A Test Bed for a Finite Difference Time Domain Micromagnetic Program with Eddy Currents

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Abstract

The inclusion of eddy currents into micromagnetic programs is important for the proper analysis of dynamic effects in conducting magnetic media. This subject has received little attention in the past although it can cause significant errors in device calculations. This paper introduces a computational test bed for eddy current calculations and discusses some interesting analytic cases in this simplified geometry.

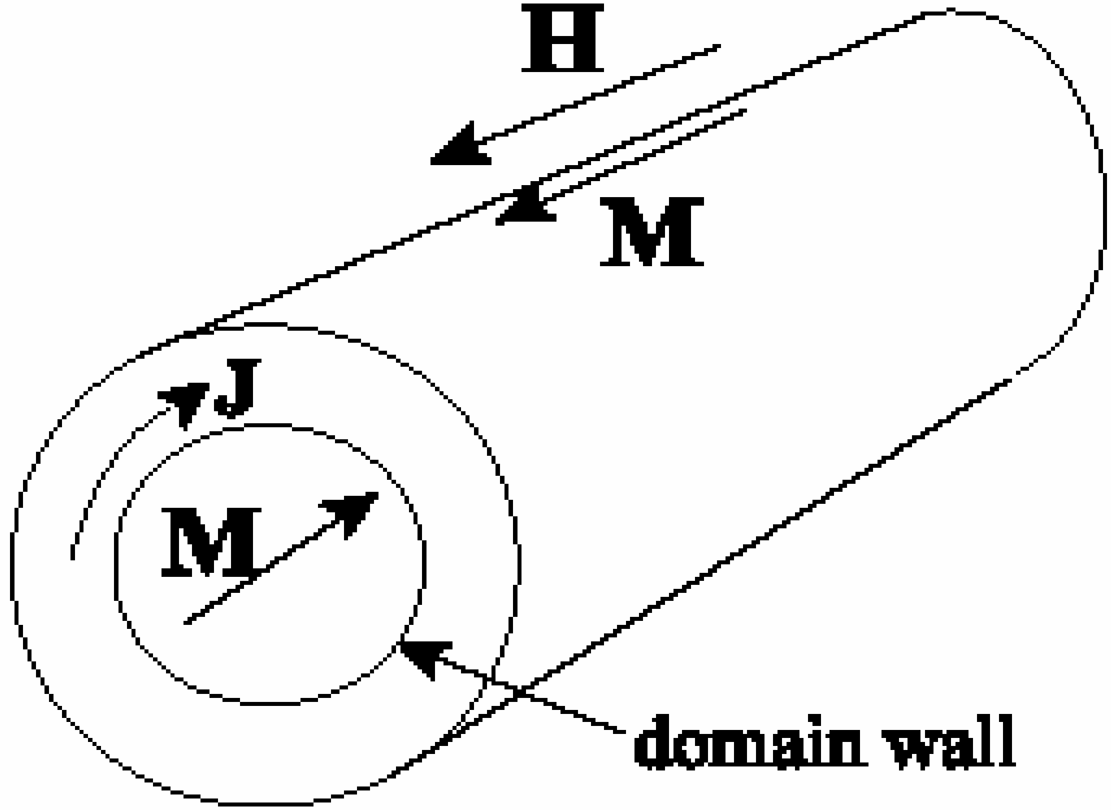
Introduction

- A solution of micromagnetic problems with eddy currents has been proposed by Della Torre and Eicke and implemented by Torres, et al.
- This approach requires the simultaneous solution of the coupled equation for eddy currents and for magnetization reversal.
- Here we present a program using finite difference time domain, FDTD, calculations for a simplified problem to be used as a test bed for more general programs.
- We intend to use this program to verify the accuracy of a more general programs.
- We also present some results that we will use to test the final model.

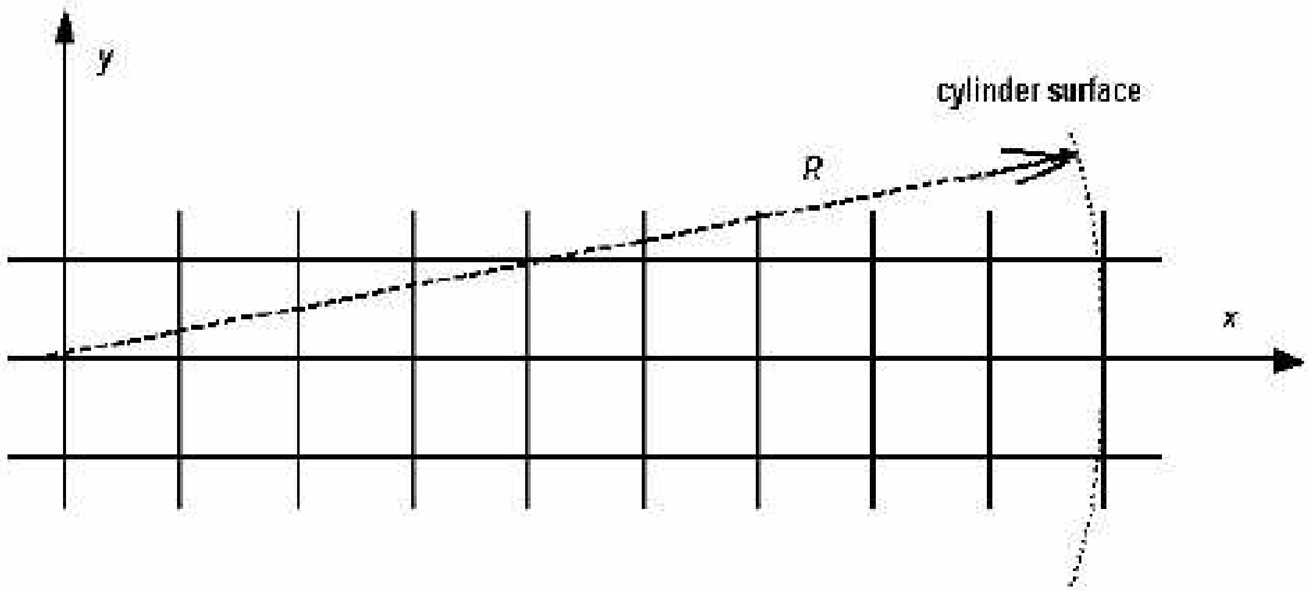
- An applied magnetic field changes the magnetization, which induces eddy currents, which in turn changes the applied field.
- Since eddy currents are bound by the shape of the material, the electric field close to the surface must be tangent to it.
- Surface charges are induced to steer the currents.
- To avoid having to compute these charges, we chose a circular cylinder shape.
- By symmetry the electric field is circular.this symmetry permits all quantities to vary with the radius only.

The model

- The model is an infinite circular cylinder of radius R
- We assume a perfect crystal of uniaxial material with easy axis, z, coinciding with the cylinder's axis.
- We assume magnetization is initially uniform in the z-direction.
- To break the symmetry, we offset the surface magnetization by a small angle nucleating a Bloch wall that propagates towards the center.
- The moving wall induces eddy currents that impede the wall's progress.
- Due to symmetry as the magnetization changes it will remain cylindrically symmetric.


H
M
J
M
domain wall

- The natural coordinate system one would use if one were attempting to solve the problem analytically would by the cylindrical coordinate system.
- However, it has a singularity at the z-axis which presents problems as the magnetization approaches saturation in the reverse direction.
- We elected to use a Cartesian coordinate system illustrated below



- Only the nodes along the *x*-axis are computed, using FDTD
- The nodes on adjacent rows are computed by rotating interpolated values.
- There is a singularity along the axis of the coordinate system the presents problems in trying to saturate the rod.
- We used this coordinate system to ease the transition to calculating arbitrary shaped specimen

- Magnetization can be computed by either the Landau-Lifshitz-Gilbert equation to study high speed phenomena
- or by minimizing the energy of the system at each calculation step.

- In this calculation, we force the magnetization to lie in the y-z plane, so that only Bloch-type walls may be formed, so we can't have precessing spins.
- We are forced to use the energy minimization approach.
- This limits us to relatively slow applied field changes compared to magnetization changes.

The magnetization will be of the form

$$M(r) = M_S(\cos\alpha\, \mathbf{1}_z + \sin\alpha\, \mathbf{1}_y),$$

By Faraday's law, the curl of that field is given by

$$\text{curl } \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = -\mu_0\left(\frac{\partial \mathbf{H}}{\partial t} + \frac{\partial \mathbf{M}}{\partial t}\right).$$

Thus, the electric field is given by

$$\mathbf{E}(r) = -\frac{\mu_0 \mathbf{1}_y}{r}\int_0^{R_0}\left[\frac{\partial H_z(\rho,t)}{\partial t} + \frac{\partial M_z(\rho,t)}{\partial t}\right]\rho\, d\rho.$$

This electric field will induce the eddy currents according to Ohm's law $\mathbf{J} = \sigma\, \mathbf{E}$.

Ampere's law reduces the applied field at the surface, $H_z(R) = H_{app}$, in the interior by eddy currents.

Thus the field at any point, $H_z(r)$ is given by

$$H_z(r) = H_{app} - \int_r^R J_y \, d\rho .$$

The magnetization is computed by minimizing the total energy, which is the sum of the exchange energy, the anisotropy energy and the Zeeman energy.

The exchange energy per unit computational cell is

$$w_{ex} = -A\ \mathbf{M} \cdot \nabla^2 \mathbf{M} = \frac{A\,(6\,M^2 - \mathbf{M}\cdot\Sigma\mathbf{M})}{h^2},$$

h is the distance between nodes in the grid

Σ indicates a sum over nearest neighbor nodes.

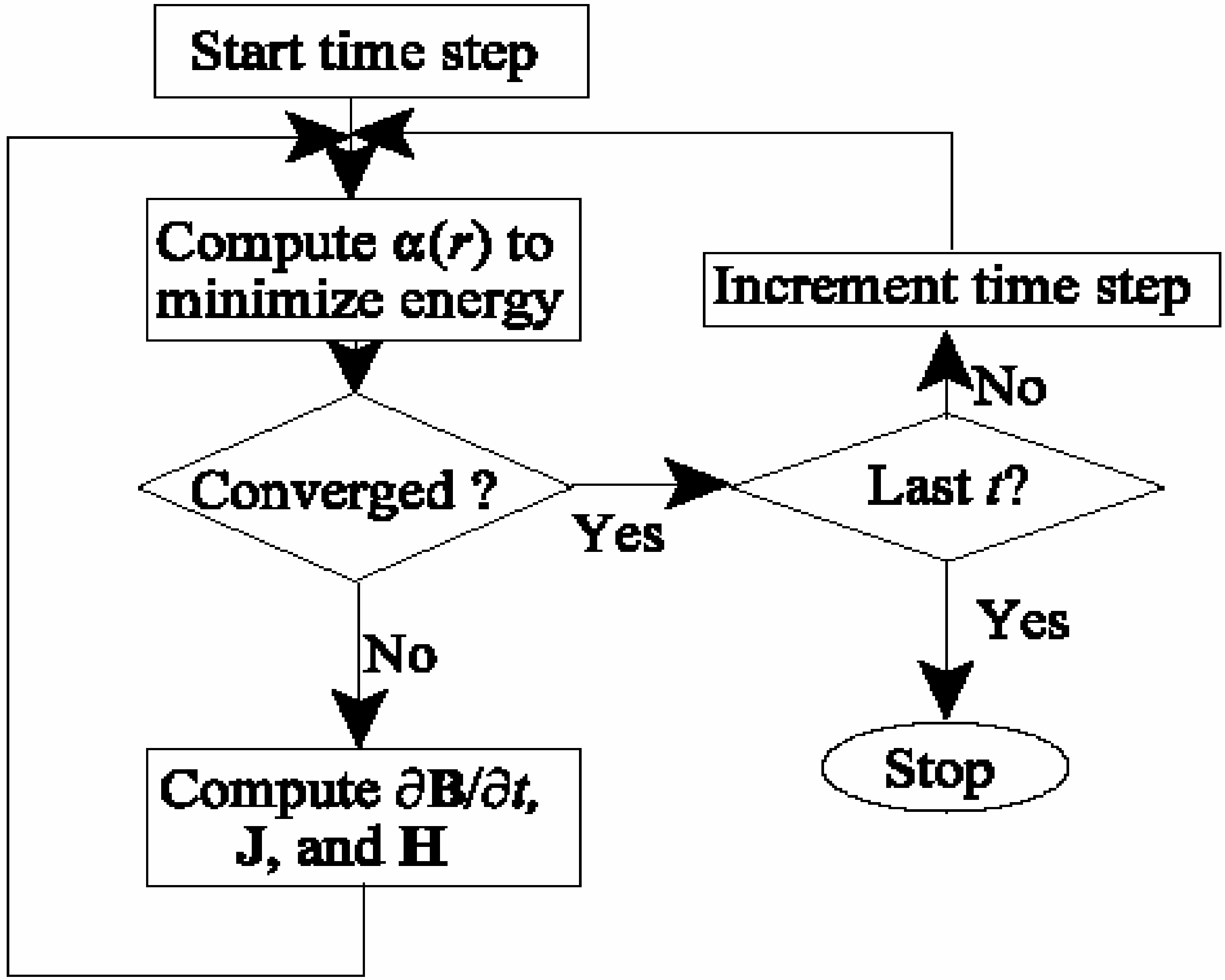
The anisotropy energy for uniaxial anisotropy is

$$w_{anis} = K \sin^2 \alpha\,.$$

Finally, the Zeeman energy is given by

$$w_{Zeeman} = -\mu_0 H_z(r)\, M_z(r) = -\mu_0 M_S H_z(r) \cos[\alpha(r)]\,.$$

- At each time step the patern is recomputed by varying $\alpha(r)$ at each node using the current value of **H**.
- The change in magnetization and then the electric field are computed.
- With this electric field, the eddy currents are computed and the *H* is recomputed.
- This procedure is recomputed until *H* converges.
- Then we proceed to the next time step until finished.


Start time step
Compute $\alpha(r)$ to minimize energy
Converged ?
Yes
No
Compute $\partial\mathbf{B}/\partial t$, J, and H
Last t?
No
Yes
Increment time step
Stop

This problem is defined by the material parameters: A, K, and M_S, the geometric parameter R and the applied field function of time.

We prefer to use the domain wall width of a planar Bloch wall, l_w,

$$l_w = \pi \sqrt{\frac{A}{K}},$$

and the wall energy density w_w per unit area of a planar Bloch wall

$$w_w = 4\sqrt{AK}.$$

An analytical example

- If l_w is negligible compared to $r_w < R$, then the problem reduces to a domain level problem.
- Furthermore, we can neglect the effect of wall curvature on both l_w and w_w.
- This problem has a single unknown, the radius, r_w, of the wall, and the magnetization changes when the wall moves.

The line integral of **E is**

$$\oint_C \mathbf{E} \cdot \mathbf{dl} = \begin{cases} 4\pi\mu_0 M_S v r_w & \text{if } r > r_w \\ 0 & \text{if } r < r_w \end{cases}$$

where v is the velocity of the domain wall.

Hence, the current density is given by

$$\mathbf{J} = \begin{cases} \dfrac{2\sigma\mu_0 M_S v r_w}{\mathbf{r}} \mathbf{1}_y & \text{if } r > r_w \\ 0 & \text{if } r < r_w \end{cases}$$

The field at the wall is

$$\mathbf{H}(r_w) = \left[H_{app} - \int_R^{r_w} \frac{2\,\sigma\,\mu_0\,M_S\,v\,r_w}{r}\,dr \right] \mathbf{1}_z$$

$$= [H_{app} + 2\,\sigma\,\mu_0\,M_S\,v\,r_w\,\ln(R/r_w)]\,\mathbf{1}_z.$$

Then, we can set the field equal to zero and solve for the wall's velocity

$$v = -\frac{H_{app}}{2\,\sigma\,\mu_0\,M_S\,r_w\,\ln(R/r_w)}$$

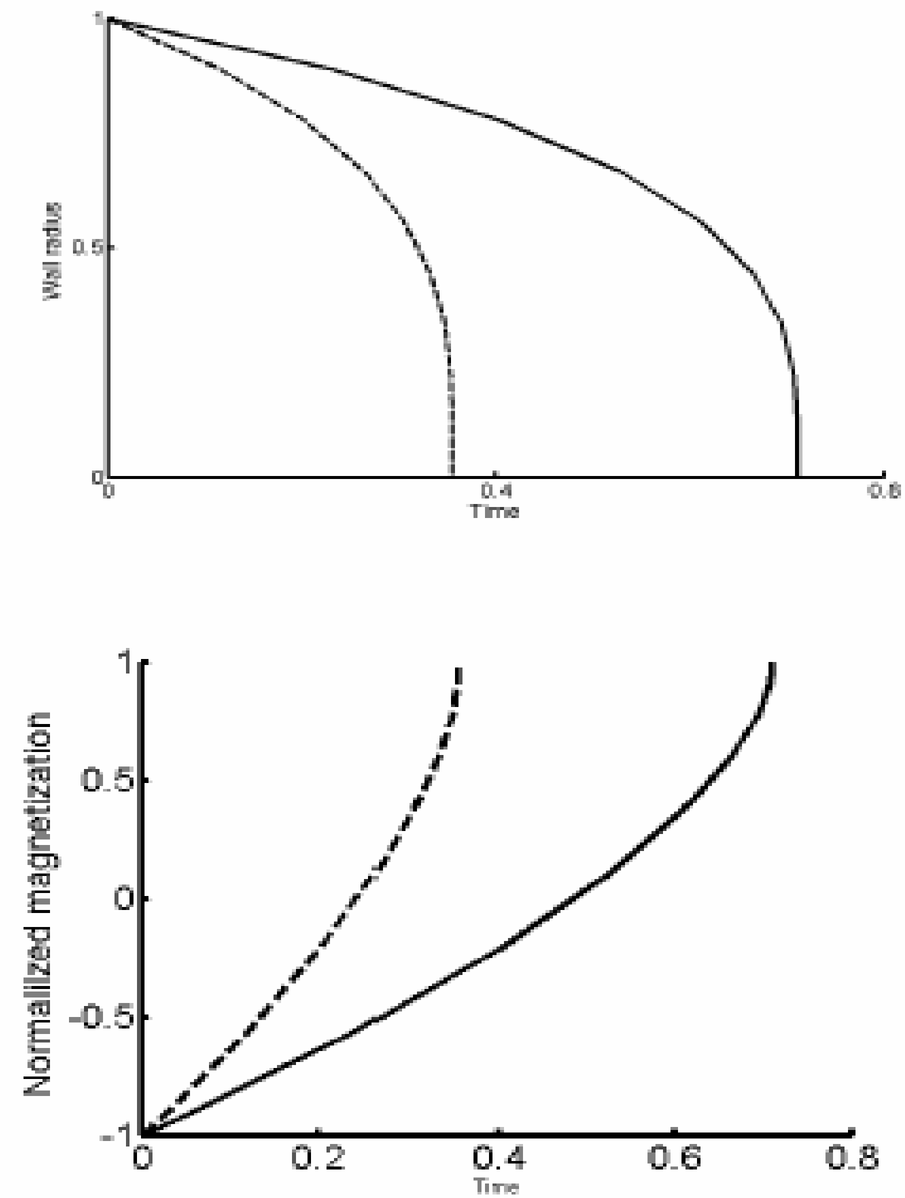
H_w, tending to push the wall towards the center. This is due to the fact that the total wall energy per unit length, W_w, is directly proportional to the length of the circumference of the wall.

$$W_w = 2\pi r_w w_w.$$

So that

$$H_w = \frac{1}{\mu_0 M_S}\frac{dW_w}{dr_w} = \frac{2\pi w_w}{\mu_0 M_S} + \frac{2\pi r_w}{\mu_0 M_S}\frac{dw_w}{dr_w}.$$

Even if there is no applied field but the wall energy is uniform, if $H_w > H_C$, then the wall will shrink to the center as was observed in iron single crystals.



Wall radius
1
0.5
0
0
0.4
0.8
Time
Normalized magnetization
1
0.5
0
-0.5
-1
0
0.2
0.4
0.6
0.8
Time

Coercivity

We can simulate a critical field by applying a ripple to the wall energy

$$\boldsymbol{w_w = w_0 + \Delta w \sin(\beta r_w)},$$

where the wavelength of the ripple, $\lambda = 2\pi/\beta$, is much larger than the wall width.

The peak ripple slope is dw_w/dr_w is $\beta\Delta w$.

Thus, to keep the wall propagating, the wall field must be greater than $\beta\Delta w$.

We have created an effective critical field of $\beta\Delta w$.

For smaller fields, the wall hangs up periodically at points separated by λ.

For very short wavelength ripple, integrating the wall energy over the ripple gives zero, and no critical field.

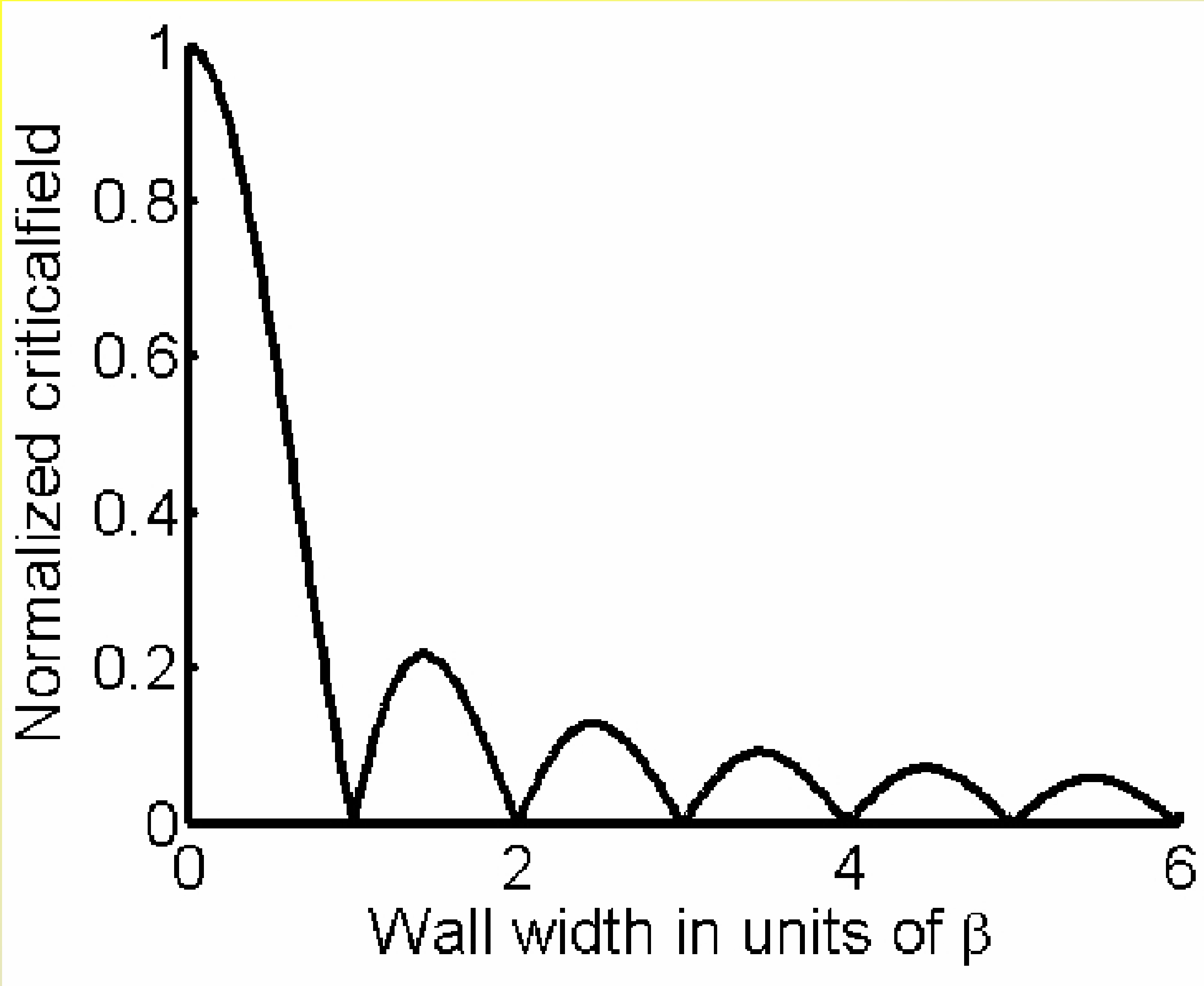
Assuming the equal angle model

$$\alpha = \begin{cases} 0 & \text{if} \quad r > r_w + l_w/2 \\ 2\pi(r_w - r)/l_w & \text{if} \quad r_w - l_w/2 < r < r_w + l_w/2 \\ \pi & \text{if} \quad r < r_w - l_w/2, \end{cases}$$

one can compute the critical field as a function of wall width.

The average critical field over the wall is

$$<H_C> = \left| \frac{\Delta w}{l} \int_{-l/2}^{l/2} \cos(\beta r)\, dr \right| = \left| \frac{2\Delta w}{\beta l} \sin\frac{\beta l}{2} \right|.$$



1
0.8
0.6
0.4
0.2
0
0
2
4
6
Normalized criticalfield
Wall width in units of β

If the applied field reverses, there are two possibilities:

- the wall could reverse direction
- or a new wall could be nucleated at the surface and start propagating inward.

- The choice of which occurs is determined by the size of the nucleating field compared to the size of the propagating field.

- In the analytical example, we simply postulate that there is no wall until the an arbitrary nucleating field is reached.

- If that field is small enough, there is the possibility that there may be more than one wall active at any time.
- However, the outermost wall will see the largest field since the inner walls will have additional shielding by the eddy currents between the walls.

Energy loss considerations

- There are two losses in this model: hysteresis and eddy current losses.

- The hysteresis loss per unit length is given by

$$w_{hyst} = \int_0^R 2\pi r \mu_0 H_C \frac{\partial M}{\partial t} dr.$$

- For the thin wall with a constant applied field, this reduces to

$$w_{hyst} = 4\pi r \mu_0 H_C M_S v.$$

The eddy current loss per unit length is given by

$$w_{ec} = \int_0^R 2\pi\sigma J^2 r \, dr.$$

For a thin wall with a constant applied field, this reduces to

$$w_{ec} = \int_{r_x}^{R} \left(\frac{2\sigma\mu_0 M_S v r_w}{r} \right)^2 dr = \left(\frac{2\sigma\mu_0 M_S v r_w}{r} \right)^2 \ln(R/r_w).$$

We note that in this equation, both v and r_w are functions or time, even if the applied field is held constant.

Conclusions

- We have presented a simplified model of the introduction of eddy currents into a micromagnetic calculation.
- We also discussed the special analytic calculation in the case of a thin domain wall.
- Coercivity can be added by adding a wall energy ripple.
- This paper is intended to provide a limiting calculation to test a more general model.
- The question remains how to include wall mass.