A Nash-Cournot Equilibrium Model for the North American Natural Gas Sector*

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Outline of Presentation

- My Background
- North American Market Background
- Equilibrium Model
- Numerical Results
- Future Work and References
Overview of Research

- **Research: Main Topics**
  - Mathematical modeling in engineering-economic systems using optimization and equilibrium analysis usually involving some infrastructural elements
    - Models of energy markets and risk (natural gas and electricity)
    - Transportation/traffic flow
    - “Smart Growth” land development
    - Wastewater treatment
    - Wireless telecommunications networks
  - Development of algorithms for solving equilibria in energy & transportation systems
  - Development of general purpose algorithms for equilibrium models (using the nonlinear complementarity format)
  - Operations research areas: Multiobjective optimization, nonlinear programming, complementarity theory, statistics, integer programming
From well-head to burner-tip

My Background
North American Market
Equilibrium Model
Numerical Results
Future Work
- Seasonality in demand
- Pipeline, storage, peak gas
- Pipelines
- 110 Interstate Pipelines, (51 classified as majors) with 190,000+ miles of Transmission Lines
Key events (US) (Chambers, Sturm)

Before 1985

- regulated interstate gas pipelines provided a bundled service that included
  - transportation
  - transportation-related services (e.g., storage)
  - the natural gas itself
- Customers paid the cost of gas based on long-term contracts between the pipelines and unaffiliated gas producers
- Customers paid on a “pass-through” basis, i.e., no return on the commodity allowed for the pipelines (unlike electric power)
- Thus, pipelines made no profit on the purchase and sale of gas
Key events (US)
  - Unbundling of services by interstate pipelines
  - Natural gas buyer can choose to buy gas from a supplier at one location, transport it along a pipeline a short distance (lower transportation rate), and receive the volumes
  - Promoting wholesale competition through open access, non-discriminatory transmission services by public utilities
  - Recovery of stranded costs by public utilities and transmitting utilities
  - Standards of conduct developed for pipelines and marketer affiliates
Key events (US)

This new marketplace may permit certain abuses of market power

- Interstate pipelines have a natural monopoly but highly regulated by FERC
- Production is more or less a perfectly competitive market due to the large volume of producers
- Marketer/shippers are unregulated by FERC maybe they have some market power?

Why straightforward system optimization will not work

Need for a game-theoretic format (e.g., Nash-Cournot) for some players
Want to account for imperfect competition (e.g., oligopoly) to measure the influence of market power along the natural gas supply chain.
Develop short term model to characterize the new natural gas industry (no new capacity)

- **Pipeline companies**
  - Maximize net revenues: regulated rate revenues + congestion revenues subject to capacity bounds

- **Production companies**
  - Maximize net profits subject to drilling restrictions
  - Perfect competition in the production market (reasonable for North America), price-takers

- **Storage reservoir operators**
  - Maximize net profits subject to extraction, injection, and volumetric restrictions
  - Injection and extraction in different seasons
  - Storage reservoir operators use “seasonal arbitrage”
  - Perfect competition in the storage market, price takers for production and transportation
– **Marketers/shippers**
  - Maximize net profits
  - Nash-Cournot players in the “marketer market”, thus marketers can exert market power via inverse demand functions
  - Price-takers in the storage, production, peak gas, and transportation markets

– **Peak gas suppliers**
  - Maximize net profits subject to peak supply capacity restrictions
  - Perfect competition in the peak supply market
  - Peak supply only in the high demand season, substitute for storage and pipeline gas

– **Consumers**
  – Residential, commercial, industrial, electric power sectors
  – Inverse demand functions as part of the marketer problems
- Market clearing
  - Total supply = total demand in various markets
- Use multiple seasons
  - Season 1 (low demand), April-October, \( \text{days}_1 = 214 \)
  - Season 2 (high demand), November-March, excluding January, \( \text{days}_2 = 120 \)
  - Season 3, (very high or peak demand), e.g., January, \( \text{days}_3 = 31 \)
My Background

North American Market

Equilibrium Model

Numerical Results

Future Work

Sample Network
1 pipeline link

P=producer node
S=storage node
M=marketer node
RD,CD,ID,ED= demand nodes
PK=peak supply node

Consumption region
Sample Network

Low Demand Season

P=producer node
S=storage node
M=marketer node
RD,CD,ID,ED= demand nodes
PK=peak supply node

Consumption region

My Background
North American Market
Equilibrium Model
Numerical Results
Future Work
Sample Network

High Demand Season

P = producer node
S = storage node
M = marketer node
RD, CD, ID, ED = demand nodes
PK = peak supply node

My Background
North American Market
Equilibrium Model
Numerical Results
Future Work

Consumption region
Sample Network

Very High

Demand Season

- P = producer node
- S = storage node
- M = marketer node
- RD, CD, ID, ED = demand nodes
- PK = peak supply node

My Background

North American Market

Equilibrium Model

Numerical Results

Future Work
Pipeline Aggregation 1: Regional Aggregation

Region 7

TX → LA: 320
MS → AL: 1756

Region 6

Region 10

FL: 1969

7 → 6: 1300
6 → 10: 1969

Pipeline Aggregation 1: Regional Aggregation

My Background

North American Market

Equilibrium Model

Numerical Results

Future Work

TX

320

LA

1300

MS

1756

AL

1969

FL

7

1300

6

1969

10
Pipeline Aggregation 2: Pipeline Combination

pipelines 1 (Solid)

pipelines 2 (dashed)

Aggregated pipelines
Pipeline Aggregation 3: Link Capacity Constraints

<table>
<thead>
<tr>
<th>OD Pair</th>
<th>Path Flows</th>
<th>Pipelines Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (1,2)</td>
<td>1. 1-&gt;2</td>
<td>both</td>
</tr>
<tr>
<td>2. (1,3)</td>
<td>2. 1-&gt;2-&gt;3</td>
<td>both</td>
</tr>
<tr>
<td>3. (1,4)</td>
<td>3. 1-&gt;2-&gt;4</td>
<td>1-&gt;2 both, 2-&gt;4 solid</td>
</tr>
<tr>
<td>4. (1,3)</td>
<td>4. 1-&gt;2-&gt;3-&gt;4</td>
<td>1-&gt;2 both, 2-&gt;3 both, 3-&gt;4 solid</td>
</tr>
<tr>
<td>5. (2,3)</td>
<td>5. 2-&gt;3</td>
<td>2-&gt;3 both</td>
</tr>
<tr>
<td>6. (2,4)</td>
<td>6. 2-&gt;4</td>
<td>2-&gt;4 solid</td>
</tr>
<tr>
<td>7. (2,3)</td>
<td>7. 2-&gt;3-&gt;4</td>
<td>2-&gt;3 both, 3-&gt;4 solid</td>
</tr>
<tr>
<td>8. (3,4)</td>
<td>8. 3-&gt;4</td>
<td>3-&gt;4 solid</td>
</tr>
</tbody>
</table>

Denoting $f_i$ as the path flow for $i = 1,..,8$, the following constraints are needed:

\[
\begin{align*}
\left( f_1 + f_2 + f_3 + f_4 \right) & \leq 16 \text{ arc } 1,2 \\
\left( f_2 + f_4 + f_5 + f_7 \right) & \leq 17 \text{ arc } 2,3 \\
\left( f_3 + f_6 \right) & \leq 20 \text{ arc } 2,4 \\
\left( f_4 + f_7 + f_8 \right) & \leq 2 \text{ arc } 3,4
\end{align*}
\]
Pipeline Aggregation 4: Additional Restriction

- Assume that gas cannot go back and forth between the two different pipelines, the flow on paths would have to stay on the same pipeline.
- Additional constraints are needed to capture pipeline specific information.
Pipeline Aggregation 5: Additional Restriction 1

- $f_2$: flow 1->2->3 use both pipelines
- For pipeline 1: $f_{2-1} \leq \min(10, 8)$
- For pipeline 2: $f_{2-2} \leq \min(6, 9)$
- Hence, $f_2 \leq 14$ instead of $f_2 \leq 16$
Pipeline Aggregation 5: Additional Restriction 2

- $f_3$ (flow 1->2->4) and $f_4$ (flow 1->2->3->4) would stay in pipeline 1 and use the arc (1,2) of pipeline 1 in common
- So $f_3 + f_4 \leq 10$ would be used to enforce this condition
Pipeline Operator’s Problem (Linear Program)

- Maximize congestion revenues
  s.t.
  - bounds on capacity
  - post-processor for regulated revenues
  - Other constraints that are pipeline-specific (not shown here)

\[
\text{Max } \sum_{y \in Y} \sum_{s=1}^{3} \text{days}_s \tau_{asy} f_{asy}
\]

s.t.
\[
f_{asy} \leq \bar{f}_a \left( \rho_{asy} \right) \quad \forall s, y
\]
\[
0 \leq f_{asy} \quad \forall s, y
\]
Pipeline Operator’s Problem (Linear Program)

- KKT conditions are both necessary and sufficient for optimality
- These conditions are

\[ 0 \leq -\text{days}_s \tau_{asy} + \rho_{asy} \perp f_{asy} \geq 0 \quad \forall s, y \]

\[ 0 \leq \bar{f}_a - f_{asy} \perp \rho_{asy} \geq 0 \quad \forall s, y \]
Producer’s Problem (Convex Program)

- Maximize production revenues less production costs
  s.t.
  - bounds on production rates
  - bounds on volume of gas produced

\[
\begin{align*}
\max & \quad \sum_{y \in Y} \sum_{s=1}^{3} \text{days}_s \pi_{n_{sy}} q_{csy} - \text{days}_s c_{pr}^s (q_{csy}) \\
\text{s.t.} & \quad q_{csy} \leq \bar{q}_c \\
& \quad \left(\lambda_{csy}\right) \quad \forall s, y \\
& \quad \sum_{y \in Y} \sum_{s=1}^{3} \text{days}_s q_{csy} \leq \prod_c \\
& \quad \left(\mu_c\right) \\
& \quad 0 \leq q_{csy} \quad \forall s, y
\end{align*}
\]
Producer’s Problem  
(Convex Program)

- If cost function is convex, KKT conditions are both necessary and sufficient for optimality
- Necessity since polyhedral constraints
- These conditions are

\[ 0 \leq -\text{days}_s \pi_{nsy} + \text{days}_s c^\text{pr}_c (q_{c_{sy}})' + \lambda_{c_{sy}} + \text{days}_s \mu_c \perp q_{c_{sy}} \geq 0 \quad \forall s, y \]
\[ 0 \leq q_c - q_{c_{sy}} \perp \lambda_{c_{sy}} \geq 0 \quad \forall s, y \]
\[ 0 \leq prod_c - \sum_{y \in Y} \sum_{s=1}^{3} \text{days}_s q_{c_{sy}} \perp \mu_c \geq 0 \]
Storage Reservoir Operator’s Problem
(Convex Program)

- Maximize net revenues from marketers less injection, long-distance transportation and congestion costs
  s.t.
  - volumetric bound on working gas
  - maximum extraction rate bound
  - maximum injection rate bound
  - annual injection-extraction balancing
  - nonnegativity of injection and extraction
Storage Reservoir Operator’s Problem (Convex Program)

- If cost function is convex, KKT conditions are both necessary and sufficient for optimality
- Necessity since polyhedral constraints

Max

\[
\sum_{y \in T} \left[ \text{days}_2 \gamma_{n2y} x_{r2y} + \text{days}_3 \gamma_{n3y} x_{r3y} - \text{days}_1 c^r \left( \sum_{a \in A(n)} g_{ary} \right) - \sum_{a \in A(n)} \text{days}_1 \left( \tau_{aly} + \tau_{aly}^{\text{reg}} + \pi_{n2(a)y} \right) g_{ary} \right]
\]

s.t.

\[
days_2 x_{r2y} + days_3 x_{r3y} - \sum_{a \in A(n)} g_{ary} \left( 1 - \text{loss}_a \right) \left( 1 - \text{loss}_r \right) = 0 \quad (\delta_{ry}) \quad \forall y
\]

\[
x_{ary} \leq \bar{x}_r \quad (\omega_{sry}) \quad s = 2, 3, \forall y
\]

\[
\sum_{a \in A(n)} g_{ary} \leq \bar{g}_r \quad (\xi_{ry}) \quad \forall y
\]

\[
\sum_{s=2,3} \text{days}_s x_{ary} \leq \bar{k}_r \quad (\zeta_{ry}) \quad \forall y
\]

\[
0 \leq g_{ary} \forall a \in A(n), x_{r2y}, x_{r3y} \quad \forall y
\]
Marketer/Shipper’s Problem (Convex Program)

- Maximize net demand sector revenues less, local delivered costs from storage and peak supply, long-distance cost from producers, congestion costs (inverse demand equations by sectors used)

s.t.
- pipeline gas consistency
- storage gas consistency
- nonnegativity of gas supplies (pipeline, storage, peak)
Marketer/Shipper’s Problem (Convex Program)

- If revenue functions concave, KKT conditions are both necessary and sufficient for optimality
- Necessity since polyhedral constraints

\[
\begin{align*}
\text{Max} & \sum_{y=1}^{3} \sum_{k \in K} \left[ \text{days}_y \theta_{n_1y}^k \left( h_{m_1y}^k + h_{m_1y}^{k*} \right) + \text{days}_y \theta_{n_2y}^k \left( h_{m_2y}^k + h_{m_2y}^{k*} + u_{m_2y}^k + u_{m_2y}^{k*} \right) + \text{days}_y \theta_{n_3y}^k \left( h_{m_3y}^k + h_{m_3y}^{k*} + u_{m_3y}^k + u_{m_3y}^{k*} + \gamma_{n_3y}^k + \gamma_{n_3y}^{k*} \right) \right] \\
& \quad - \sum_{y=1}^{3} \left[ \sum_{a=1}^{n} \sum_{a(n)} \text{days}_s \left( r_{a, y} + \gamma_{a, y} + \pi_{n_2(a)y} \right) h_{a, msy}^k \right] + \text{days}_s \gamma_{n_2y} u_{m_2y} + \text{days}_s \gamma_{n_3y} u_{m_3y} + \text{days}_s \beta_{my} v_{my} \right]
\end{align*}
\]

s.t.

\[
\begin{align*}
\sum_{k \in K} \text{days}_s h_{msy}^k &= \sum_{a \in A(n)} \text{days}_s \left( 1 - \text{loss}_a \right) h_{amsy}^k \quad (\alpha_{msy}) \quad \forall s, y \\
\sum_{k \in K} \text{days}_s u_{msy}^k &= \text{days}_s u_{msy} = 0 \quad (\phi_{msy}) \quad \forall s = 2, 3, \forall y \\
\sum_{k \in K} \text{days}_s v_{my}^k &= \text{days}_s v_{my} = 0 \quad (\phi_{my}) \quad \forall y \\
0 &\leq h_{msy}^k \quad \forall k, s, y \\
0 &\leq h_{amsy}^k \quad \forall a \in A(n), s, y \\
0 &\leq u_{msy}^k \quad \forall k, s = 2, 3, y \\
0 &\leq u_{msy} \quad \forall s = 2, 3, y \\
0 &\leq v_{my}^k, 0 \leq v_{my} \quad \forall y
\end{align*}
\]
Peak Gas Operator’s Problem
(Convex Program)

- Maximize net revenues from marketers less peak gas costs
  s.t.
  - maximum peak gas supply upper bound
  - nonnegative peak gas supply and deliveries
Peak Gas Operator’s Problem
(Convex Program)

\[
\text{Max } \sum_{y \in Y} \text{days}_3 \left( \beta_{ny} w_{py} - c_p^{pg} \left( w_{py} \right) \right)
\]

s.t.
\[
w_{py} \leq \overline{w}_p \left( \sigma_{py} \right) \quad \forall y
\]
\[
0 \leq w_{py} \quad \forall y
\]
Applying Karush-Kuhn-Tucker Optimality conditions for the optimization problems faced by the
- Pipeline operators, producers, storage operators, marketers, peak gas suppliers
- Market clearing conditions
- Existence & uniqueness results for mixed NCP version as well as model formulation discussion
  - For numerical study, convex, quadratic cost functions + Linear demand equations → mixed linear complementarity problem
North American Numerical Study

- Total 14 Regions
- US portion based on US DOE natural gas regions

<table>
<thead>
<tr>
<th>New England</th>
<th>Mid Atlantic</th>
<th>E. North Central</th>
<th>W. North Central</th>
<th>South Atlantic</th>
<th>E. South Central</th>
<th>W. South Central</th>
<th>Mountain</th>
<th>Pacific</th>
<th>Florida</th>
<th>AZ/NM</th>
<th>CA</th>
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</tbody>
</table>

3535

My Background

North American Market

Equilibrium Model

Numerical Results

Future Work
National Petroleum Council (NPC) Study
http://www.npc.org/

- Investigations of the ongoing and future operations
- Requirements of the U.S. oil and gas industries
- Statistical studies descriptive of these industries
- Delineations of the U.S. oil and gas resource base
- Comprehensive analyses of the domestic energy
- Supply/Demand Situation
- Examine other evolving market conditions that may affect the potential for natural gas demand, supplies and delivery through 2025
- The current policy direction - unaltered - will likely lead to difficult conditions in the natural gas market, but industries, government, and consumers will react
- Therefore, this study assumes action beyond the status quo
## NPC Study

**Two Paths Beyond Status Quo**

<table>
<thead>
<tr>
<th>1. Reactive Path Scenario</th>
<th>2. Balanced Future Scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public Policies Remain in Conflict, Encouraging Consumption while Inhibiting Supply … Resulting in Higher Prices and Volatility.</td>
<td>Public policies aligned: alternate fuels and new natural gas supply sources compete to ensure lowest consumer cost.</td>
</tr>
</tbody>
</table>
NPC Study Potential Price Ranges

Sources: NPC 2003
### NPC Study Interpretation

#### Summary of Demand and Supply Annual Percentage Changes for Each Case

<table>
<thead>
<tr>
<th></th>
<th>Demand Sectors Growth or Decline</th>
<th>Supply Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Res.</strong></td>
<td><strong>Com.</strong></td>
</tr>
<tr>
<td><strong>Reactive Path</strong></td>
<td>0.75%</td>
<td>0.81%</td>
</tr>
<tr>
<td><strong>Balanced Future</strong></td>
<td>0.51%</td>
<td>0.89%</td>
</tr>
</tbody>
</table>

*Demand percentage changes are actual figures from the NPC study, supply values are estimated based on graphs.*

- Base Cases, 2002: Nash-Cournot and Perfectly Competitive Marketers
- Balanced Future Cases, 2008: Nash-Cournot & Perfectly Competitive Marketers
- Reactive Path Cases, 2008: Nash-Cournot & Perfectly Competitive Marketers
Market Participants by Region

<table>
<thead>
<tr>
<th>Region</th>
<th>Production</th>
<th>Storage</th>
<th>Marketers</th>
<th>Peak Gas</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. New England</td>
<td>No</td>
<td>No</td>
<td>2</td>
<td>Yes</td>
</tr>
<tr>
<td>2. Mid Atlantic</td>
<td>Yes</td>
<td>Yes</td>
<td>2</td>
<td>Yes</td>
</tr>
<tr>
<td>3. East North Central</td>
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<tr>
<td>4. West North Central</td>
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<tr>
<td>5. South Atlantic</td>
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<tr>
<td>6. East South Central</td>
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<td>7. West South Central</td>
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<td>8. Mountain</td>
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<td>9. Pacific</td>
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<td>10. Florida</td>
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<td>11. Arizona/New Mexico</td>
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<td>12. California</td>
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<td>13. Eastern Canada</td>
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<td>14. Western Canada</td>
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</table>
Model Calibration Accuracy

- Investigations of the calibration dataset used
  - Gas Demand Quantity for all 4 sectors
  - Gas Price (Production, City Gate and End User)
  - Capacity (Pipeline, Production, Storage, and Peak Gas)
  - Transportation Costs
  - Sources of Calibration Information Used (Yr. 2002):
    - Energy Information Administration (EIA) of the U.S. Department of Energy (DOE)
    - Natural Resources Canada (NRCAN)
### Calibration Accuracy for Base Case 2002*

#### Calibration Price Accuracy Table:

<table>
<thead>
<tr>
<th>Region</th>
<th>Production</th>
<th>City Gate</th>
<th>Res. Demand</th>
<th>Comm. Demand</th>
<th>Ind. Demand</th>
<th>Power Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td>7.49 %</td>
<td>1.50 %</td>
<td>0.10 %</td>
<td>0.62 %</td>
<td>0.44 %</td>
<td>2.47 %</td>
</tr>
<tr>
<td>Canada</td>
<td>10.49 %</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

#### Calibration Quantity Accuracy Table:

<table>
<thead>
<tr>
<th>Region</th>
<th>Production</th>
<th>Res. Demand</th>
<th>Comm. Demand</th>
<th>Ind. Demand</th>
<th>Power Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td>4.66 %</td>
<td>-0.55 %</td>
<td>-0.62 %</td>
<td>-0.90 %</td>
<td>-0.63 %</td>
</tr>
<tr>
<td>Canada</td>
<td>-2.22 %</td>
<td>0.72 %</td>
<td>0.22%</td>
<td>0.14 %</td>
<td>0.69 %</td>
</tr>
</tbody>
</table>

* Calibration Accuracy Based on Comparison Between Base Case and EIA & NRCAN Data
## Supply and Demand Functions Used

<table>
<thead>
<tr>
<th>Function</th>
<th>Function Forms</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Producer Costs</td>
<td>Quadratic</td>
<td>( \alpha_0 + \alpha_1 x + \frac{1}{2} \alpha_2 x^2 )</td>
</tr>
<tr>
<td>(Producer ( c ) for season ( s ) and year ( y ))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Storage Operator Costs</td>
<td>Quadratic</td>
<td>( \beta_0 + \beta_1 x + \frac{1}{2} \beta_2 x^2 )</td>
</tr>
<tr>
<td>(Operator ( r ) for season ( s ) and year ( y ))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak Gas Operator Costs</td>
<td>Quadratic</td>
<td>( \gamma_0 + \gamma_1 x + \frac{1}{2} \gamma_2 x^2 )</td>
</tr>
<tr>
<td>(Operator ( p ) for season ( s ) and year ( y ))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inverse Demand</td>
<td>Linear</td>
<td>( A - B \theta )</td>
</tr>
<tr>
<td>(Sector ( k ) for season ( s ) and year ( y ))</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Computational Statistics

- Computational and Modeling Aspects
  - LCP with 4298 variables all together
  - Solver: GAMS/PATH
  - Computer: 2.80 GHz Intel® Pentium® 4 Processor and 1.0GB of memory
  - Typical solution times for each case
  - About 25 seconds to read the input from an EXCEL file
  - 10 to 100 seconds for GAMS/PATH to solve the model depending on the parameter settings and cases solved
  - About 8 seconds to write the output to another EXCEL file
  - About 3-4 months to calibrate the Base Case!
## North American Market

### Equilibrium Model

1. **Producers**
   - **Wellhead Prices ($/Mcf)**
     - BC-NC: $3.49
     - BC-PC: $4.39
     - % diff.: -20.62%
   - **Production (MMcf)**
     - BC-NC: 21,449,980
     - BC-PC: 22,410,085
     - % diff.: -4.28%
   - **Profits (1000$)**
     - BC-NC: 40,999,255
     - BC-PC: 64,320,640
     - % diff.: -36.26%

2. **Storage Operators**
   - **Gas Prices ($/Mcf)**
     - BC-NC: $3.96
     - BC-PC: $5.08
     - % diff.: -22.05%
   - **Extraction (MMcf)**
     - BC-NC: 1,806,400
     - BC-PC: 2,854,332
     - % diff.: -36.71%
   - **Profits (1000$)**
     - BC-NC: 70,325
     - BC-PC: 159,069
     - % diff.: -55.79%

3. **Peak Gas Operators**
   - **Gas Prices ($/Mcf)**
     - BC-NC: $4.22
     - BC-PC: $5.20
     - % diff.: -18.85%
   - **Supply (MMcf)**
     - BC-NC: 241,644
     - BC-PC: 241,644
     - % diff.: 0.00%
   - **Profits (1000$)**
     - BC-NC: 673,754
     - BC-PC: 908,682
     - % diff.: -25.85%

4. **Marketers**
   - **Profits (1000$)**
     - BC-NC: 39,050,713
     - BC-PC: 0
     - % diff.: n/a

5. **End-user Prices**
   - **RD**
     - BC-NC: $7.98
     - BC-PC: $5.22
     - % diff.: 52.70%
   - **CD**
     - BC-NC: $6.79
     - BC-PC: $5.18
     - % diff.: 30.99%
   - **ID**
     - BC-NC: $4.54
     - BC-PC: $4.46
     - % diff.: 1.76%
   - **ED**
     - BC-NC: $3.88
     - BC-PC: $4.11
     - % diff.: -5.66%

6. **Consumption (MMcf)**
   - **RD**
     - BC-NC: 5,070,051
     - BC-PC: 6,752,150
     - % diff.: -24.91%
   - **CD**
     - BC-NC: 3,359,012
     - BC-PC: 4,326,044
     - % diff.: -22.35%
   - **ID**
     - BC-NC: 7,791,256
     - BC-PC: 7,666,899
     - % diff.: 1.62%
   - **ED**
     - BC-NC: 5,332,594
     - BC-PC: 3,744,228
     - % diff.: 42.42%

7. **Pipeline**
   - **Regulated Income (1000$)**
     - BC-NC: 8,477,208.21
     - BC-PC: 9,395,139.17
     - % diff.: -9.77%
   - **Congestion Income (1000$)**
     - BC-NC: 7,896,513.94
     - BC-PC: 6,611,806.11
     - % diff.: 19.43%
## Balanced Future Nash Cournot (NC) vs. Balanced Future Perfect Competition (PC)

<table>
<thead>
<tr>
<th></th>
<th>BF-NC</th>
<th>BF - PC</th>
<th>% diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Producers</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wellhead Prices ($/Mcf)</td>
<td>$3.60</td>
<td>$4.45</td>
<td>-19.10%</td>
</tr>
<tr>
<td>Production (MMcf)</td>
<td>21,596,952</td>
<td>22,834,094</td>
<td>-5.42%</td>
</tr>
<tr>
<td>Profits (1000$)</td>
<td>42,648,106</td>
<td>64,262,676</td>
<td>-33.63%</td>
</tr>
<tr>
<td><strong>Storage Operators</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gas Prices ($/Mcf)</td>
<td>$4.03</td>
<td>$5.10</td>
<td>-20.98%</td>
</tr>
<tr>
<td>Extraction (MMcf)</td>
<td>1,532,182</td>
<td>2,478,187</td>
<td>-38.17%</td>
</tr>
<tr>
<td>Profits (1000$)</td>
<td>48,105</td>
<td>152,930</td>
<td>-68.54%</td>
</tr>
<tr>
<td><strong>Peak Gas Operators</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gas Prices ($/Mcf)</td>
<td>$3.57</td>
<td>$4.72</td>
<td>-24.36%</td>
</tr>
<tr>
<td>Supply (MMcf)</td>
<td>1,076,855</td>
<td>1165085.298</td>
<td>-7.57%</td>
</tr>
<tr>
<td>Profits (1000$)</td>
<td>1,514,677</td>
<td>2,827,067</td>
<td>-46.42%</td>
</tr>
<tr>
<td><strong>Marketers</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Profits (1000$)</td>
<td>42,832,340</td>
<td>0</td>
<td>n/a</td>
</tr>
<tr>
<td><strong>End-user Prices</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RD</td>
<td>$8.06</td>
<td>$5.26</td>
<td>53.39%</td>
</tr>
<tr>
<td>CD</td>
<td>$7.03</td>
<td>$5.22</td>
<td>34.60%</td>
</tr>
<tr>
<td>ID</td>
<td>$4.56</td>
<td>$4.52</td>
<td>0.97%</td>
</tr>
<tr>
<td>ED</td>
<td>$4.19</td>
<td>$4.20</td>
<td>-0.15%</td>
</tr>
<tr>
<td><strong>Consumption (MMcf)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RD</td>
<td>5,330,381</td>
<td>7,169,293</td>
<td>-25.65%</td>
</tr>
<tr>
<td>CD</td>
<td>3,712,096</td>
<td>4,871,733</td>
<td>-23.80%</td>
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<tr>
<td>ID</td>
<td>6,351,427</td>
<td>5,438,908</td>
<td>16.78%</td>
</tr>
<tr>
<td>ED</td>
<td>7,138,690</td>
<td>6,358,119</td>
<td>12.28%</td>
</tr>
<tr>
<td><strong>Pipeline</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regulated Income (1000$)</td>
<td>8,504,341.30</td>
<td>9,594,763.95</td>
<td>-11.36%</td>
</tr>
<tr>
<td>Congestion Income (1000$)</td>
<td>9,120,153.98</td>
<td>9,030,300.89</td>
<td>1.00%</td>
</tr>
</tbody>
</table>
### Base Case, Balanced Future, and Reactive Path

#### Nash Cournot Cases

<table>
<thead>
<tr>
<th></th>
<th>Base Case</th>
<th>Balanced Future</th>
<th>Reactive Path</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Producers</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wellhead Prices ($/Mcf)</td>
<td>3.49</td>
<td>3.21%</td>
<td>3.50%</td>
</tr>
<tr>
<td>Production (MMcf)</td>
<td>21,449,980</td>
<td>0.69%</td>
<td>0.66%</td>
</tr>
<tr>
<td>Profits (1000$)</td>
<td>40,999,255</td>
<td>4.02%</td>
<td>4.46%</td>
</tr>
<tr>
<td><strong>Storage Operators</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gas Prices ($/Mcf)</td>
<td>3.96</td>
<td>1.77%</td>
<td>1.77%</td>
</tr>
<tr>
<td>Extraction (MMcf)</td>
<td>1,806,400</td>
<td>-15.18%</td>
<td>-13.87%</td>
</tr>
<tr>
<td>Profits (1000$)</td>
<td>70,325</td>
<td>-31.60%</td>
<td>-25.52%</td>
</tr>
<tr>
<td><strong>Peak Gas Operators</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gas Prices ($/Mcf)</td>
<td>4.22</td>
<td>-15.40%</td>
<td>-12.09%</td>
</tr>
<tr>
<td>Supply (MMcf)</td>
<td>241,644</td>
<td>345.64%</td>
<td>311.45%</td>
</tr>
<tr>
<td>Profits (1000$)</td>
<td>673,754</td>
<td>124.81%</td>
<td>136.36%</td>
</tr>
<tr>
<td><strong>Marketers</strong></td>
<td></td>
<td></td>
<td></td>
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<td>RD</td>
<td>$7.98</td>
<td>1.11%</td>
<td>2.55%</td>
</tr>
<tr>
<td>CD</td>
<td>$6.79</td>
<td>3.48%</td>
<td>3.32%</td>
</tr>
<tr>
<td>ID</td>
<td>$4.54</td>
<td>0.52%</td>
<td>0.04%</td>
</tr>
<tr>
<td>ED</td>
<td>$3.88</td>
<td>8.15%</td>
<td>9.26%</td>
</tr>
<tr>
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<td>33.87%</td>
<td>38.50%</td>
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<td>7,896,513.94</td>
<td>15.50%</td>
<td>16.35%</td>
</tr>
</tbody>
</table>

Case Comparison, % diff. with Base Case
Balanced Future and Reactive Path Cases Not Much Different for ID, ED

Comparison of Industrial and Electric Power Demand (MMcf)
Future Work and References

- Adding stochasticity to the market player problems
  - Model formulation and solution (Denver 2004 INFORMS meeting, marketers have chance constraints, ongoing work to consider recourse with the spot market)
  - Mathematical analysis including existence & uniqueness results (some improvements for deterministic case, stochastic case ongoing)
  - Decomposition methods (e.g., Benders, Dantzig-Wolfe)

- Using micro-level approach for demand and/or supply functions
  - Certain modules are “black boxes”, hard to generate data
  - US DOE NEMS model, ICF Consulting’s GSAM model
Future Work and References

- **Related Papers: North American market**

- **Related Papers: European market**