Micromagnetics on curved geometries using rectangular cells: error correction and analysis

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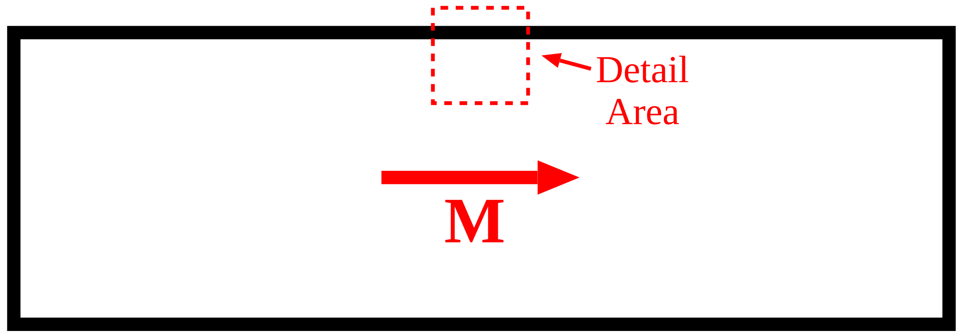
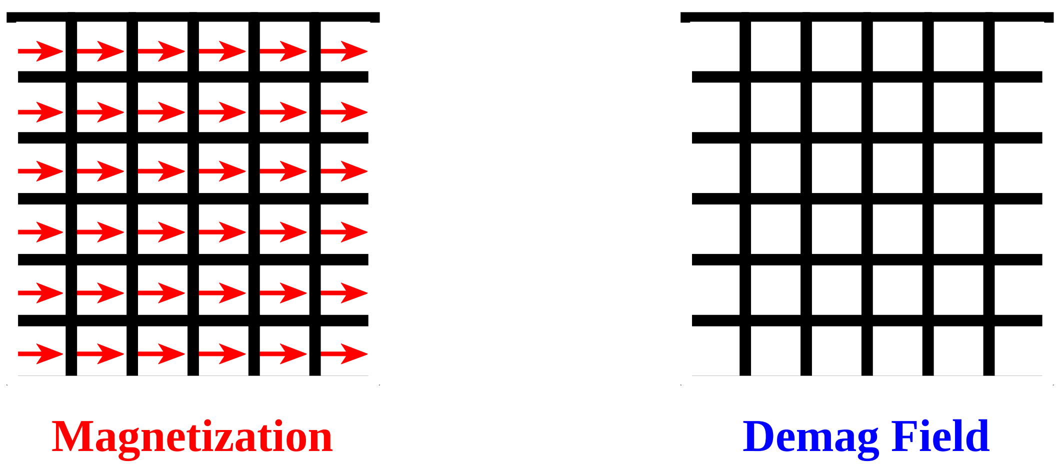
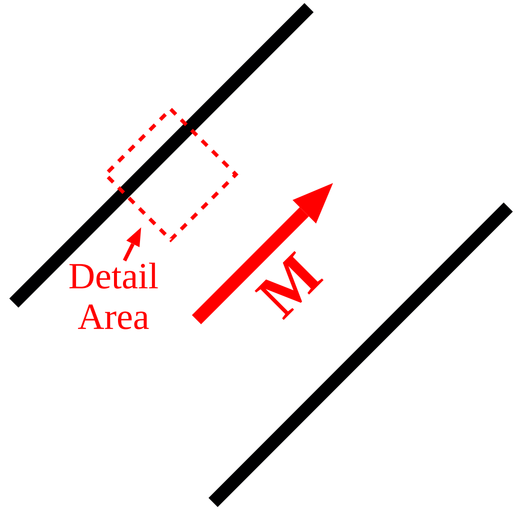
Uniformly Magnetized Strip



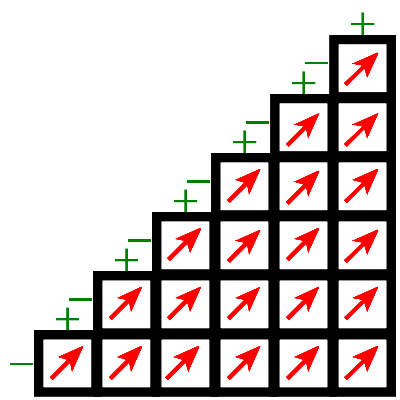
Detail



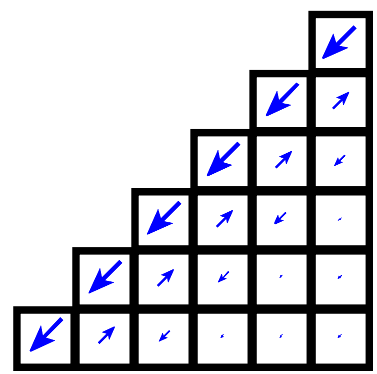
Uniformly Magnetized Strip, Rotated



Detail

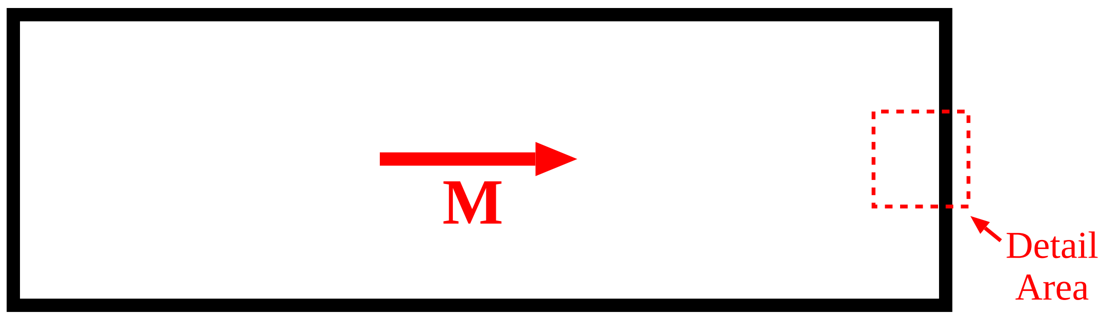


Magnetization

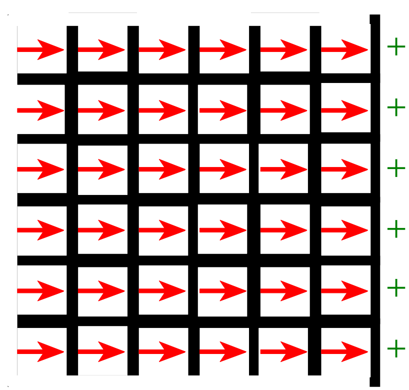


Demag Field

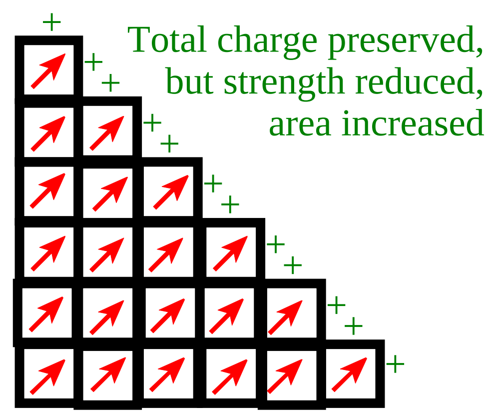
Uniformly Magnetized Strip



Magnetization Detail

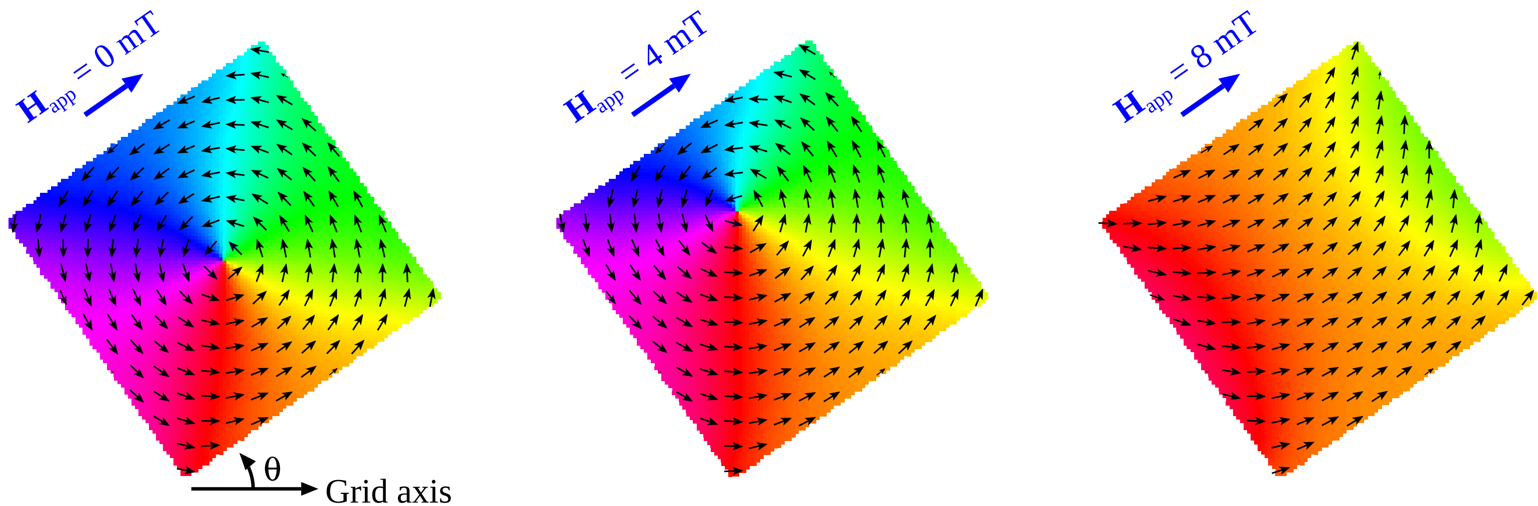


Grid Aligned



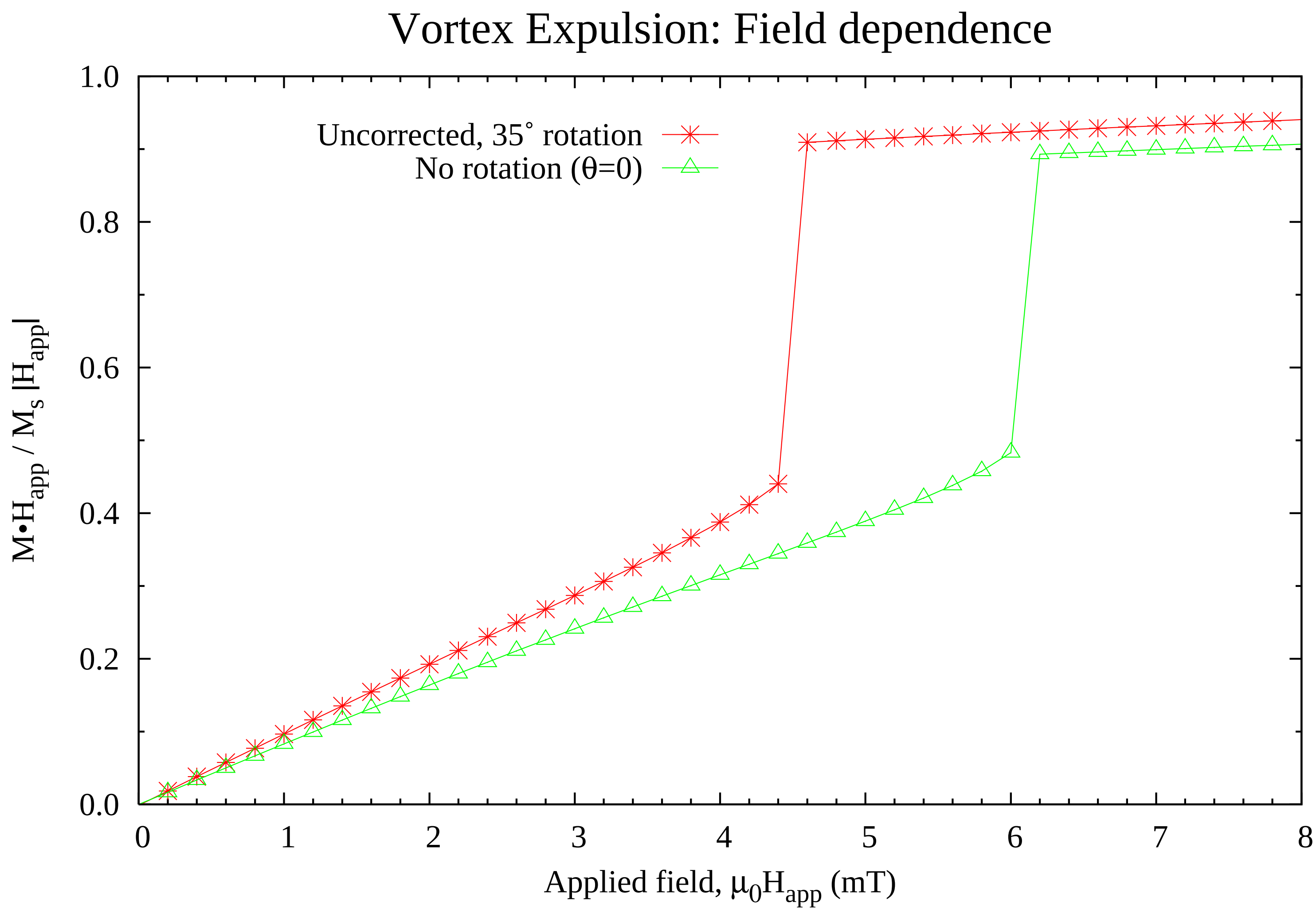
Rotated

Vortex Expulsion Test

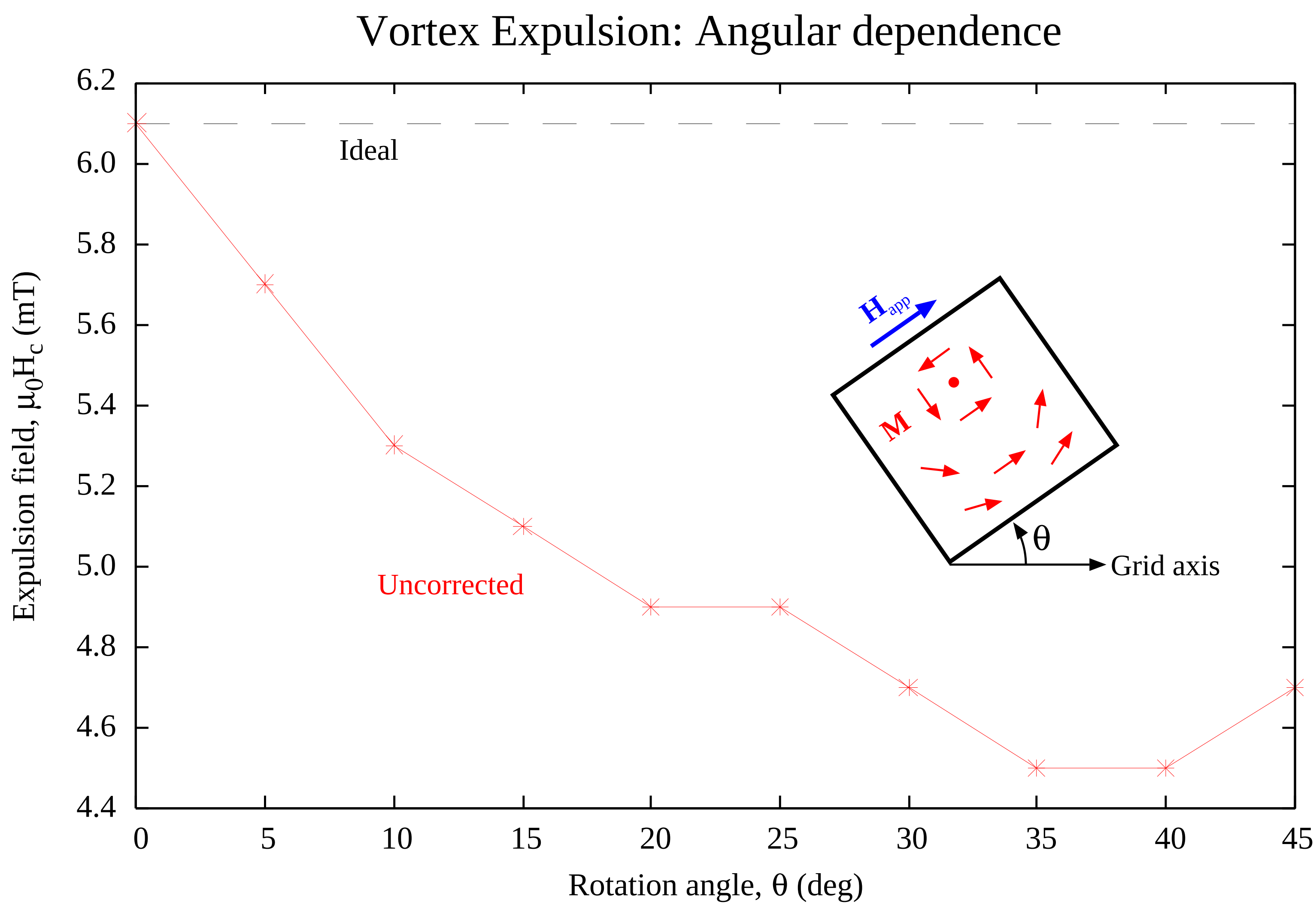


220 nm x 220 nm x 2.5 nm Py square
Cellsize Δ = 2.5 nm (cubes)

- Compute M vs. H_{app}
- Compute expulsion field H_c vs. grid angle θ

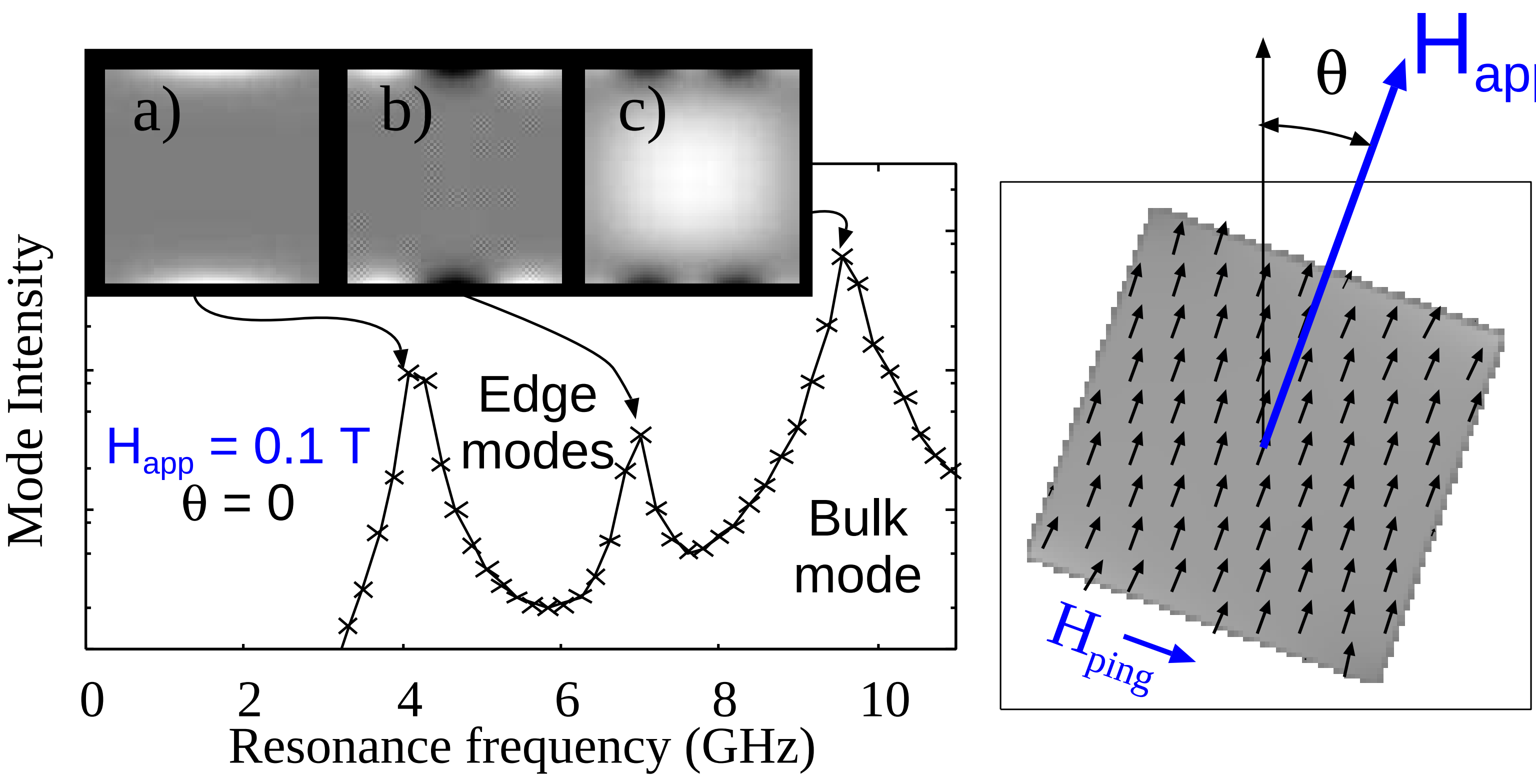


Vortex Expulsion: Field dependence
Uncorrected, 35° rotation
No rotation (θ=0)
M•H_app / M_s |H_app|
Applied field, μ_0H_app (mT)
0.0
0.2
0.4
0.6
0.8
1.0
0
1
2
3
4
5
6
7
8

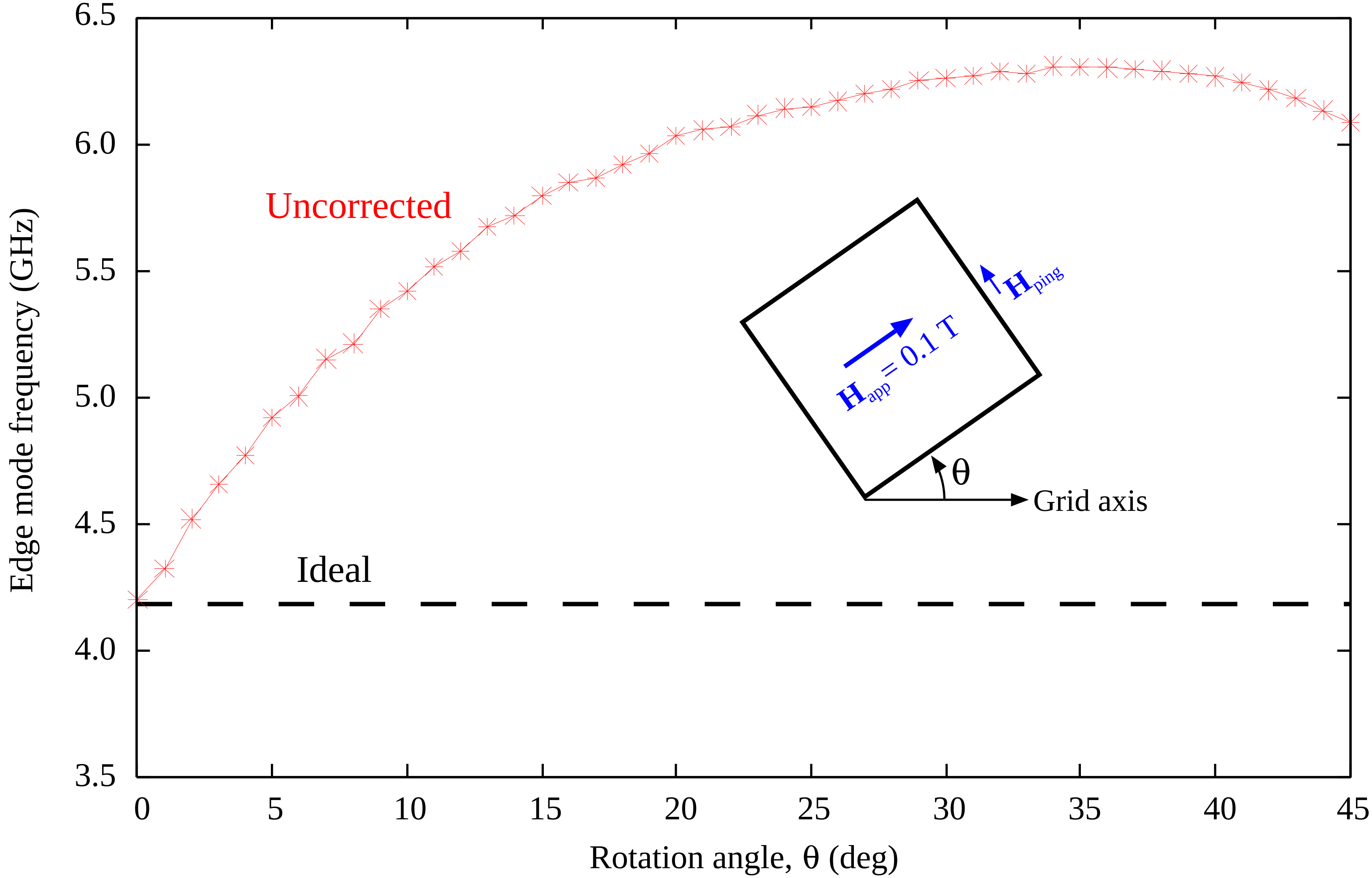


Vortex Expulsion: Angular dependence
6.2
6.0
5.8
5.6
5.4
5.2
5.0
4.8
4.6
4.4
Expulsion field, $\mu_0 H_c$ (mT)
0
5
10
15
20
25
30
35
40
45
Rotation angle, θ (deg)
Ideal
Uncorrected
$\mathbf{H}_{app}$
M
θ
Grid axis

Edge mode test



Corrections: Angular dependence



Edge mode test: key points

- Edge mode sensitive only to edge effects
- Quantitative
- Robust quantity, does not involve critical field
- Experimentally accessible

Discrete demag field

In general:

$$\mathbf{H}_{\mathrm{demag,i}} = -\sum_j N_{i,j}\mathbf{M}_j.$$

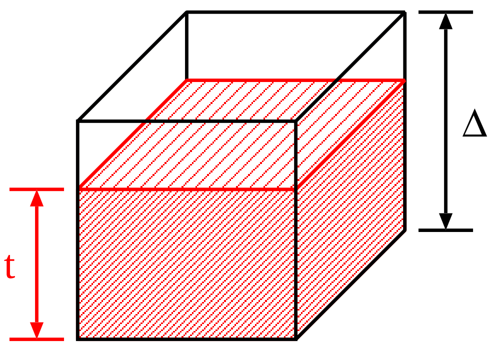
For uniform grid:

$$\mathbf{H}_{\mathrm{demag,i}} = -\sum_j N_{i-j}\mathbf{M}_j.$$

Here FFT can be used to evalute $\mathbf{H}_{\mathrm{demag}}$.

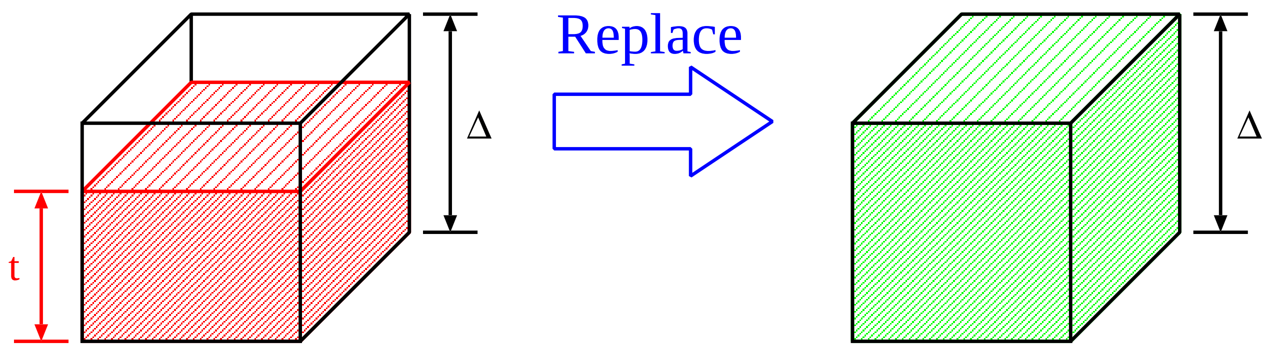
(Note: Uniform **grid**; $|M_j|$'s can vary cell-to-cell.)

PROBLEM: Partially filled cell has different geometry, so FFT can't be used to compute demag field.



Partially filled cell,
height=t, $|\mathbf{M}|=M_s$

SOLUTION: Use full cell so all cells have same geometry, but reduce M_s so far-field demag is correct.

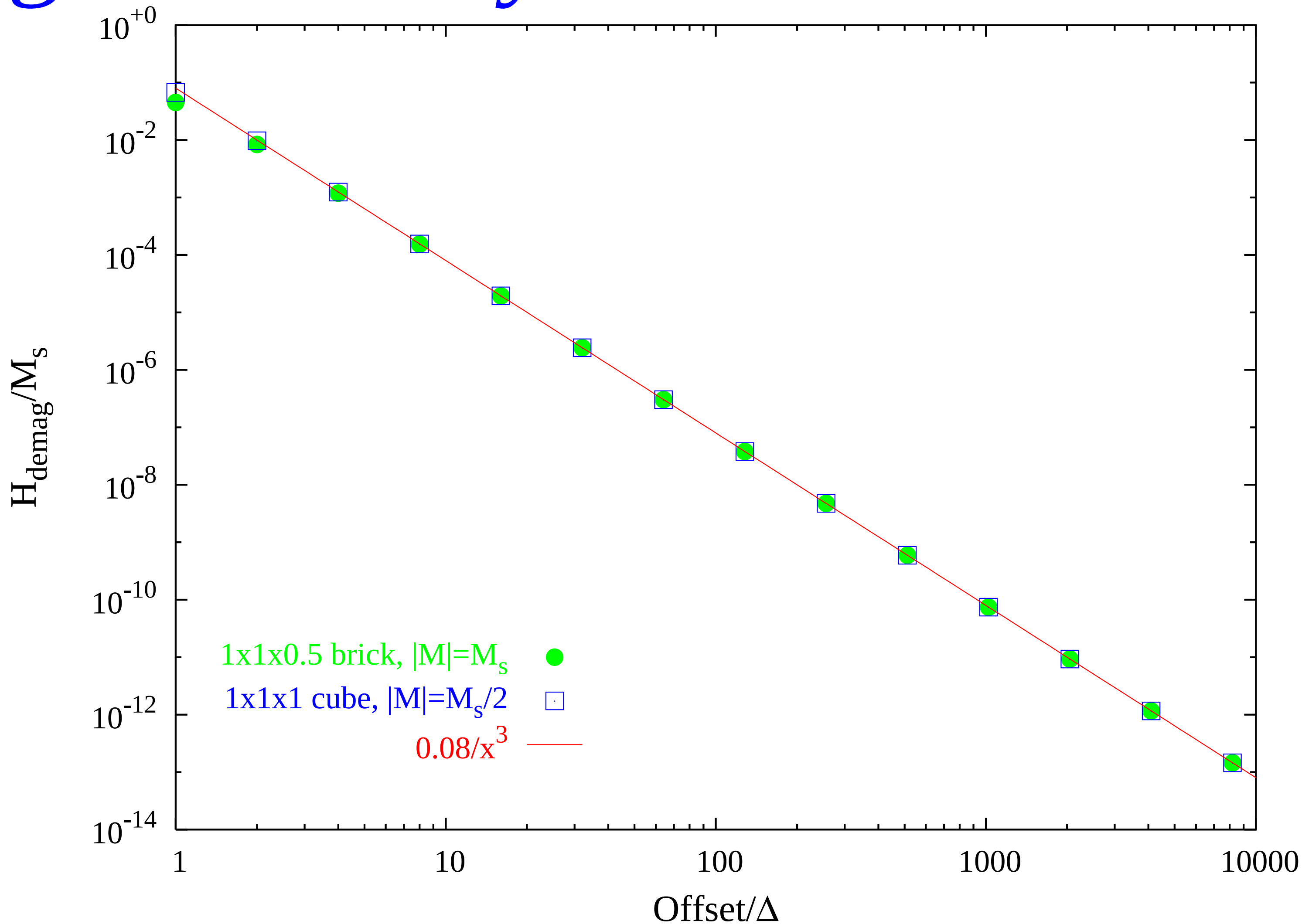


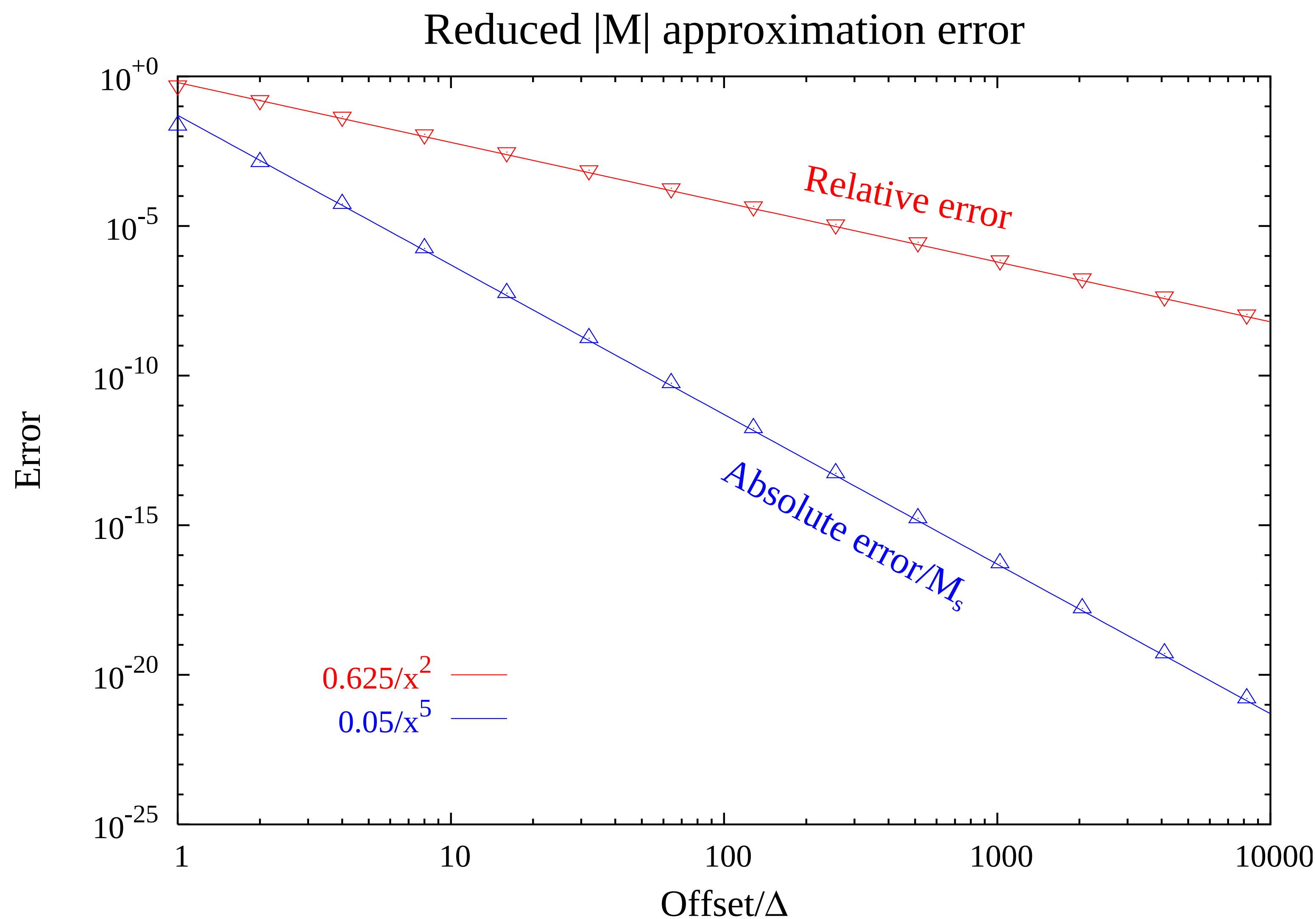
Partially filled cell, height=t, $|\mathbf{M}|=M_s$

Full cell, height=Δ
Reduced magnetization, $|\mathbf{M}|=(t/\Delta)M_s$

Porter & Donahue, "Generalization of a two-dimensional micromagnetic model to non-uniform thickness," *JAP*, **89**, 7257 (2001).

Single cell stray field





Reduced |M| approximation error
Error
Offset/Δ
10+0
10-5
10-10
10-15
10-20
10-25
1
10
100
1000
10000
Relative error
Absolute error/Ms
0.625/x2
0.05/x5

$\mathbf{H}_{\mathrm{demag}}$ *decomposition*

$$\begin{aligned}\mathbf{H}_{\mathrm{demag,i}} &= -\sum_{j} N_{i,j}\mathbf{M}_j \\ &= -\sum_{j\in\Omega_{\mathrm{local}}} N_{i,j}\mathbf{M}_j - \sum_{j\in\Omega_{\mathrm{far}}} N_{i-j}\mathbf{M}_j.\end{aligned}$$

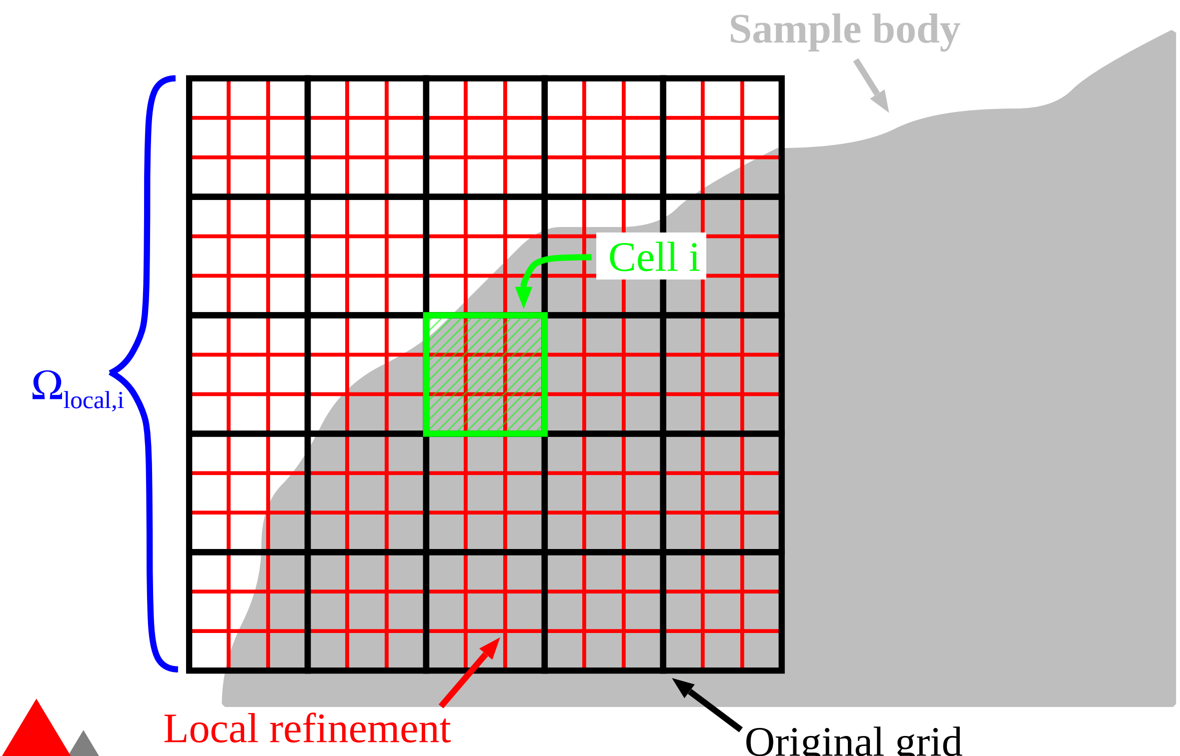
Handle Ω_{far} via modified M_s and FFT, Ω_{local} some other way.

García-Cervera, Gimbutas, & E, “Accurate numerical methods for micromagnetics simulations with general geometries,” *J. Comp. Physics*, **184**, 37 (2003).

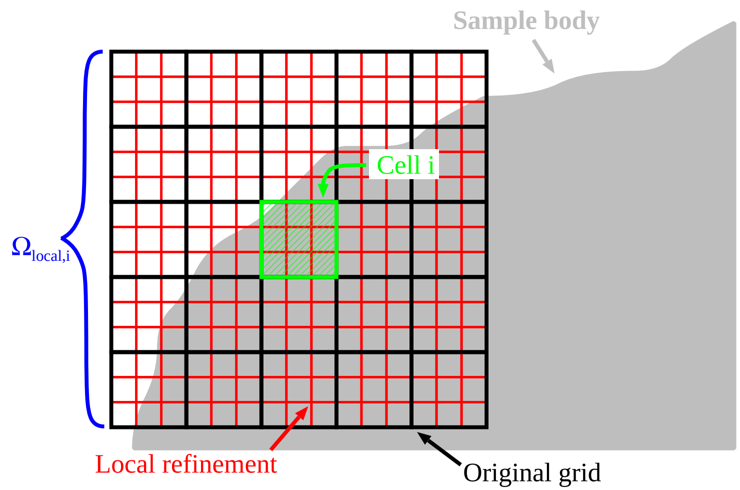
Local field computation

Problem: Computing $\mathbf{H}_{\text{demag}}$ on Ω_{local} not easy.

Idea: Use existing demag code on a local, refined grid.



Local field computation



- Compute $N^{\text{fine}}_{i'-j'}$ for fine mesh on Ω_{local} (once)
- For i, j near boundary, compute $\left\langle \mathbf{H}^{\text{fine}}_{\text{demag}} \right\rangle_{i,j}$
- $\mathbf{H}^{\text{fine}}_{\text{demag}} - \mathbf{H}^{\text{coarse}}_{\text{demag}}$ define correction factors $K_{i,j}$
- NOTE: Done once during initialization!

Local field computation

During simulation run:

- Compute $\mathbf{H}_{\mathrm{demag}}$ as usual, with volume-modified $|M|$.
- For cells near boundary, include local corrections

$$\mathbf{H}_{\mathrm{corr,i}} = -\sum_{j\in\Omega_{\mathrm{local_i}}} K_{i,j}\mathbf{M}_j$$

- Correction is $O(N_{\mathrm{boundary}})$

Local correction, pushed

$$\mathbf{H}_{\mathrm{corr,i}} = -\sum_{j \in \Omega_{\mathrm{local_i}}} K_{i,j} \mathbf{M}_j$$

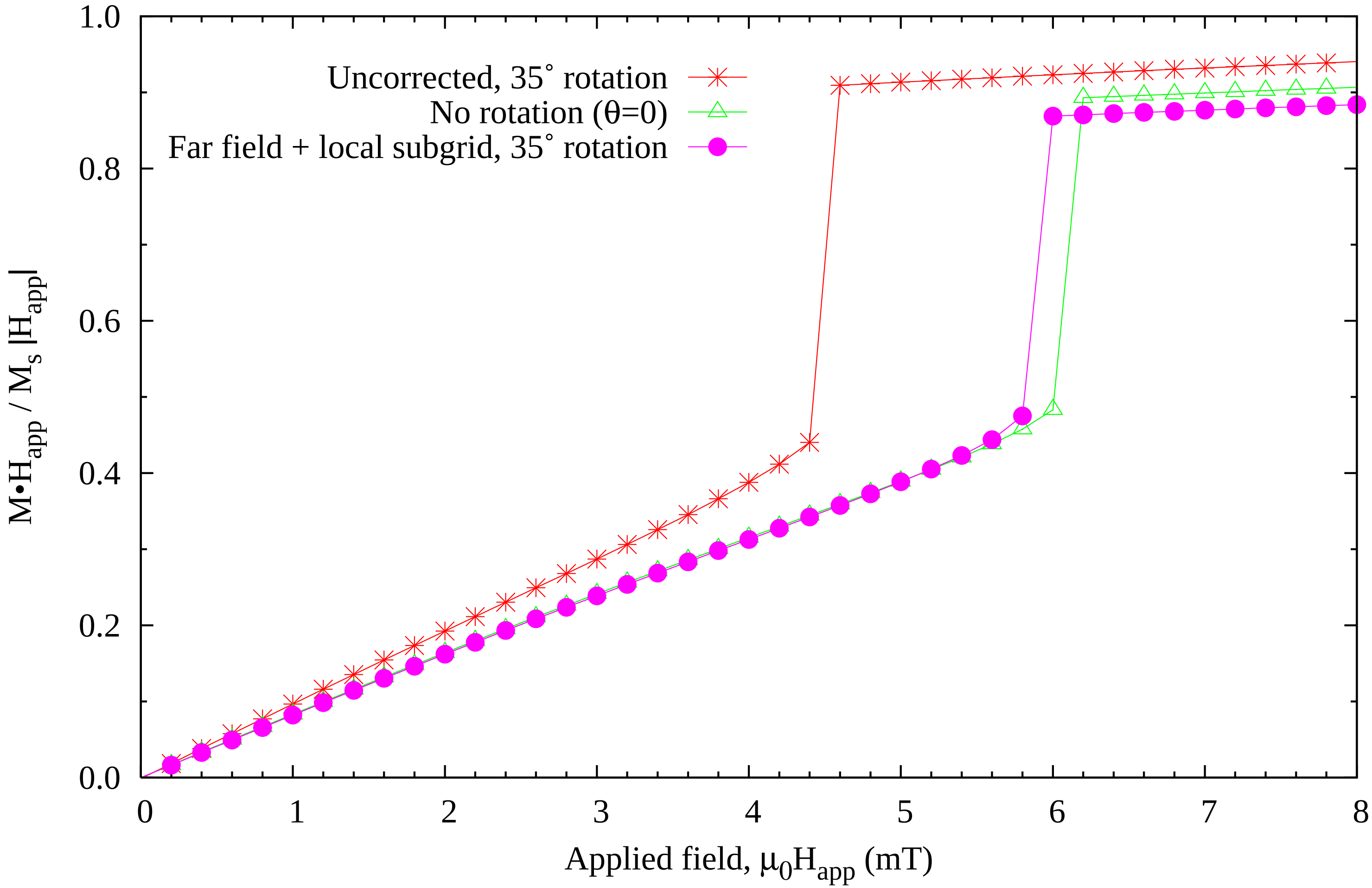
$$= \sum_{j \in \Omega_{\mathrm{local_i}}} K_{i,j} |\mathbf{M}_j| (\mathbf{m}_i - \mathbf{m}_j) - \sum_{j \in \Omega_{\mathrm{local_i}}} K_{i,j} |\mathbf{M}_j| \mathbf{m}_i$$

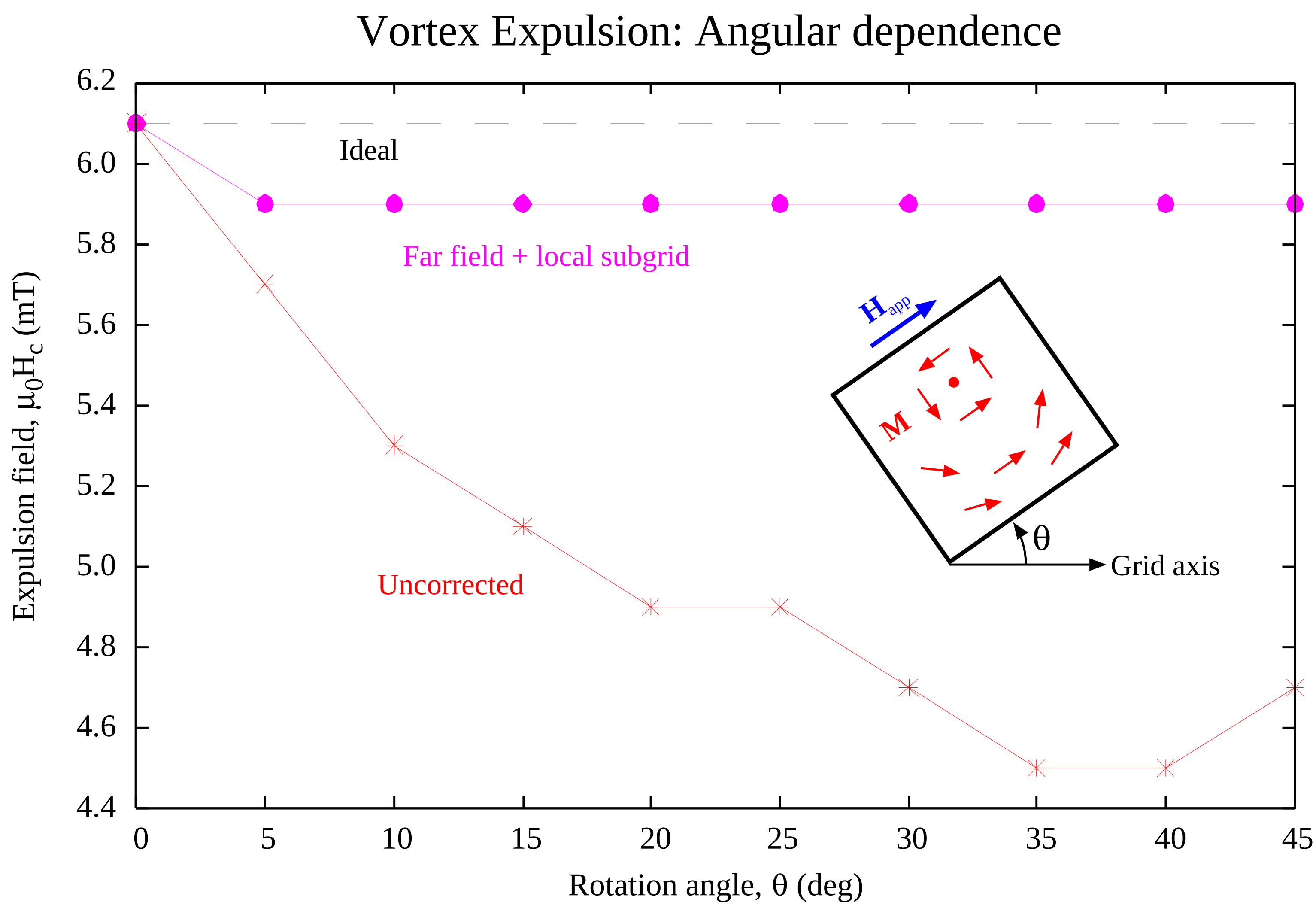
$$\approx \ -K_i \mathbf{M}_i \qquad \textbf{(if } |\mathbf{m}_i - \mathbf{m}_j| \ll 1)$$

where

$$K_i = \sum_{j \in \Omega_{\mathrm{local}_i}} \frac{|\mathbf{M}_j|}{|\mathbf{M}_i|} K_{i,j}.$$

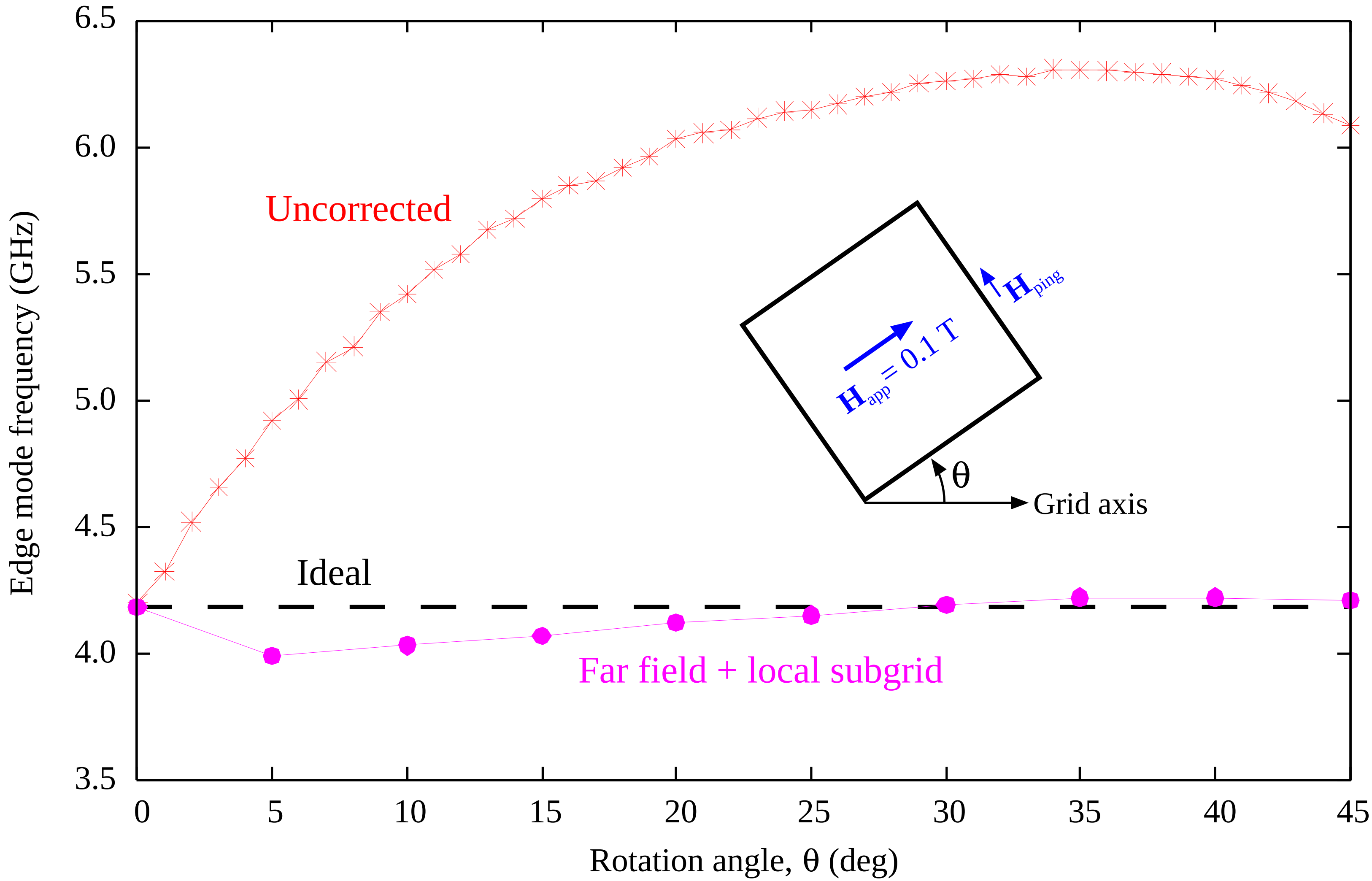
Vortex Expulsion: Field dependence





Vortex Expulsion: Angular dependence
Expulsion field, $\mu_0 H_c$ (mT)
Rotation angle, θ (deg)
6.2
6.0
5.8
5.6
5.4
5.2
5.0
4.8
4.6
4.4
0
5
10
15
20
25
30
35
40
45
Ideal
Far field + local subgrid
Uncorrected
H_{app}
M
θ
Grid axis

Corrections: Angular dependence



Summary

- Staircase artifact can be significant.
- Far field (FFT) with local correction (K_{ij} or K_i) decomposition effective and efficient.
- K_{ij} terms computed once via usual demag code on local mesh.
- Edge mode frequency test quantitative and numerically robust.