Analysis of Switching in Uniformly Magnetized Bodies

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Overview

- Assume uniform $\mathbf{M}$
  - ▷ 3D form of Stoner-Wohlfarth model
  - ▷ "Biaxial anisotropy of second degree" [1]
- Analysis of magnetization reversal
  - ▷ 1D parameterization of stationary points
  - ▷ Classification of stationary points
- Quadratically convergent iterative algorithms
  - ▷ Switching field H_s
  - ▷ Equilibrium magnetization(s) $\mathbf{M}$, given $\mathbf{H}$
- Direct calculation of coercivity H_c when $\neq H_s$
- As $|H| \uparrow H_s$:
  - ▷ Precession frequency: $f \propto (H_s - |H|)^{(1/4)}$
  - ▷ Susceptibility: $\chi \propto (H_s - |H|)^{-(1/2)}$

Energy Components...

$$
\begin{aligned}
E_{\mathrm{exch}} &= \int_V \frac{A}{M_s^2}\left(|\nabla M_x|^2 + |\nabla M_y|^2 + |\nabla M_z|^2\right) d^3r \\
E_{\mathrm{anis}} &= -\int_V \frac{K_1}{M_s^2}(\mathbf{M}\cdot\mathbf{u})^2\, d^3r \\
E_{\mathrm{Zeeman}} &= -\mu_0 \int_V \mathbf{M}\cdot\mathbf{H}_{\mathrm{applied}}\, d^3r \\
E_{\mathrm{demag}} &= -\frac{\mu_0}{2}\int_V \mathbf{M}\cdot\mathbf{H}_{\mathrm{demag}}\, d^3r
\end{aligned}
$$

where

$$
\begin{aligned}
\mathbf{H}_{\mathrm{demag}}(r) &= -\frac{1}{4\pi}\int_V \nabla\cdot\mathbf{M}(\mathbf{r}')\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}\, d^3r' \\
&\quad +\frac{1}{4\pi}\int_S \hat{\mathbf{n}}\cdot\mathbf{M}(\mathbf{r}')\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}\, d^2r'
\end{aligned}
$$

... for Uniform M

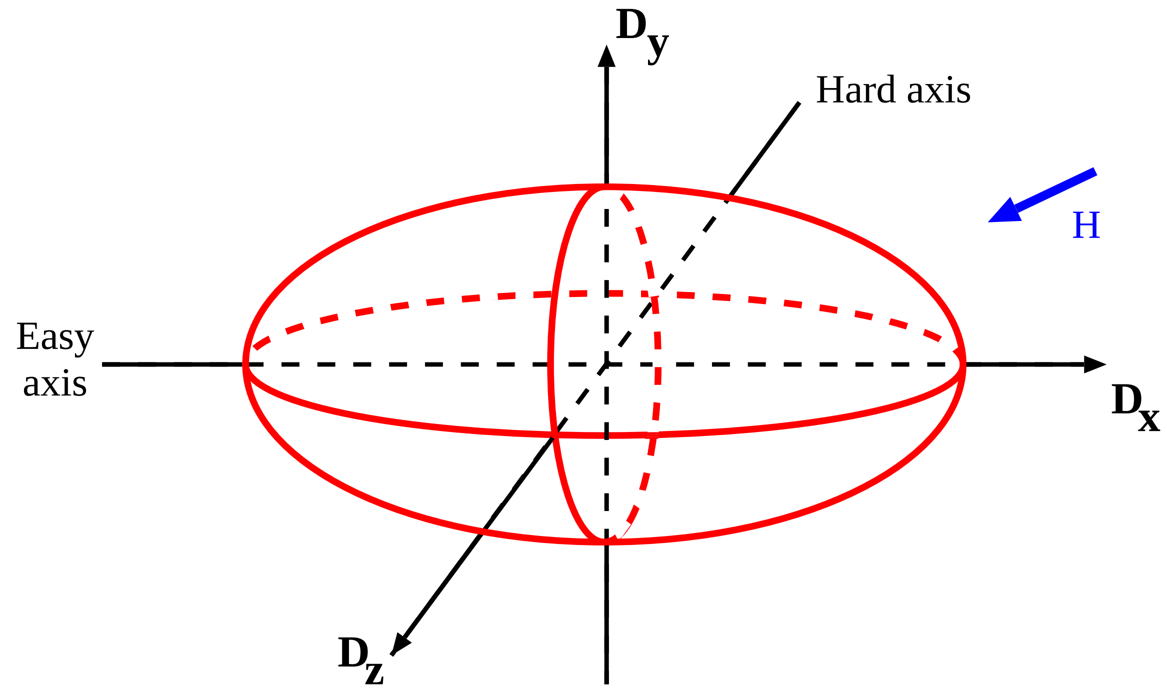
$$
\begin{aligned}
E_{\text{exch}} &= 0 \\
E_{\text{anis}} &= \mathbf{M}^{\mathrm{T}} \left[-\frac{1}{M_s^2} \int_V K_1 \mathbf{u}\mathbf{u}^{\mathrm{T}} \, d^3 r \right] \mathbf{M} \\
E_{\text{Zeeman}} &= \left[-\mu_0 \int_V \mathbf{H}_{\text{applied}}^{\mathrm{T}} \, d^3 r \right] \mathbf{M} \\
E_{\text{demag}} &= \mathbf{M}^{\mathrm{T}} K \mathbf{M}
\end{aligned}
$$

where

$$
K = -\frac{\mu_0}{8\pi} \int_V \int_S \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} \hat{\mathbf{n}}^{\mathrm{T}} \, d^2 r' \, d^3 r
$$

$$
\boxed{\Rightarrow \quad E_{\text{total}} = (\mu_0/2)\mathbf{M}^{\mathrm{T}} D \mathbf{M} - \mu_0 \mathbf{H}_{\text{applied}}^{\mathrm{T}} \mathbf{M}}
$$

Lagrange Analysis



$$E = (\mu_0/2)\mathbf{M}^T D\mathbf{M} - \mu_0\mathbf{H}^T\mathbf{M}$$

- Choice of coordinates:
- $D = \begin{bmatrix} D_x & 0 & 0 \\ 0 & D_y & 0 \\ 0 & 0 & D_z \end{bmatrix}, \qquad D_x < D_y < D_z$
- Let $\mathbf{M} = M\mathbf{m}$ and $\mathbf{H} = H\mathbf{h}$.
- Fix M and $\mathbf{h}$. Vary H.
- How does $\mathbf{m}$ rotate in response?

Lagrange Analysis

- Constrained optimization problem
- Apply multiplier $(\mu_0\lambda)$ to constraint $|\mathbf{M}|^2 = M^2$

$$\nabla_{\mathbf{M}} E = \mu_0 D\mathbf{M} - \mu_0 \mathbf{H} = (\mu_0\lambda)\mathbf{M}$$

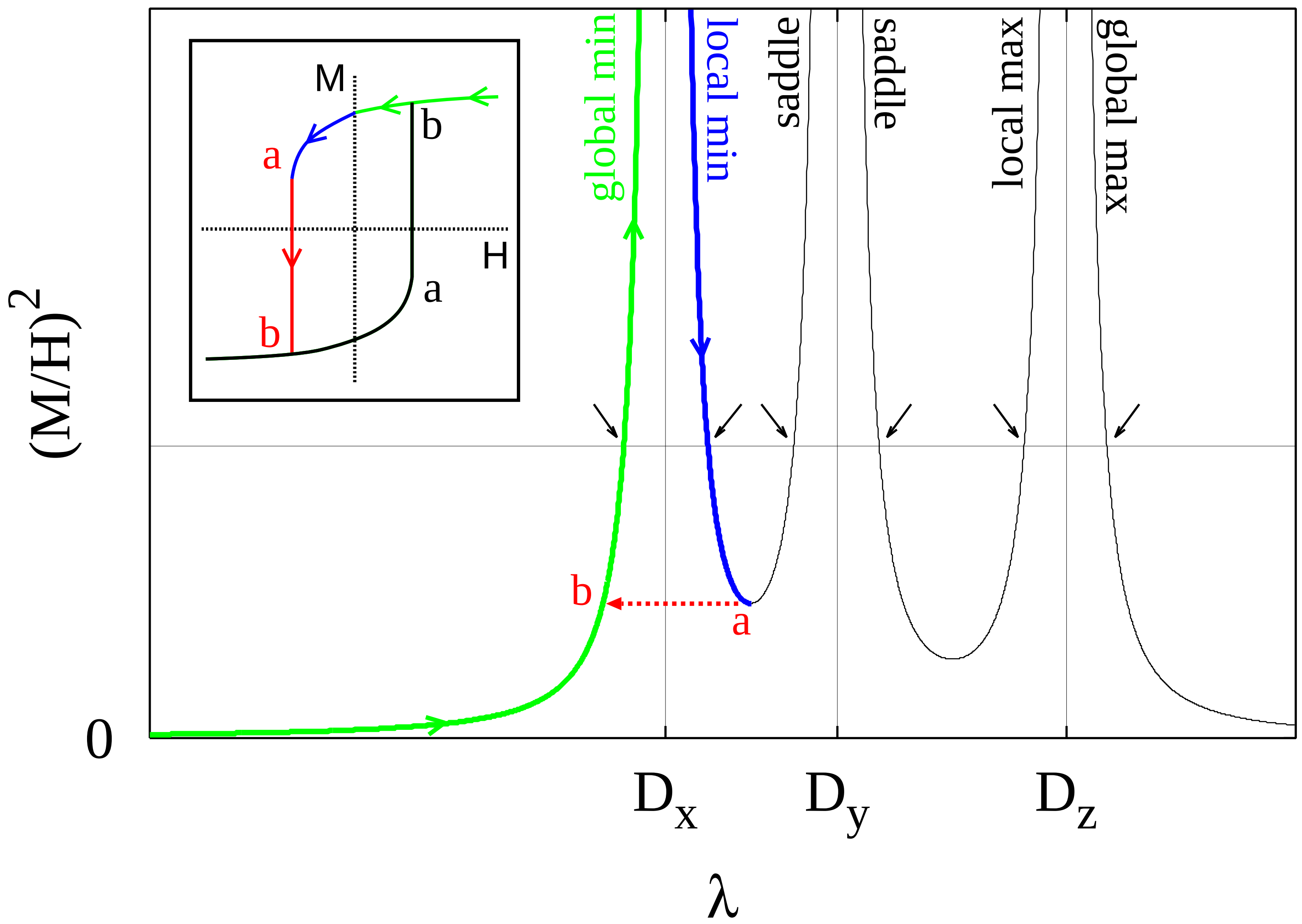
- Solve for $\mathbf{M}$

$$\mathbf{M}(\lambda) = M\mathbf{m}(\lambda) = (D - \lambda I)^{-1}\mathbf{H}$$

- Substitute into constraint

$$\begin{aligned}\left(\frac{M}{H}\right)^2 &= \frac{h_x^2}{(D_x-\lambda)^2} + \frac{h_y^2}{(D_y-\lambda)^2} + \frac{h_z^2}{(D_z-\lambda)^2} \\ &\triangleq g(\lambda)\end{aligned}$$

Plot of $g(\lambda)$



Classification of Stationary Points

- Let λ^* be solution of $g(\lambda^*) = (M/H)^2$. Define:

$$\Phi(\mathbf{R}) \stackrel{\triangle}{=} \mathbf{R}^T \left(D - \lambda^* I\right) \mathbf{R}$$

- Let $\mathbf{m}^* = \mathbf{m}(\lambda^*)$. Then,

$$\Delta E = E(\mathbf{m}) - E(\mathbf{m}^*) = \frac{\mu_0}{2} M^2 \, \Phi(\mathbf{m} - \mathbf{m}^*)$$

- $\lambda^* < D_x$: $\Delta E > 0$; global minimum
- $\lambda^* > D_z$: $\Delta E < 0$; global maximum
- $D_x < \lambda^* < D_z$: examine $\Phi(\mathbf{R})$ near $\mathbf{R} = 0$.

$$\Phi(\mathbf{R}) = R^2 \Phi(\mathbf{r}); \qquad |\mathbf{r}| = 1, \; \mathbf{r} \cdot \mathbf{m} = 0.$$

- Let $a_1 = \min \Phi(\mathbf{r})$.
- Let $a_2 = \max \Phi(\mathbf{r})$.

Classification of Stationary Points

- Another constrained optimization problem

$$a_1 a_2 = \frac{H^2}{2M^2}(D_x - \lambda^*)(D_y - \lambda^*)(D_z - \lambda^*)g'(\lambda^*)$$

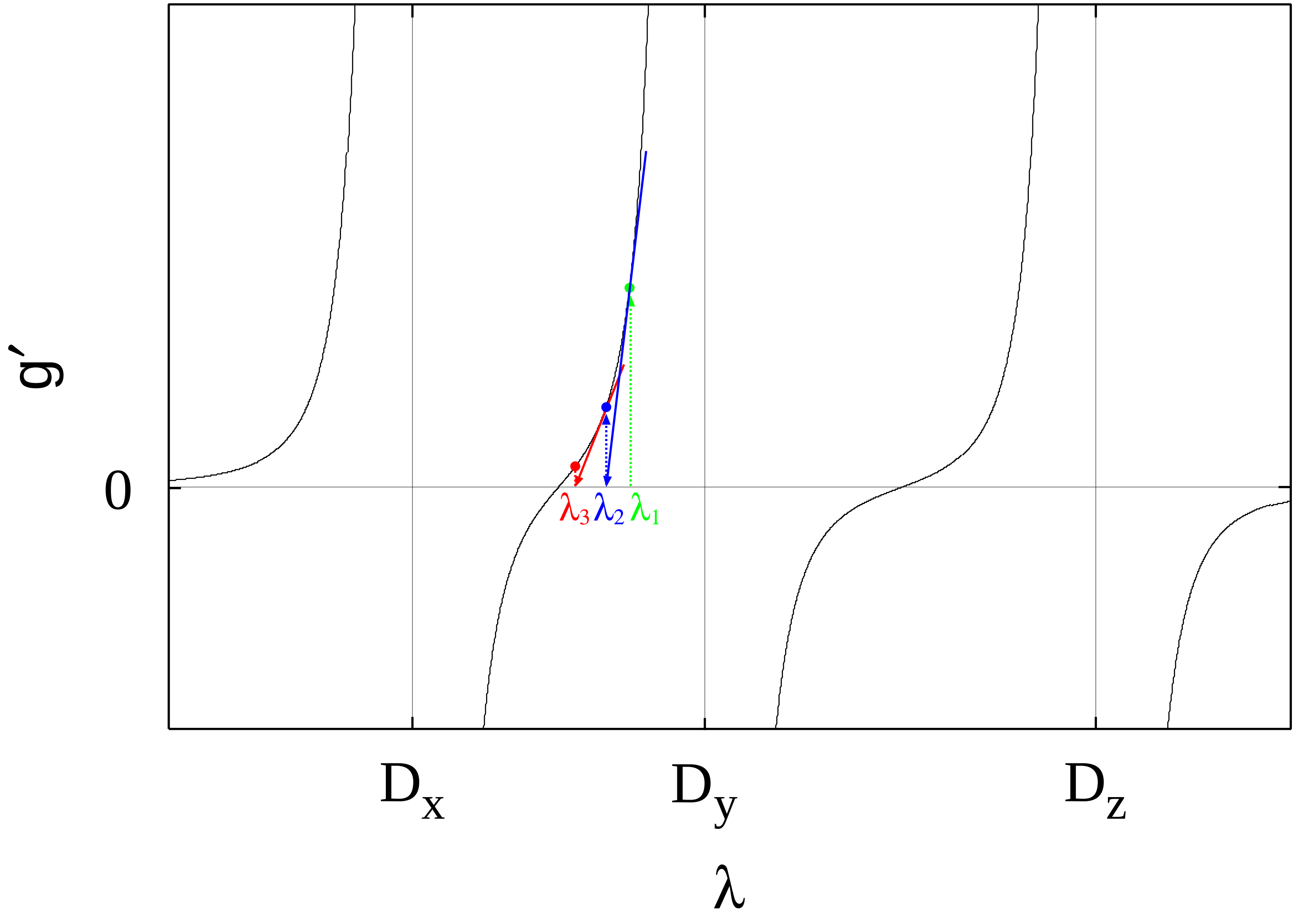
$$a_1 + a_2 = D_x + D_y + D_z - 3\lambda^* - \frac{\mathbf{H}^T \mathbf{m}^*}{M}$$

- "Interlacing property" [2, 3]

$$D_x - \lambda^* \le a_1 \le D_y - \lambda^* \le a_2 \le D_z - \lambda^*$$

- $D_x < \lambda^* < D_y$: $a_2 > 0$
  - ▷ $g'(\lambda^*) < 0 \Rightarrow 0 < a_1 < a_2 \Rightarrow$ local minimum
  - ▷ $g'(\lambda^*) > 0 \Rightarrow a_1 < 0 < a_2 \Rightarrow$ saddle point
- $D_y < \lambda^* < D_z$: $a_1 < 0$
  - ▷ $g'(\lambda^*) < 0 \Rightarrow a_1 < 0 < a_2 \Rightarrow$ saddle point
  - ▷ $g'(\lambda^*) > 0 \Rightarrow a_1 < a_2 < 0 \Rightarrow$ local maximum

Plot of $g'(\lambda)$



Switching Field H_s

- Switching ($\texttt{a} \to \texttt{b}$) when $|\mathbf{H}| = H_s$
- Point $\texttt{a}$: $(\lambda_s, (M/H_s)^2)$, where

$$D_x < \lambda_s < D_y; \qquad g'(\lambda_s) = 0$$

- Newton's method iteration converges to λ_s

$$\lambda_{n+1} = \lambda_n - g'(\lambda_n)/g''(\lambda_n)$$

$$\lambda_{n+1} = \lambda_n - \frac{1}{3}\frac{\mathbf{h}^T (D - \lambda_n I)^{-3}\,\mathbf{h}}{\mathbf{h}^T (D - \lambda_n I)^{-4}\,\mathbf{h}}$$

- From λ_s and $g(\cdot)$, compute H_s

Equilibrium Magnetizations

- Each λ^* solves $g(\lambda^*) - (M/H)^2 = 0$
- Newton's method iteration converges to λ^*

$$\lambda_{n+1} = \lambda_n - \left(g(\lambda_n) - (M/H)^2\right)/g'(\lambda_n)$$

$$\lambda_{n+1} = \lambda_n - \frac{\mathbf{h}^T\,(D - \lambda_n I)^{-2}\,\mathbf{h} - (M/H)^2}{2\,\mathbf{h}^T\,(D - \lambda_n I)^{-3}\,\mathbf{h}}$$

- From λ^* and $\mathbf{H}$, compute $\mathbf{m}^*$
- Different λ intervals for $\mathbf{m}^*$ min, max, etc.

Coercive Field H_c

- When does $\mathbf{M} \cdot \mathbf{H}$ change sign?
  - ▷ Applied field direction near easy axis
    - ∗ Irreversible sign change
    - ∗ Only during switching (at H_s)
  - ▷ Applied field direction far from easy axis
    - ∗ Reversible sign change
    - ∗ Coercive field $|\mathbf{H}_c| < H_s$ exists

$$\mathbf{H}_c \cdot \mathbf{M}(\lambda_c) = 0$$

$$\frac{h_x^2}{D_x - \lambda_c} + \frac{h_y^2}{D_y - \lambda_c} + \frac{h_z^2}{D_z - \lambda_c} = 0$$

- Quadratic equation in λ_c
- Roots: $D_x < \lambda_1 < D_y < \lambda_2 < D_z$
- From λ_1 and $g(\cdot)$, compute H_c

Precession Frequency f

- LLG dynamics near equilibrium
- Precession frequency f is a function of the energy surface curvature[4]:

$$f = \frac{\gamma\sqrt{1+\alpha^2}M\sqrt{a_1 a_2}}{2\pi}$$

where

$$\gamma = \text{gyromagnetic ratio}$$
$$\alpha = \text{damping coefficient}$$

- For λ near λ_s

$$g'(\lambda) = 2M\sqrt{\frac{g''(\lambda_s)(H_s - |H|)}{H_s^3}} + \mathcal{O}(H_s - |H|)$$

- Thus as $|H| \uparrow H_s$

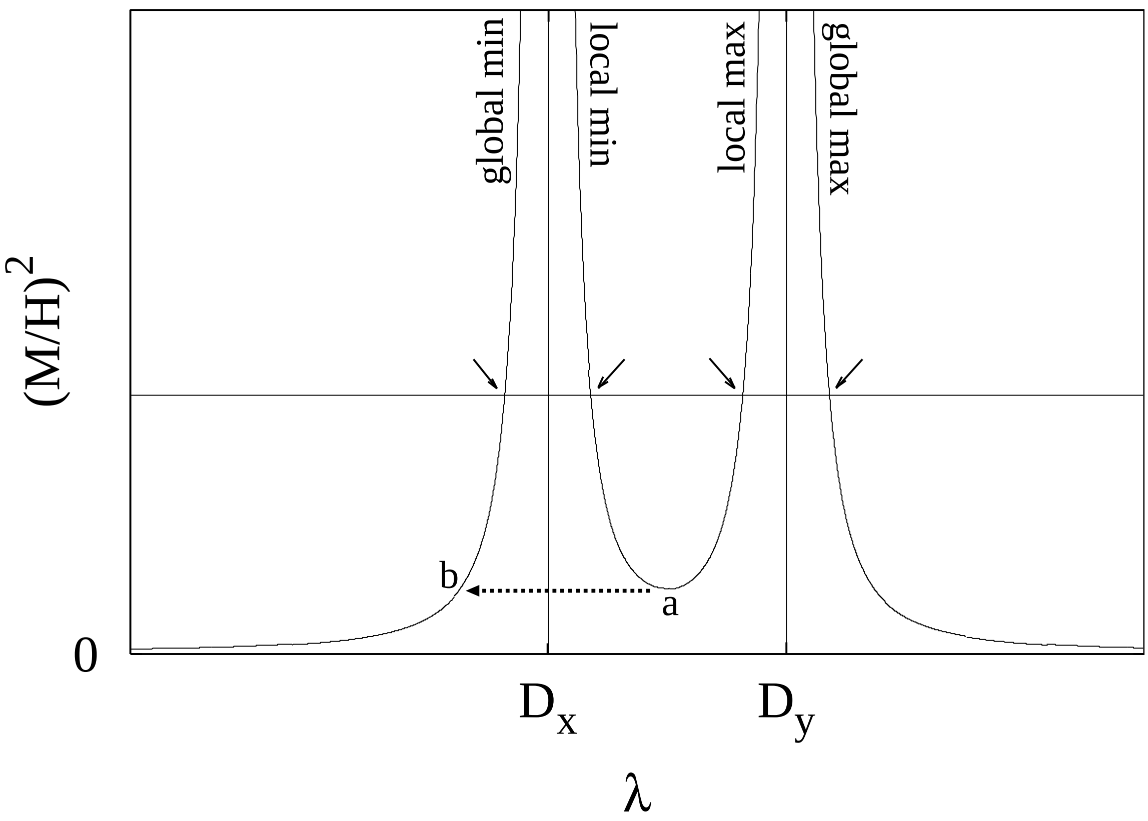
$$\begin{aligned} f(H) &\approx \frac{\gamma}{2\pi}\sqrt{(1+\alpha^2)\, M \det\left(D - \lambda_s I\right)} \\ &\quad \times \sqrt[4]{g''(\lambda_s) H_s (H_s - |H|)} \end{aligned}$$

- Likewise, susceptibility χ as $|H| \uparrow H_s$

$$\chi \propto (H_s - |H|)^{-(1/2)}$$

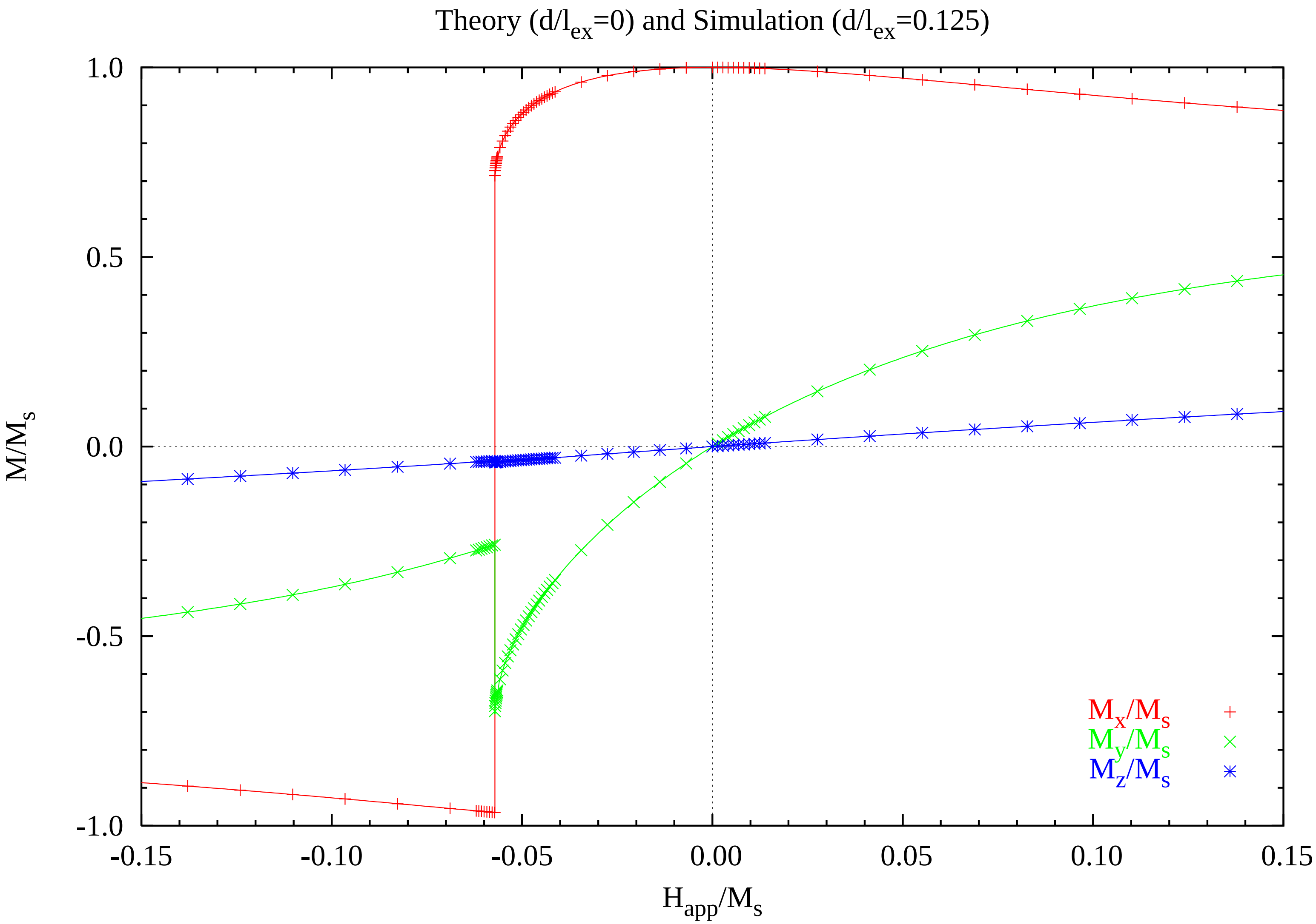
Special Cases

- Classic Stoner-Wohlfarth Model (2D)
  - ▷ Simplify $g(\cdot)$ to two terms.
  - ▷ Any 2 of D_x, D_y, D_z equal; or
  - ▷ Any 1 of h_y, h_z equal to 0

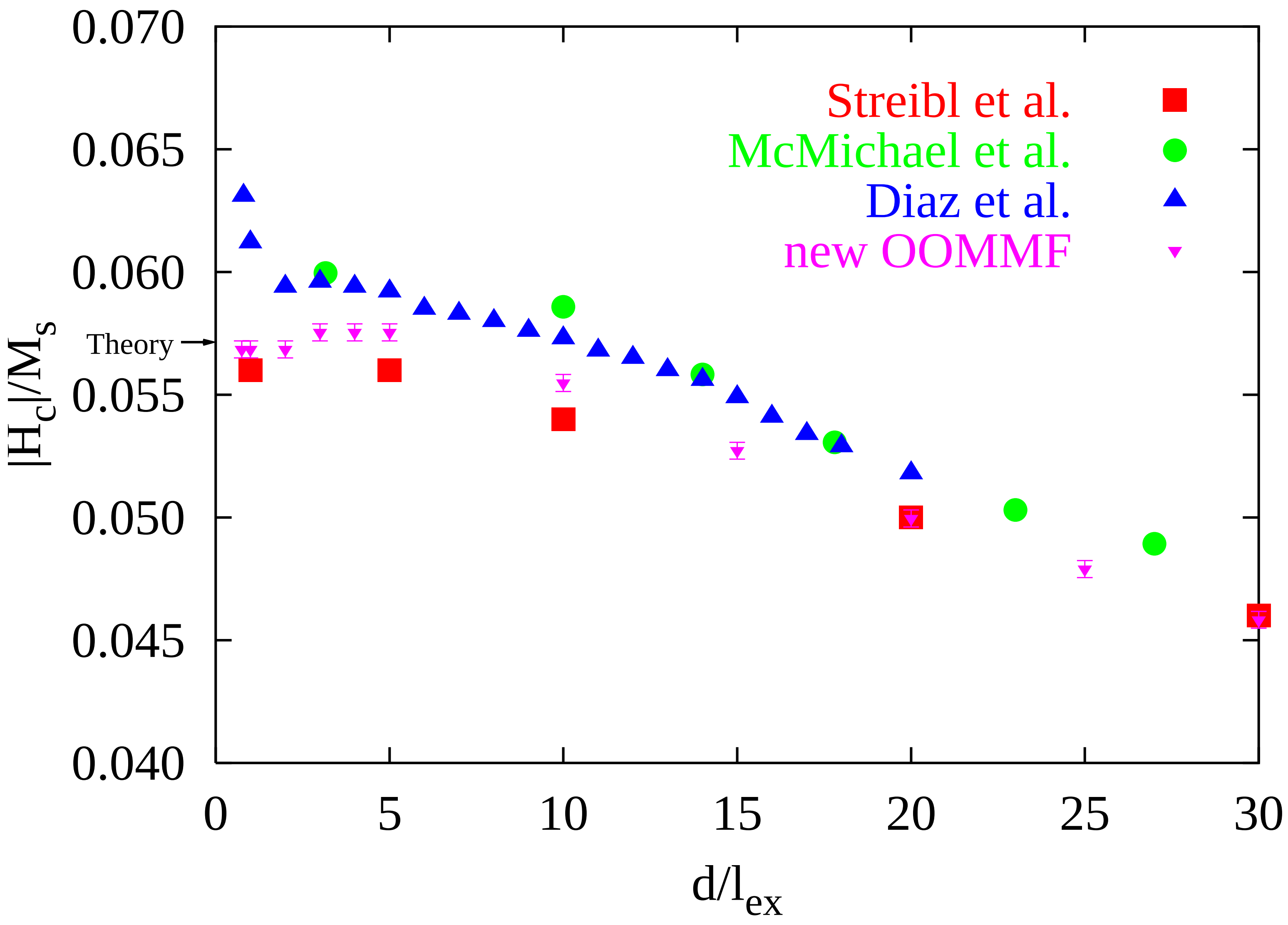


- Isotropic case
  - ▷ All 3 of D_x, D_y, D_z equal

μMAG Std. Prob. 2: Reversal



μMAG Std. Prob. 2: Coercive Fields[5, 6, 7]



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