

MATHEMATICAL & COMPUTATIONAL SCIENCES DIVISION  
SEMINAR SERIES

**Carbon Dioxide, Global Warming,  
and Michael Crichton's  
"State of Fear"**

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Tuesday, Sept. 13, 2005, 15:00-16:00

NIST North (820), Room 145

## Abstract

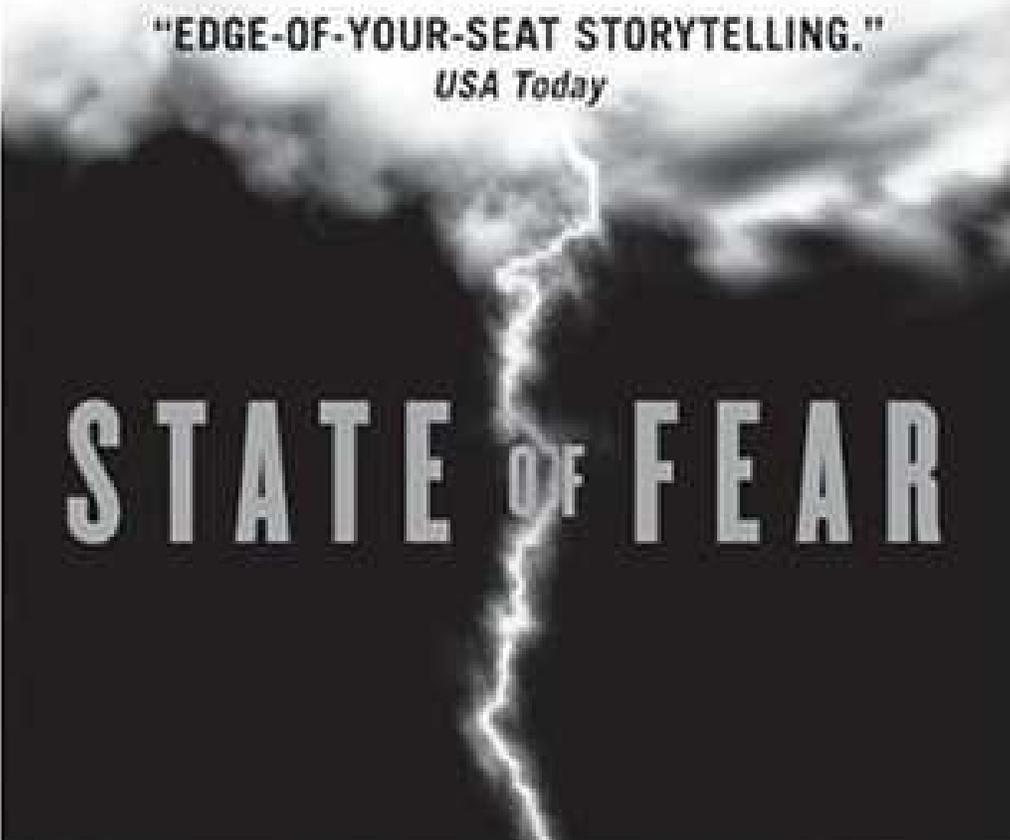
In his recent novel, **State of Fear** (HarperCollins, 2004), Michael Crichton questioned the reality of global warming and its connection to increasing atmospheric carbon dioxide levels. He bolstered his arguments by including plots of historical temperature records and other environmental variables, together with footnotes and appendices that purport to document them. Although most of his arguments were flawed, he did introduce at least one legitimate question by pointing out that in the years 1940-1970, global temperatures were decreasing while atmospheric carbon dioxide was increasing. I resolve this apparent contradiction by constructing a suite of simple mathematical models for the temperature time series. Each model consists of an accelerating baseline plus a 64.7 year sinusoidal oscillation. This cycle, which was first reported by Schlesinger and Ramankutty [Nature, Vol 367 (1994) pp. 723-726], appears also, with its sign reversed, in the time series record of fossil fuel carbon dioxide emissions. This suggests a negative temperature feedback in fossil fuel production. The acceleration in the temperature baseline is demanded by the data, but the temperature record is not yet long enough to precisely specify both the form and the rate of the acceleration. The most interesting model has a baseline derived from a power law relation between temperature changes and changes in the atmospheric carbon dioxide level. And the increase in atmospheric carbon dioxide is easily modelled by the cumulative accretion of a fixed fraction of each year's fossil fuel emissions, so the power law model posits a direct connection between the emissions and the warming. For all of the temperature models, the cycle was decreasing more rapidly than the baseline was rising in the years 1940-1970, and in 1880-1910. We have recently entered another declining phase of the cycle, but the temperature hiatus this time will be far less dramatic because the accelerating baseline is rising more rapidly now.

**NEW YORK TIMES BESTSELLER**

# **MICHAEL CRICHTON**

**"EDGE-OF-YOUR-SEAT STORYTELLING."**

*USA Today*



**STATE OF FEAR**

**In Paris, a physicist dies after performing  
a laboratory experiment for a beautiful visitor.**

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**In the jungles of Malaysia, a mysterious buyer  
purchases deadly cavitation technology, built to  
his specifications.**

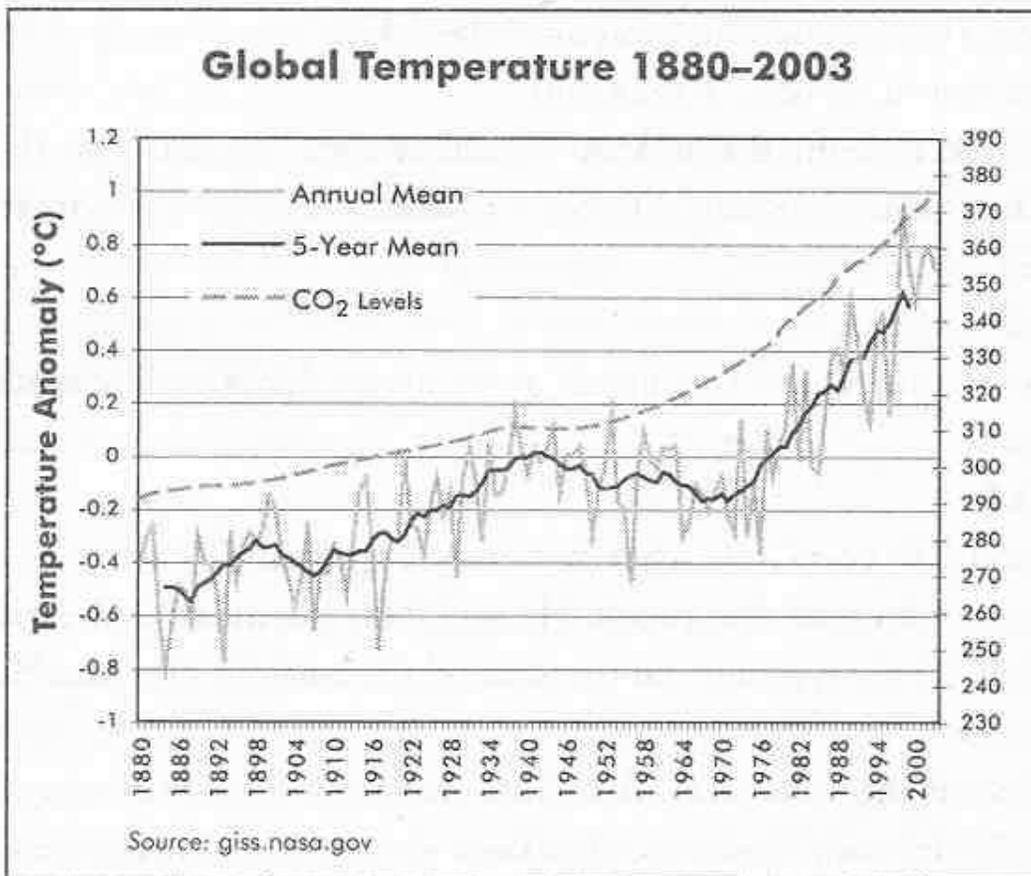
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**In Vancouver, a small research submarine is leased  
for use in the waters off New Guinea.**

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**And in Tokyo, an intelligence agent tries to  
understand what it all means.**

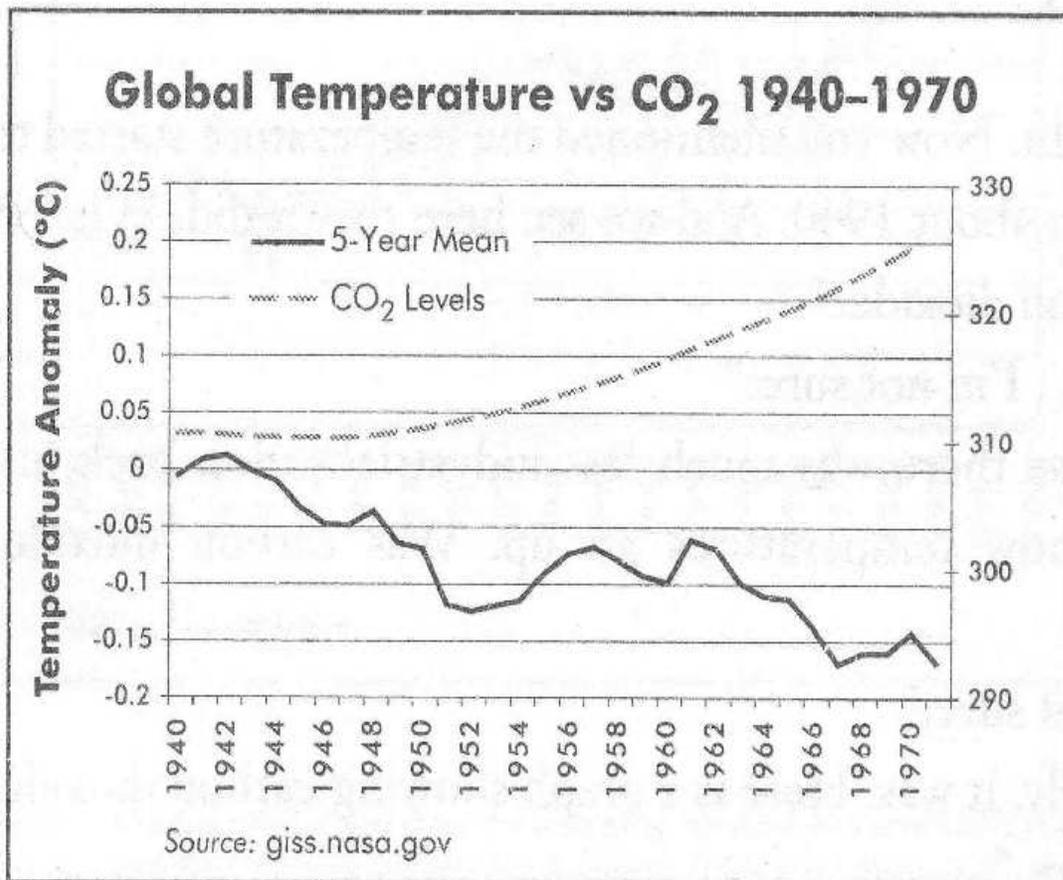
Michael Crichton, "State of Fear,"  
HarperCollins (2004) pp. 86-87.

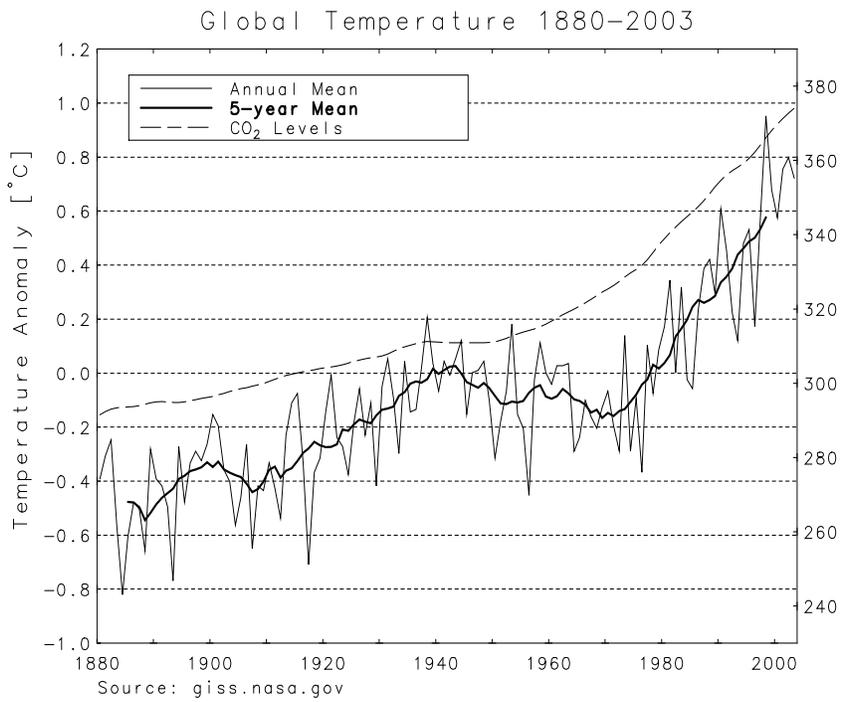
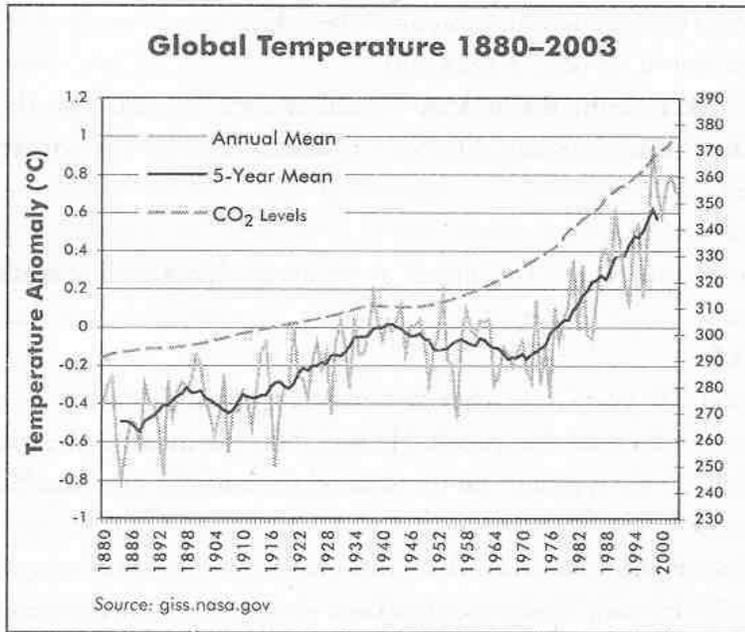


“So, if rising carbon dioxide is the cause of rising temperatures, why didn't it cause temperatures to rise from 1940 to 1970?”

“Now I want to direct your attention to the period from 1940 to 1970. As you see, during that period the global temperature actually went down. You see that?”

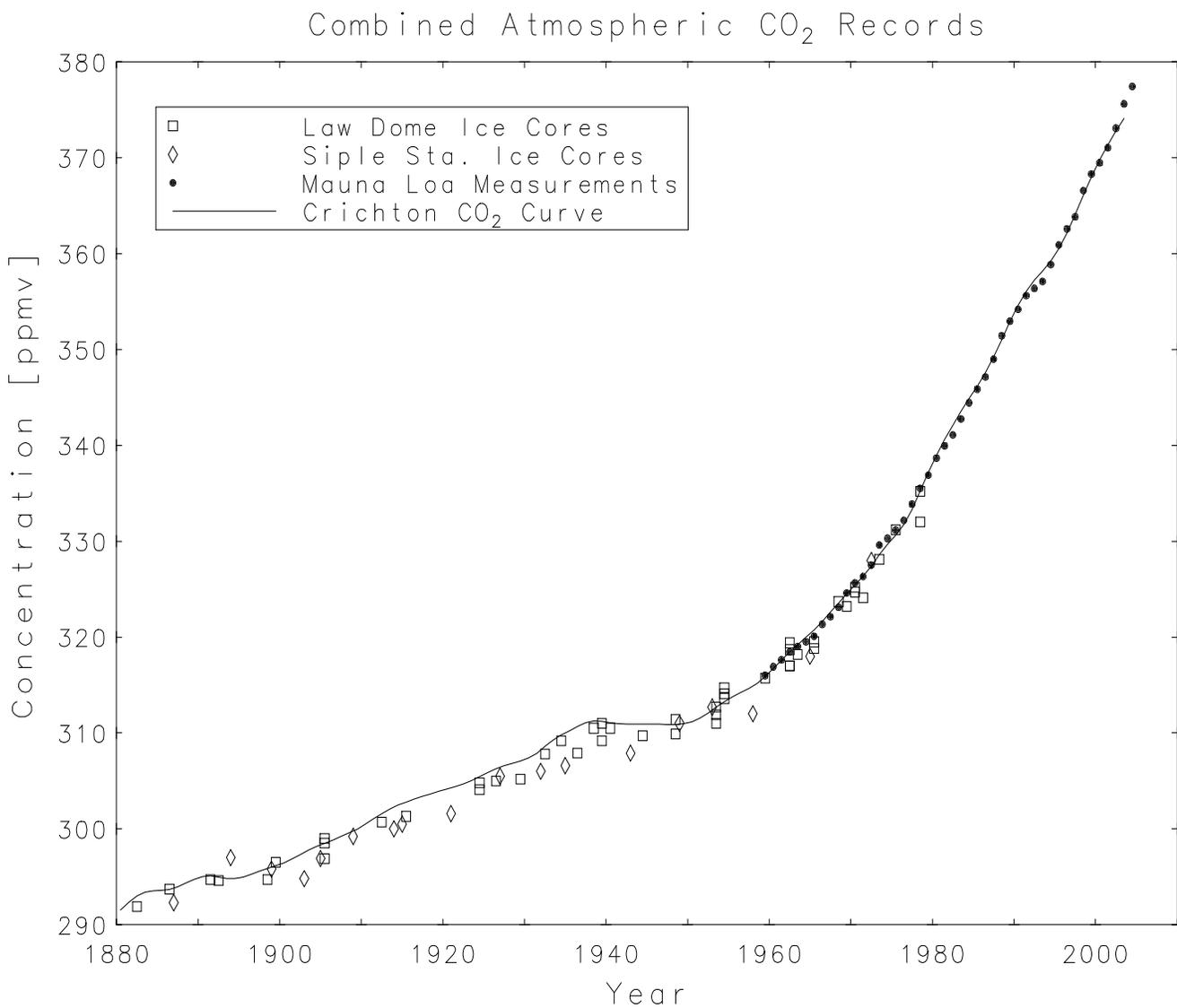
“Yes ...”

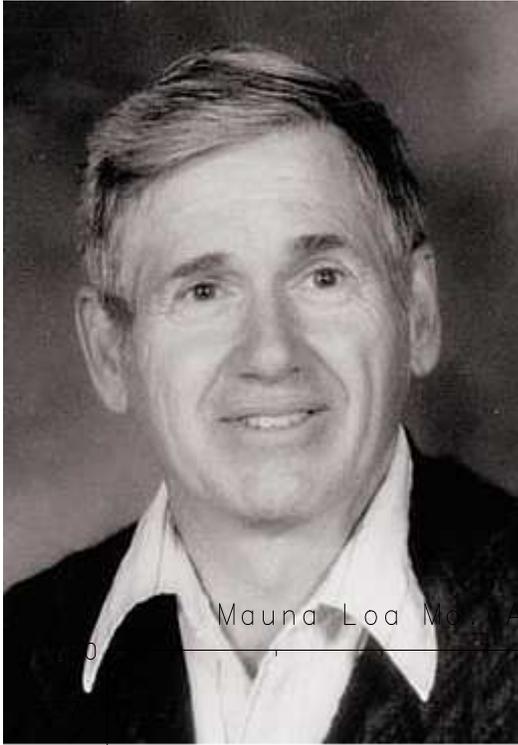




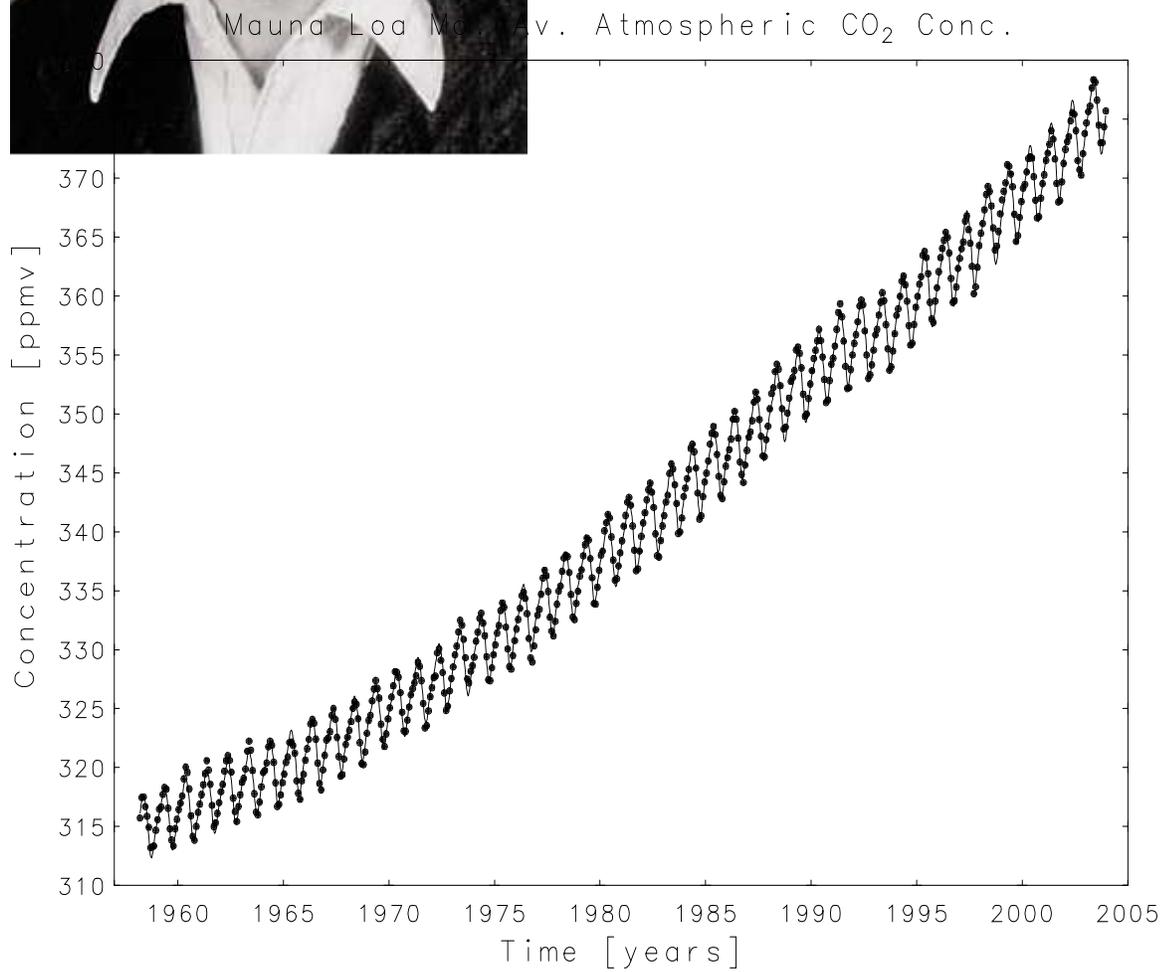
# Atmospheric CO<sub>2</sub> concentration data from CDIAC, Oak Ridge National Lab.

High precision Mauna Loa measurements by  
C. D. Keeling, et. al.

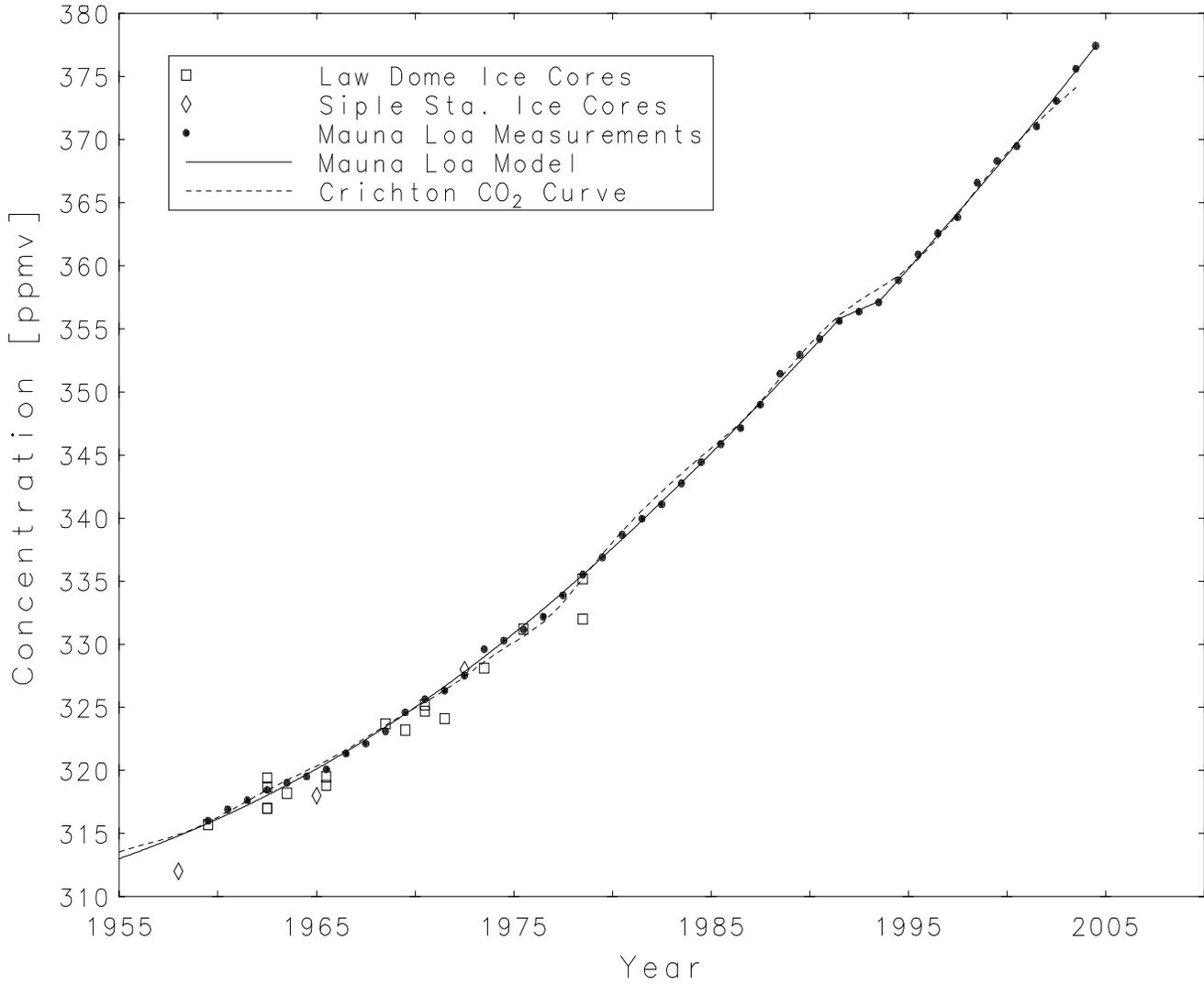




**Charles David Keeling**  
April 1928 – June 2005

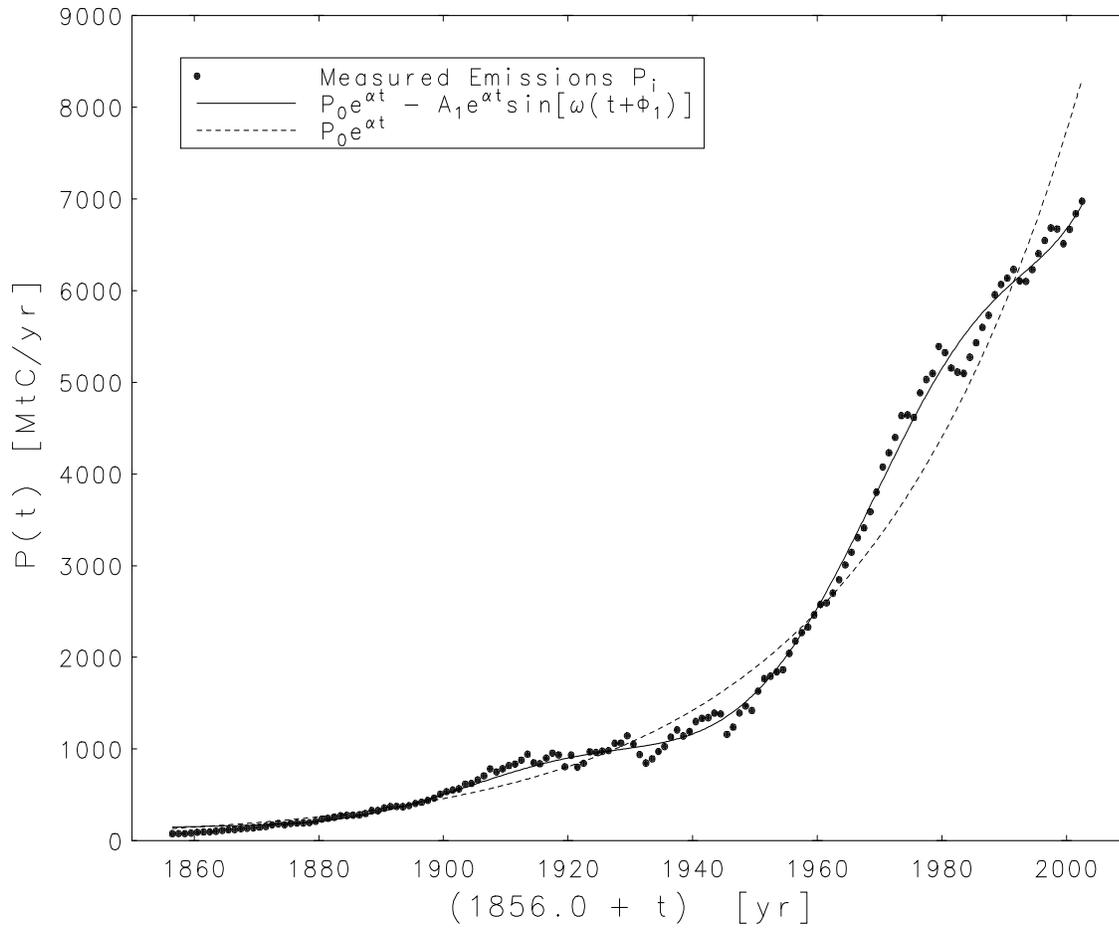


Combined Atmospheric CO<sub>2</sub> Records



Choose  $t = 0$  at epoch 1856.0

Global Fossil Fuel Emissions 1856–2002



$$P(t) = P_0 e^{\alpha t} - A_1 e^{\alpha t} \sin \left[ \frac{2\pi}{\tau} (t + \phi_1) \right]$$

$$\hat{\alpha} = 0.02824 \pm .00029$$

$$\hat{\tau} = 64.7 \pm 1.4$$

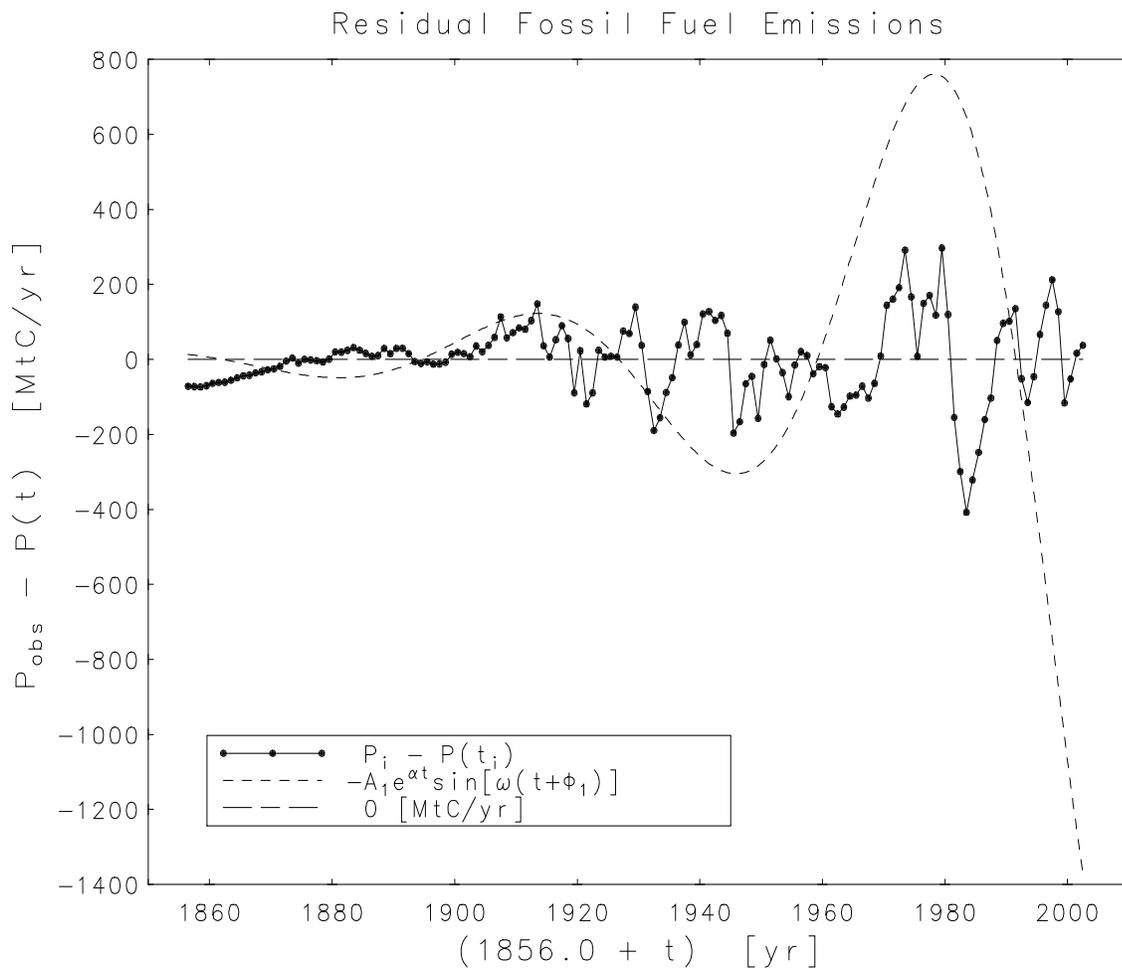
$$\hat{P}_0 = 132.7 \pm 4.4$$

$$\hat{A}_1 = 25.1 \pm 1.1$$

$$\hat{\phi}_1 = -6.1 \pm 2.4$$

$$\omega \equiv \frac{2\pi}{\tau} = 0.0971 \text{ [rad/yr]}$$

$$P(t_i) = P_0 e^{\alpha t_i} - A_1 e^{\alpha t_i} \sin[\omega(t_i + \phi_1)]$$



## Model for the Atmospheric CO<sub>2</sub> Concentration

$$c(t) = c_0 + \gamma \int_0^t P(t') dt' + \delta S(t)$$

where

$$P(t') = P_0 e^{\alpha t'} - A_1 e^{\alpha t'} \sin[\omega(t' + \phi_1)]$$

$$S(t) \equiv \begin{cases} 0, & t \leq t_P \\ \frac{1}{2}(t - t_P), & t_P < t < (t_P + 2) \\ 1, & (t_P + 2) \leq t \end{cases}$$

$$t_P = 1991.54 - 1856.0 = 135.54$$

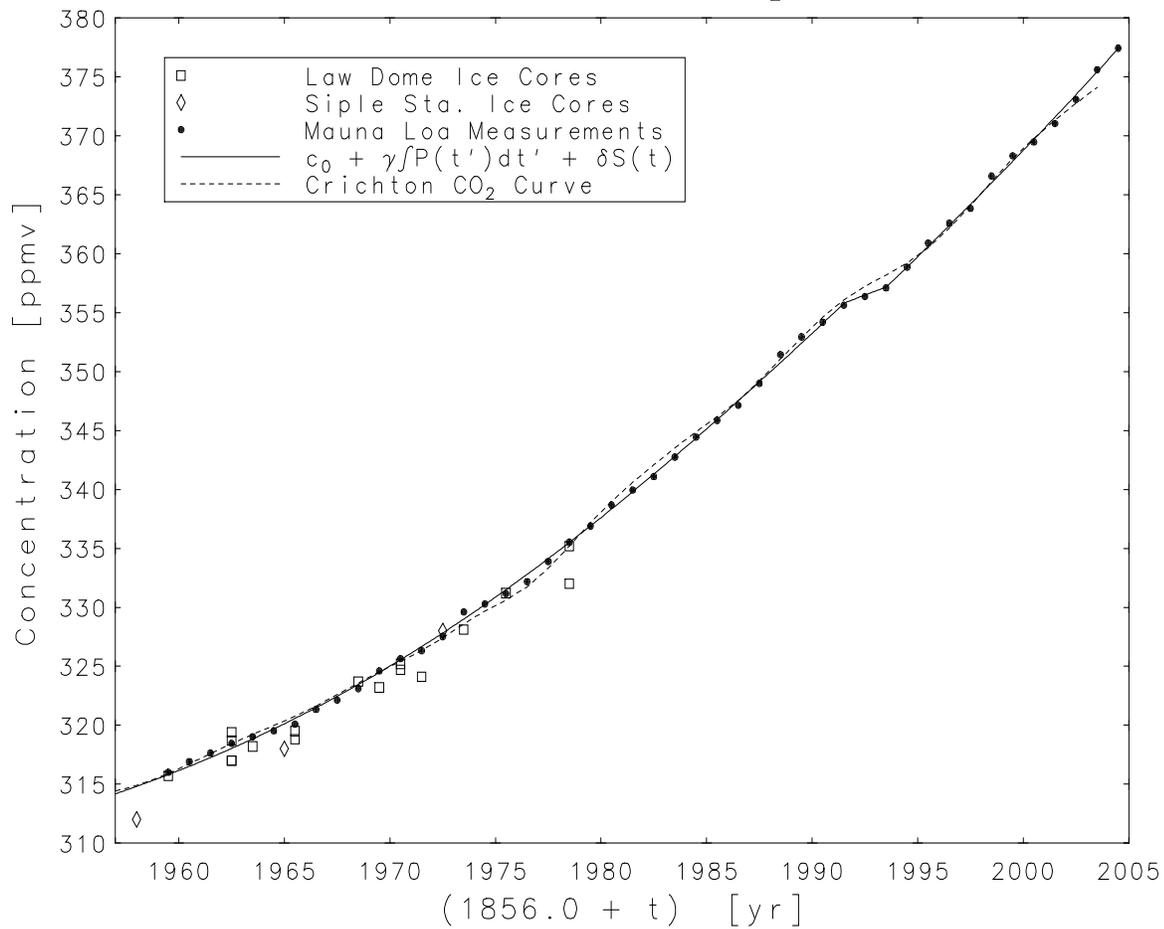
Mount Pinatubo erupted on June 15, 1991

$$1 \text{ [ppmv]} = 2130 \text{ [MtC]}$$

Lianhong Gu, et al, "Response of a Deciduous Forest to the Mount Pinatubo Eruption: Enhanced Photosynthesis," *Science*, 299 (2003) 2035-2038.

$$c(t) = c_0 + \gamma \int_0^t P(t') dt' + \delta S(t)$$

Combined Atmospheric CO<sub>2</sub> Records

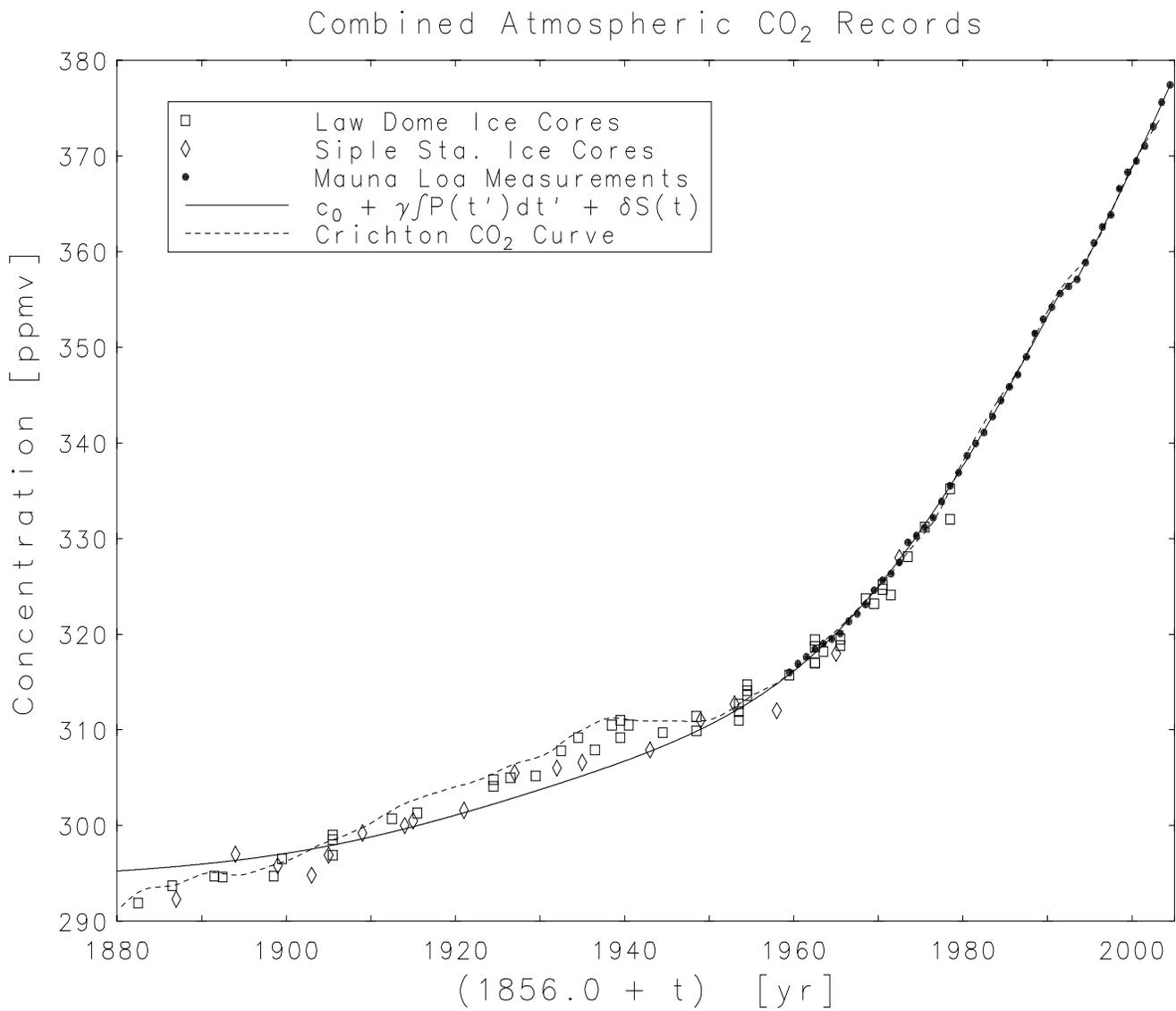


$$\hat{c}_0 = 294.10 \pm .19 \text{ [ppmv]}$$

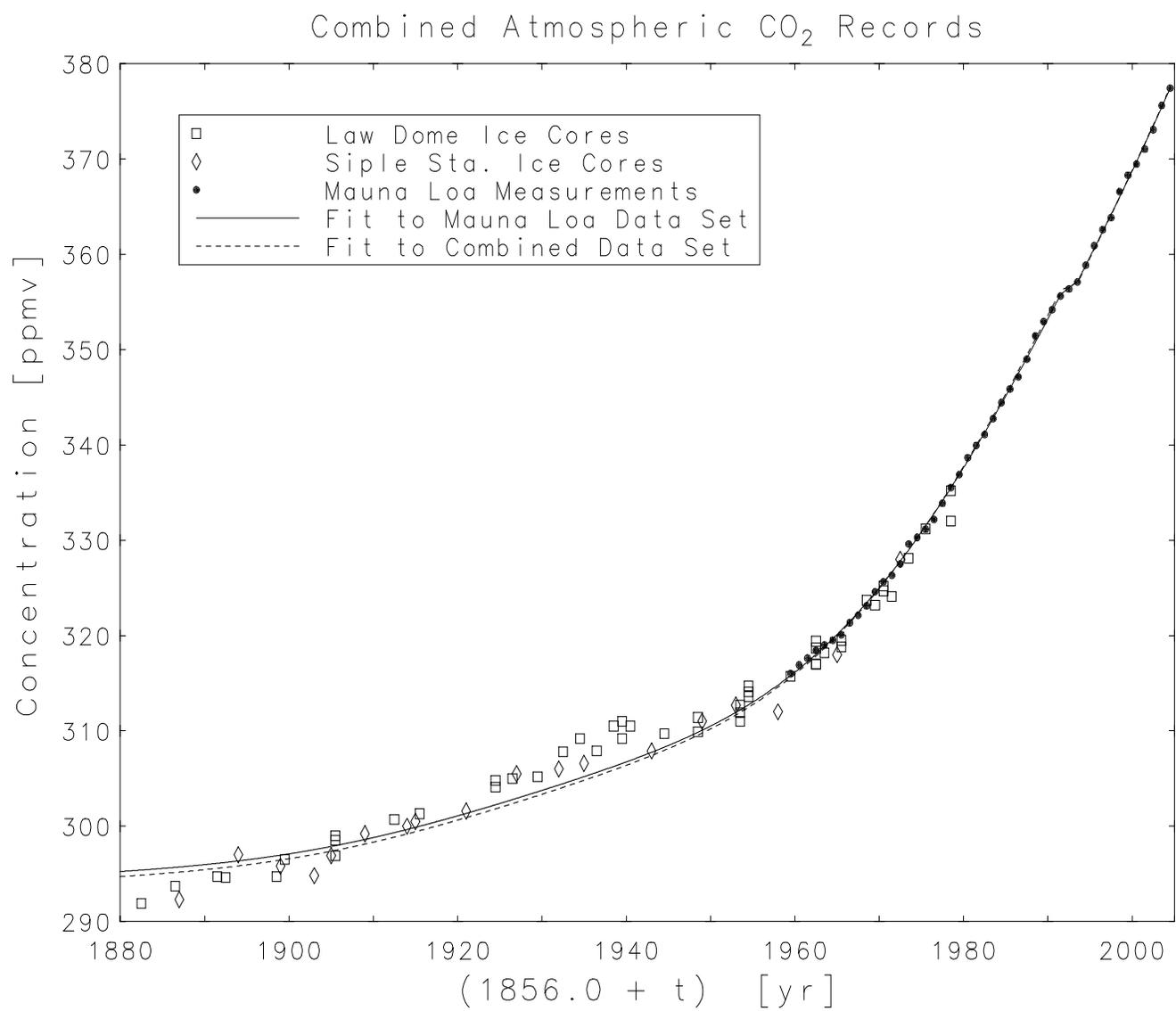
$$\hat{\gamma} = 0.5926 \pm .0026$$

$$\hat{\delta} = -2.05 \pm .20 \text{ [ppmv]}$$

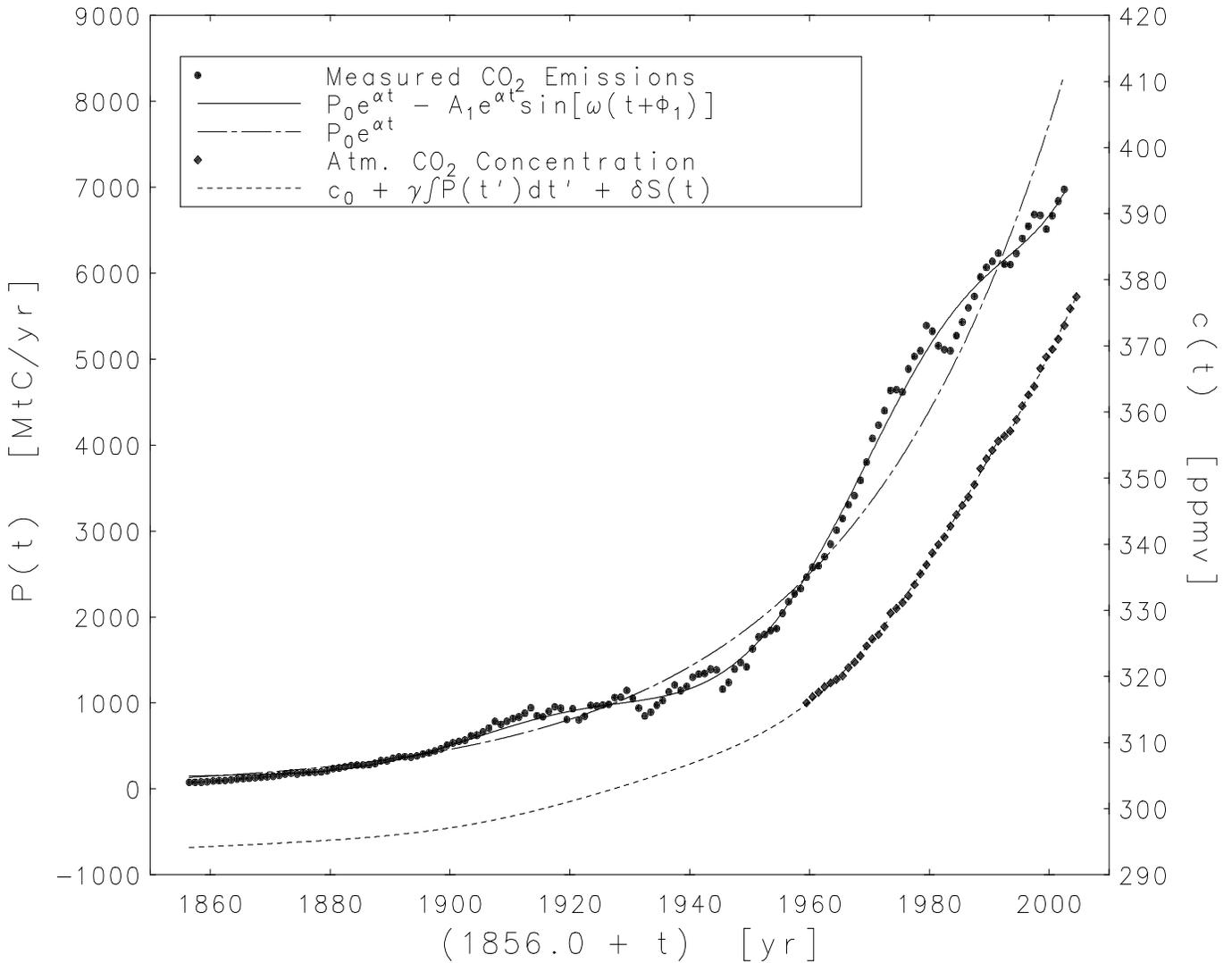
## Extrapolating the Fit Backward



## Fitting the Combined Data Set



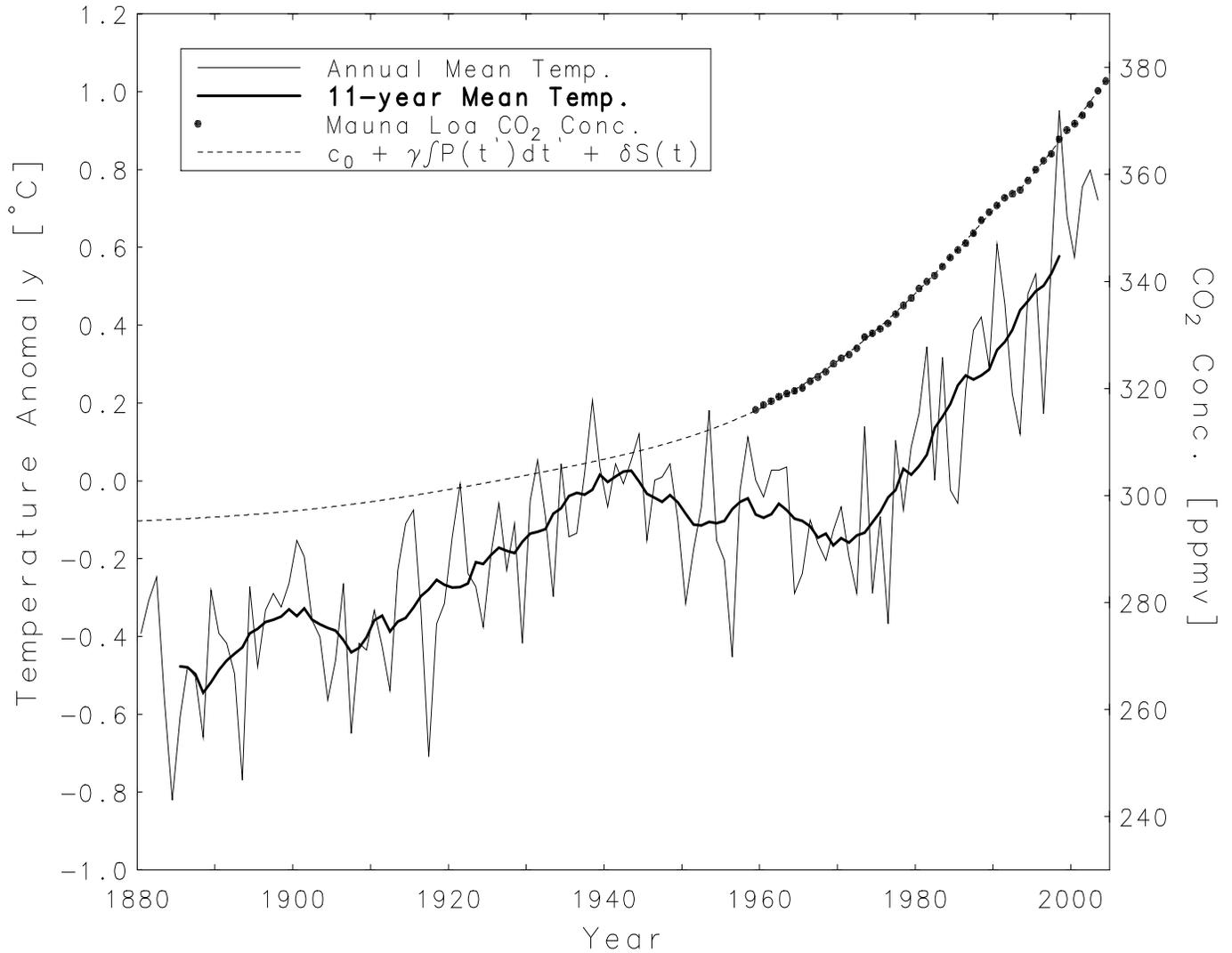
Fossil Fuel Emiss. and Atm. CO<sub>2</sub> Conc.



$$P(t) = P_0 e^{\alpha t} - A_1 e^{\alpha t} \sin[\omega(t + \phi_1)]$$

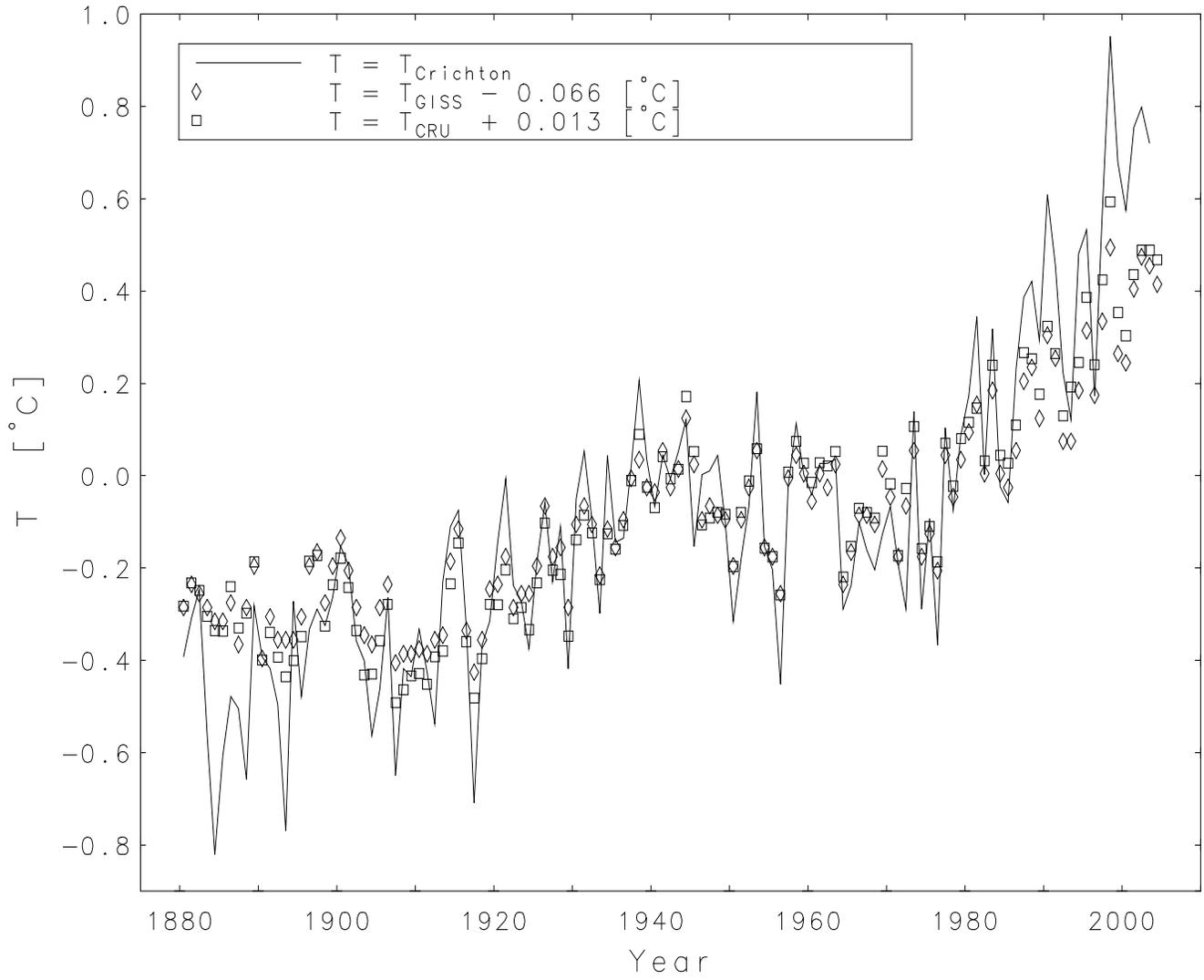
$$c(t) = c_0 + \gamma \int_0^t P(t') dt' + \delta S(t)$$

Crichton Temp. Anomalies and Mauna Loa CO<sub>2</sub> Conc.

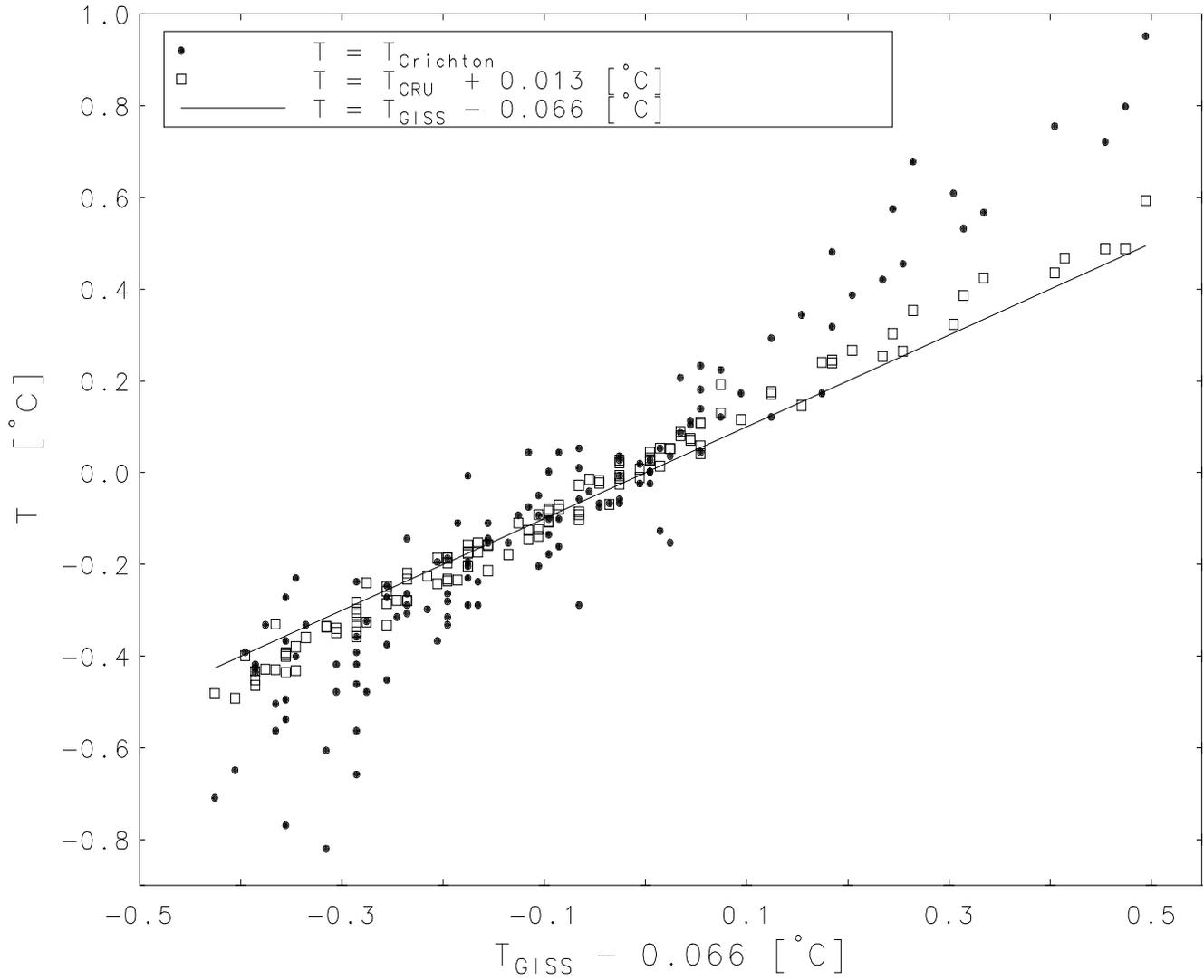


$$\left[ \begin{array}{c} \text{Temperature} \\ \text{"anomaly"} \\ \text{for year } t_i \end{array} \right] \equiv \left[ \begin{array}{c} \text{Average} \\ \text{Temperature} \\ \text{in year } t_i \end{array} \right] - \left[ \begin{array}{c} \text{Average Temp.} \\ \text{for some} \\ \text{reference period} \end{array} \right]$$

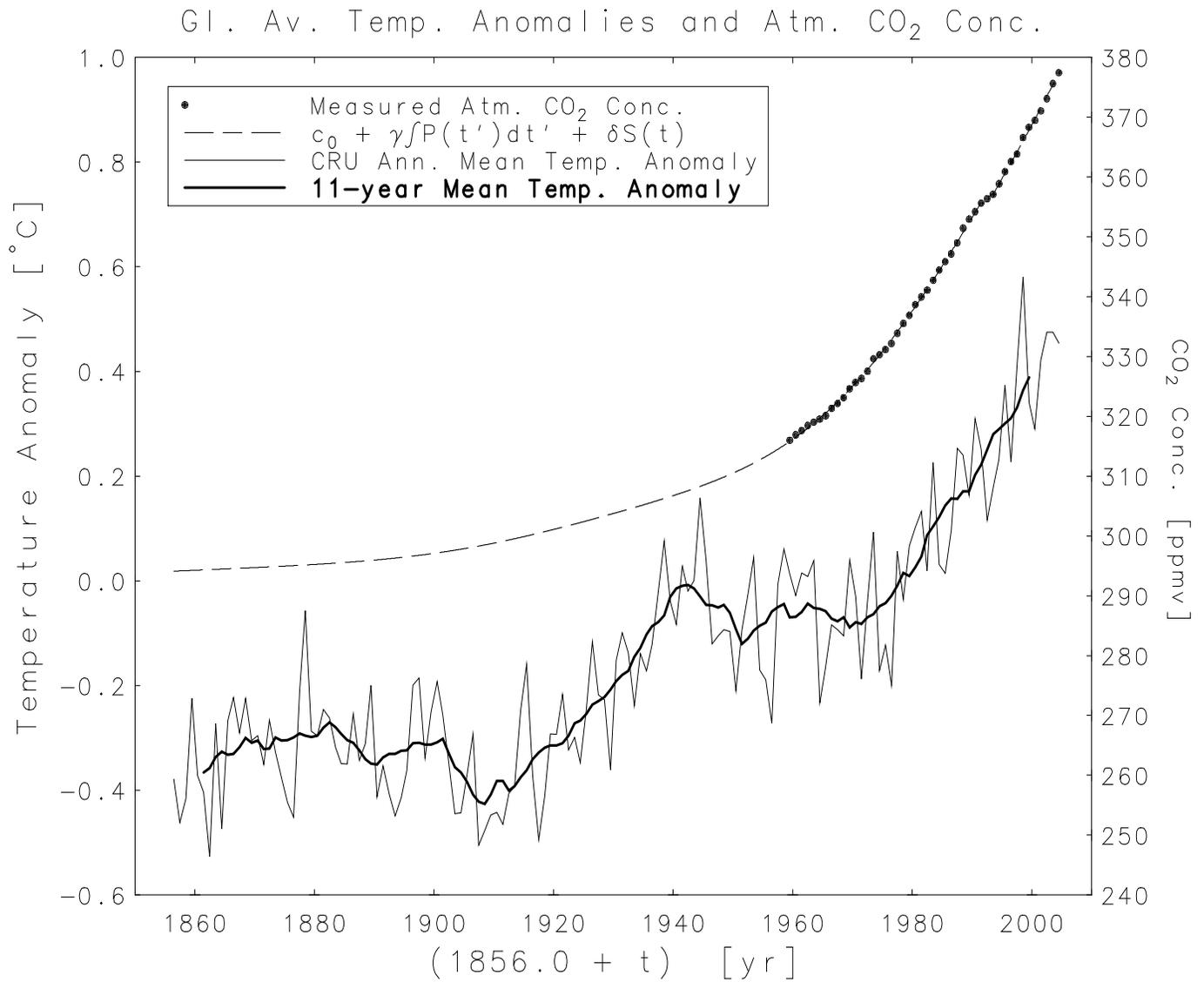
# Crichton, GISS, and CRU Temperature Anomalies



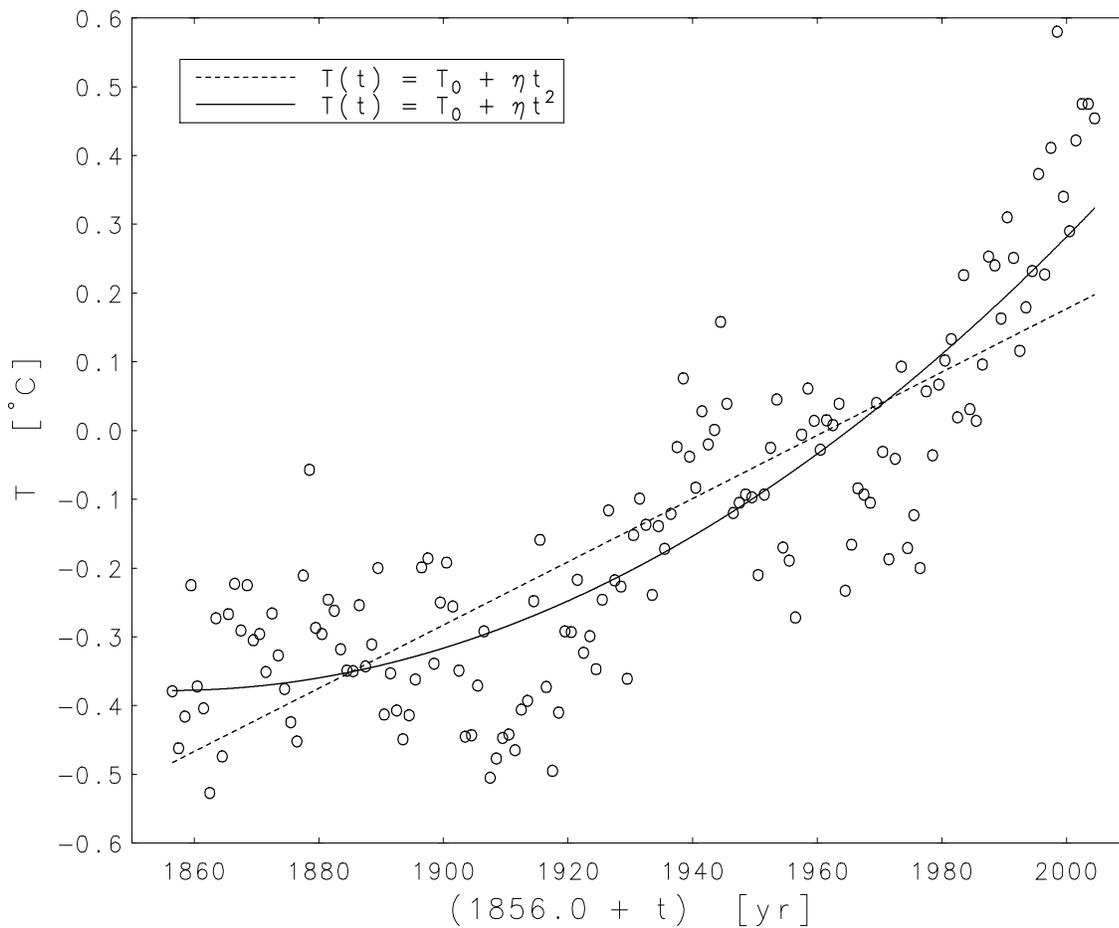
Temperature Anomalies: Crichton and CRU vs. GISS



## Improved and Corrected Crichton Plot

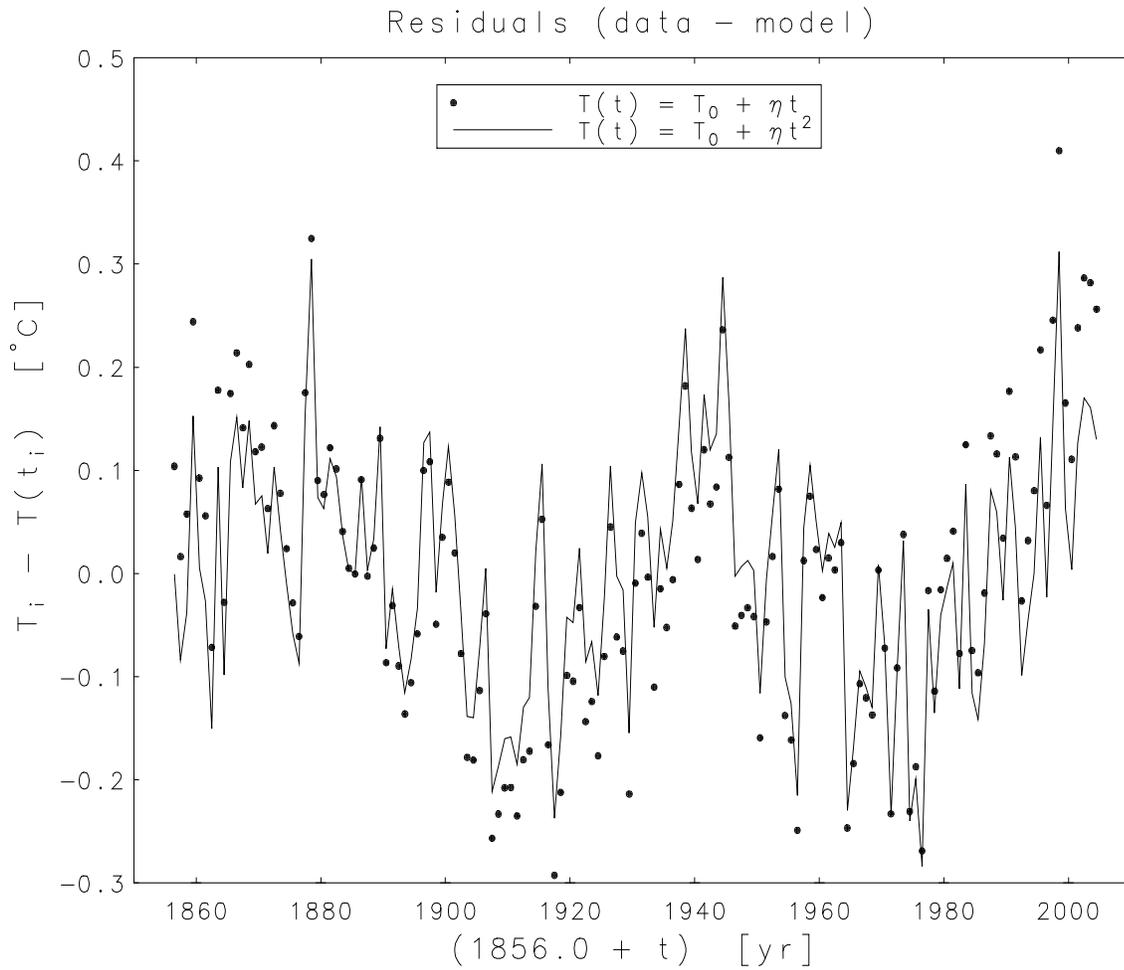


Ann. Global Av. Temp. Anomalies (1856–2004)



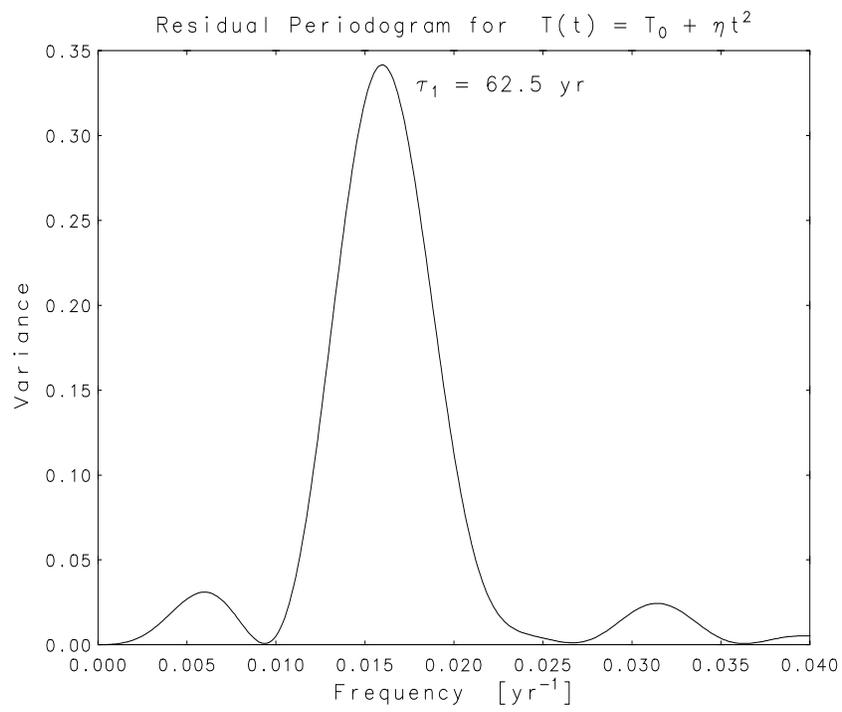
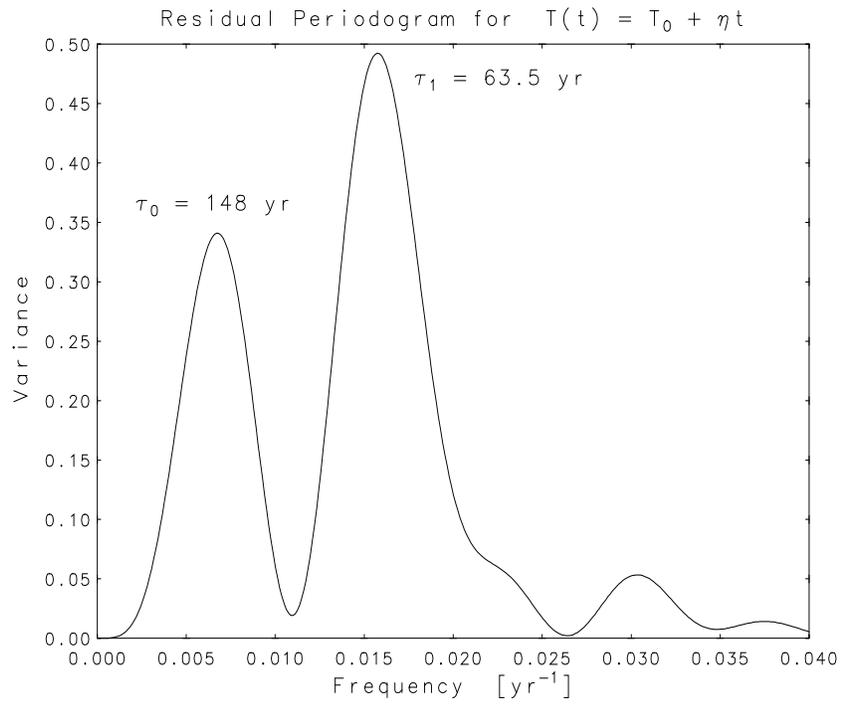
Stat.	$T(t) = T_0 + \eta t$	$T(t) = T_0 + \eta t^2$
SSR	2.7572	1.9798
$100R^2$	67.89%	76.94%

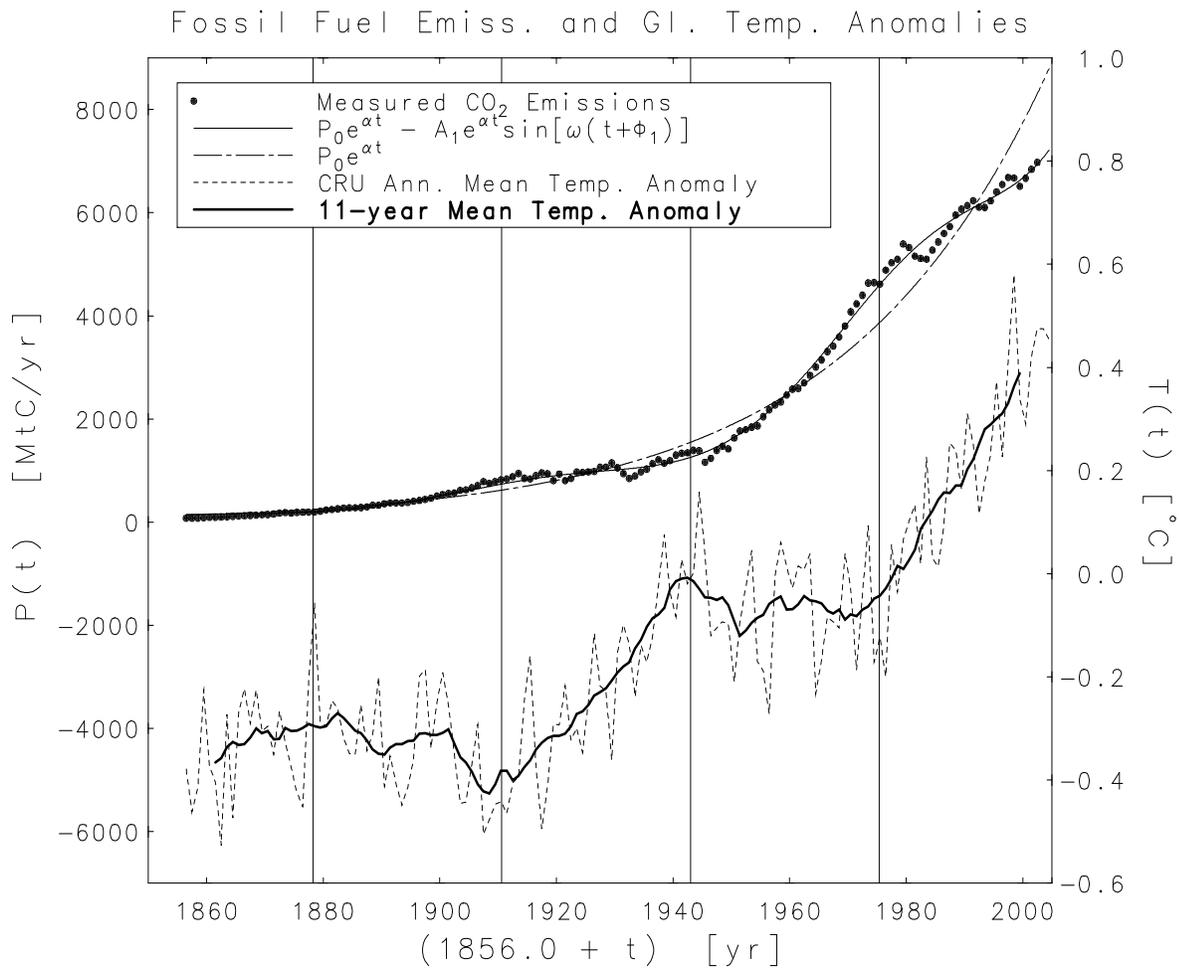
$$R^2 = 1 - \frac{\text{SSR}}{\text{CTSS}}, \quad \text{CTSS} = \sum_{i=1}^m (T_i - \bar{T})^2$$



The data demand a concave upward baseline.

**The warming is accelerating!**





Schlesinger and Ramankutty, “An oscillation in the global climate system of period 65-70 years,” *Nature*, 367 (1994) 723-726.

“These oscillations have obscured the greenhouse warming signal...”

“...the oscillation arises from predictable internal variability of the ocean-atmosphere system.”

## A Gaiaen Feedback?

$$T(t) = T_0 + \eta t^2 + A_3 \sin \left[ \frac{2\pi}{\tau_1} (t + \phi_1) \right]$$

$$P(t) = P_0 e^{\alpha t} - A_1 e^{\alpha t} \sin \left[ \frac{2\pi}{\tau_1} (t + \phi_1) \right]$$

Could the presence of the 65-year cycle in both records, with sign reversed, be caused by an inverse temperature feedback?

$$\left\{ \begin{array}{l} \text{cooler} \\ \text{warmer} \end{array} \right\} T(t) \Rightarrow \left\{ \begin{array}{l} \text{more} \\ \text{less} \end{array} \right\} \text{demand for } P(t)$$

B. W. Rust and B. L. Kirk, "Modulation of Fossil Fuel Production by Global Temperature Variations," *Environment International*, 7 (1982) 419-422.

$$\frac{dP}{dt} = \left( \alpha - \beta \frac{dT}{dt} \right) P, \quad P(0) = P_0$$

$$P(t) = P_0 e^{\alpha t} - A_1 e^{\alpha t} \sin [\omega(t + \phi_1)]$$

$$c(t) = c_0 + \gamma \int_0^t P(t') dt' + \delta S(t)$$

$$T(t) = T_0 + \eta t + A_3 \sin [\omega(t + \phi_1)]$$

$$T(t) = T_0 + \eta t^2 + A_3 \sin [\omega(t + \phi_1)]$$

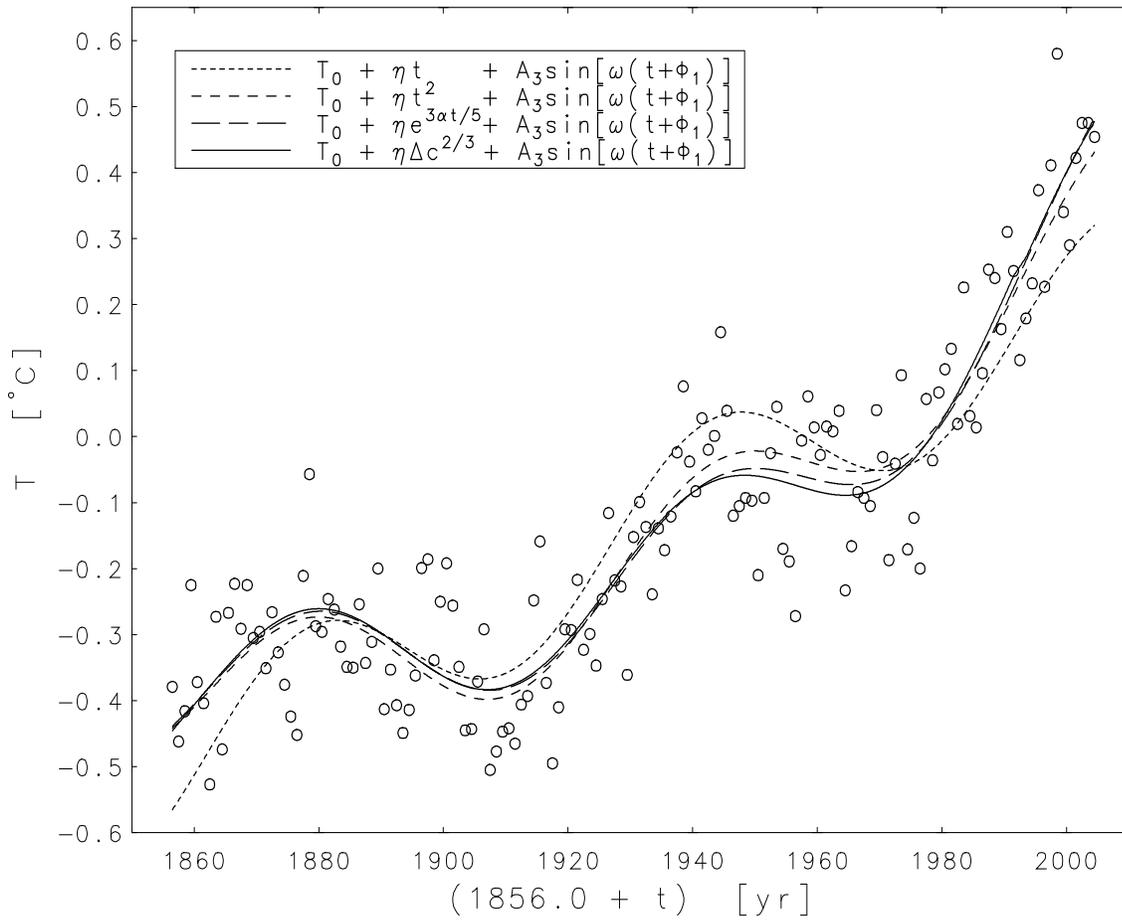
$$T(t) = T_0 + \eta \exp\left(\frac{3\alpha}{5}t\right) + A_3 \sin [\omega(t + \phi_1)]$$

$$T(t) = T_0 + \eta [\Delta c]^{2/3} + A_3 \sin [\omega(t + \phi_1)]$$

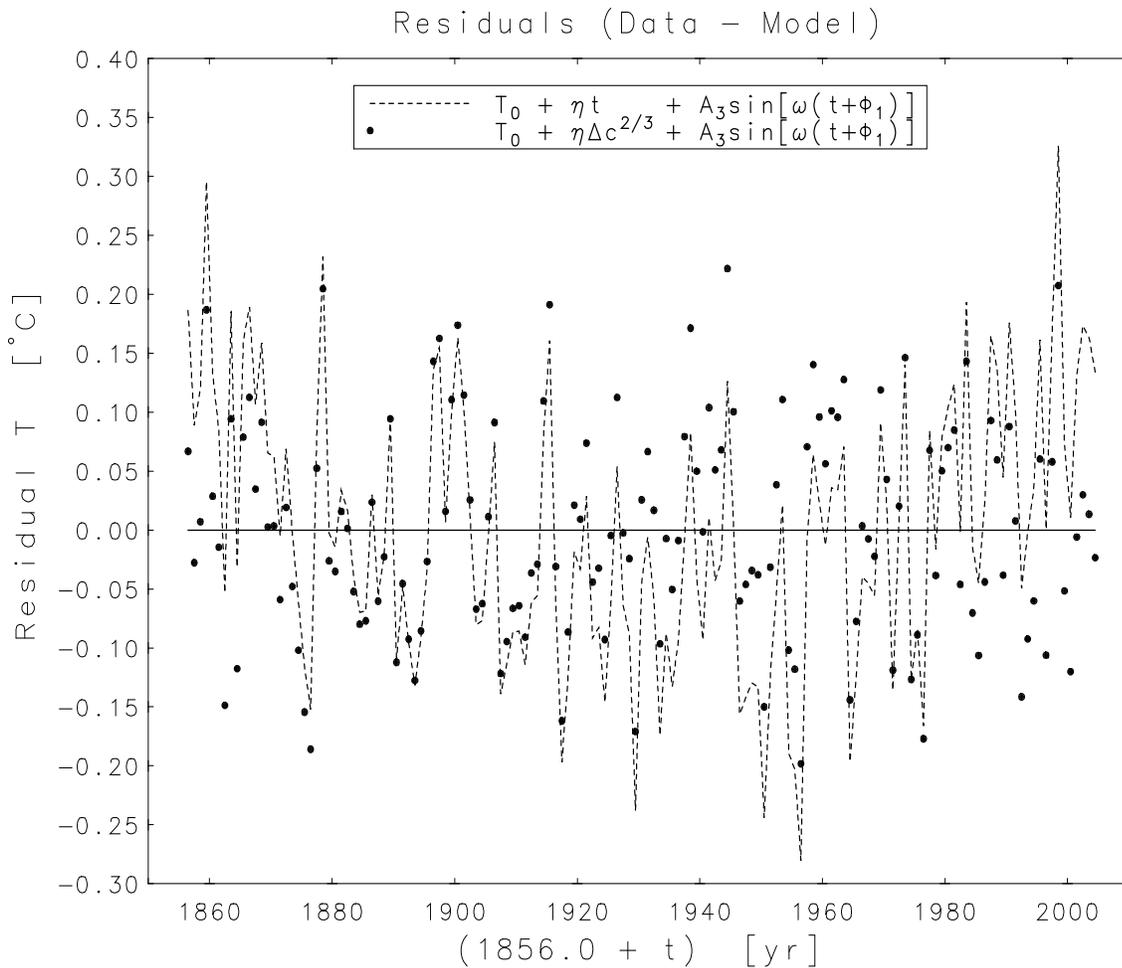
$$\Delta c \equiv c(t) - c_0$$

$$= \gamma \int_0^t P(t') dt' + \delta S(t)$$

Ann. Global Av. Temp. Anomalies (1856–2004)



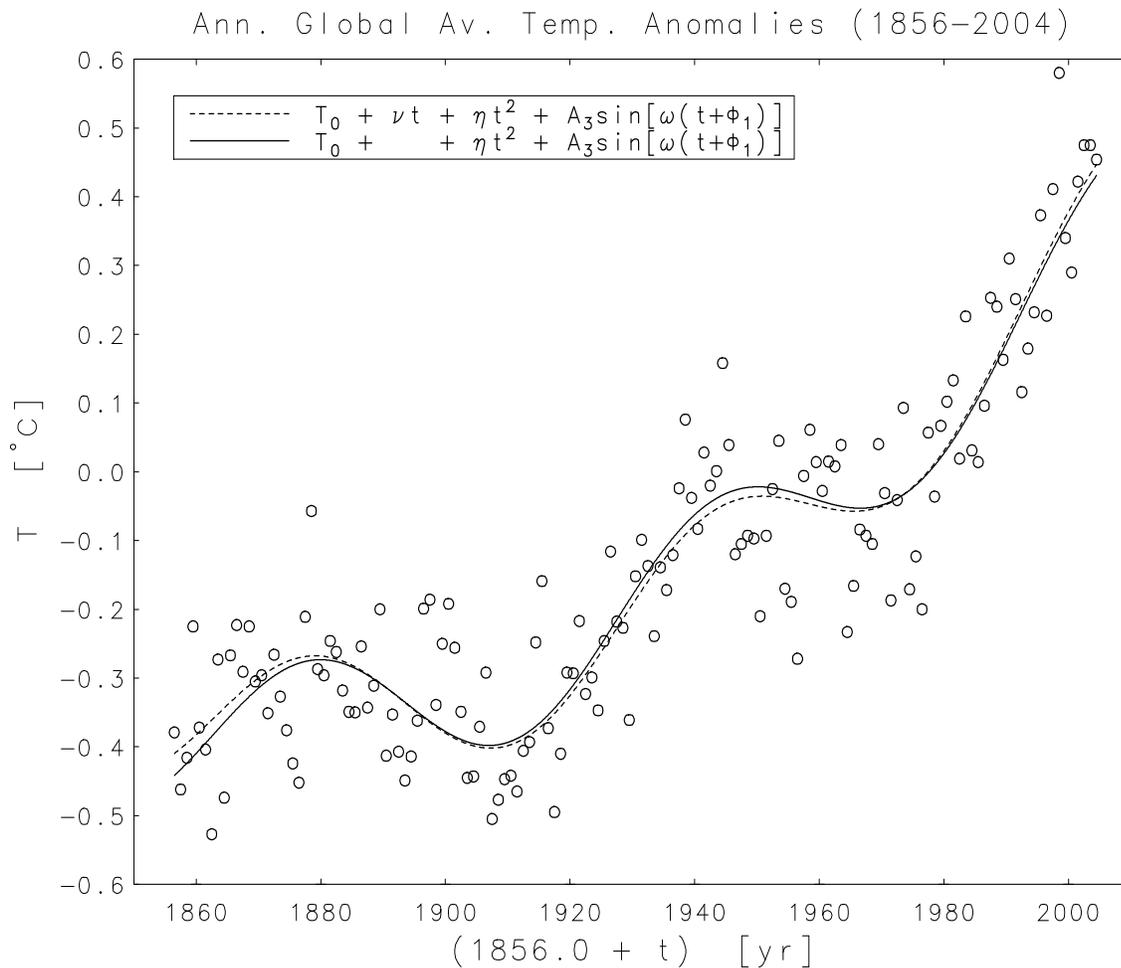
Stat.	$T_0 + \eta t$	$T_0 + \eta t^2$	$T_0 + \eta e^{3\alpha t/5}$	$T_0 + \eta \Delta c^{2/3}$
SSR	1.8965	1.2891	1.2604	1.2630
$100R^2$	77.91%	84.99%	85.32%	85.29%



Note concave upward pattern in straight-line residuals!

**The warming is accelerating!**

$$T(t) = T_0 + \nu t + \eta t^2 + A_3 \sin[\omega(t + \phi_1)]$$



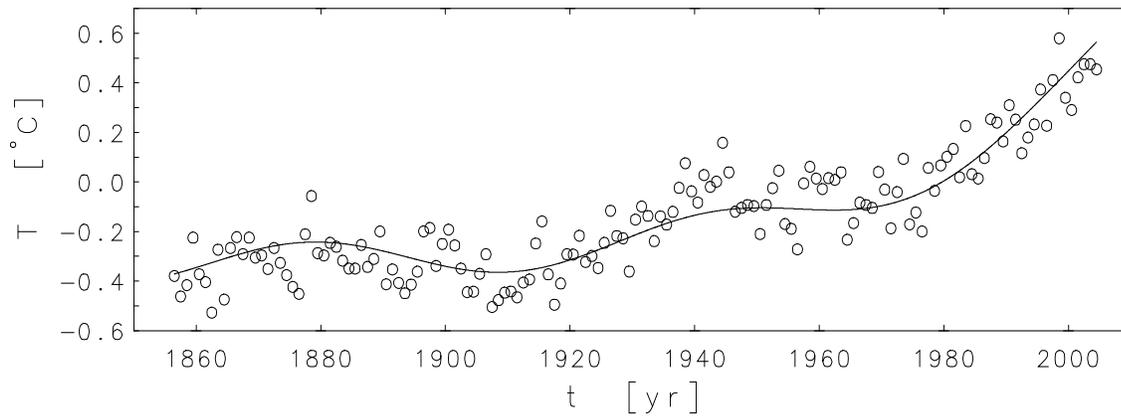
$$\hat{\nu} = (-1.08 \pm .73) \times 10^{-3} \implies H_0 : \nu = 0$$

F-test accepts  $H_0$  at the 95% level

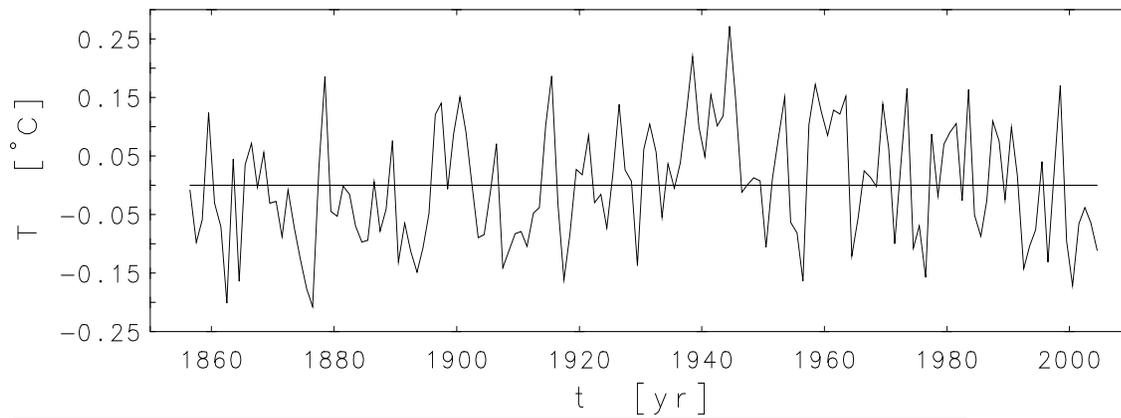
The data demand a monotone increasing baseline

$$T(t) = T_0 + \eta e^{\alpha t} + A_3 \sin[\omega(t + \phi_1)]$$

Ann. Global Av. Temp. Anomalies (1856–2004)

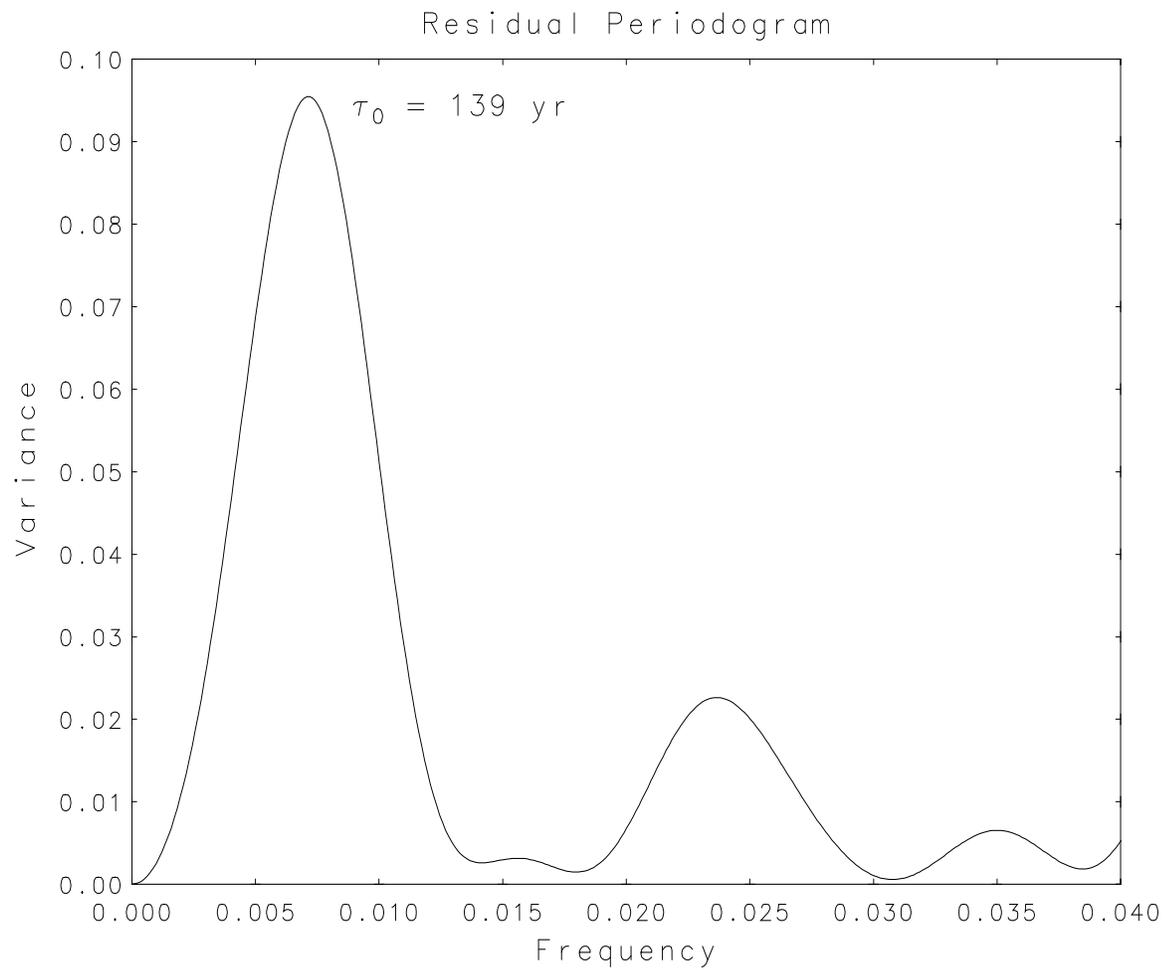


Residuals (data – model)



Stat.	$\alpha = 0.0168$	$\frac{3\alpha}{5} = 0.0169$
SSR	1.46	1.26
$100R^2$	83.0%	85.3%

$$T(t) = T_0 + \eta e^{\alpha t} + A_3 \sin[\omega(t + \phi_1)]$$



$$T(t) = T_0 + \eta e^{\nu t} + A_3 \sin[\omega(t + \phi_1)]$$

$$\left. \begin{array}{l} \hat{\eta} = 0.071 \pm .024 \\ \hat{\nu} = 0.0168 \pm .0022 \end{array} \right\} \hat{\rho}(\eta, \nu) = -0.995$$

$$\frac{3\alpha}{5} = 0.0169 \implies \eta = 0.0690 \pm .0024$$

Stat.	$\hat{\nu} = 0.0168$	$\frac{3\alpha}{5} = 0.0169$
SSR	1.260319	1.260355
$100R^2$	85.3214%	85.3210%

$$T(t) = T_0 + \eta [\Delta c]^\nu + A_3 \sin [\omega(t + \phi_1)]$$

$$\left. \begin{array}{l} \hat{\eta} = (3.2 \pm 2.5) \times 10^{-4} \\ \hat{\nu} = 0.645 \pm .063 \end{array} \right\} \hat{\rho}(\eta, \nu) = -0.9989$$

$$\frac{2}{3} = 0.6666667 \implies \eta = (2.490 \pm .087) \times 10^{-4}$$

Stat.	$\hat{\nu} = 0.645$	$\frac{2}{3} = 0.6666667$
SSR	1.2620	1.2630
$100R^2$	85.302%	85.290%

## The World's Simplest Climate Model

(With apologies to Johannes Kepler)

*“The third power of change in tropospheric temperature is proportional to the square of change in atmospheric CO<sub>2</sub> concentration”*

$$[T(t) - T_0]^3 = \eta [c(t) - c_0]^2$$

$$T(t) = T_0 + \eta [c(t) - c_0]^{2/3}$$

*“But an interaction between the oceans and the atmosphere imposes a cycle with period  $\tau \approx 65$  year on the temperatures which is independent of the CO<sub>2</sub> concentration”*

$$T(t) = T_0 + \eta [c(t) - c_0]^{2/3} + A_3 \sin \left[ \frac{2\pi}{\tau} (t + \phi_1) \right]$$

$$T(t) = T_0 + \eta [c(t) - c_0]^{2/3} + A_3 \sin \left[ \frac{2\pi}{\tau} (t + \phi_1) \right]$$

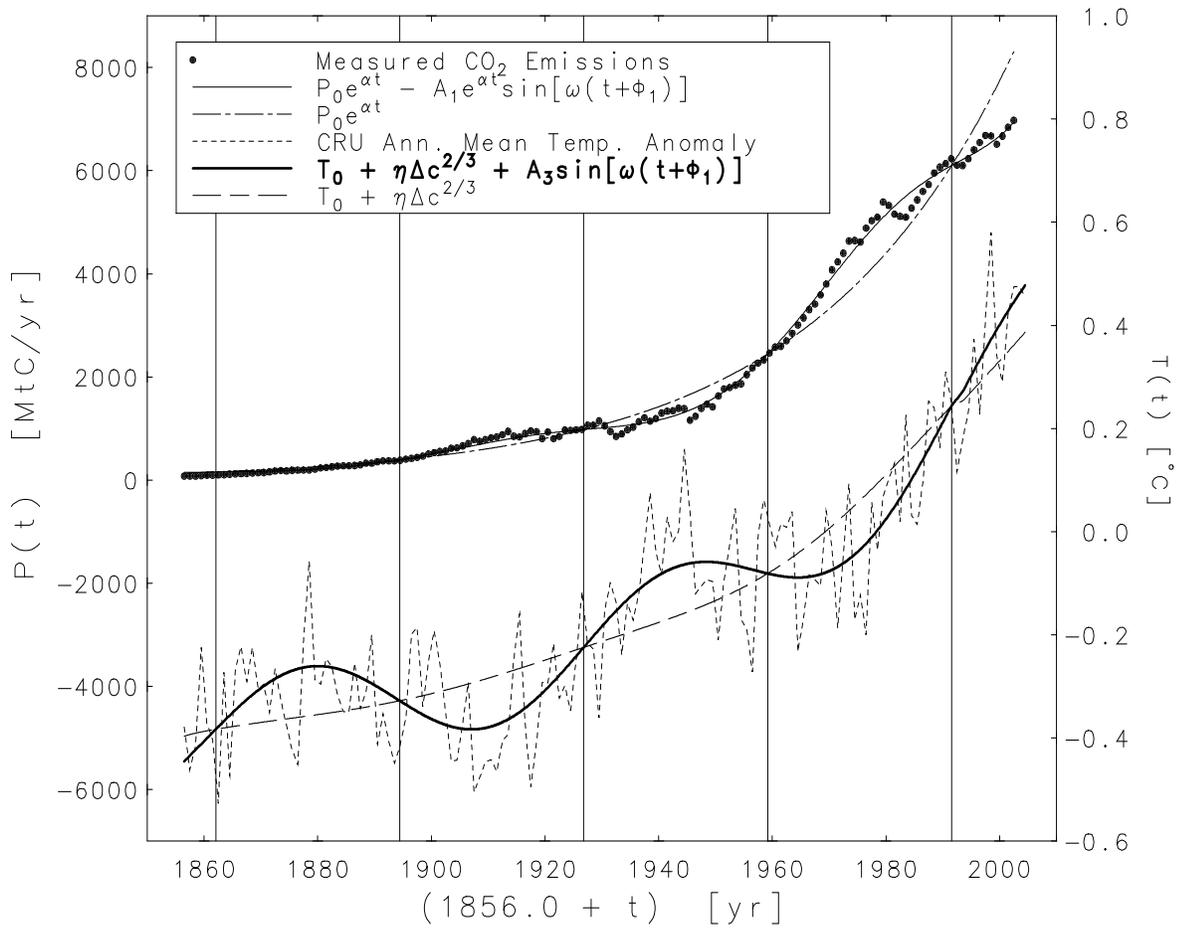
$$c(t) = c_0 + \gamma \int_0^t P(t') dt' + \delta S(t)$$

$$T(t) = T_0 + \eta \left[ \gamma \int_0^t P(t') dt' + \delta S(t) \right]^{2/3} \\ + A_3 \sin \left[ \frac{2\pi}{\tau} (t + \phi_1) \right]$$

$$P(t) = P_0 e^{\alpha t} - A_1 e^{\alpha t} \sin \left[ \frac{2\pi}{\tau} (t + \phi_1) \right]$$

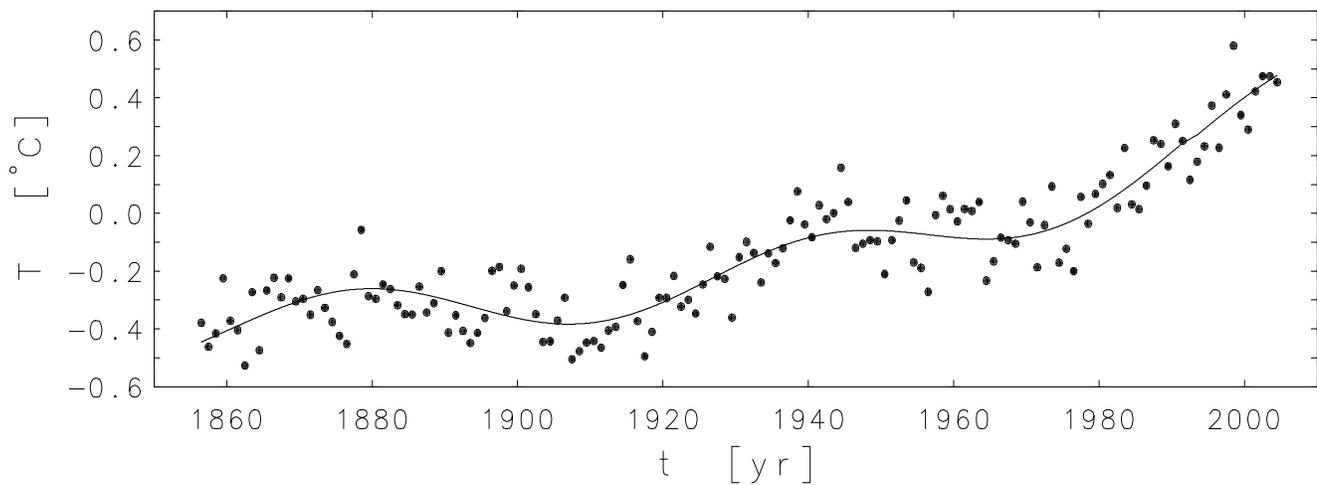
$$T(t) = T_0 + \eta \left[ \gamma \int_0^t P(t') dt' + \delta S(t) \right]^{2/3} + A_3 \sin \left[ \frac{2\pi}{\tau} (t + \phi_1) \right]$$

Fossil Fuel Emiss. and Gl. Temp. Anomalies

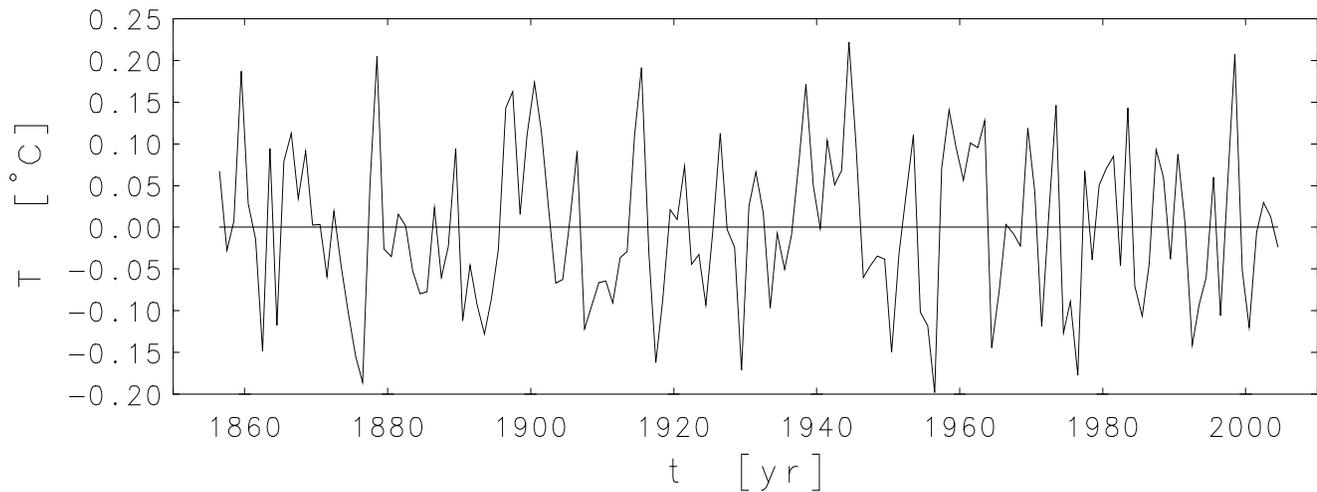


$$T(t) = T_0 + \eta \left[ \gamma \int_0^t P(t') dt' + \delta S(t) \right]^{2/3} + A_3 \sin \left[ \frac{2\pi}{\tau} (t + \phi_1) \right]$$

Ann. Global Av. Temp. Anomalies (1856–2004)

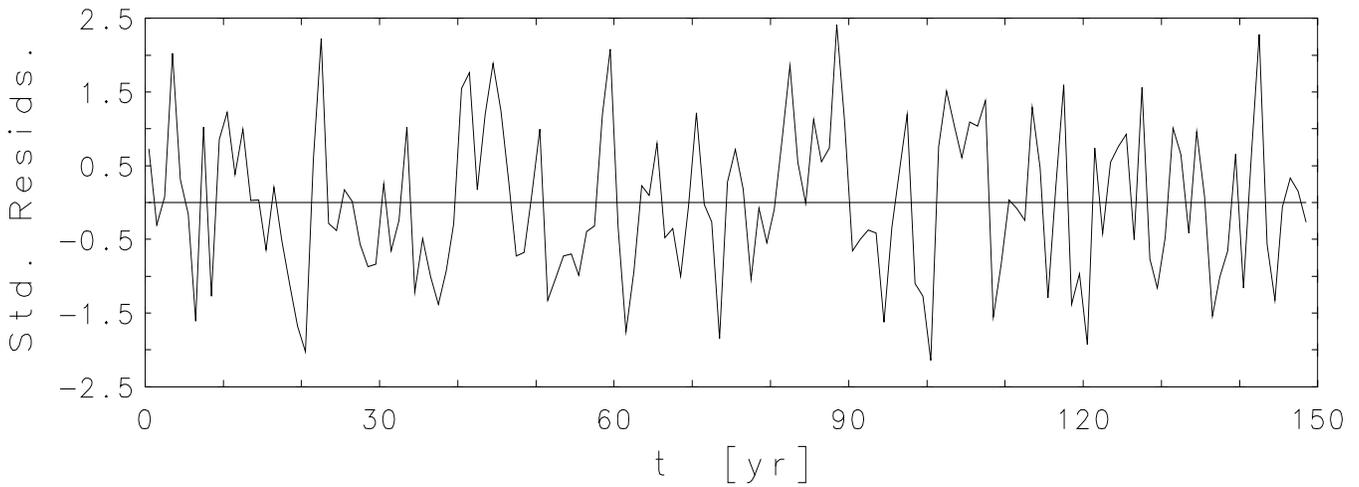


Residuals (data - model)

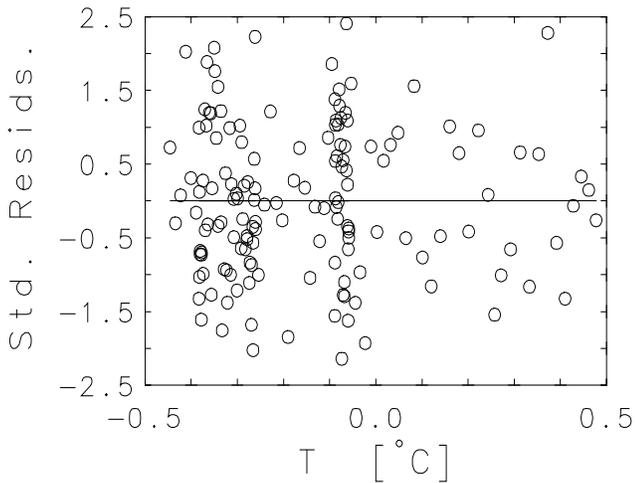


$$T(t) = T_0 + \eta \left[ \gamma \int_0^t P(t') dt' + \delta S(t) \right]^{2/3} + A_3 \sin \left[ \frac{2\pi}{\tau} (t + \phi_1) \right]$$

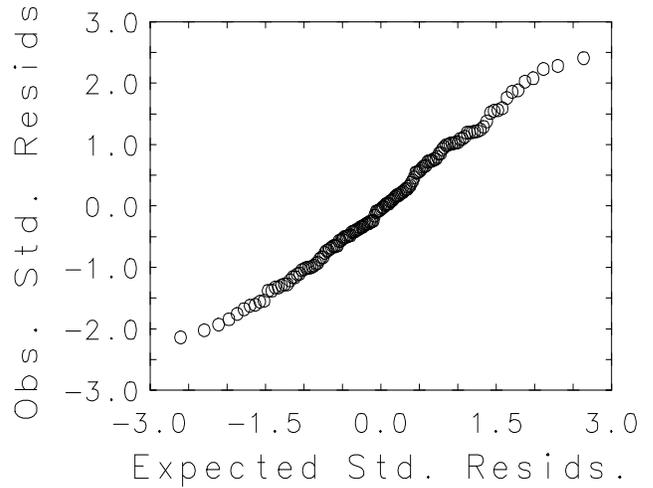
Std Resids vs Ind Var



Std Resids vs Dep Var

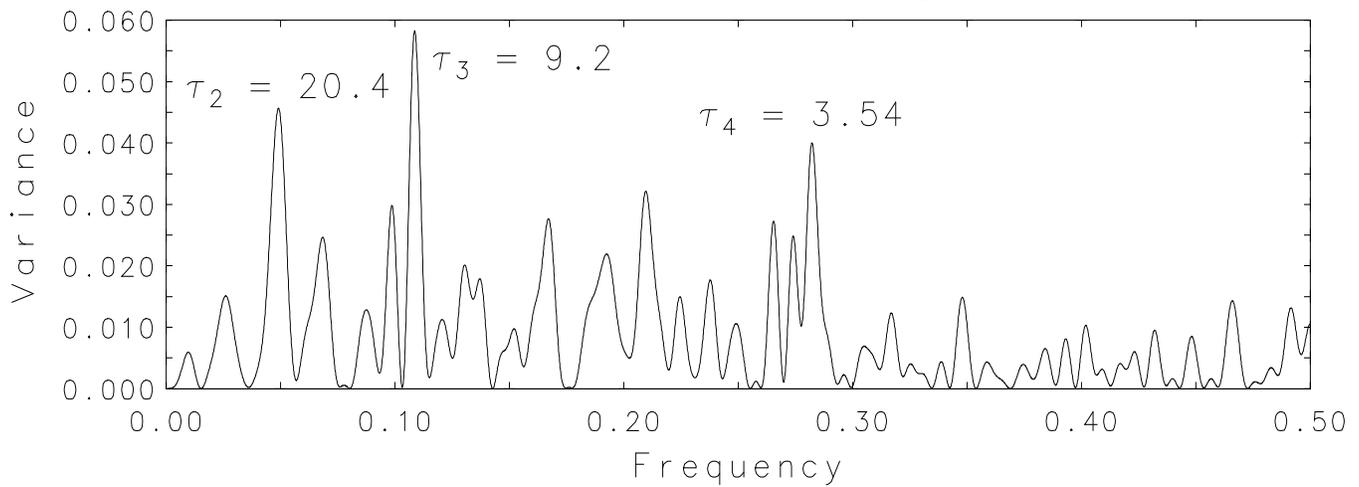


Normal Prob. Plot

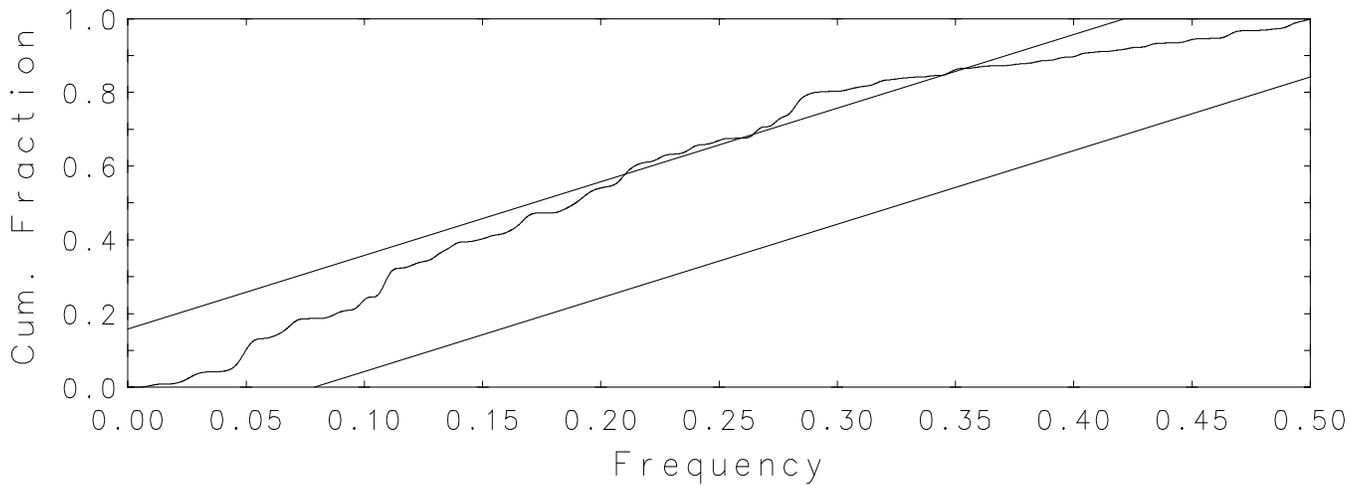


$$T(t) = T_0 + \eta \left[ \gamma \int_0^t P(t') dt' + \delta S(t) \right]^{2/3} + A_3 \sin \left[ \frac{2\pi}{\tau} (t + \phi_1) \right]$$

Residual Periodogram

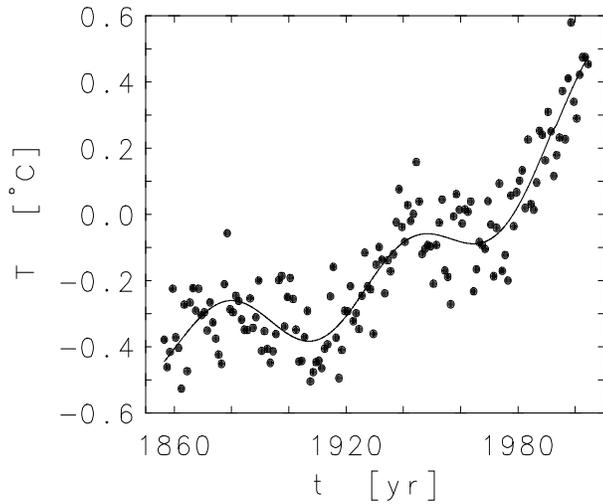


Cumulative Periodogram

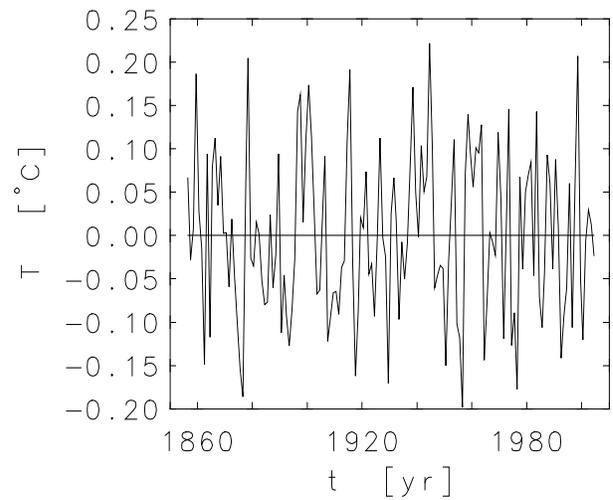


$$T(t) = T_0 + \eta \left[ \gamma \int_0^t P(t') dt' + \delta S(t) \right]^{2/3} + A_3 \sin \left[ \frac{2\pi}{\tau} (t + \phi_1) \right]$$

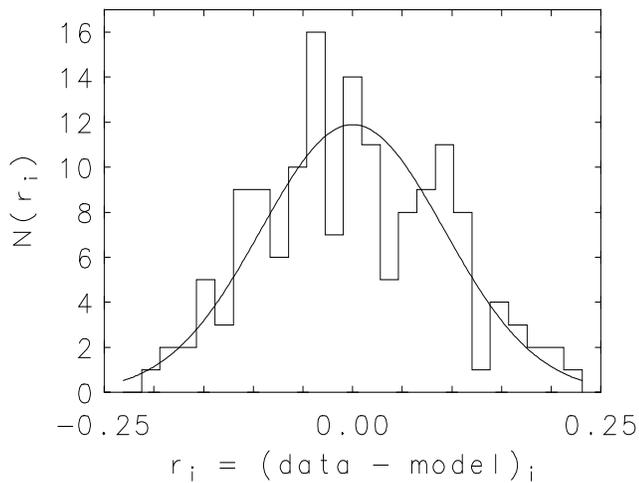
Ann. Global Av. Temp. Anomalies (1856–2004)



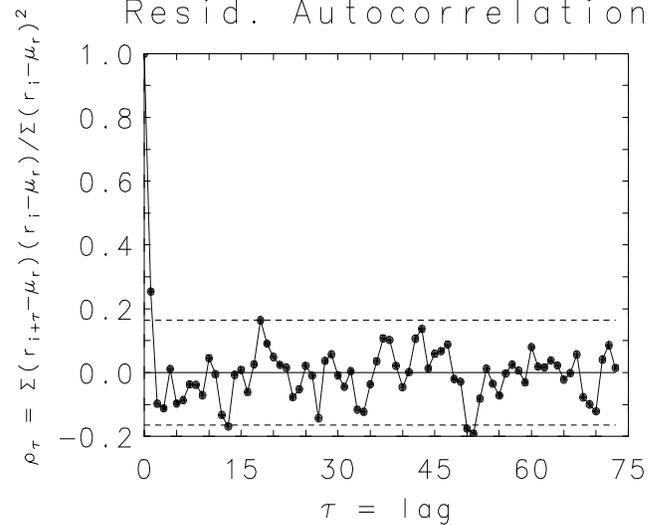
Residuals (data - model)



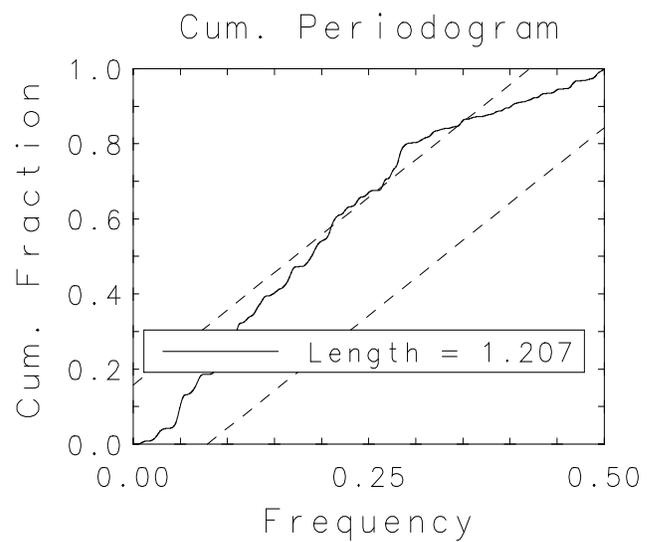
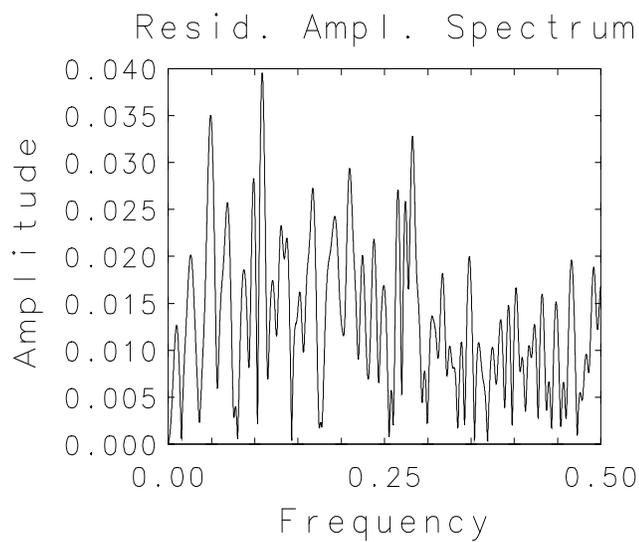
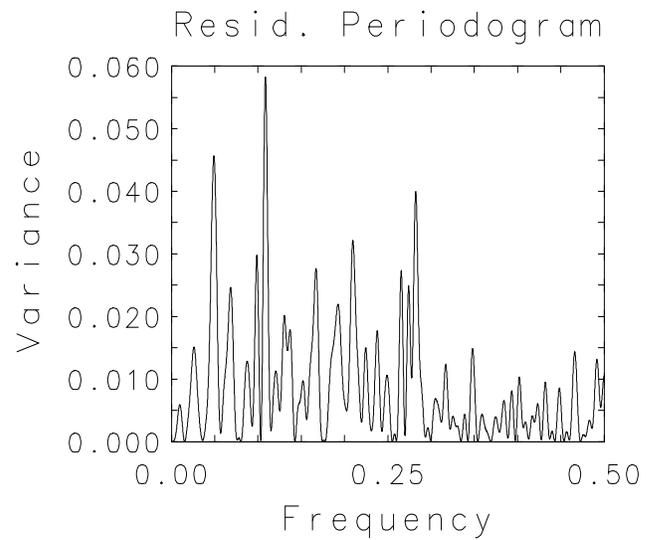
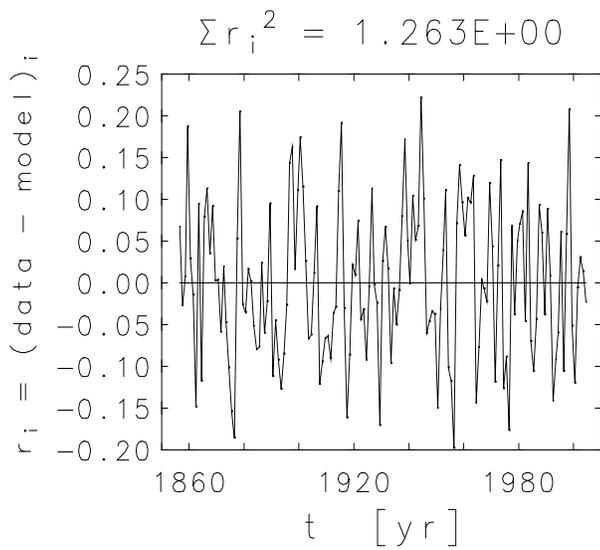
$$\Pr\{\chi^2 > 15.1229\} = 0.515655$$



Resid. Autocorrelation

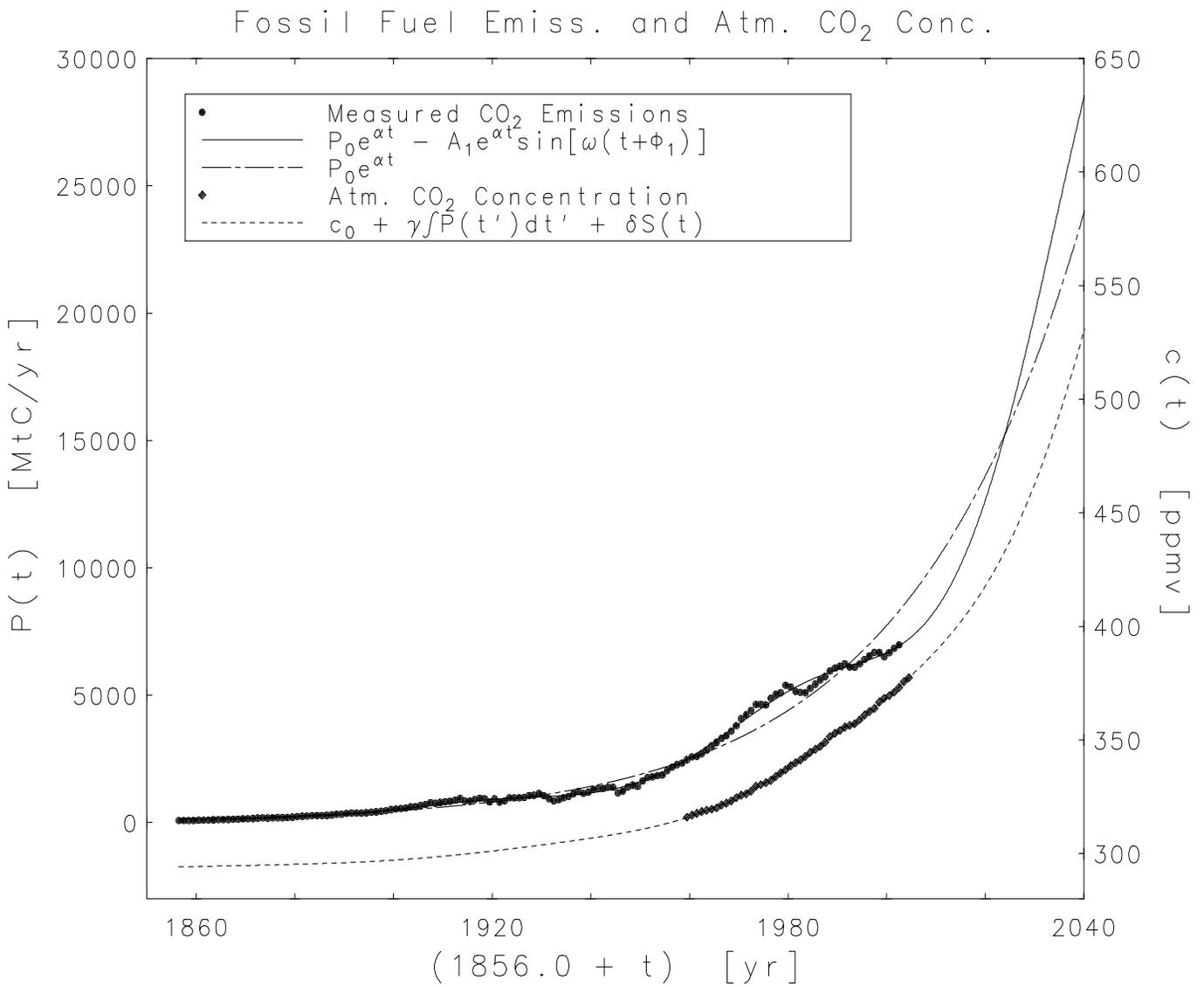


$$T(t) = T_0 + \eta \left[ \gamma \int_0^t P(t') dt' + \delta S(t) \right]^{2/3} + A_3 \sin \left[ \frac{2\pi}{\tau} (t + \phi_1) \right]$$

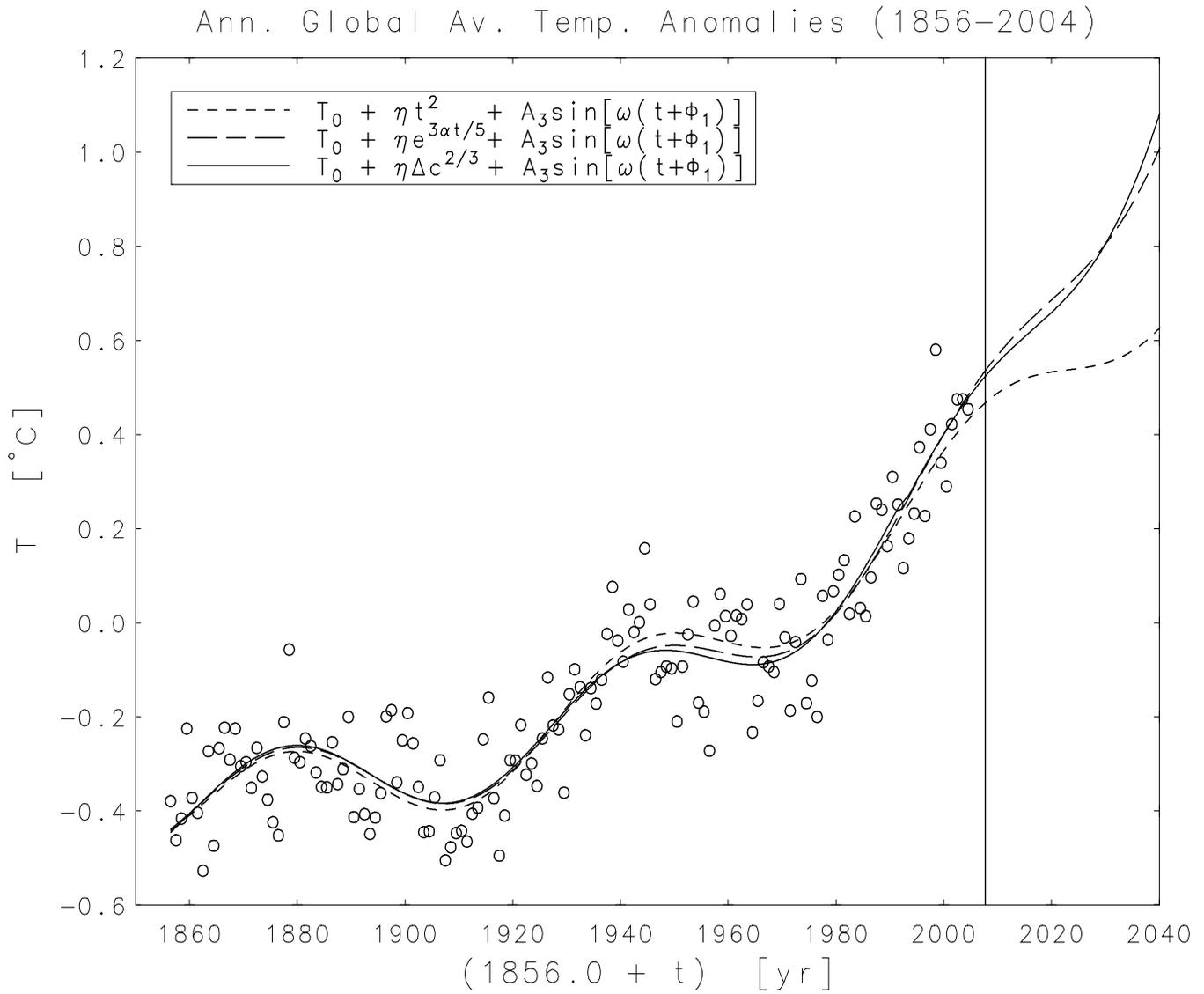


## Extrapolating to epoch 2100.0 yields

$$P(2100) \approx 140,000 \text{ [MtC/yr]} \approx 20 \times P(2002)$$



# The next “cooling” period is September 2007 – March 2040



Kerry Emanuel, *Nature*, Vol. 436 (4 August 2005) pp. 686-687

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**nature**

Vol 436/4 August 2005 doi:10.1038/nature03906

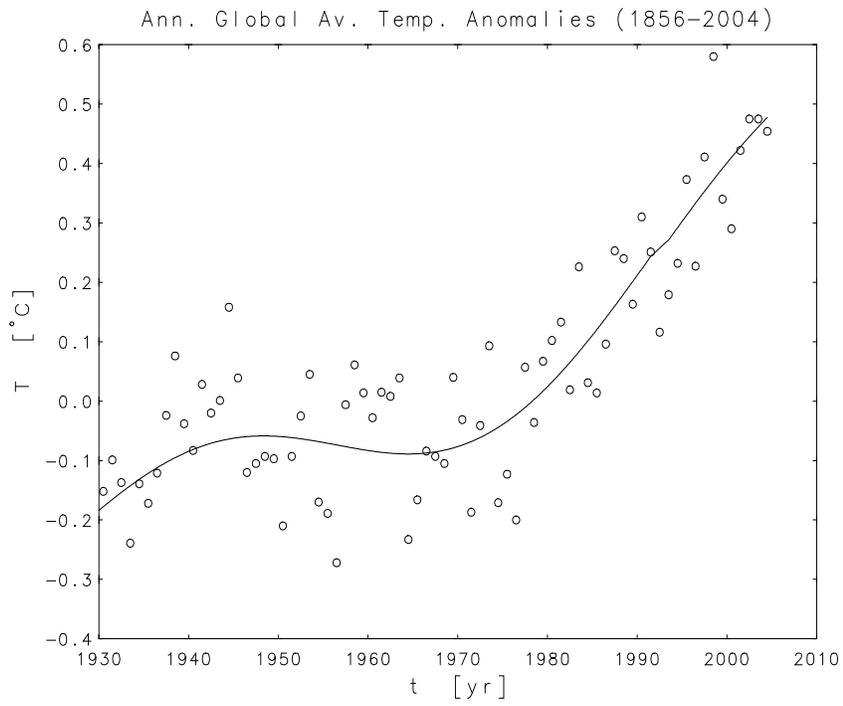
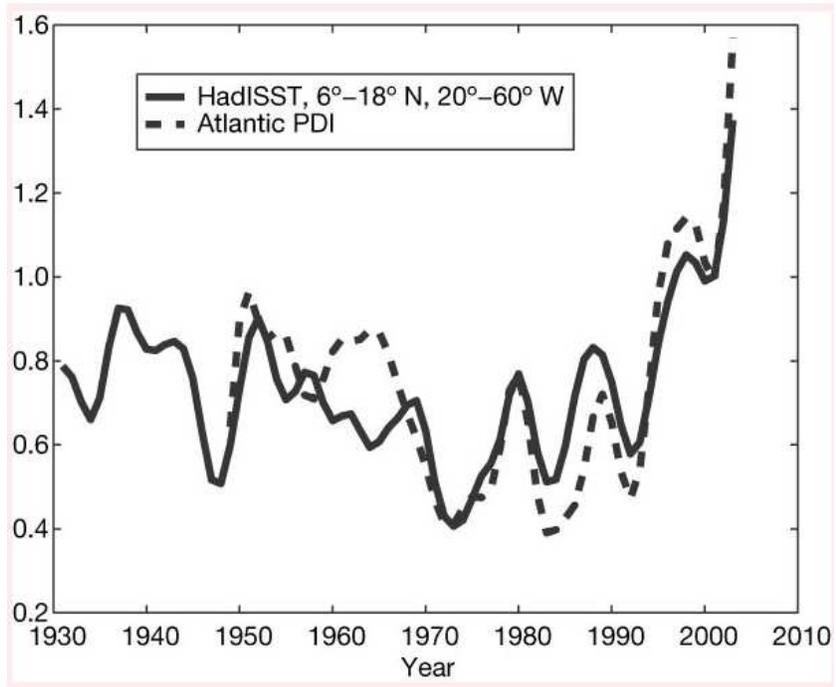
## LETTERS

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### **Increasing destructiveness of tropical cyclones over the past 30 years**

Kerry Emanuel<sup>1</sup>

“Here I define an index of the potential destructiveness of hurricanes based on the total dissipation of power, integrated over the lifetime of the cyclone, and show that this index has increased markedly since the mid-1970s. I find that the record of net hurricane power dissipation is highly correlated with tropical sea surface temperature, reflecting well-documented climate signals, including multi-decadal oscillations in the North Atlantic and North Pacific, and global warming.”



$$\frac{dP}{dt} = \left( \alpha - \beta \frac{dT}{dt} \right) P , \quad P(0) = P_0$$

$$c(t) = c_0 + \gamma \int_0^t P(t') dt'$$

$$T(t) = T_0 + \eta [c(t) - c_0] + A \sin \left[ \frac{2\pi}{\tau} (t + \phi) \right]$$


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$$\frac{dP}{dt} = \left( \alpha - \beta \frac{dT}{dt} \right) P , \quad P(0) = P_0$$

$$\frac{dc}{dt} = \gamma P(t) , \quad c(0) = c_0$$

$$\frac{dT}{dt} = \eta \frac{dc}{dt} + \frac{2\pi A}{\tau} \cos \left[ \frac{2\pi}{\tau} (t + \phi) \right] , \quad T(0) = T_0$$


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$$\frac{dP}{dt} = \alpha P - \beta \left\{ \eta' P + A' \cos \left[ \frac{2\pi}{\tau} (t + \phi) \right] \right\} P , \quad P(0) = P_0$$

$$\frac{dc}{dt} = \gamma P , \quad c(0) = c_0$$

$$\frac{dT}{dt} = \eta' P + A' \cos \left[ \frac{2\pi}{\tau} (t + \phi) \right] , \quad T(0) = T_0$$

$$\eta' \equiv \gamma \eta , \quad A' \equiv \frac{2\pi A}{\tau}$$

