Simulation of radiographs

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INTRODUCTION

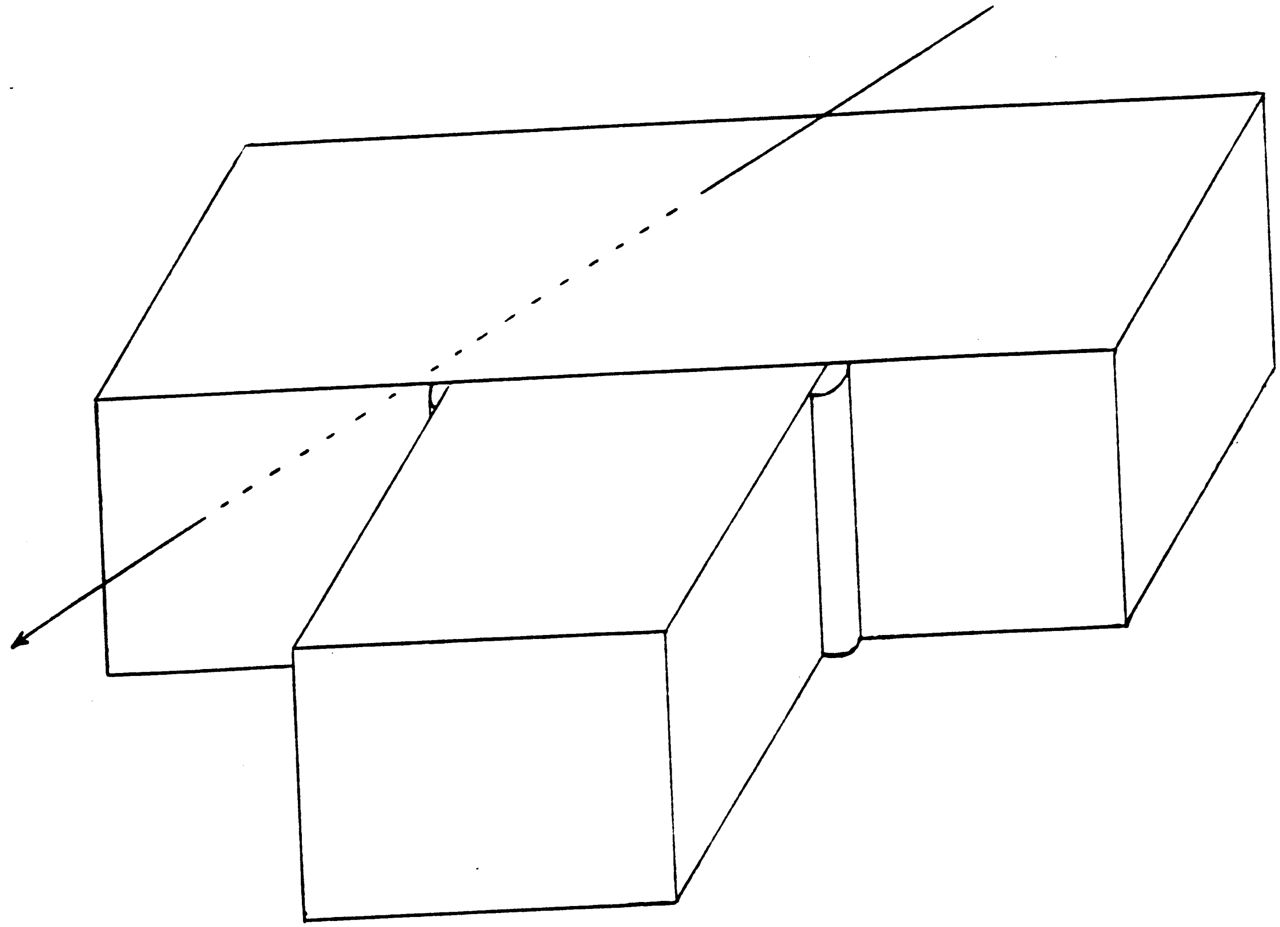
The resulting intensity of an X-ray of initial intensity I_0 passing along line L through a body with linear attenuation coefficient $\mu(x)$ is given by

$$I = I_0 \exp\left\{- \int_L \mu(x)\, dx\right\}$$

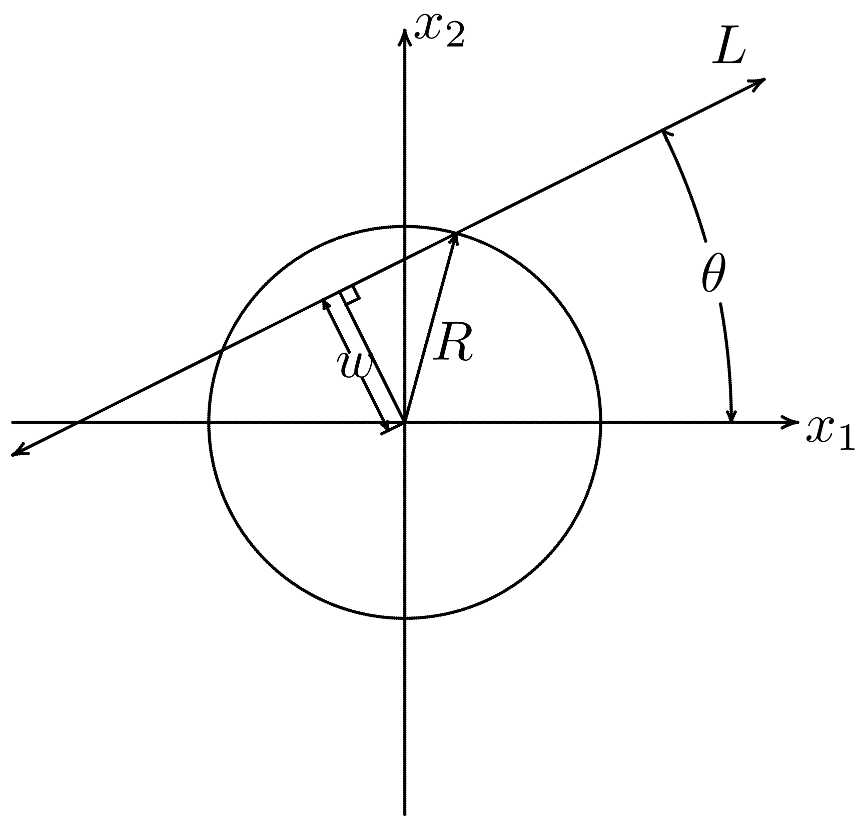
If the body is homogeneous (i.e., $\mu(x)$ is constant inside the body, 0 outside), then this reduces to

$$I = I_0 \exp\{-\mu \| L \cap \mathrm{Body} \|\}$$

In particular, we will call $\mu \| L \cap \mathrm{Body} \|$ the **linear attenuation** due to the body.

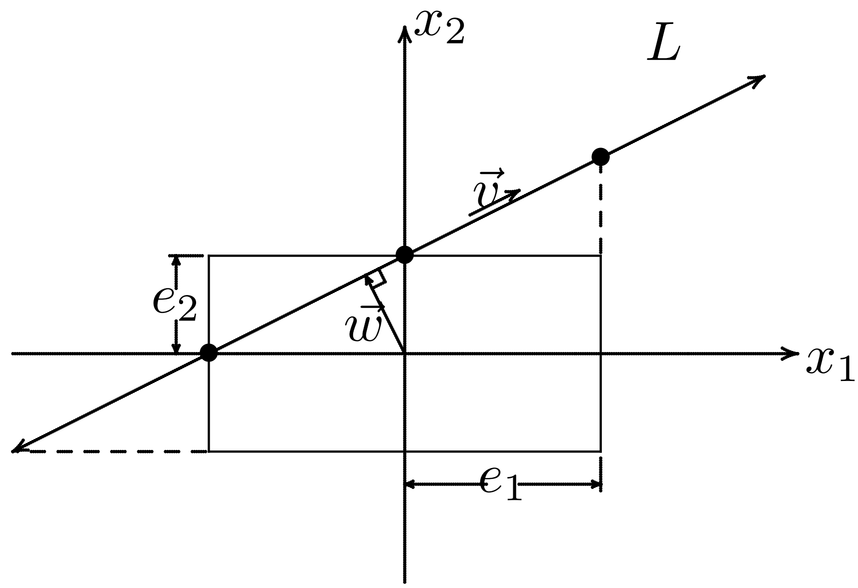


T-joint with trace of X-ray path.



Length of intersection of line L with circle centered at the origin with radius R is

$$\Phi(L_{\theta,w}) = 2\sqrt{R^2 - w^2}$$



Parameterize line L by

$$(x_1(s), x_2(s)) = L_{\vec{v},\vec{w}}(s) = s\vec{v} + \vec{w}$$

where $\vec{v}$, $\vec{w}$ satisfy $\|\vec{v}\| = 1$ and $\langle \vec{v}, \vec{w} \rangle = 0$.

We then have the following 4 conditions on s:

$$\begin{aligned} -e_1 \le x_1(s) &= sv_1 + w_1 \le e_1 \\ -e_2 \le x_2(s) &= sv_2 + w_2 \le e_2 \end{aligned}$$

Solving yields

$$
\begin{aligned}
s_{\text{min}} &= \max\left[\min\left(\frac{\pm e_1 - w_1}{v_1}\right), \min\left(\frac{\pm e_2 - w_2}{v_2}\right)\right] \\
s_{\text{max}} &= \min\left[\max\left(\frac{\pm e_1 - w_1}{v_1}\right), \max\left(\frac{\pm e_2 - w_2}{v_2}\right)\right]
\end{aligned}
$$

The length of the intersection of L with the rectangle is

$$\Phi(L_{\vec{v},\vec{w}}) = \max(s_{\text{max}} - s_{\text{min}}, 0)$$

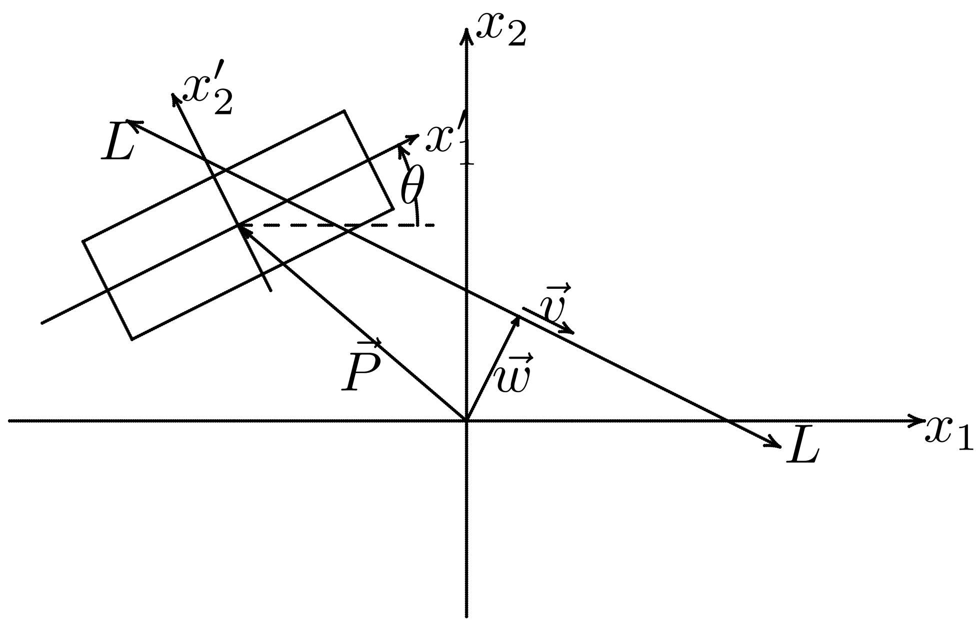


Illustration of intersection of line L with a rotated and translated rectangle.

Let U be the orthogonal transformation given by

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

and let T be the rigid body motion

$$T\vec{w} = U(\vec{w} - \vec{P})$$

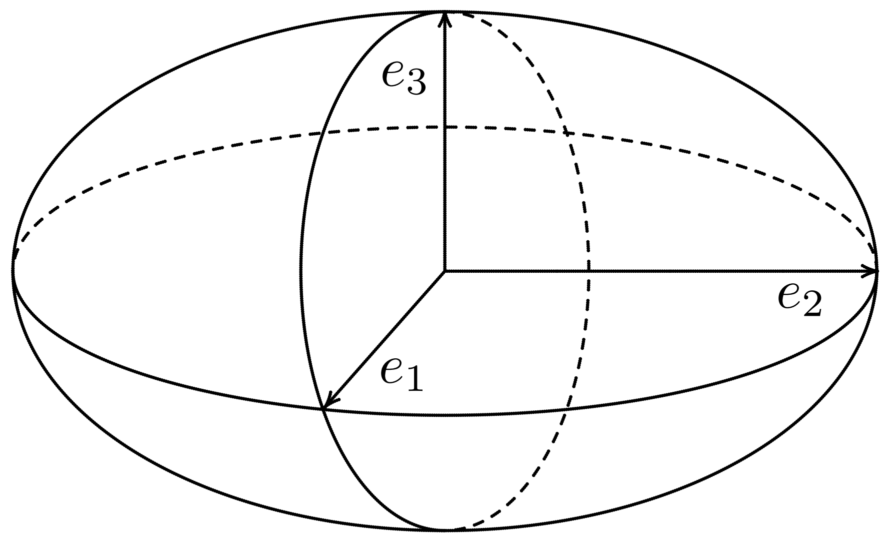
Then the line $L_{\vec{v},\vec{w}}$ becomes in the new coordinate system $L_{\vec{v}',\vec{w}'}$, where

$$\begin{aligned} \vec{v}' &= U\vec{v} \\ \vec{w}' &= T\vec{w} - \langle T\vec{w}, \vec{v}'\rangle\vec{v}' \end{aligned}$$

BASE ELEMENT TYPES:

- Sphere
- Ellipsoid*
- Cylinder
- Elliptical Cylinder*
- Box*
- Free Form*
- Erehwon

where * indicates those element types supporting cut-planes.



Ellipsoid:

$$(x_1/e_1)^2 + (x_2/e_2)^2 + (x_3/e_3)^2 \leq 1$$

Let

$$\begin{aligned}\vec{v}' &= (v_1/e_1, v_2/e_2, v_3/e_3)^t \\ \vec{w}' &= (w_1/e_1, w_2/e_2, w_3/e_3)^t\end{aligned}$$

And further define

$$\begin{aligned}a &= \langle \vec{v}', \vec{v}' \rangle \\ b &= \langle \vec{v}', \vec{w}' \rangle \\ c &= \langle \vec{w}', \vec{w}' \rangle - 1 \\ \Delta &= \sqrt{b^2 - ac}\end{aligned}$$

Then the line-ellipsoid intersection points are at

$$s_1 = \frac{-b - \Delta}{a} \qquad s_2 = \frac{-b + \Delta}{a}$$

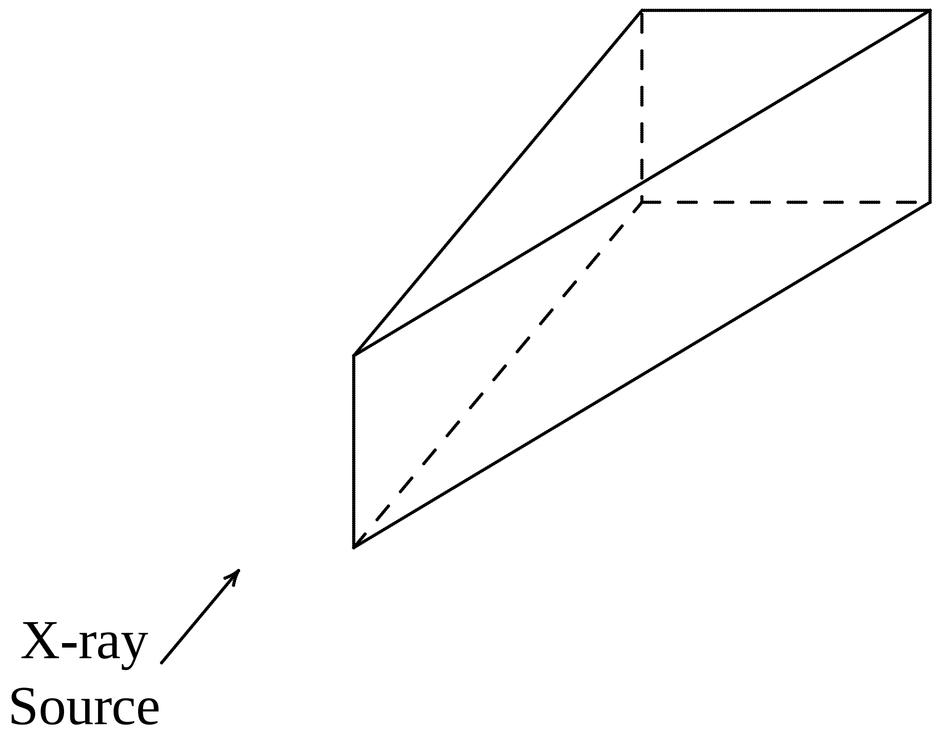
```
/******************************* ELLIPSOID *********************************/
float ellipsoid(ELEMENT *ell,LINE *ray)
   /* This routine returns the length of the intersection between      */
   /* the ellipsoid ell and the line *ray.  The object ell definition  */
   /* includes the size, density and orientation of the ellipsoid.     */
   {
   int i;
   float eff_length,a,b,c,det,t1,t2;
   LINE newray;

   /* Crude out-of-bounds check */
   for(i=0;i<3;i++) if(fabs(ray->offset[i])<=ell->param[i]) break;
   if(i>2) return 0.0;

   /* Convert to elliptical coordinates */
   for(i=0;i<3;i++) {
      newray.dir[i]=ray->dir[i]/ell->param[i];
      newray.offset[i]=ray->offset[i]/ell->param[i];
   }

   a=dot(newray.dir,newray.dir);
   b=dot(newray.dir,newray.offset);
   c=dot(newray.offset,newray.offset)-1.0;

   /* Compute crossing times */
   det=b*b-a*c;
   if(det<TOO_SMALL*TOO_SMALL) return 0.0;   /* No intersection! */
   det=sqrt(det);
   t1=(-b-det)/a; t2=(-b+det)/a;

   /* Incorporate "cut plane" restrictions */
   plane_limits(&t1,&t2,ell,ray);

   /* Compute total length, including density */
   if(t2<t1) eff_length=0;
    else     eff_length=(t2-t1)*ell->density;

   return eff_length;
   }
```

Subroutine for calculation of ellipsoid linear attenuation.

Simulated radiograph of an ellipsoid with cut-plane.

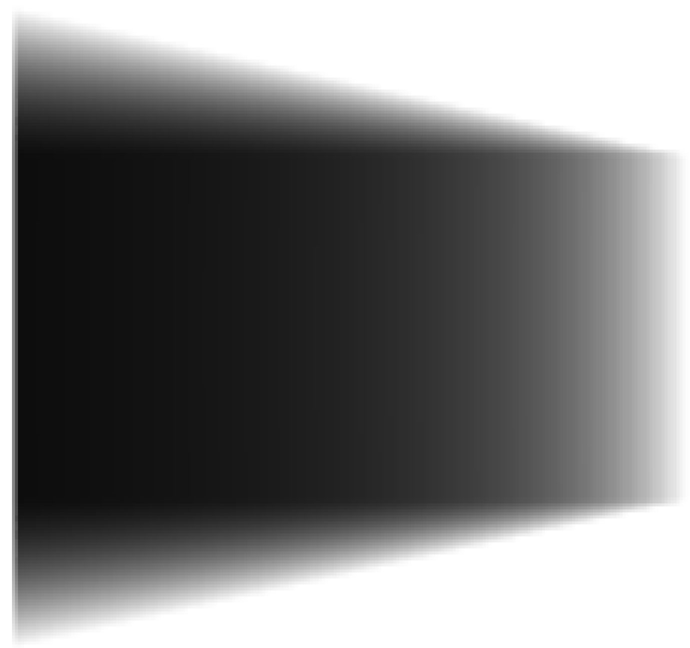
```
ELLIPSOID
1  0           0
0  0.707107  -0.707107
0  0.707107   0.707107
0 100 0
.3
500 100 50
0 0.707107 0.707107 50 1
```

Data file for ellipsoid with 1 cut-plane.

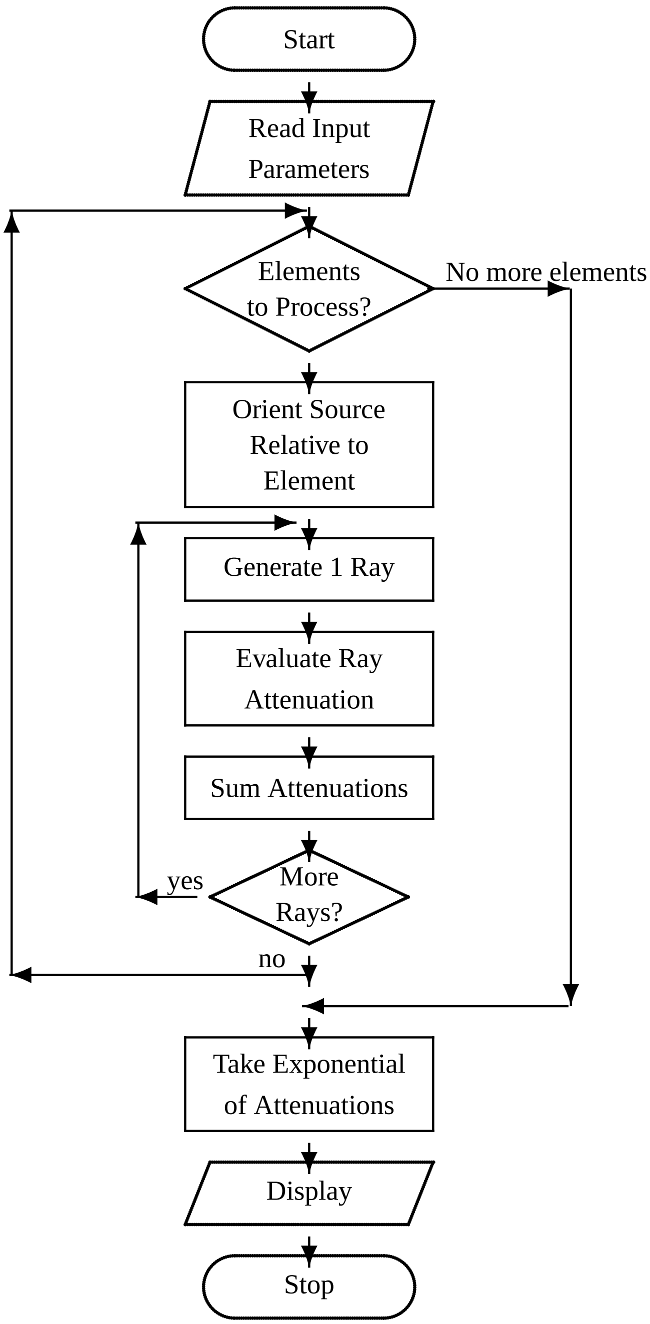


Triangular prism constructed as a “free form” element.

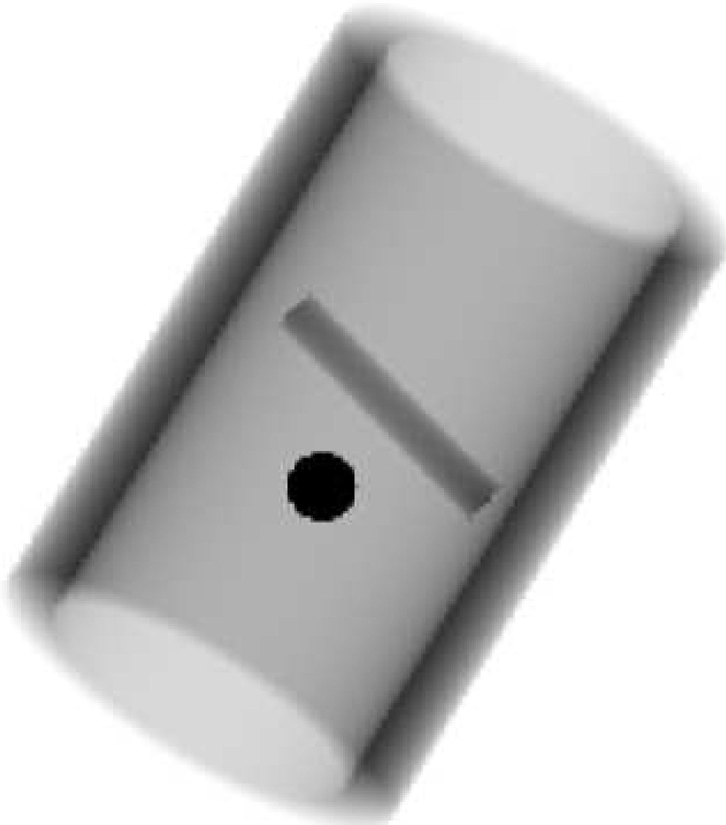
Simulated radiograph of triangular prism with parallel beam geometry.



Simulated radiograph of triangular prism with cone beam geometry.



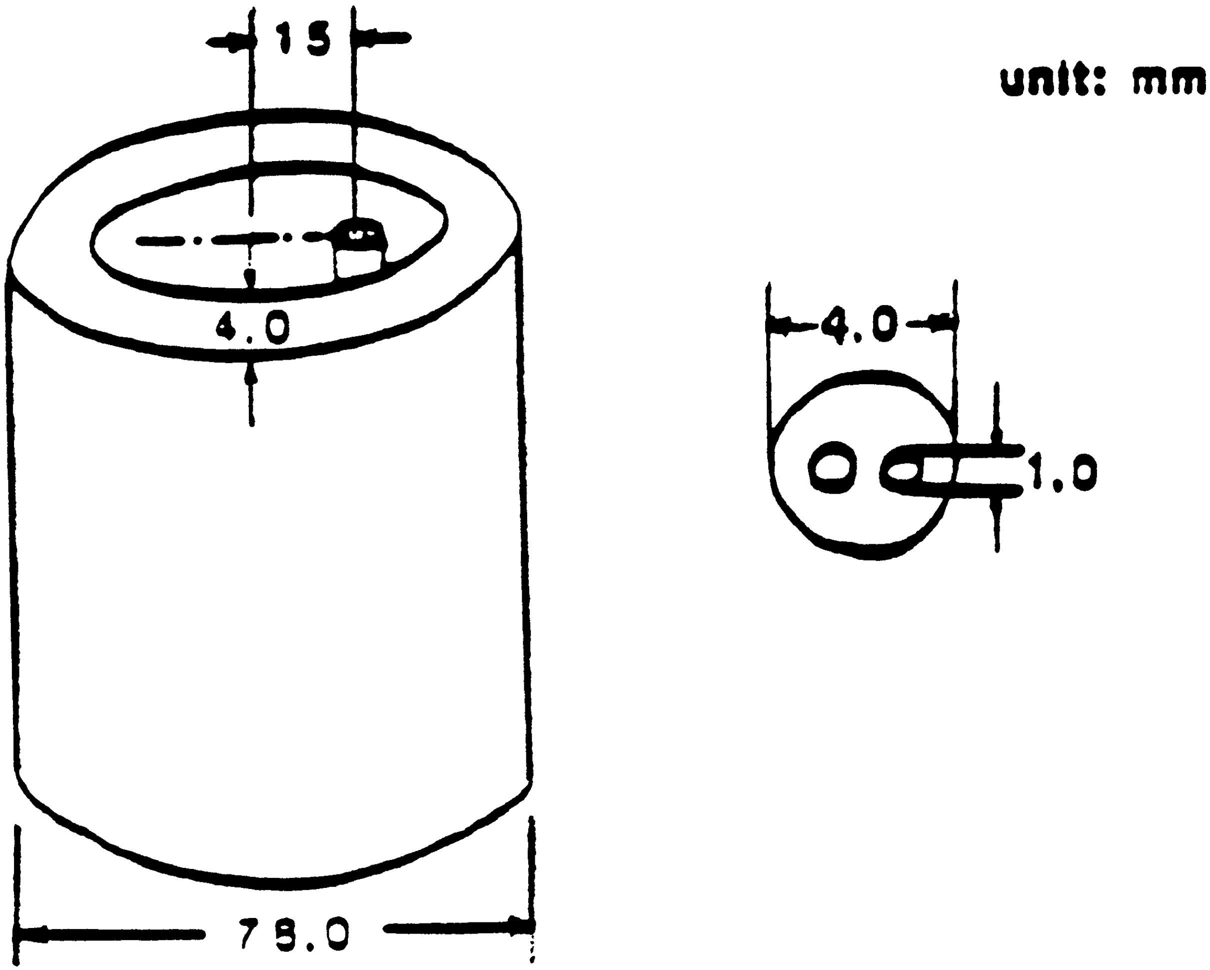
Flowchart for radiograph simulation package.



Simulated radiograph of a pipe with internal spherical and cylindrical inclusions.

```
CYLINDER
 0.95      -0.15612    0.27042
 0.0        0.86602    0.5
-0.31225   -0.475   0.82272
0  0  0
1.5
100 150

CYLINDER
 0.95      -0.15612    0.27042
 0.0        0.86603    0.5
-0.31225   -0.475      0.82272
0 0 0
-1.5
75 150

CYLINDER
1     0         0
0     0.70711  -0.70711
0     0.70711   0.70711
-5 10 5
3.5
10 60

SPHERE
1     0    0
0     1    0
0     0    1
0 -20 -30
25
15
```

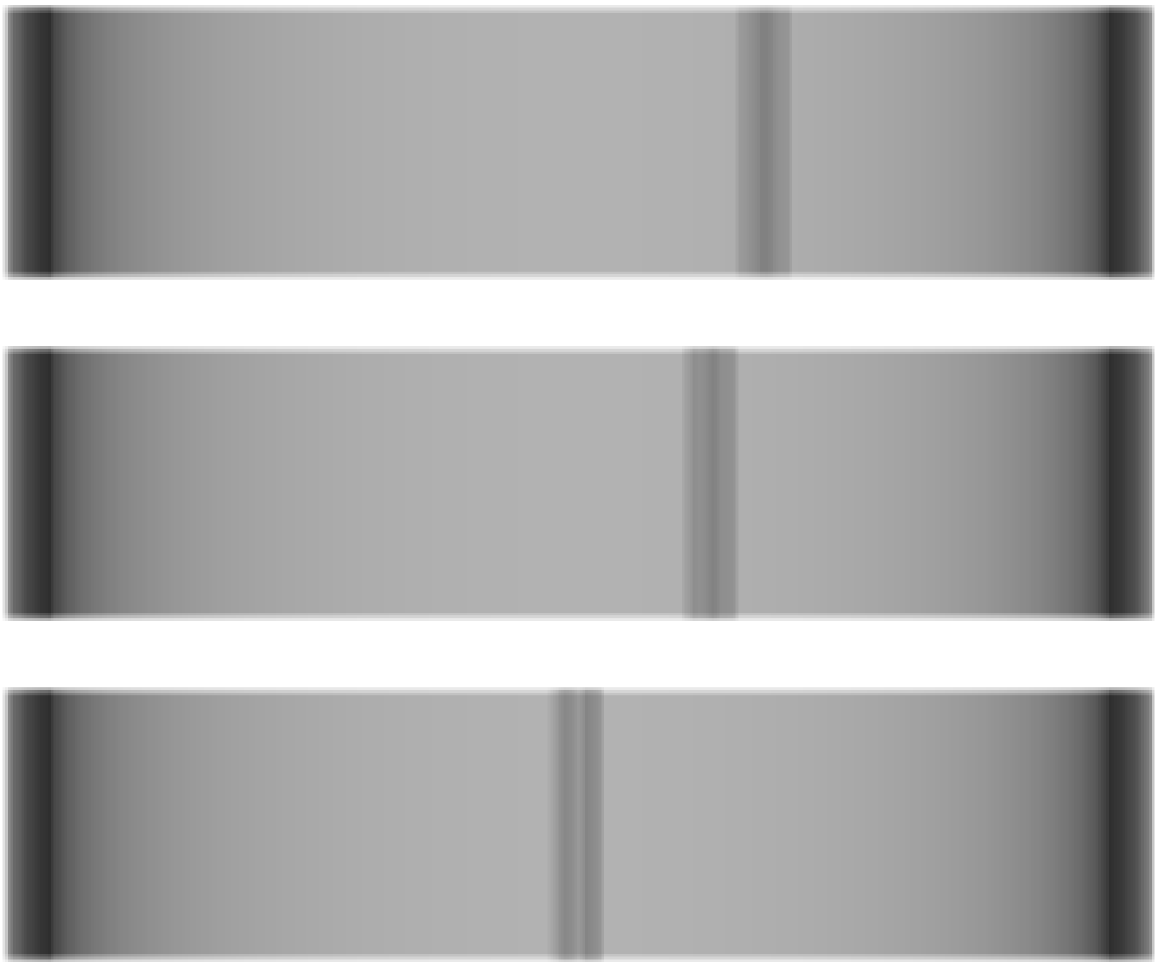
Data file for pipe simulation with inclusions.



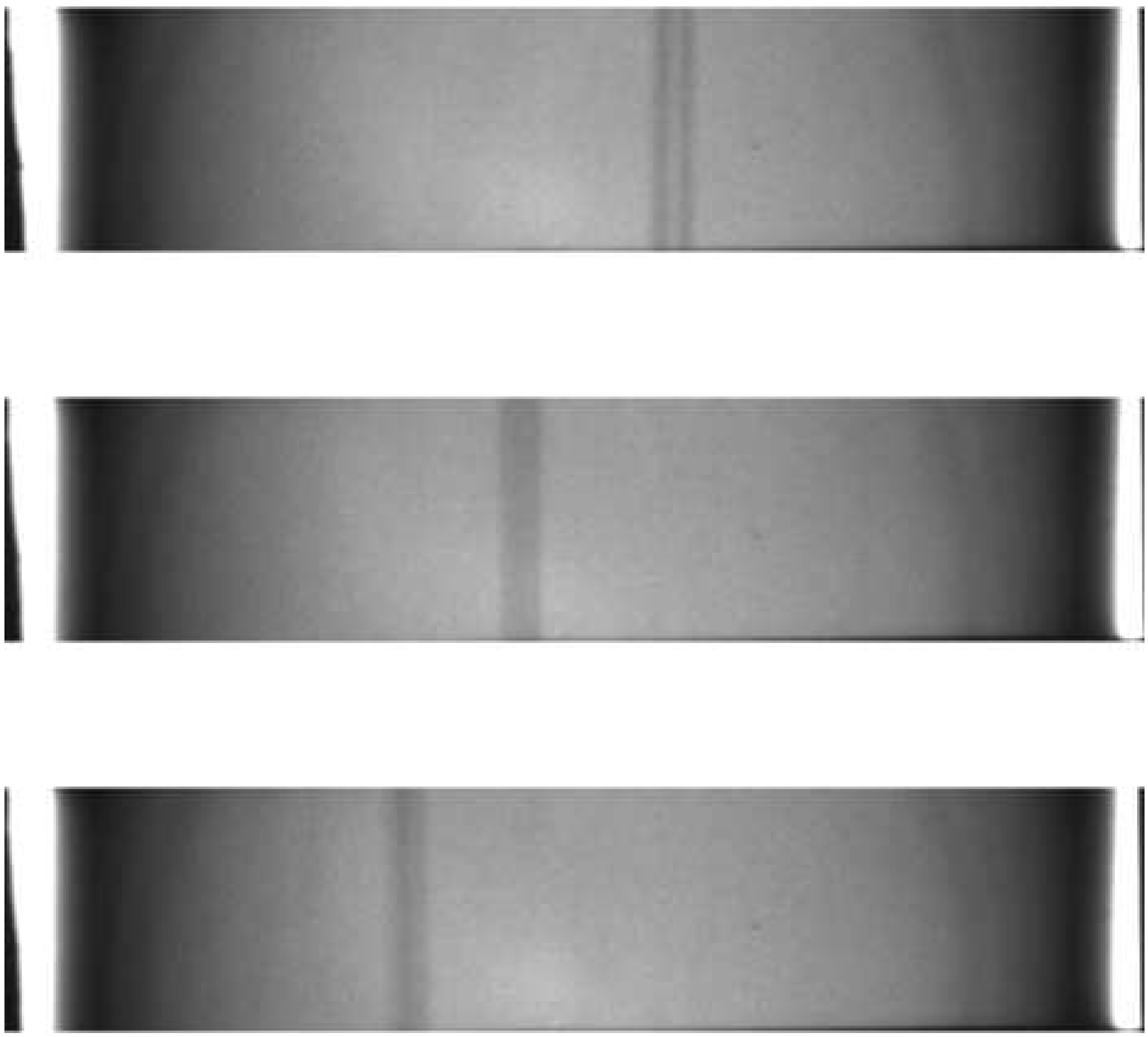
Schematic of experimental sample

```
CYLINDER
1 0 0
0 1 0
0 0 1
0 0 0
.01
218 50

CYLINDER
1 0 0
0 1 0
0 0 1
0 0 0
-.01
200 50

CYLINDER
1 0 0
0 1 0
0 0 1
0 70 0
.015
10 50

CYLINDER
1 0 0
0 1 0
0 0 1
0 65 0
-.015
2.5 50

CYLINDER
1 0 0
0 1 0
0 0 1
0 75 0
-.015
2.5 50
```

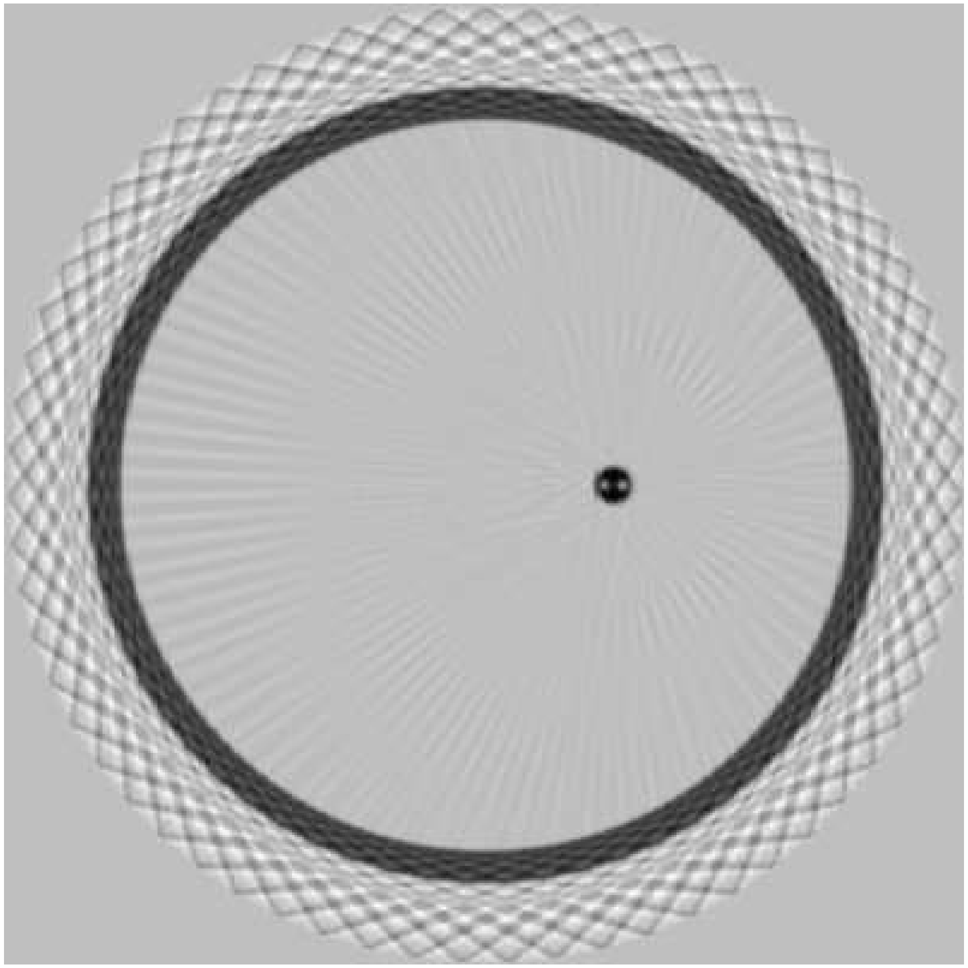
Data file for simulation of experimental sample.



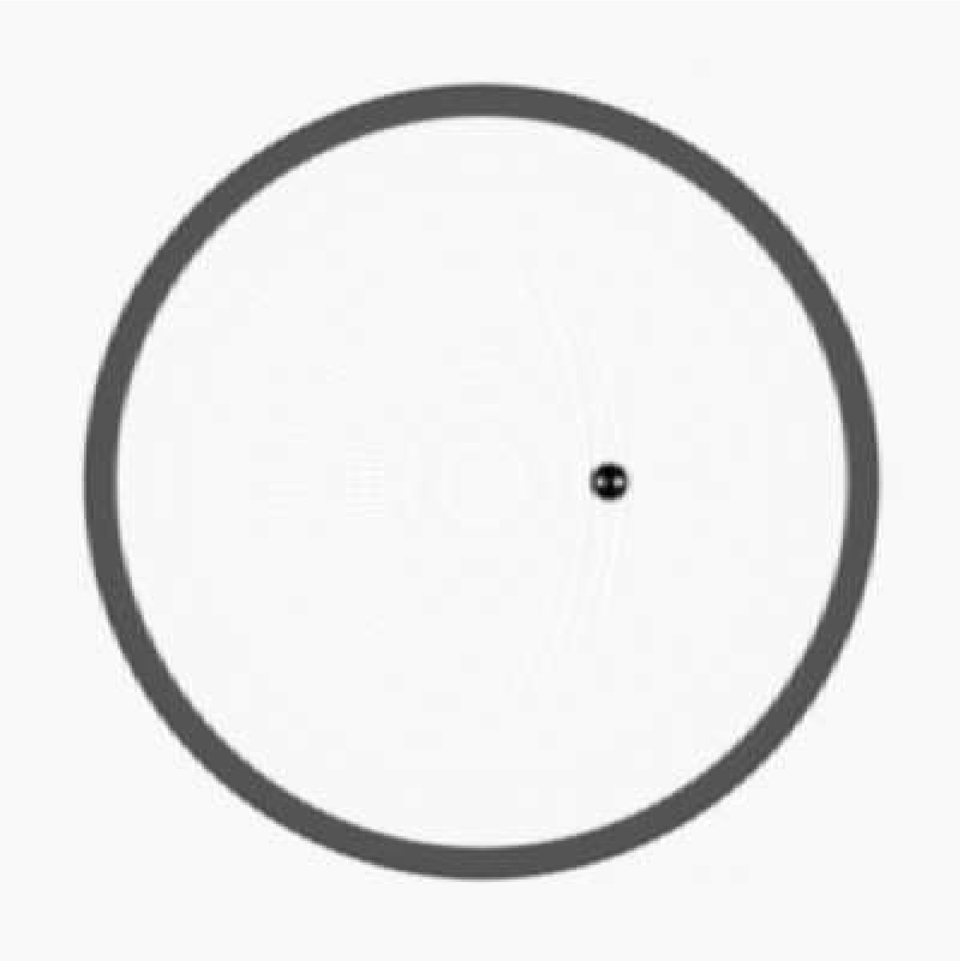
Simulated projections: 0°, 45°, 90°.



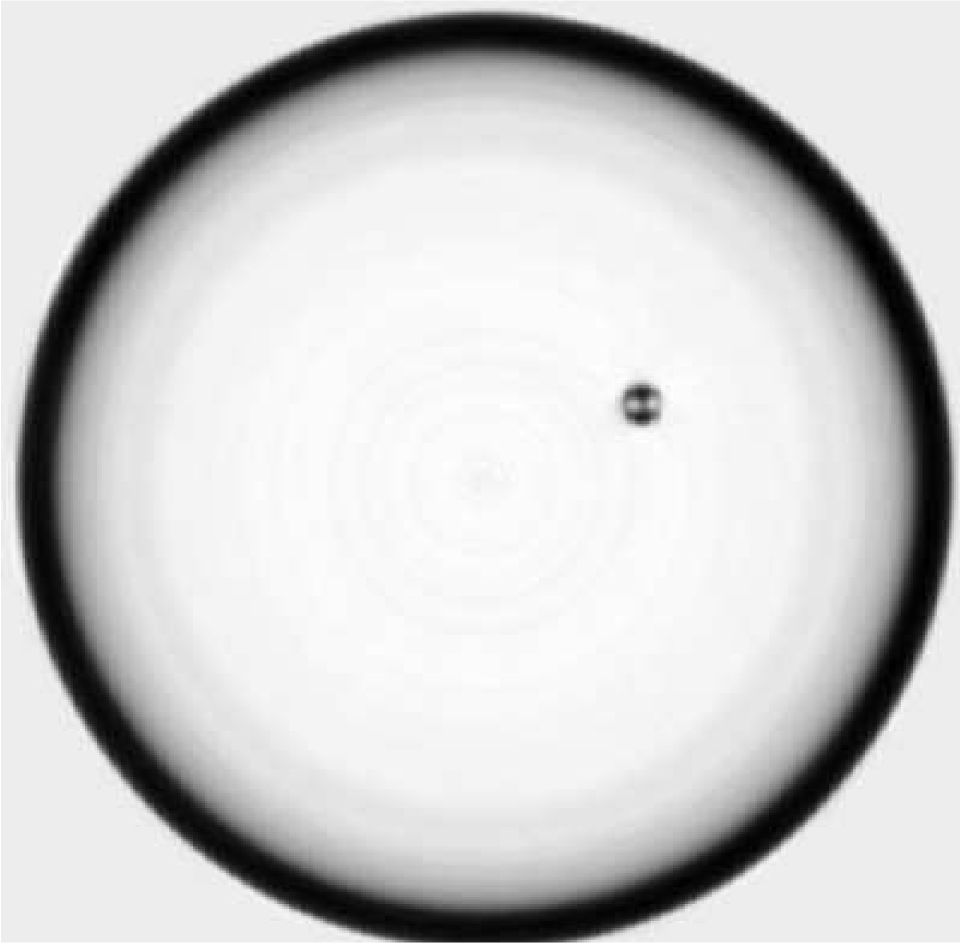
Experimental projections: 0°, 45°, 90°.



Reconstruction from simulated data, 65 projections.



Reconstruction from simulated data, 315 projections.



Reconstruction from experimental data, 315 projections.

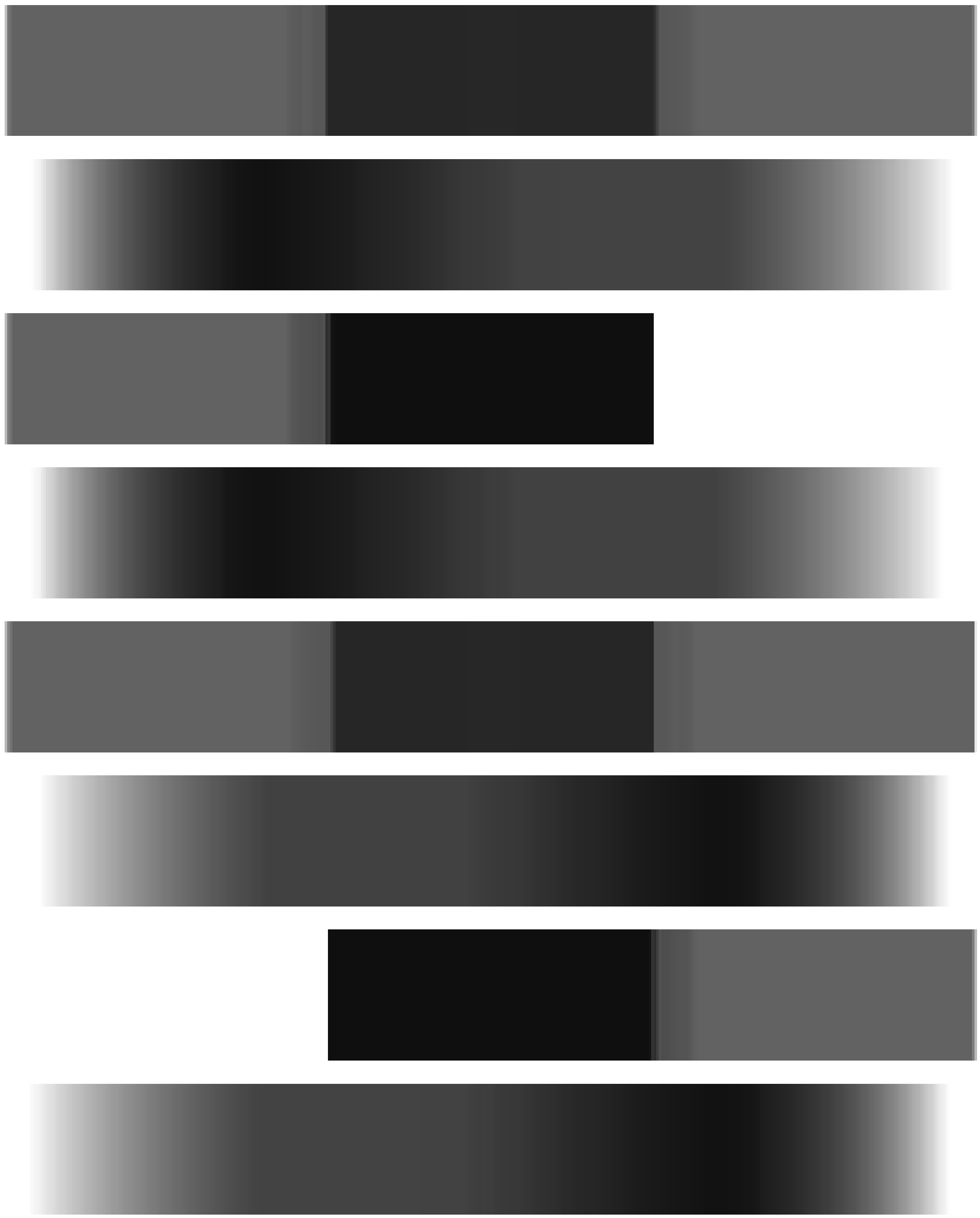
Simulated T-joint, top view.

```
BOX
 1.0   0.0          0.0
 0.0   1.0          0.0
 0.0   0.0          1.0
0  0  0
0.0075
64 190 100

BOX
 1.0   0.0          0.0
 0.0   1.0          0.0
 0.0   0.0          1.0
0  0  0
0.0075
190 64 100
1 0 0 64.25 -1

ELLIPTICAL CYLINDER
 1.0   0.0          0.0
 0.0   1.0          0.0
 0.0   0.0          1.0
64 64 0
0.0075
16 16 100
1 0 0 0.5 -1
0 1 0 0 -1
```

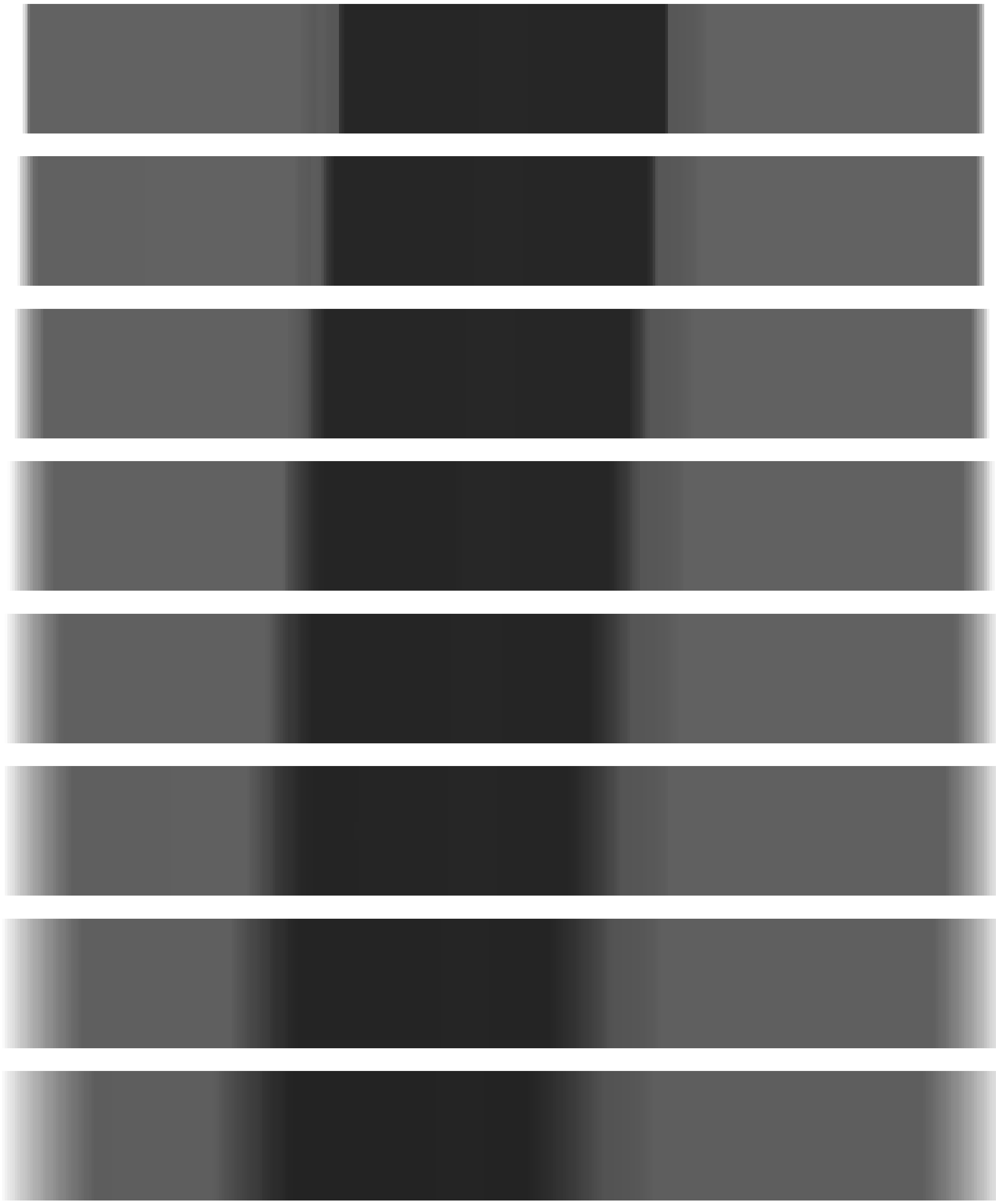
Data file for T-joint simulation, Page 1/2.

```
ELLIPTICAL CYLINDER
 1.0   0.0         0.0
 0.0   1.0         0.0
 0.0   0.0         1.0
64 -64 0
0.0075
16 16 100
 1  0 0 0.5 -1
 0 -1 0 0 -1

BOX
 1.0   0.0         0.0
 0.0   1.0         0.0
 0.0   0.0         1.0
64.5 0 0
-0.0075
2 10 100

ELLIPTICAL CYLINDER
 1.0   0.0         0.0
 0.0   1.0         0.0
 0.0   0.0         1.0
72 -72 0
-0.0075
3 2 100
```

Data file for T-joint simulation, Page 2/2.

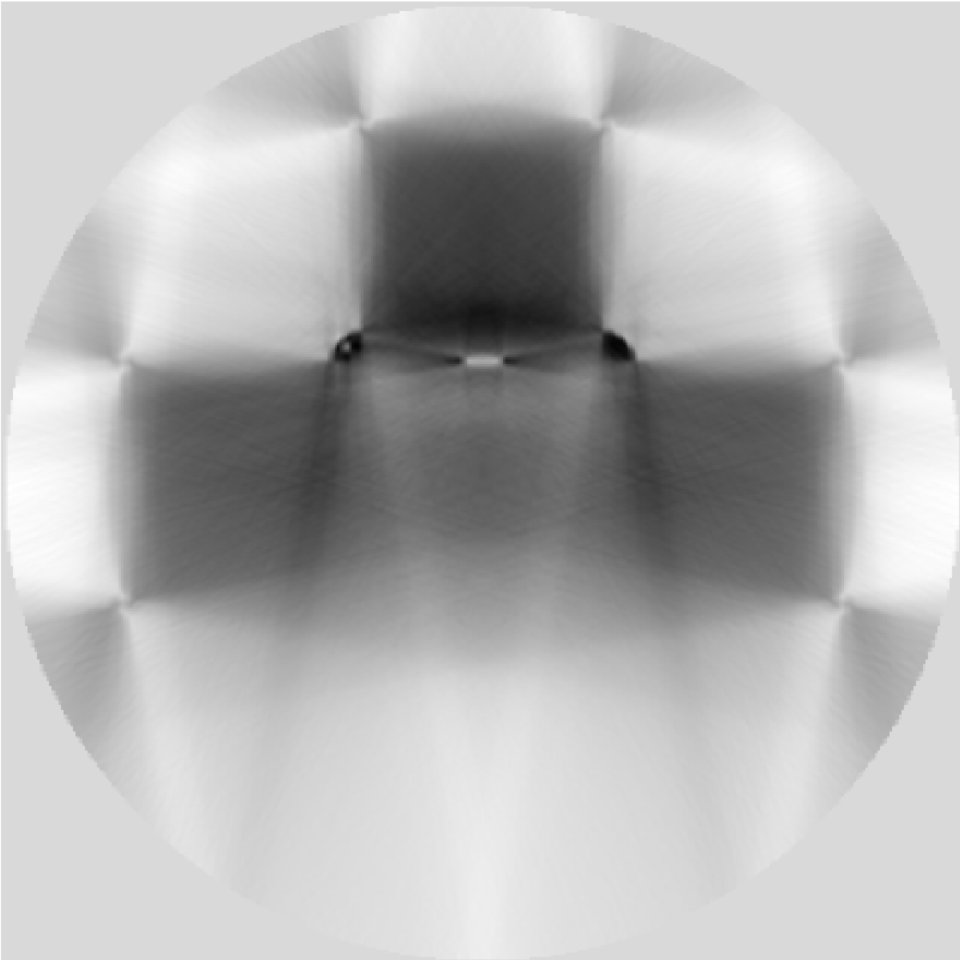


Projections at 45° increments.



Projections at 2.3° increments.

T-joint reconstruction from simulated data, 315 projections.



Reconstruction from simulated data, 20° missing angle.

Reconstruction from simulated data, 20°missing angle, lower 20% of frequency range filled with ideal data.