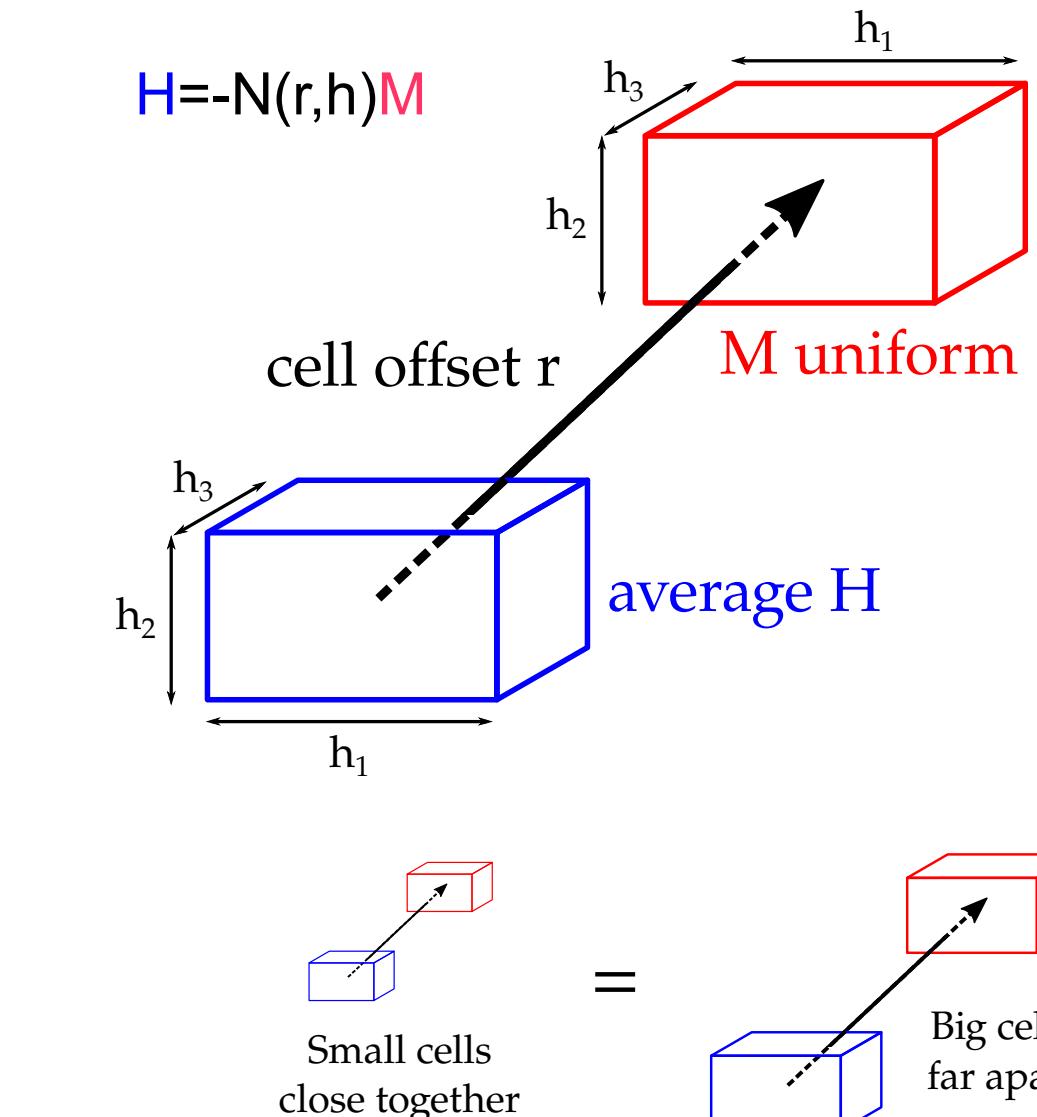


Quantitative Evaluation and Reduction of Error in Computation of the Demagnetization Tensor

Michael Donahue and Donald Porter

Mathematical Software Group, Applied and Computational Mathematics Division, Information Technology Laboratory, NIST

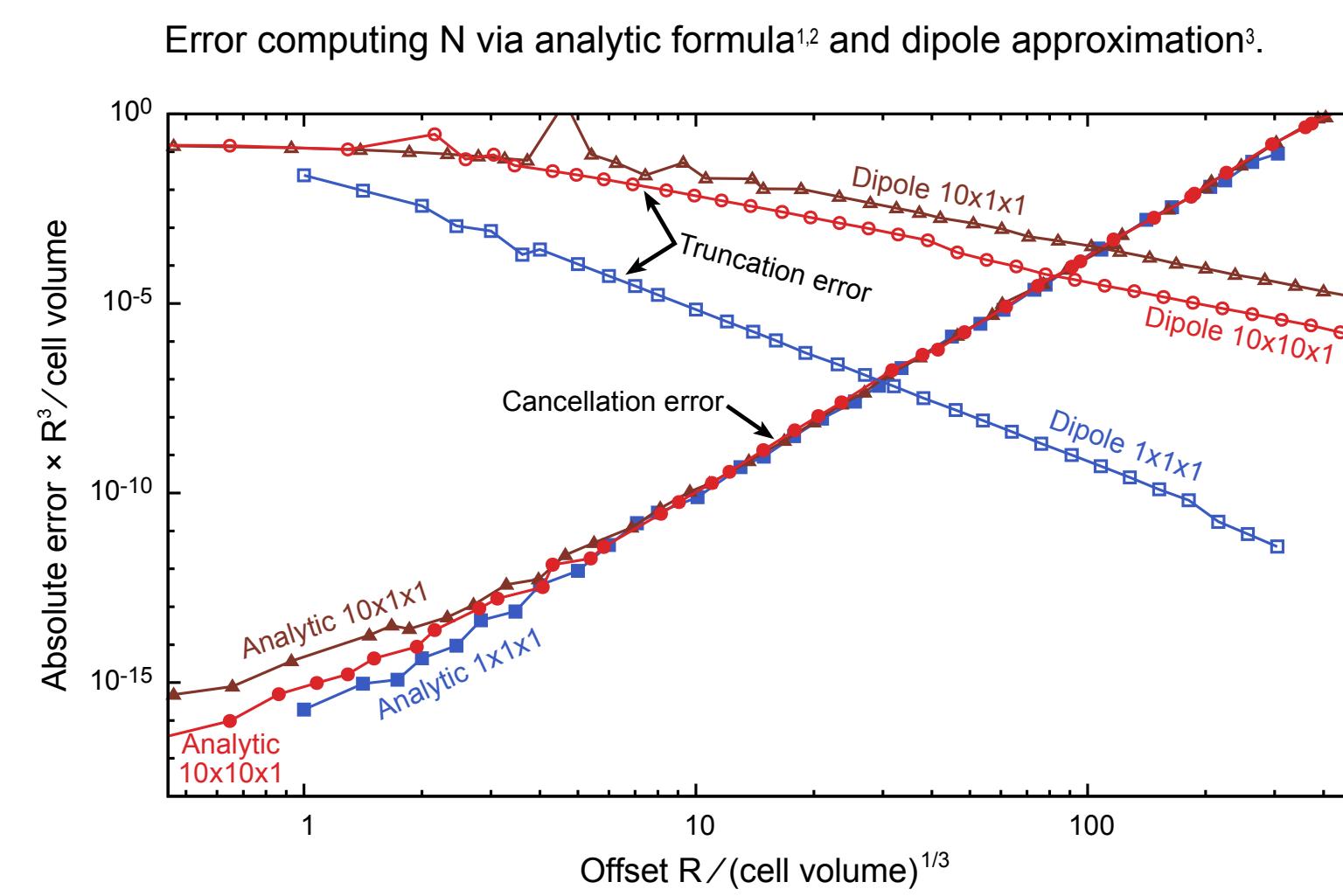
Background Work



$$-4\pi V_i N_{uv} = \iiint_{V_i} \iiint_{V_j} K_{uv}(\mathbf{r} + \mathbf{y} - \mathbf{x}) d^3y d^3x.$$

$$= \underbrace{\phi_{uv}(\mathbf{r} + \mathbf{y} - \mathbf{x})}_{\text{antiderivative}} \Big|_{V_i} \Big|_{V_j}$$

Six differences in antiderivative evaluation
⇒ six digits accuracy lost per R -decade.



Setting $\mathbf{s} = \mathbf{y} - \mathbf{x}$,

$$\begin{aligned} -4\pi V_i N_{12} &= \int_{V_i} \int_{V_j} K_{12}(\mathbf{r} + \mathbf{s}) d^3y d^3x \\ &= \int_{V_i} \int_{V_j} \frac{3(r_1 + s_1)(r_2 + s_2)}{[(r_1 + s_1)^2 + (r_2 + s_2)^2 + (r_3 + s_3)^2]^{5/2}} d^3y d^3x \\ &= \frac{3}{\|\mathbf{r}\|^5} \int_{V_i} \int_{V_j} (r_1 + s_1)(r_2 + s_2) \\ &\quad \times \sum_{k=0}^{\infty} (-1)^k \frac{(2k+3)!!}{(k!)^2} \left(\frac{2\mathbf{r} \cdot \mathbf{s} + \mathbf{s}^2}{\mathbf{r}^2} \right)^k d^3y d^3x. \end{aligned}$$

Collect powers of $O(1/\|\mathbf{r}\|^{2k+1})$ to get polynomials in \mathbf{x} and \mathbf{y} that can be integrated algebraically. Handle other N_{ij} terms the same way.

Error in asymptotic method due to truncation of asymptotic series:

$$E_{\text{asymptotic}} \approx \frac{VR^2}{5h_{\max}^3(R^2 - h_{\max}^2)} \left(\frac{h_{\max}}{R} \right)^{n+2}$$

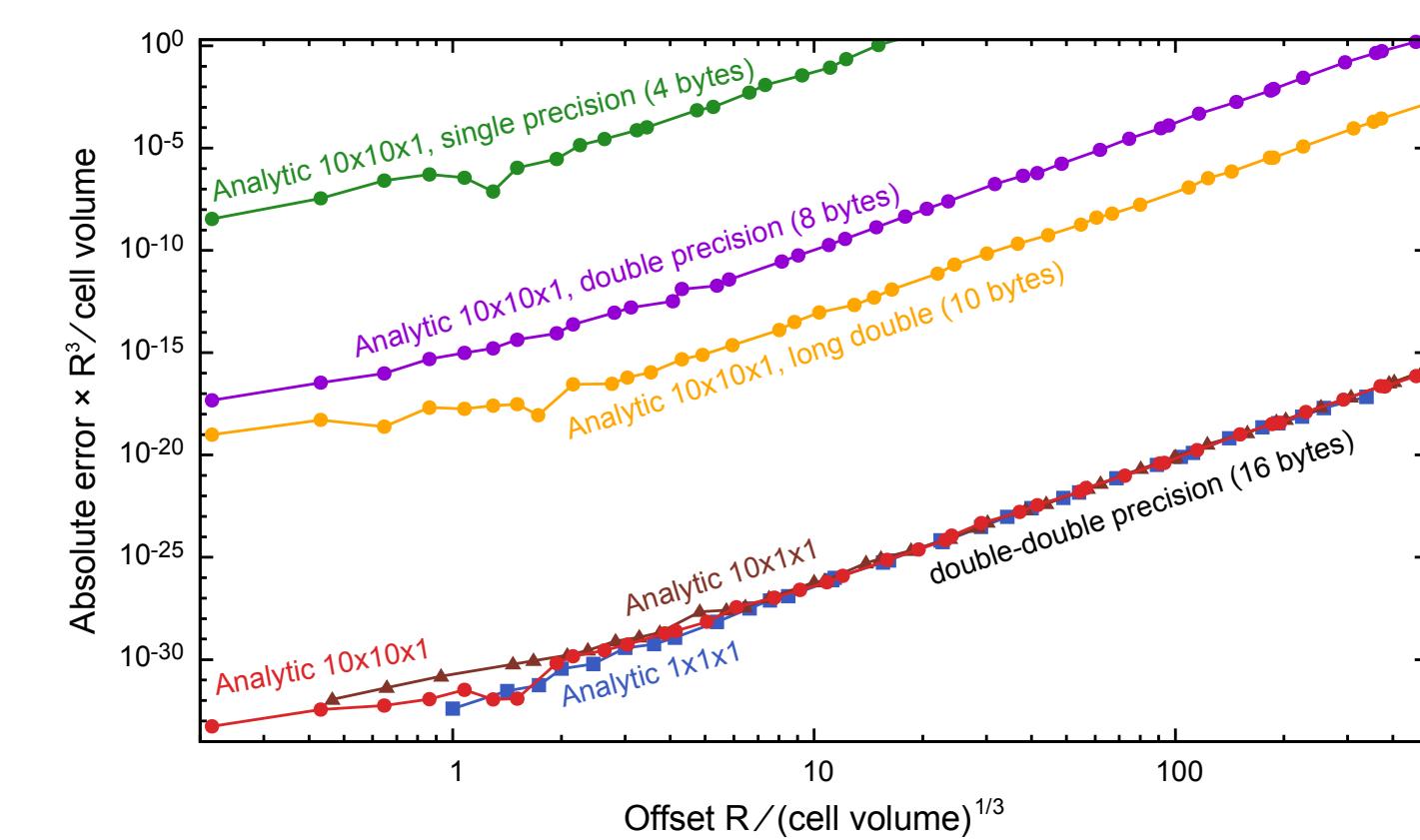
for cell volume V , max cell edge h_{\max} , asymptotic method order n .

Error in analytic evaluation due to floating point cancellation:

$$E_{\text{analytic}} \approx \epsilon R^3/V$$

with floating point precision ϵ , cell offset R , cell volume V .

High Precision Floating Point



SLOW!

Use only when necessary.

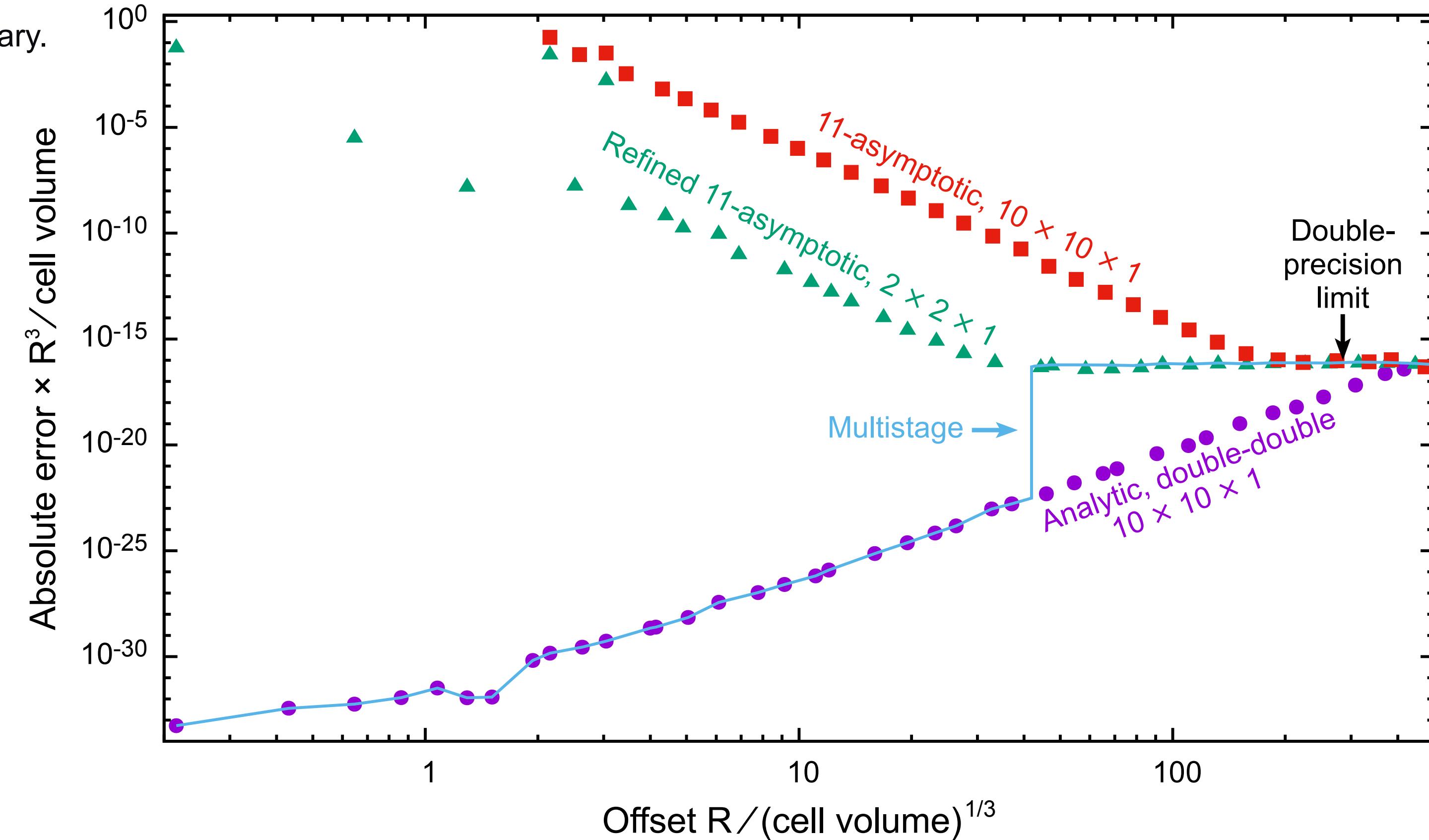
References

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- (3) K.M. Lebecki, M.J. Donahue, and M.W. Gutowski, "Periodic boundary conditions for demagnetization interactions in micromagnetic simulations," Journal of Physics D – Applied Physics, 41, 175005 (2008).

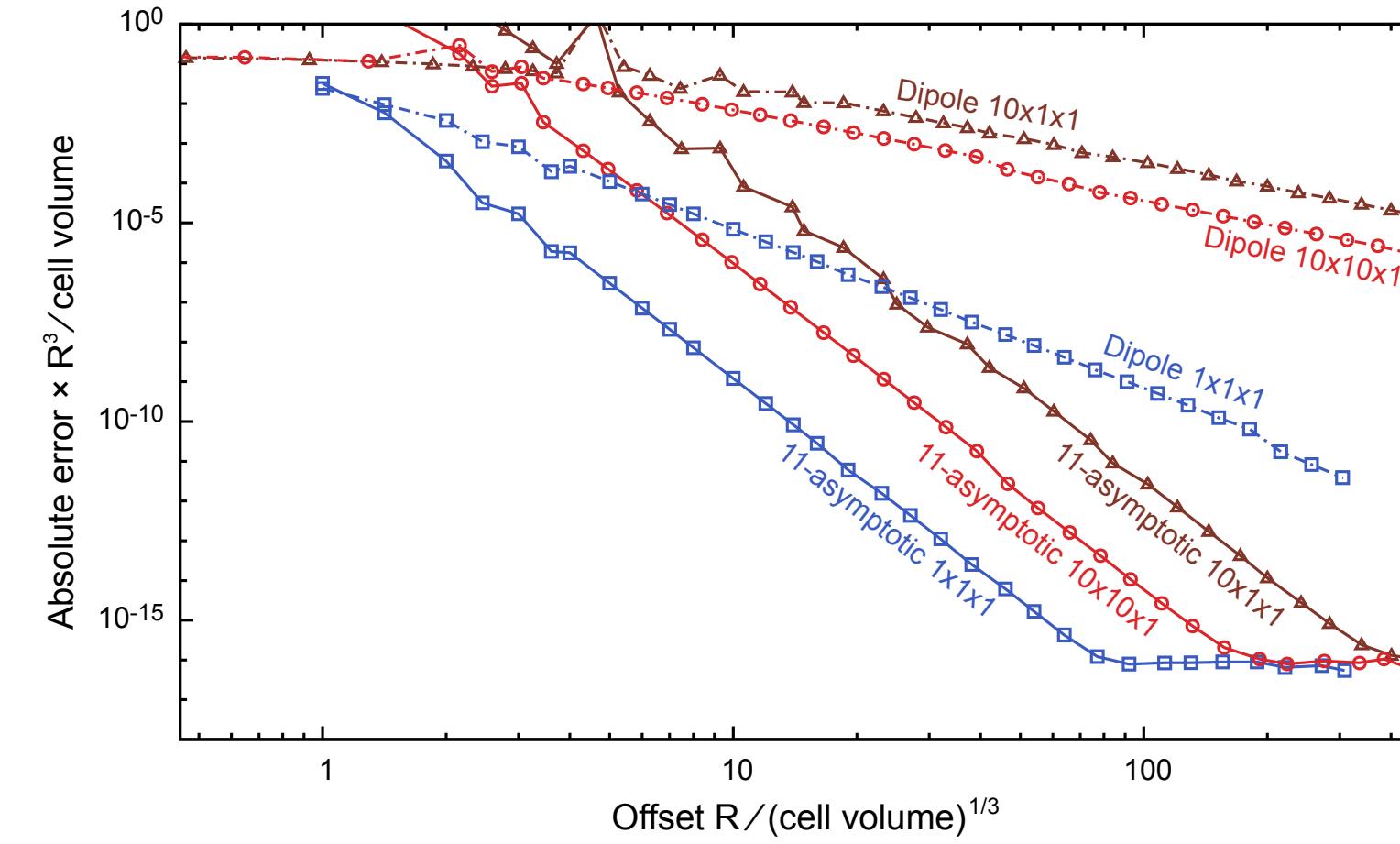
Multistage Hybrid Method

= High Precision Analytic Expression Evaluation

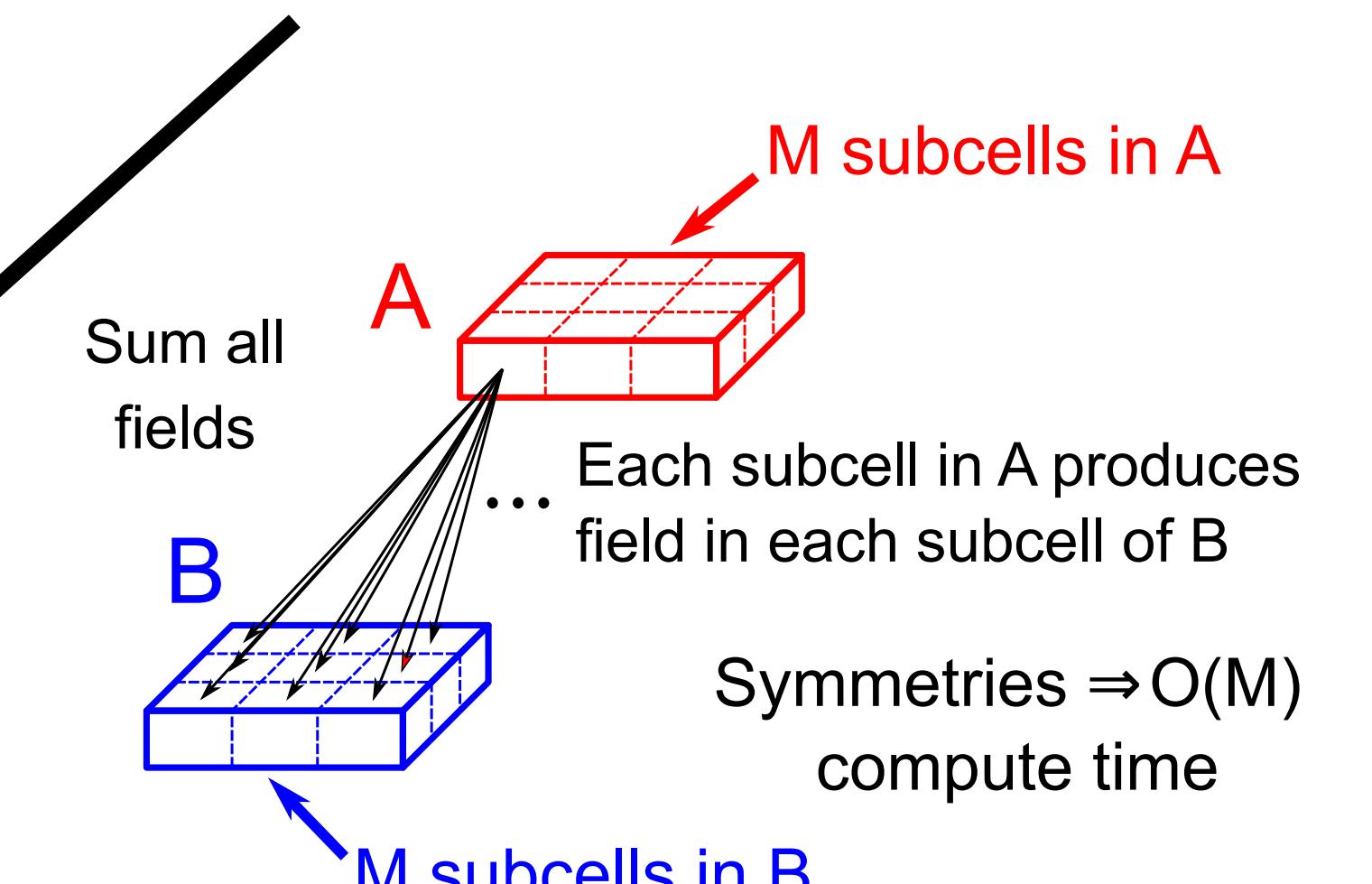
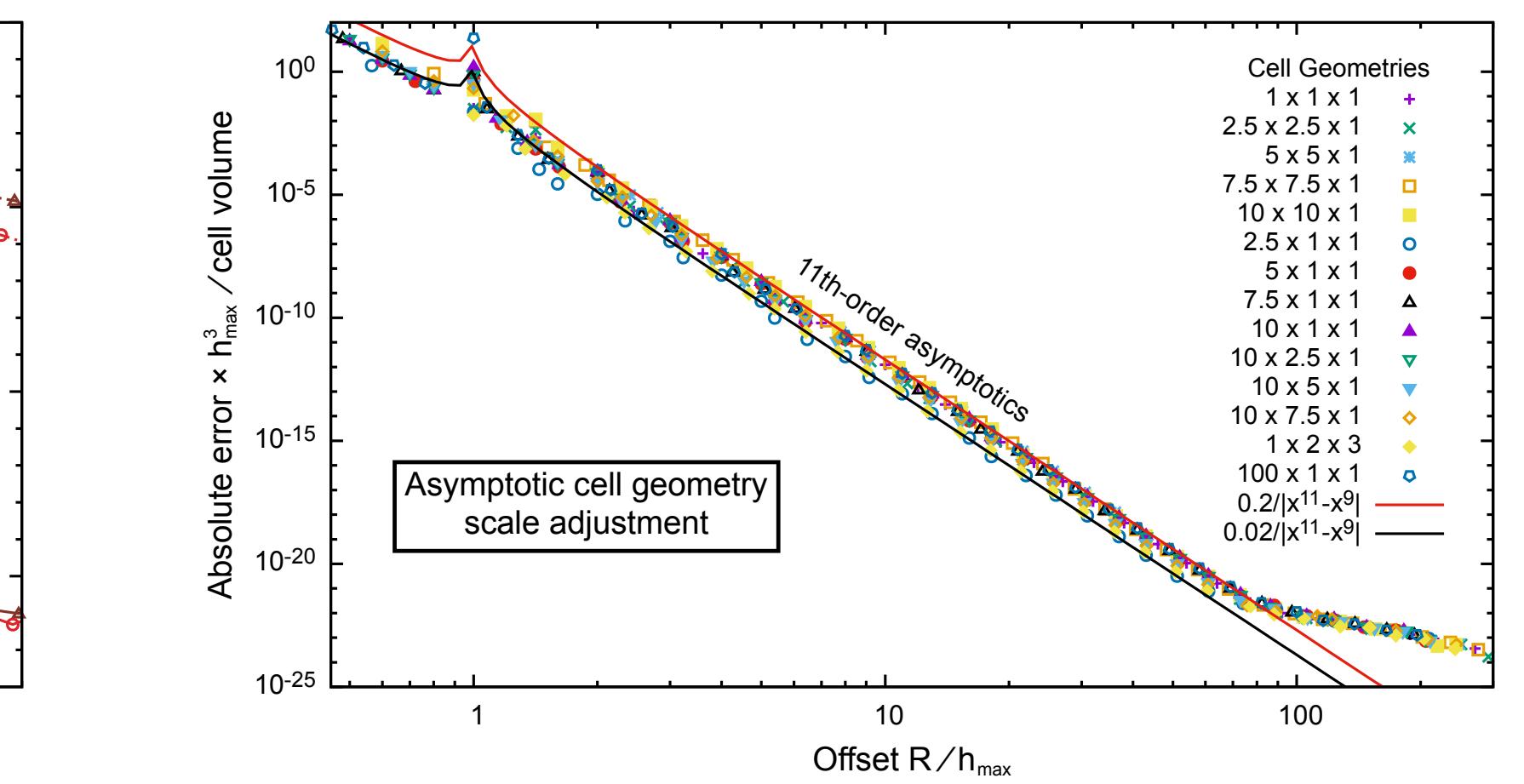
+ Refined Resolution High Order Asymptotics



High Order Asymptotics



Errors increase for cells less like cubes. Subdivision reduces error with tolerable cost.



Conclusions

- The demag tensor is used to compute self-magnetostatic fields in a micromagnetic simulation. The initialization of tensor values is a one-time cost.
- Numeric evaluation of analytical expressions for the demag tensor suffers from large cancellation error. Six decimal digits of accuracy are lost for each 10-fold increase of cell offset R . Higher precision calculations can address this.
- Dipole approximations of the tensor values converge only slowly with R , especially for non-cubic cells.
- High-order asymptotic expressions for the demag tensor are fast to compute and accurate for large R , but are sensitive to cell geometry. Accuracy can be improved by subdividing cells, but with an $O(M)$ speed penalty, where M is the number of cells in the subdivision.
- A multistage hybrid method using the analytic expressions with extra-precision arithmetic in the near field (small R), high-order asymptotic expressions with subdivided cells in the mid field (medium R), and high-order asymptotic expressions without subdivided cells in the far field (large R) can compute the demag tensor with near double-precision accuracy without excessive computation time.
- An illustrative example: a dual-core laptop can compute the demag tensor for a 2 million cell simulation in 5 seconds.