Fast Iterative Solvers for Fluid Flows

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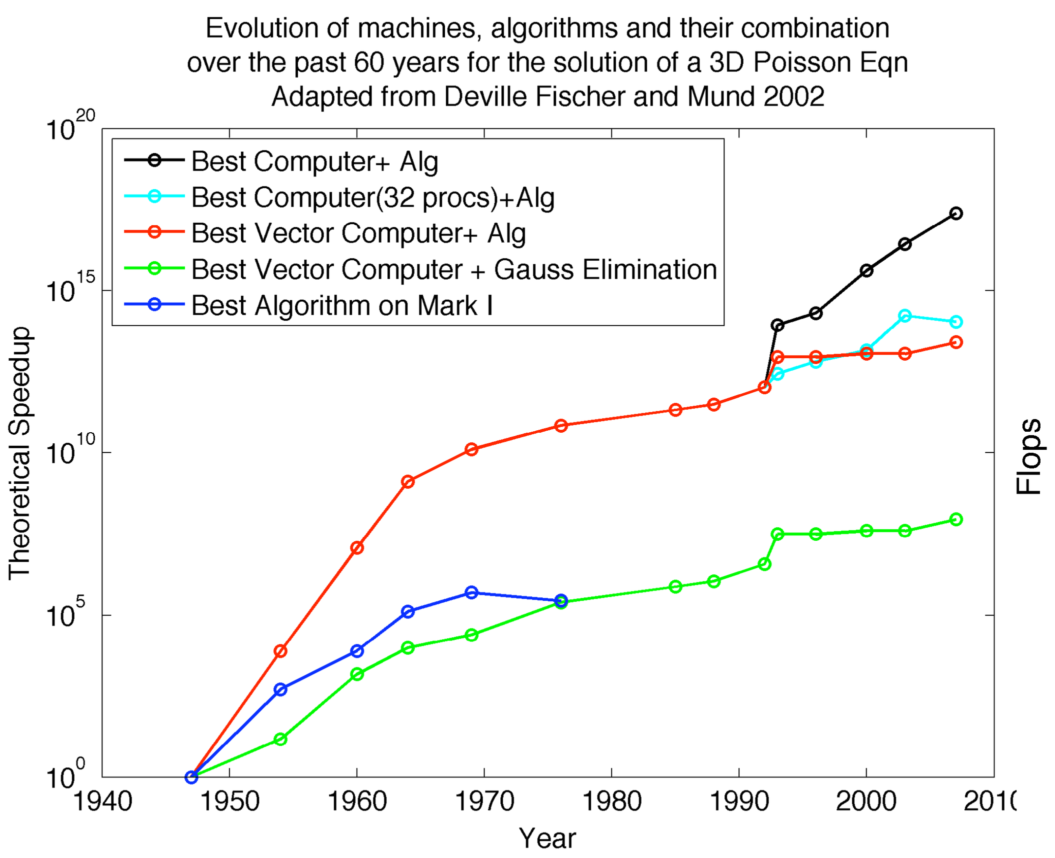
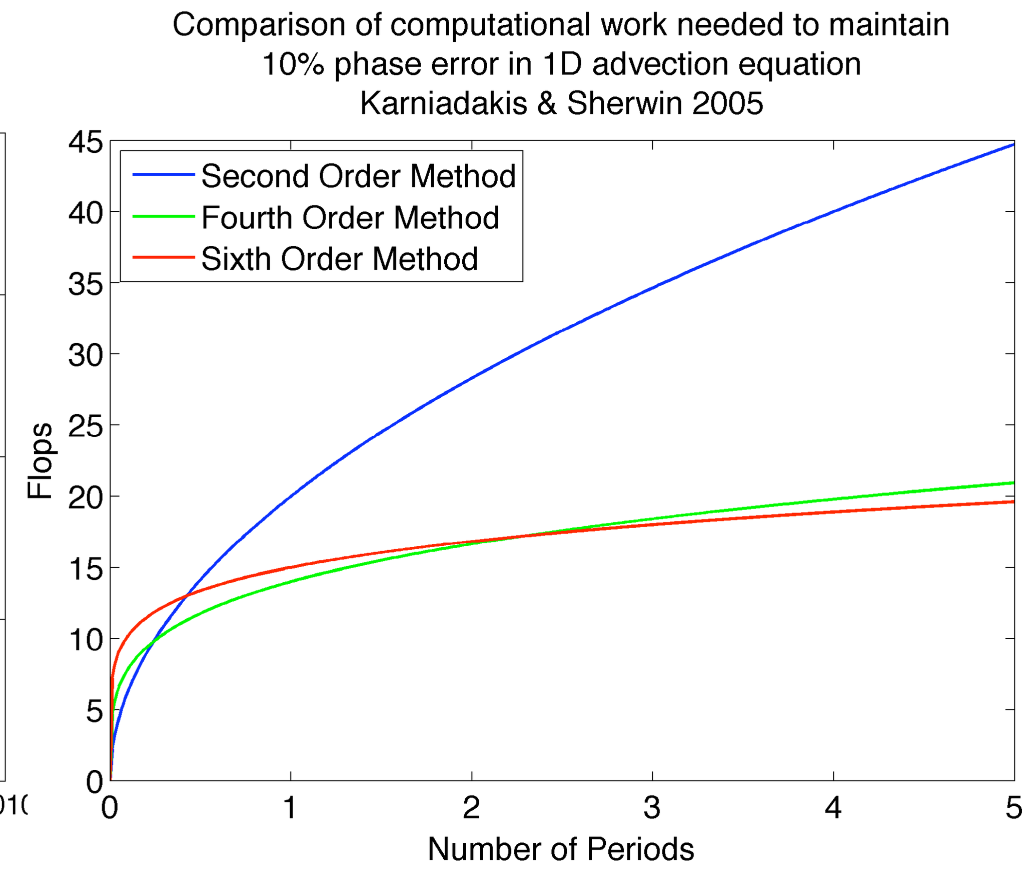
Motivation - Efficient Solvers & Discretization



Faster machines and computational algorithms can dramatically reduce simulation time.



High order based discretizations can be used to obtain accurate, efficient simulations.

Model - Steady Advection Diffusion

$$-\epsilon\nabla^2 u + (\vec{w}\cdot\nabla)u = f$$

Inertial and viscous forces occur on disparate scales causing **sharp flow features** which:

- require fine numerical grid resolution
- cause poorly conditioned systems.

These properties make solving the discrete systems computationally expensive.

Methods - Spectral Element Discretization

A spectral element discretization provides:

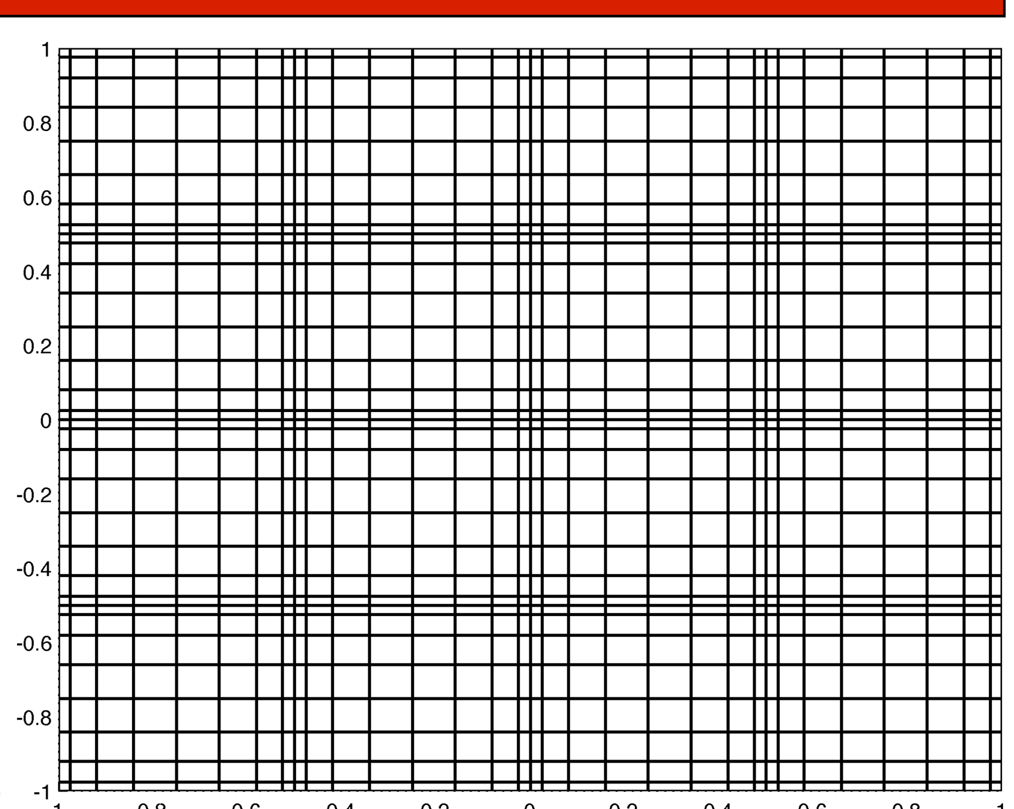
- **accurate element based discretization**
- **large volume to surface ratio**

For bi-constant winds, we can use:

- **fast diagonalization**
- **minimal memory**

$$F(\vec{w})u = Mf$$

$$\tilde{F} = \hat{M}\otimes\hat{F}(w_x) + \hat{F}(w_y)\otimes\hat{M}$$



Methods - Solver & Preconditioner

The discrete system of equations is solved iteratively using Flexible GMRES.

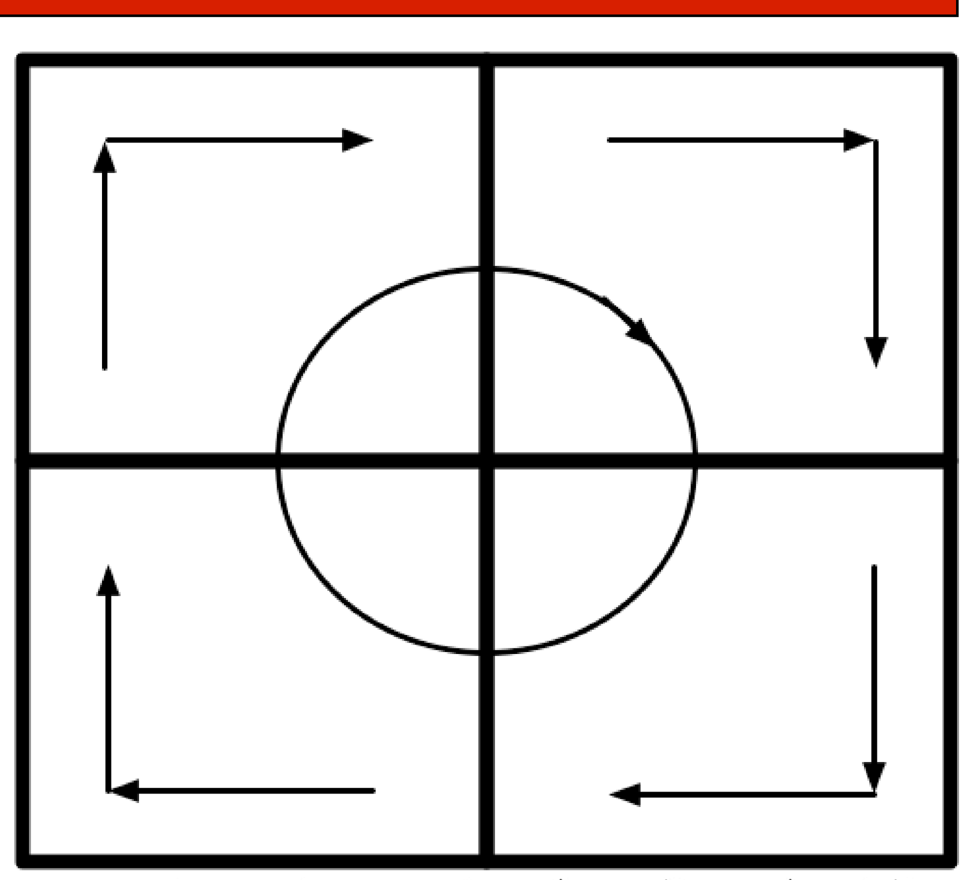
We construct a **preconditioner** based on:

- **Bi-constant wind approximations**
- **Fast Diagonalization**
- **Domain Decomposition**

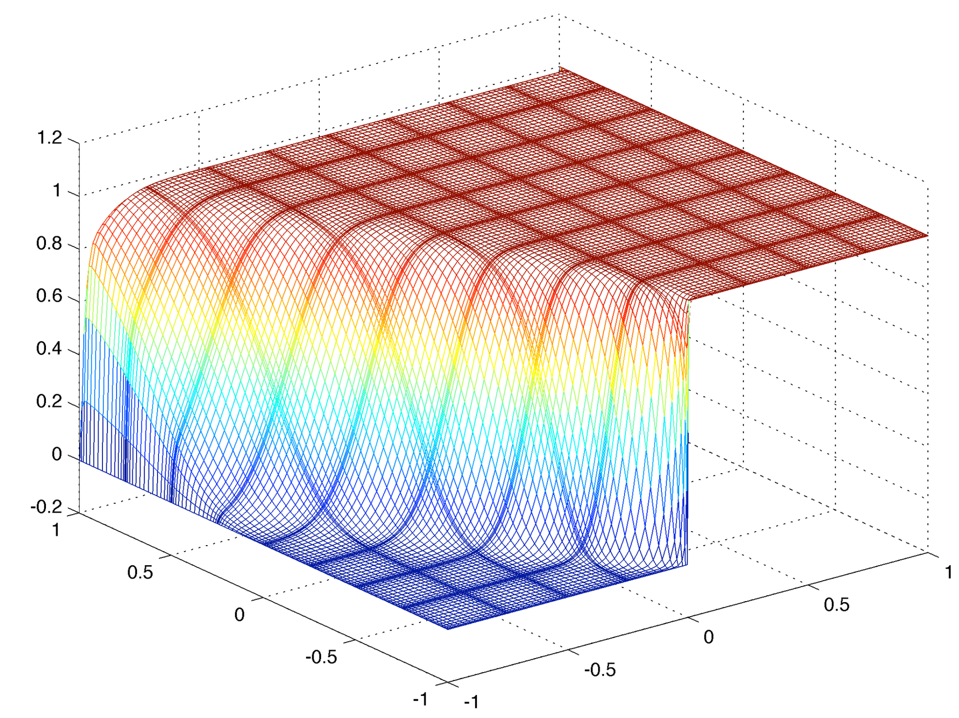
$$F(\vec{w})P_F^{-1}P_F u = Mf$$

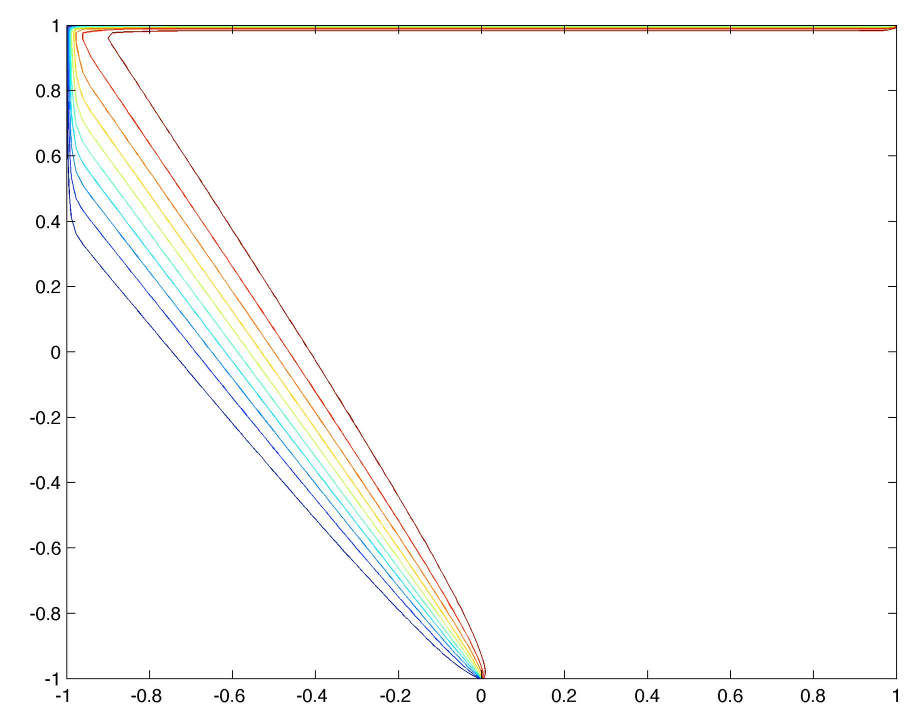
$$P_F^{-1} = R_0^T\tilde{F}_0^{-1}(\bar{w}_0)R_0 + \sum_{e=1}^{N} R_e^T\tilde{F}_e^{-1}(\bar{w}_e)R_e$$

$$\tilde{F}_e^{-1} = (\hat{M}^{-1/2}\otimes\hat{M}^{-1/2})(S\otimes T)(\Lambda\otimes I + I\otimes V)^{-1}(S^{-1}\otimes T^{-1})(\hat{M}^{-1/2}\otimes\hat{M}^{-1/2})$$



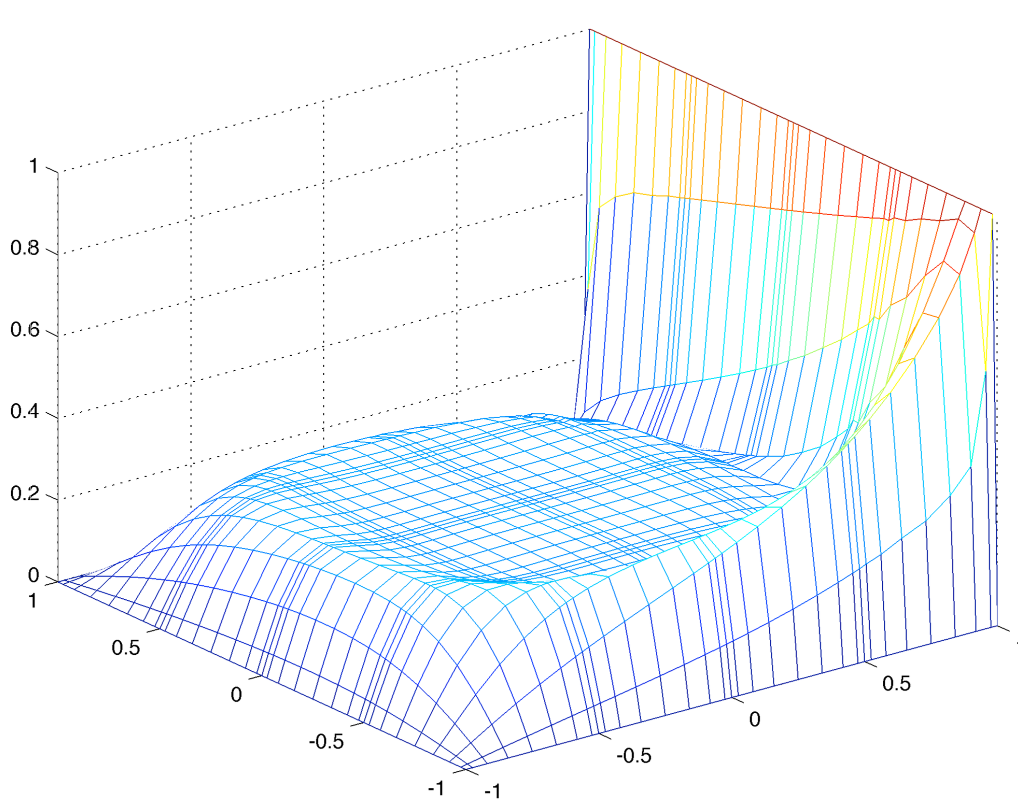
Solver Results - Bi-constant Wind



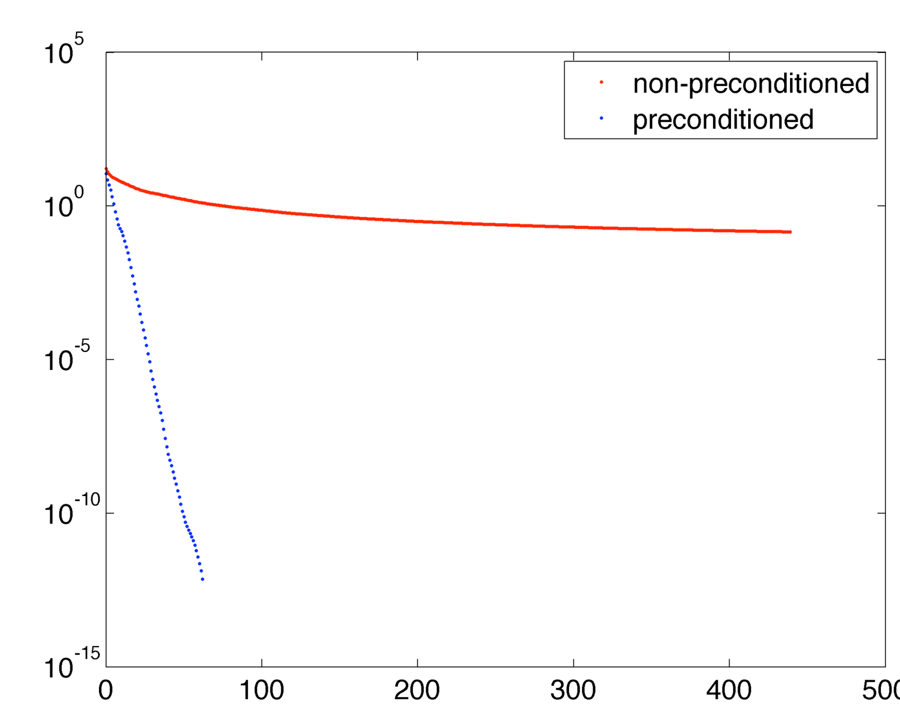


Solution and contour plots of a steady advection-diffusion flow with bi-constant wind using Domain Decomposition & Fast Diagonalization. Pc=200. Interface solve takes 150 steps to obtain 10^-5 accuracy.

Preconditioner Results - Recirculating Wind



Steady advection-diffusion flow with recirculating wind. Pc=200. Hot plate at wall results in sharp internal boundary layer.



Comparison of iteration residuals.

- 10 interface steps yeild 10% accuracy
- (P+1)[40N+(P+1)] additional flops per step

Future Directions

- Precondition Interface Solve
- 2D & 3D **Navier-Stokes**
- Flows with boundary layers

References

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