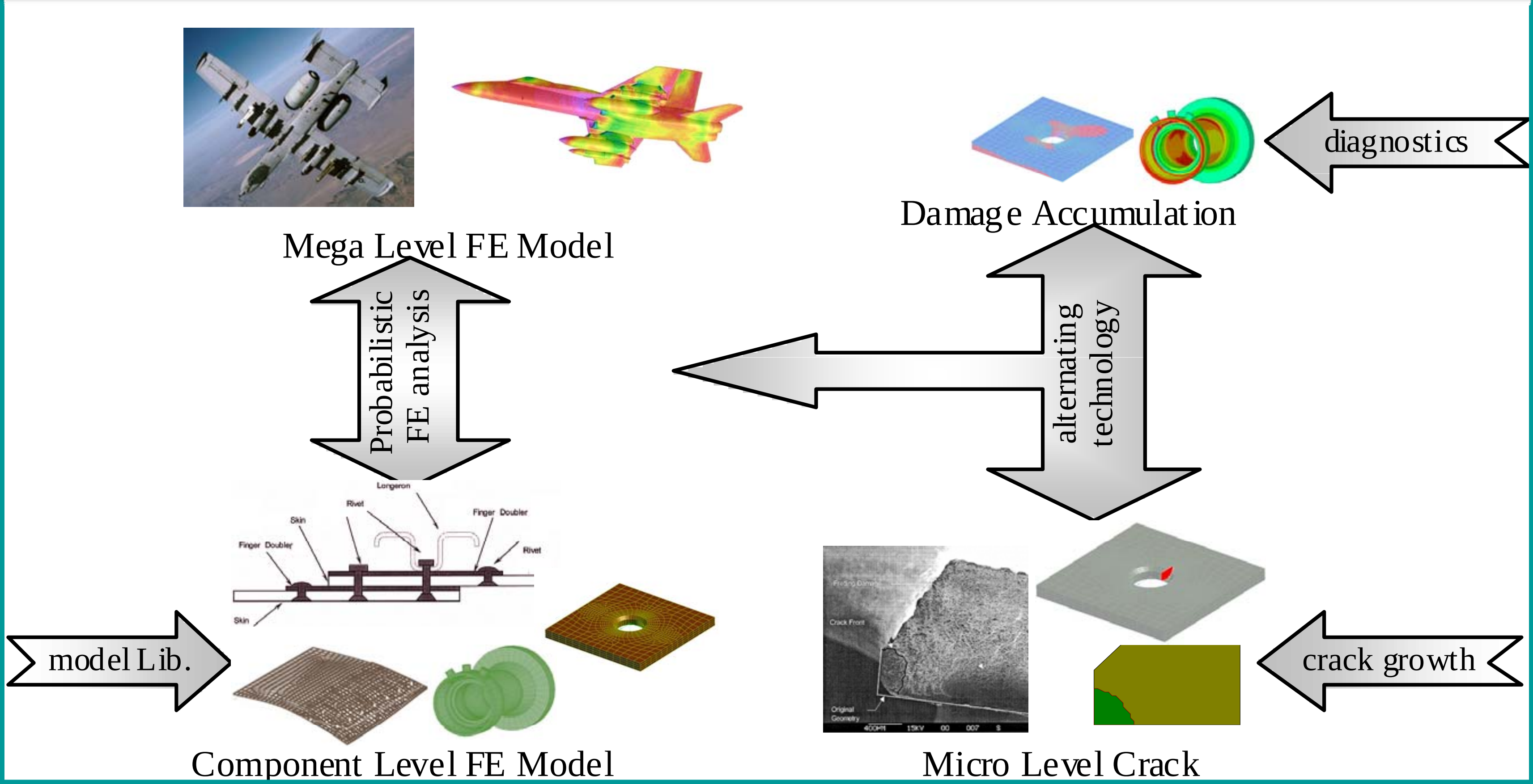
Comparison between Codes

Codes	Modeling Time	CPU Time	Accuracy	Fully Automated Growth	3D NonPlanar Crack	Complicate Model and LBCs	Link Commercial FE Codes
AGILE	Crack only	Minutes per step	<1%	YES	YES	YES	YES
BEASY	Full BEM Model with Crack	6~10 times slower	~3%	Restriction	YES	Quad Mesh	Limited
FRANC3D	Full BEM Model with Crack	Slower	~3%	Unstable	YES	NO	NO
NASGRO	Predefined Crack only	Fast	--	YES	NO	NO	NO
ABAQUS MARC	Full FEM Model with Crack	Fast	~10%	NO	YES	YES	Self
ZenCrack	Full FEM Model with Crack	Fast	~10%	Unstable	YES	Unstable	NA
XFEM	-----	-	Worse than ABAQUS	YES	NO	Not for Cracks	YES

AGILE has the BEST Accuracy & can be run on demand in a real-time fashion!

AGILE Probabilstic Prognostics Tool

Integrated Structural Health Management System



Mega Level FE Model

Probabilistic FE analysis

Damage Accumulation

diagnostics

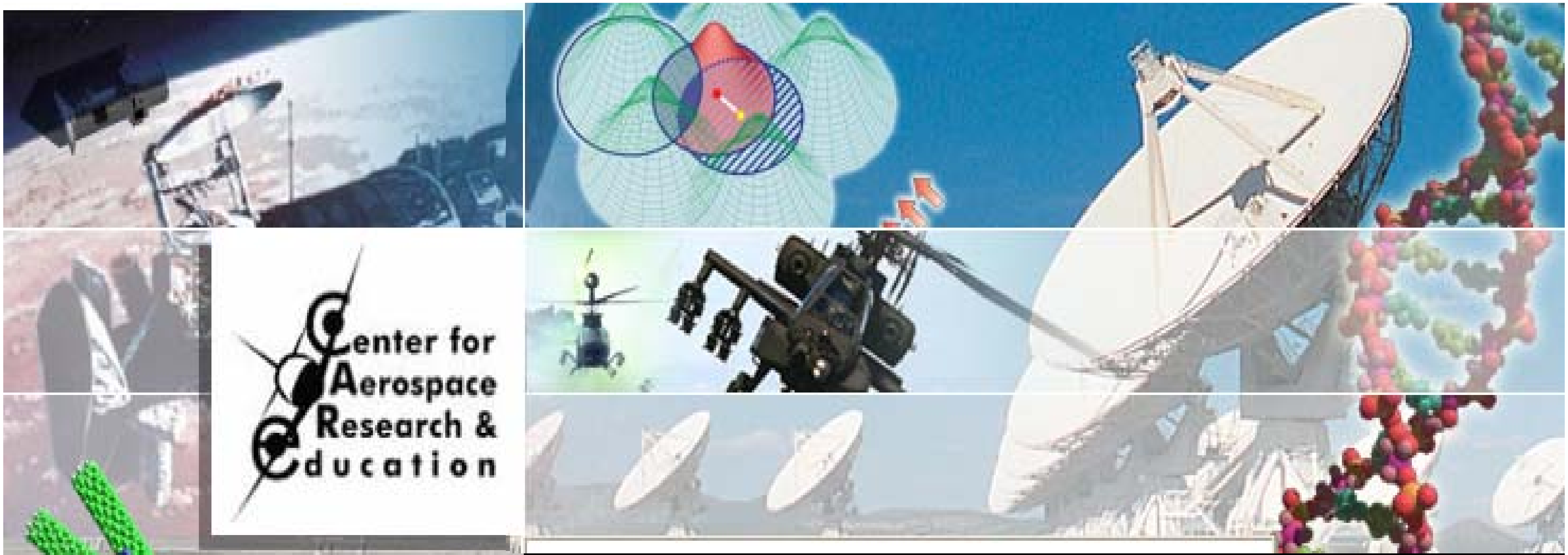
alternating technology

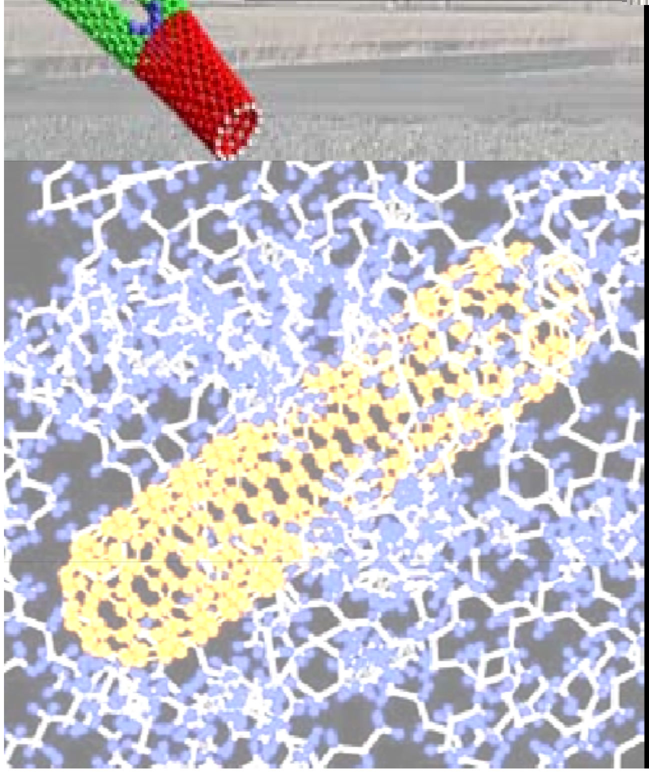
model Lib.

Component Level FE Model

Micro Level Crack

crack growth


Center for Aerospace Research & Education



Automated Global, Intermediate, & Local Evaluations *for* Damage Tolerance Analyses & Life Estimation:

AGILE *for* DTA & LE
(Status as of Dec. 2004)

Satya N. Atluri, UCI

Why AGILE?

- Simple to use:
  - Easiness of Model Creation
  - User-Friendly Graphical Interfaces
  - Least computationally intensive
  - Automatic re-solution of Intermediate model, if load-redistribution due to crack-growth occurs

What is embedded in AGILE?

- Open Architecture:
  - Various mixed mode loadings.
  - 2-D & 3-D Mixed-Mode, Non-planar fatigue-crack-growth modeling
  - Sophisticated mathematics + minimal numerics
  - Fatigue-crack-growth models.
  - Probabilistic analyses.

Support multiple load cases

- Structural components are undergoing several loading cases within one flight , including take-off & landing, lifting, carrying. The load spectrums are different.
- The life of the loading components will be estimated under the combined load cases.

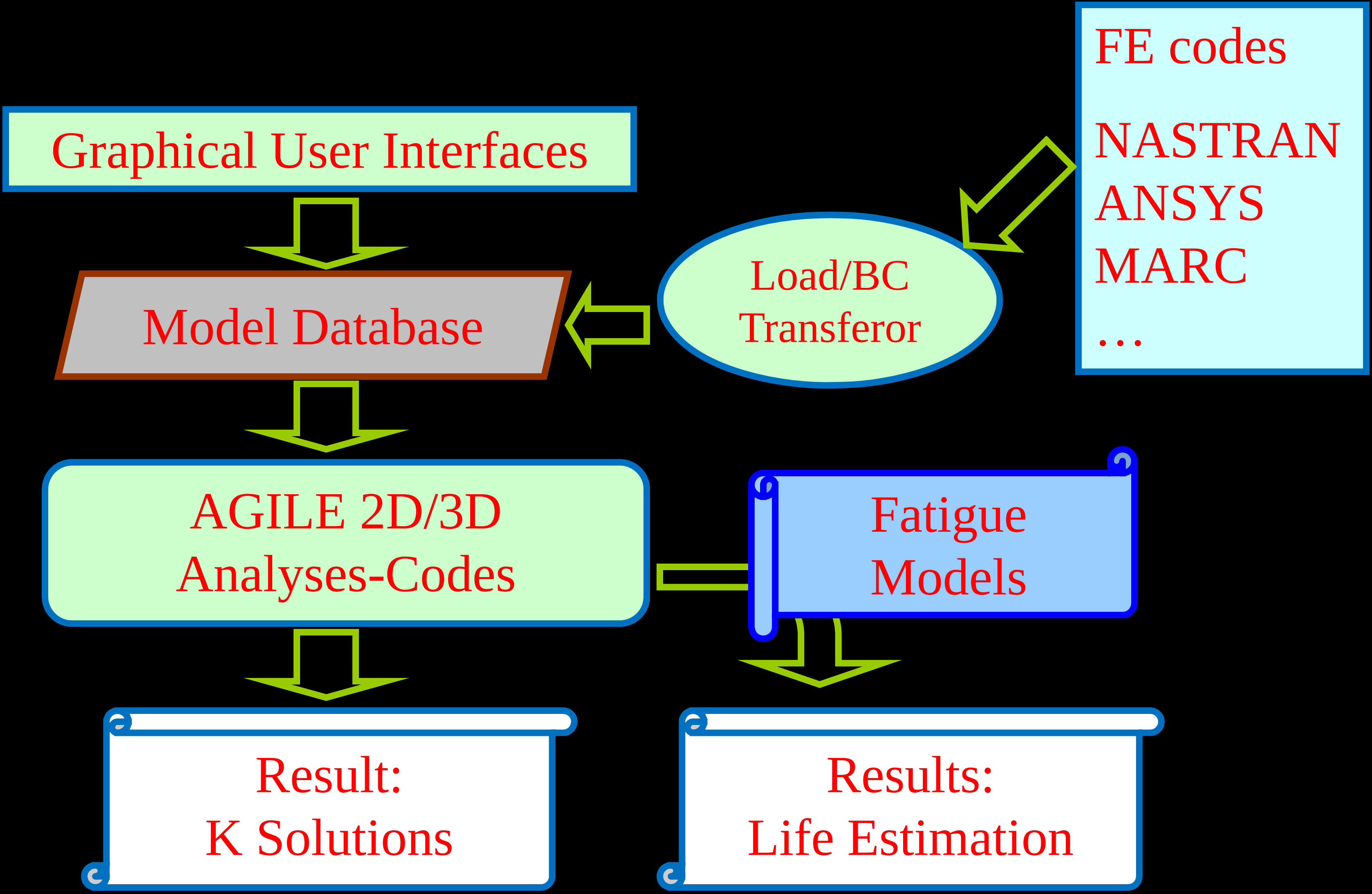
Easiness of Model Creation

- Simple FE mesh creation, without the crack surface in the FE model.
- Simple creation of crack model, as only a surface mesh in SGBEM
- Independence of the SGBEM and FE meshes:
  - leverage the existing FE models and results
  - Parametric crack analysis is very simple

Graphical User Interface
Fully integrated into PATRAN

- The proficiency of the GUI makes AGILE user-friendly and minimizes human-errors typically associated with data preparation.
- Supporting ALL AGILE model creation.
- Seamless integration with MSC.PATRAN, minimizes user training.
- Supporting PATRAN session file, i.e. recording and playing back.
- Supporting all PATRAN FE model files for NASTRAN, MARC, ABAQUS and so on.

AGILE Architecture



Graphical User Interfaces

Model Database

Load/BC Transferor

FE codes

NASTRAN
ANSYS
MARC
…

AGILE 2D/3D Analyses-Codes

Fatigue Models

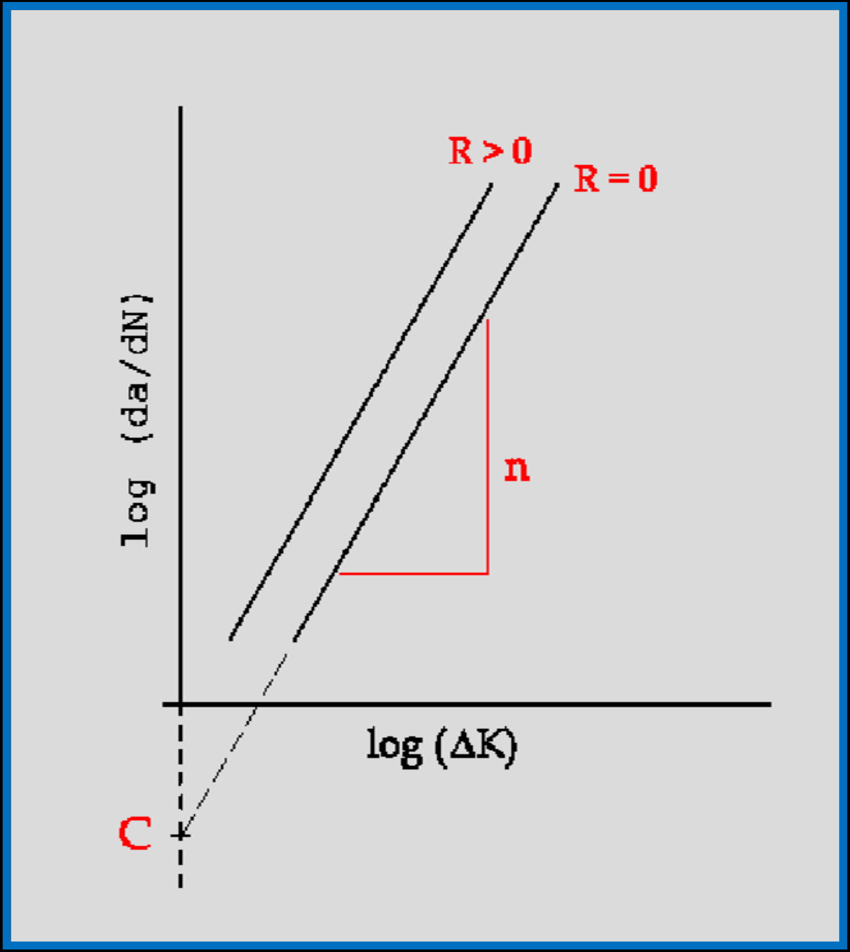
Result:
K Solutions

Results:
Life Estimation

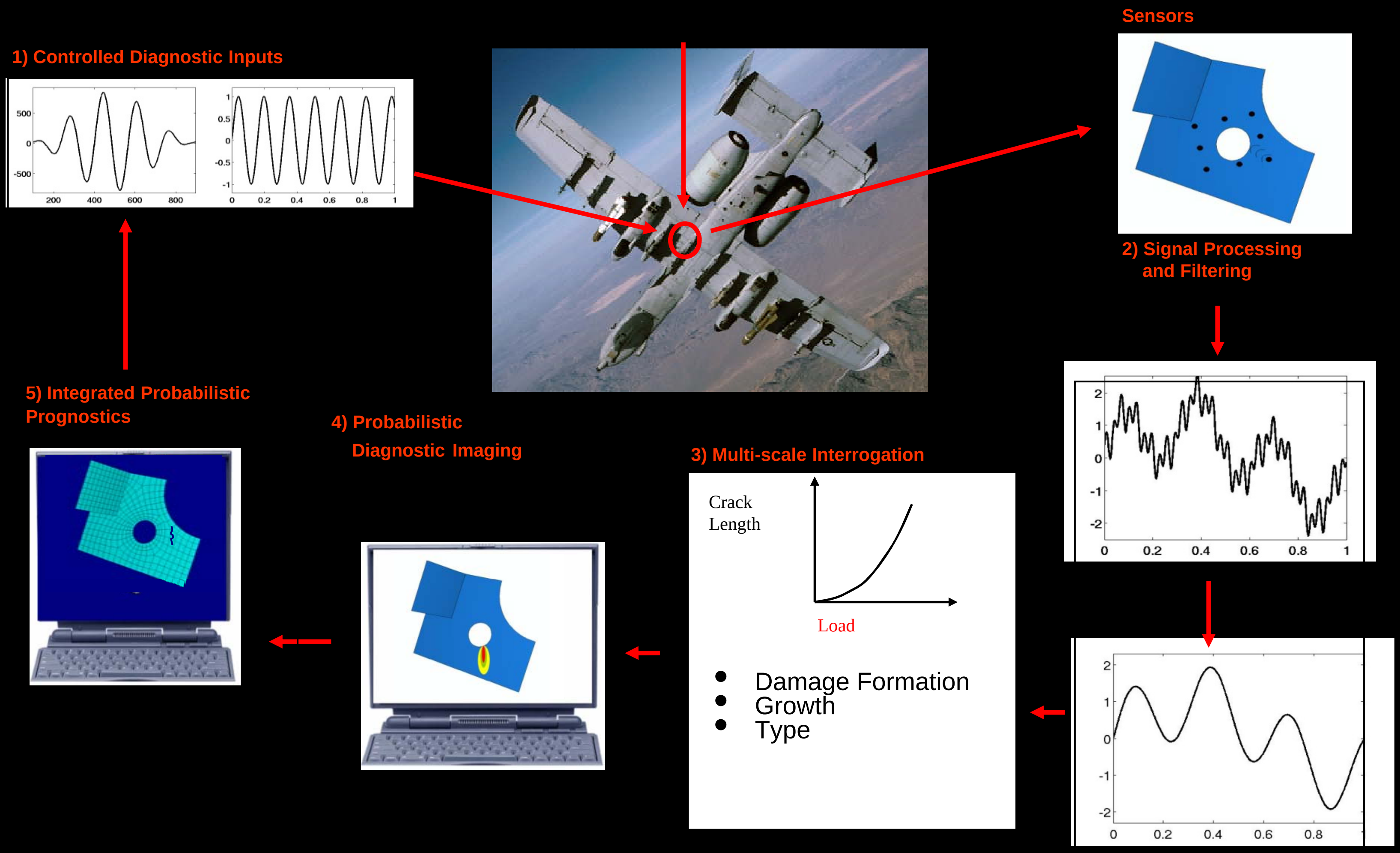
Support most crack growth models

- Paris Model
- Walker Model
- NASGRO Model

- Load Spectrum
- Analytical models for plasticity-induced Crack-closure



AGILE as an Integrated Probabilistic Prognostic Tool in an SHM System



1) Controlled Diagnostic Inputs

Sensors

2) Signal Processing and Filtering

3) Multi-scale Interrogation

- Damage Formation
- Growth
- Type

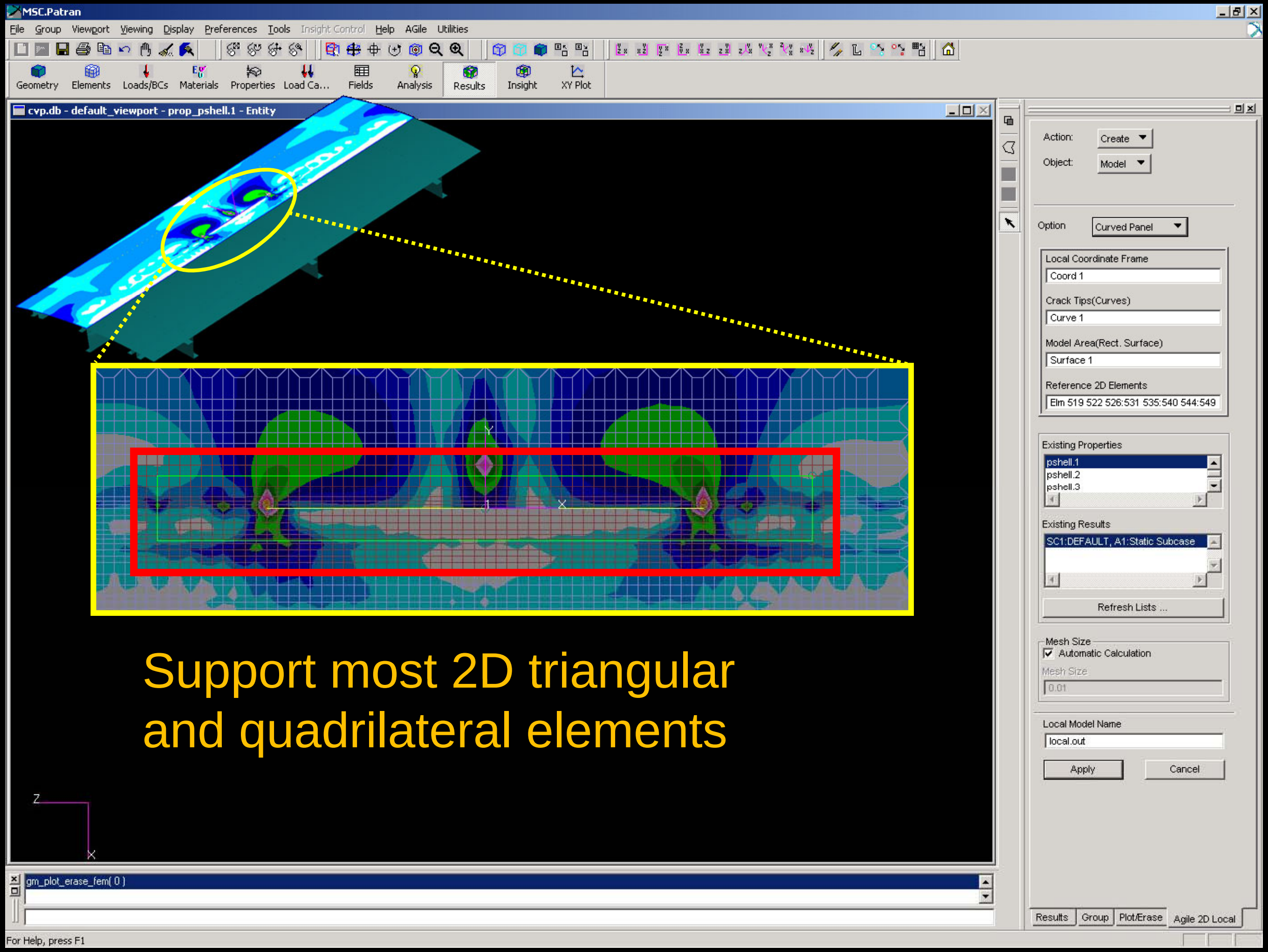
4) Probabilistic Diagnostic Imaging

5) Integrated Probabilistic Prognostics

Probabilistic Analysis

- The probabilistic information on pre-crack damage and macro-crack growth will be analyzed in terms of location, size and type of damage.
- Automatic life prediction in a probabilistic sense for structures will be implemented with probabilistic information of the real environmental conditions.
- Experimental database will be used as one possible probabilistic input, as well as other theoretical and numerical models.

AGILE-2D: Demonstration



cvp.db - default_viewport - prop_pshell.1 - Entity

Action: Create
Object: Model
Option: Curved Panel
Local Coordinate Frame: Coord 1
Crack Tips(Curves): Curve 1
Model Area(Rect. Surface): Surface 1
Reference 2D Elements: Elm 519 522 526:531 535:540 544:549

Existing Properties
pshell.1
pshell.2
pshell.3

Existing Results
SC1:DEFAULT, A1:Static Subcase

Mesh Size
☑ Automatic Calculation
Mesh Size: 0.01

Local Model Name: local.out

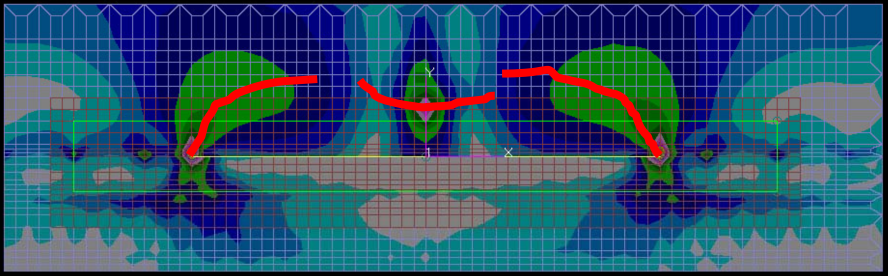
Apply
Cancel

```
gm_plot_erase_fem( 0 )
```

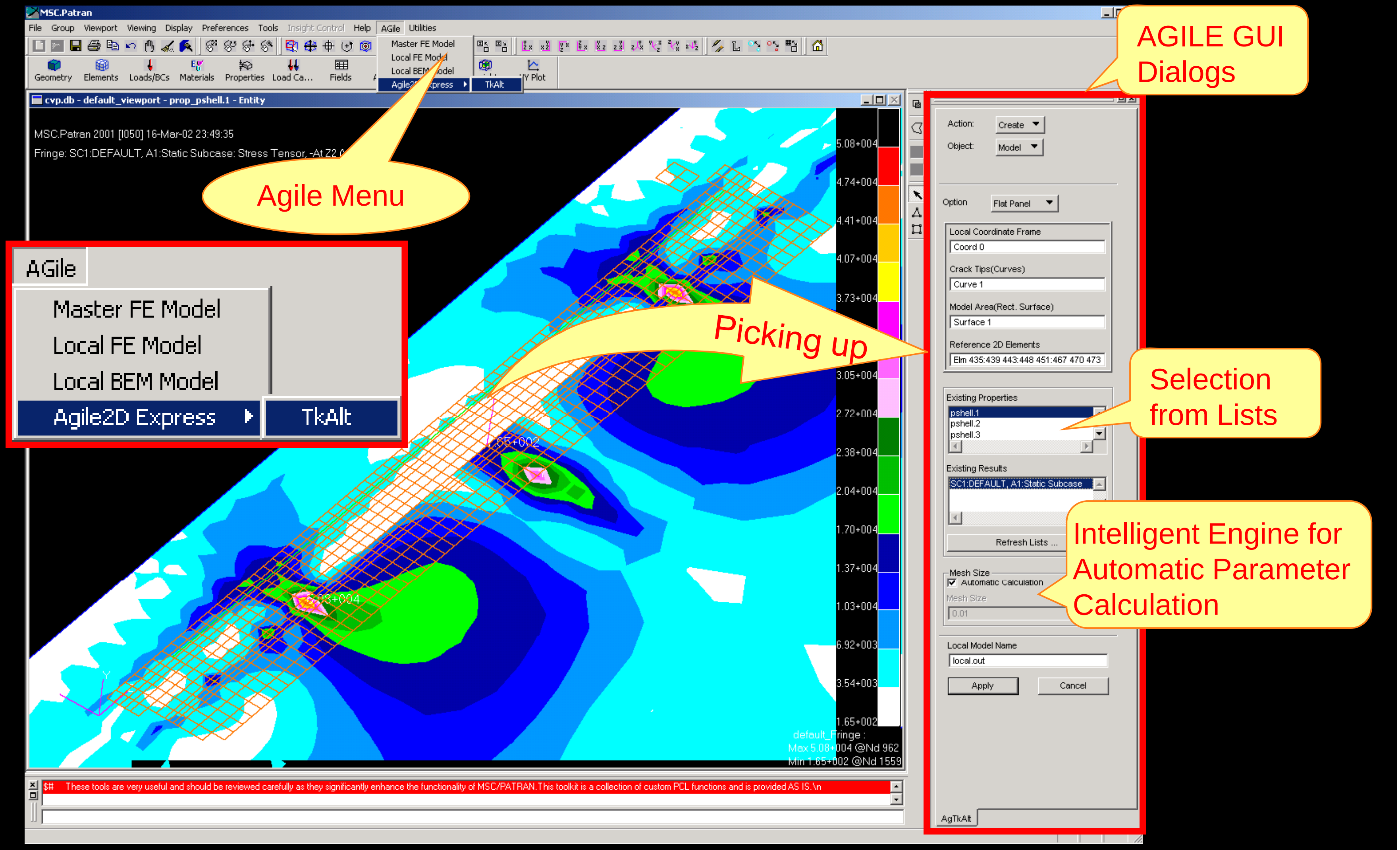
For Help, press F1

Support most 2D triangular and quadrilateral elements

Mixed Mode Crack Growth: No Changes in FE Mesh



Dialog-based Interface



MSC.Patran 2001 [1050] 16-Mar-02 23:49:35

Fringe: SC1:DEFAULT, A1:Static Subcase: Stress Tensor, -At Z2 (

default_Fringe :
Max 5.08+004 @Nd 962
Min 1.65+002 @Nd 1559

$# These tools are very useful and should be reviewed carefully as they significantly enhance the functionality of MSC/PATRAN.This toolkit is a collection of custom PCL functions and is provided AS IS.\n

Action: Create
Object: Model
Option Flat Panel
Local Coordinate Frame: Coord 0
Crack Tips(Curves): Curve 1
Model Area(Rect. Surface): Surface 1
Reference 2D Elements: Elm 435:439 443:448 451:467 470 473
Existing Properties: pshell.1, pshell.2, pshell.3
Existing Results: SC1:DEFAULT, A1:Static Subcase
Mesh Size
☑ Automatic Calculation
Mesh Size: 0.01
Local Model Name: local.out

Agile Menu

Picking up

AGILE GUI Dialogs

Selection from Lists

Intelligent Engine for Automatic Parameter Calculation