TITLE: CYLSHELL

Matrices arising from finite element discretization of a cylindrical shell.
http://math.nist.gov/MatrixMarket/data/misc/cylshell/cylshell.html

DESCRIPTION:

Source:	Reijo Kouhia, Helsinki University of Technology, Laboratory of Structural Mechanics, e-mail reijo.kouhia@hut.fi
Discipline:	Structural mechanics
Remarks:	These matrices arise from low order finite element discretizations of a cylindrical shell. Three different meshes and three different values of the relative thickness of the shell are represented. For more thorough description, see section Finite Element Model.
Accession:	Summer 1997

MATRIX CHARACTERISTICS:

In the table below $ne(\boldsymbol{A})$ denotes the number of entries in the data, $nz(\boldsymbol{A})$ is the number of nonzeros of the matrix and n is the dimension of the problem. The sparsity pattern is determined purely on the element connectivity information assuming full pattern for the local element stiffness matrix. Therefore in the uniform quadrilateral meshes there are about 6 to 8 % of zeros inside the adopted sparsity pattern. Spectral condition numbers ($\text{cond}(\boldsymbol{A}) = \lambda_{max}/\lambda_{min}$ for SPD matrices, where $\lambda_{max}, \lambda_{min}$ are the extreme eigenvalues of the matrix $\boldsymbol{A}$) are also shown in the table. The eigenvalues are computed with look-ahead Lanczos algorithm using subroutine DSLAL from QMRPACK [1] with 100 Lanczos steps, except the largest eigenvalues of the mesh `M2` matrices for which 3000 steps are used. The eigenvalues are checked with the block Lanczos solver BLZPACK [2].

case	n	$ne(\boldsymbol{A})$	$nz(\boldsymbol{A})$	λ_{max}	λ_{min}	$\text{cond}(\boldsymbol{A})$
`S1RMQ4M1`	5489	143300	133950	$6.8743215 \cdot 10^5$	$3.7969627 \cdot 10^{-1}$	$1.810479 \cdot 10^6$
`S2RMQ4M1`	5489	143300	134420	$6.8743398 \cdot 10^4$	$3.8746328 \cdot 10^{-4}$	$1.774191 \cdot 10^8$
`S3RMQ4M1`	5489	143300	134216	$6.8745249 \cdot 10^3$	$3.8936813 \cdot 10^{-7}$	$1.765559 \cdot 10^{10}$
`S1RMT3M1`	5489	112505	111570	$9.6684221 \cdot 10^5$	$3.7976650 \cdot 10^{-1}$	$2.545886 \cdot 10^6$
`S2RMT3M1`	5489	112505	111585	$9.6681138 \cdot 10^4$	$3.8747124 \cdot 10^{-4}$	$2.495182 \cdot 10^8$
`S3RMT3M1`	5489	112505	111579	$9.6688591 \cdot 10^3$	$3.8956267 \cdot 10^{-7}$	$2.481978 \cdot 10^{10}$
`S3DKQ4M2`	90449	2455670	2259087	$4.6016534 \cdot 10^3$	$2.4268619 \cdot 10^{-8}$	$1.896133 \cdot 10^{11}$
`S3DKT3M2`	90449	1921955	1888336	$8.7984364 \cdot 10^3$	$2.4269394 \cdot 10^{-8}$	$3.625322 \cdot 10^{11}$
`S3RMT3M3`	5357	106526	106240	$9.5986081 \cdot 10^3$	$3.9983548 \cdot 10^{-7}$	$2.400639 \cdot 10^{10}$

Naming convention Filenames consist of eight characters in the form `S T MO E N MX`, where

`T`	represents the power of inverse of the relative thickness $(R/t) = 10^{\text{T}}$,
`MO`	is the model type, `RM` = Reissner-Mindlin, `DK` = discrete Kirchhoff,
`E`	indicates the element type, `Q` = quadrilateral, `T` = triangular,
`N`	number of nodes in the element (3/4),
`MX`	is the mesh indicator, `M1` = uniform 30×30, `M2` = uniform 150×100, `M3` = graded with 1666 triangles.

For example, `S2RMQ4M1` corresponds to a shell with $R/t = 10^2$, and uniform 30×30 mesh with quadrilateral 4-noded elements using the Reissner-Mindlin kinematics.

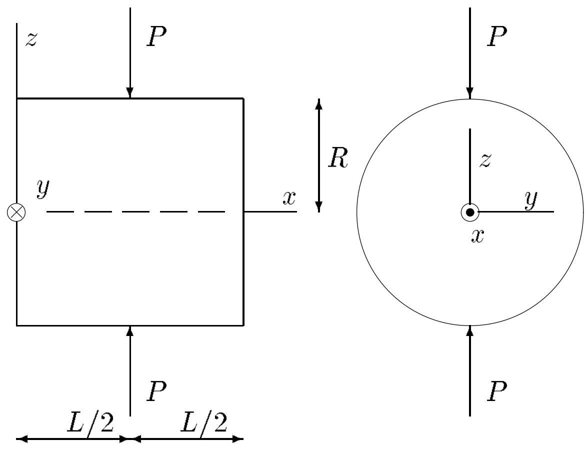

Figure 1: Pinched cylindrical shell.

FINITE ELEMENT MODEL:

Matrices in the CYLSHELL set represent low order FE discretizations of a well known shell element test, the pinched cylinder [3]. The cylinder is loaded by two normal and equal point loads applied centrally at the opposite sides of the cylindrical surface, see fig. 1. The length of the shell is chosen to be equal to its diameter ($L = 2R$). The material model is linear isotropic elastic: the Poisson's ratio is 0.3 and the Young's modulus equals to 10^6 units. Since the behaviour of the shell is markedly characterized by the value of the relative thickness of the shell t/R, three different values are represented $t/R = 10^{-1}, 10^{-2}$ and 10^{-3}. In the present FE model the radius is chosen to $R = 1$ unit.

One octant of the shell is discretized. Three different meshes are present: `M1` uniform 30×30 both quadrilateral and triangular meshes (see fig. 3), `M2` 150×100 quadrilateral and triangular meshes (150 elements in the circumferential and 100 elements in the axis direction) and mesh `M3` is a graded triangular mesh of 1666 elements with local refinements near the point load and near the free edge, see fig. 4.

The shell elements used are facet-type 3-node triangular or 4-node quadrilateral elements using the Hughes-Brezzi formulation [4] for drilling rotations. The Allman type displacement field linked to drilling rotations is used. The plate bending part of the element is based on the stabilized MITC theory [5] or the discrete Kirchhoff condensation (the DKT and the quadrilateral Lyons-Crisfield element [6]). In the MITC formulation the stabilization parameter has been 0.4 for both triangular and quadrilateral elements.

The value of the regularizing penalty parameter γ used in the formulation of Hughes and Brezzi affects the condition number of the stiffness matrix and thus the convergence of the PCG iteration. The effect of the parameter γ on the spectral condition number as well as the convergence of the PCG iteration is demonstrated in [7]. The figure 2 from ref. [7] is reprinted here as fig. 2. Adding an Allman-type displacement field [8] to the in-plane interpolation also has an effect on the convergence; however, there seems to be no definite trend in that dependency, see Table 1 (from ref. [7]). For the CYLSHELL matrices the value $\gamma = G/1000$ is chosen, where G is the shear modulus. For that value the IC(0) factorization exists without shifting for all considered cases.[1]

[1] IC(0) with respect to the given sparsity pattern.

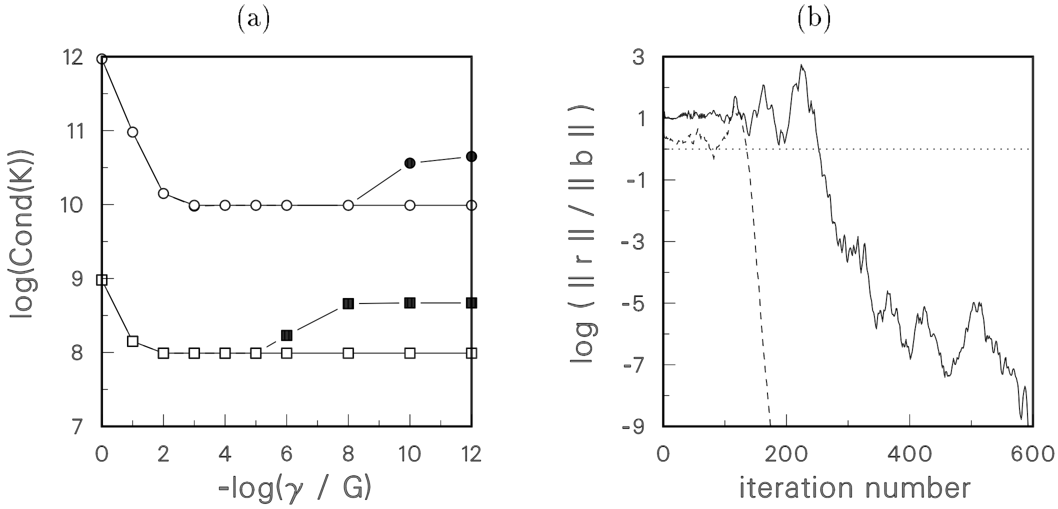

Figure 2: (a) Effect of the regularizing parameter γ on the spectral condition number of the stiffness matrix. Solid markers correspond to stiffness matrices without Allman displacement field. Upper two curves $t/R = 10^{-3}$ and lower ones $t/R = 10^{-2}$. (b) Convergence plot of the CG-IC(0) iteration: $\gamma = G$ (without Allman field), solid line $t/R = 10^{-3}$ and dashed line $t/R = 10^{-2}$. Quadrilateral 30×30 mesh. Figure from ref. [7] ($ATOL = 0, RTOL = 10^{-9}$).

Table 1: Influence of the regularizing parameter γ on the convergence of the CG-IC(0) iteration, 30×30 mesh with MITC elements. A = Allman type amendment for the in-plane interpolation. Table from ref. [7]($ATOL = 0, RTOL = 10^{-9}$).

(a) $t/R = 10^{-2}$

γ/G	Q-A	T-A	Q	T
1.0	-	-	175	293
0.5	-	-	153	283
0.25	-	306	142	273
0.2	379	301	138	269
0.1	147	288	129	259
0.01	120	262	117	2337
0.001	118*	249◇	115	1201

* CYLSHELL matrice `S2RMQ4M1`
◇ CYLSHELL matrice `S2RMT3M1`

(b) $t/R = 10^{-3}$

γ/G	Q-A	T-A	Q	T
1.0	-	-	593	-
0.5	-	-	511	-
0.25	-	-	502	-
0.2	-	-	508	-
0.1	356	-	397	1197
0.01	307	768	330	743
0.001	287*	674◇	284	636

* CYLSHELL matrice `S3RMQ4M1`
◇ CYLSHELL matrice `S3RMT3M1`

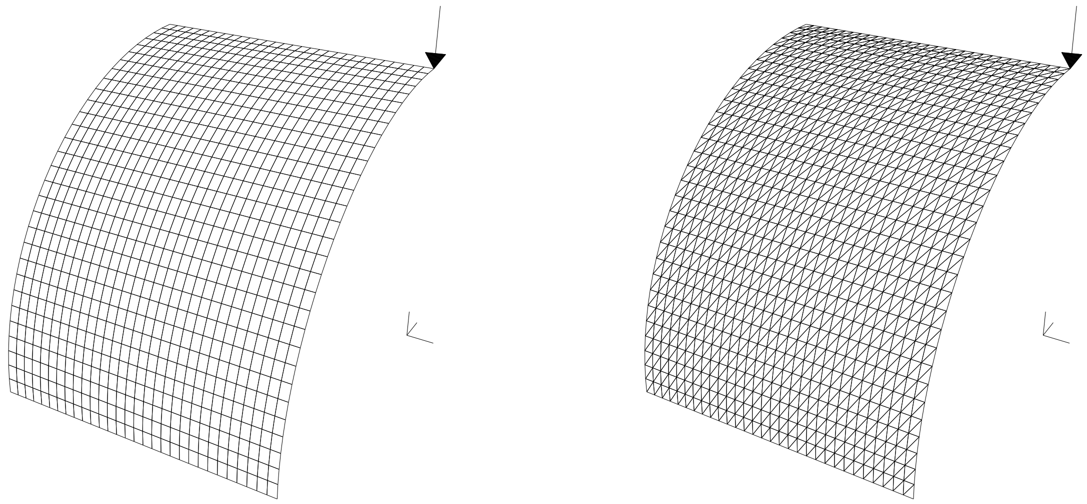

Figure 3: Uniform 30×30 quadrilateral and triangular meshes.

PERFORMANCE STATISTICS

Some results of computations with a PCG method using the no-fill IC preconditioner (with respect to the given sparsity pattern) are shown in Table 2. There is a slight difference in the number of iterations for different computers due to the strict convergence criteria and ill-conditioning. The reported computing times are obtained with a Cray C-94 vector processor, a SGI Power Challenge and a DEC AlphaServer 8400 at the Center for Scientific Computing, Espoo, Finland.

The stiffness matrix is stored by using the jagged diagonal scheme in order to take advantage of vectorization in the matrix-vector products [9]. In addition, no advantage of symmetry is taken to save storage for the stiffness matrix, as this is known to cause a degradation of the performance on supercomputers. For the dot products and the vector updates in the CG algorithm, standard BLAS are used. No rescaling of the stiffness matrix is performed.

The number of iterations shown in Table 2 correspond to computations where the right-hand side is determined from the solution vector of all ones. If the point loading is used as in [3], somewhat different iteration numbers are obtained. In Table 3 the equation number for the load is given (the last one except the case `S3RMT3M3`) as well as the deflection under the point load. This deflection is also verified with a direct solver.

In order to have a finite value for the limiting deflection for the Kirchhoff model when $t \to 0$, the load should be scaled with respect to the characteristic thickness of the shell.[2] The value used is $P = -(1/4)(E/10)(t^3/R)$, where E is the Young's modulus.

The initial guess for the PCG iteration is the zero vector and the stopping criterion is $\|\boldsymbol{r}_k\|_2 < ATOL + RTOL\|\boldsymbol{r}_0\|$, where $\boldsymbol{r}_k$, is the unpreconditioned updated residual after k iterations and $ATOL = 10^{-9}, RTOL = 10^{-8}$. The code is written in standard Fortran 77 and compiled with the -Zv option on the Cray and -O3 option on the SGI and DEC.

Results with some other preconditioners can be found in ref. [10]

[2]For the Reissner-Mindlin kinematical model, the point load is not allowed; that is, $w_P \to \infty$ when the mesh is refined, i.e. when $h \to 0$.

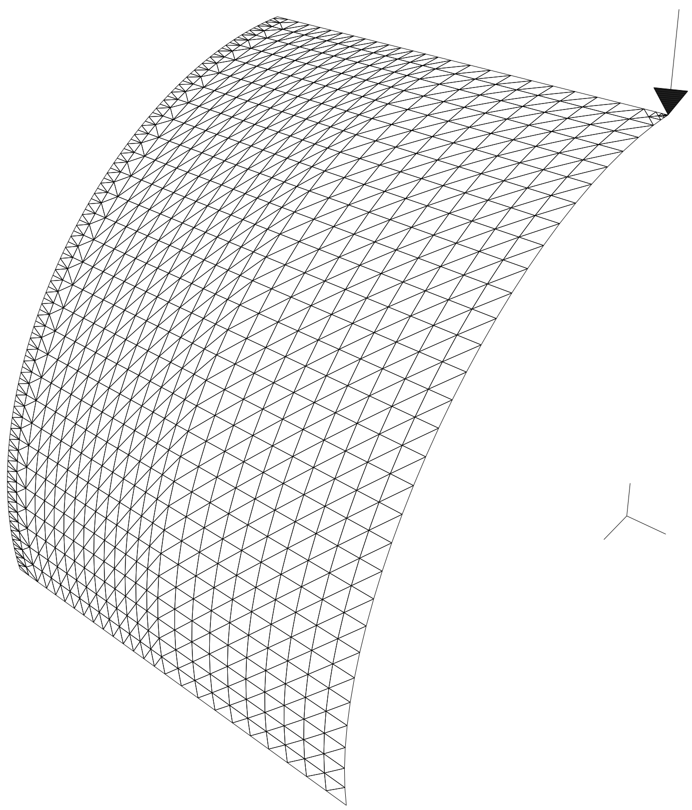

Figure 4: Graded triangular mesh 1666 elements.

Table 2: Cylshell matrices: CG-IC(0) statistics, P = preconditioner evaluation time, I = iteration time, * = not computed.

	Cray C94				DEC AlphaServer 8400				SGI PowerChallenge			
case	iter	P	I	P+I	iter	P	I	P+I	iter	P	I	P+I
`S1RMQ4M1`	91	1.40	1.90	3.30	92	0.33	4.06	4.39	92	1.00	7.31	8.31
`S2RMQ4M1`	108		2.23	3.63	110		4.65	4.98	110		9.17	10.17
`S3RMQ4M1`	161		3.24	4.64	179		6.62	6.95	179		14.10	15.10
`S1RMT3M1`	171	0.88	3.19	4.07	183	0.21	5.32	5.53	183	0.65	10.59	11.24
`S2RMT3M1`	231		4.25	5.13	234		6.48	6.69	234		14.18	14.83
`S3RMT3M1`	368		6.62	7.50	419		12.04	12.25	415		23.92	24.57
`S3DKQ4M2`	8749	24.6	2737	2762	7104	6.15	7590	7596	*			
`S3DKT3M2`	1752	15.4	507.9	523.3	2574	4.04	2198	2202	*			
`S3RMT3M3`	346	0.79	6.10	6.89	403	0.20	10.96	11.16	398	0.61	21.52	22.13

Table 3: Pinched cylindrical shells, n_P = equation number for the loaded point, w_P = deflection under the point load.

case	n_P	w_P
`S1RMQ4M1`	5489	$-4.7985 \cdot 10^{-2}$
`S2RMQ4M1`	5489	$-4.1902 \cdot 10^{-2}$
`S3RMQ4M1`	5489	$-4.0881 \cdot 10^{-2}$
`S1RMT3M1`	5489	$-4.7724 \cdot 10^{-2}$
`S2RMT3M1`	5489	$-4.1891 \cdot 10^{-2}$
`S3RMT3M1`	5489	$-4.0875 \cdot 10^{-2}$
`S3DKQ4M2`	90499	$-4.0877 \cdot 10^{-2}$
`S3DKT3M2`	90499	$-4.0875 \cdot 10^{-2}$
`S3RMT3M3`	3601	$-4.0962 \cdot 10^{-2}$

Acknowledgements

Thanks to Michele Benzi and Miroslav Tůma for providing the PCG software and Chih-Jen Lin, Jorge Moré, and Kensheng Wu for comments concerning these matrices.

REFERENCES

[1] R.W. Freund and N.M. Nachtigal. QMRPACK: a package of QMR algorithms. *ACM Transactions on Mathematical Software*, 22(1):46–77, 1996.

[2] O.A. Marques. BLZPACK: description and users guide. CERFACS Report TR/PA/95/30, 1995.

[3] H. Hakula, Y.Leino, and J. Pitkäranta. Scale resolution, locking and high-order finite element modelling of shells. *Computer Methods in Applied Mechanics and Engineering*, 133:157–182, 1996.

[4] T.J.R. Hughes and F. Brezzi. On drilling degrees of freedom. *Computer Methods in Applied Mechanics and Engineering*, 72:105–121, 1989.

[5] M. Lyly, R. Stenberg, and T. Vihinen. A stable bilinear element for the Reissner-Mindlin plate model. *Computer Methods in Applied Mechanics and Engineering*, 110:343–357, 1993.

[6] M.A. Crisfield. *Finite Elements and Solution Procedures for Structural Analysis, Volume 1: Linear Analysis*. Pineridge Press, Swansea, U.K., 1986.

[7] R. Kouhia. Using iterative solvers in non-linear continuation algorithm. In J. Aalto and T. Salmi, editors, *Proceedings of the 6th Finnish Mechanics Days*, pages 253–267, 1997.

[8] D.J. Allman. A compatible triangular element including vertex rotations for plane elasticity analysis. *Computers and Structures*, 19:1–8, 1984.

[9] Y. Saad. *Iterative Methods for Sparse Linear Systems*. PWS Publishing, 1996.

[10] M. Benzi, R. Kouhia, and M. Tůma. An assesment of some preconditioning techniques in shell problems, September 1997. Technical Report LA-UR-97-3892, Los Alamos National Laboratory, Los Alamos, MN.

Reijo Kouhia
Laboratory of Structural Mechanics
Helsinki University of Technology
P.O. Box 2100, FIN-02015 HUT, Finland
tel. -358-(0)9-4513755
fax. -358-(0)9-4513826
e-mail: reijo.kouhia@hut.fi
URL: http://www.hut.fi/~kouhia