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Atmospheric Retention of Man-made CO_2 Emissions

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Abstract. *Rust and Thijsse [9, 11] have shown that global annual average temperature anomalies $T(t_i)$ vary linearly with atmospheric CO_2 concentrations $c(t_i)$. The $c(t_i)$ can be related to man-made CO_2 emissions $F(t_i)$ by a linear regression model whose solution vector gives the unknown retention fractions $\gamma(t_i)$ of the $F(t_i)$ in the atmosphere. Gaps in the $c(t_i)$ record make the system underdetermined, but the constraints $0 \leq \gamma(t_i) \leq 1$ make estimation tractable. The $\gamma(t_i)$ are estimated by two methods: (1) assuming a finite harmonic expansion for $\gamma(t)$, and (2) using a constrained least squares algorithm [8] to compute average values of $\gamma(t)$ on suitably chosen time subintervals. The two methods give consistent results and show that $\gamma(t)$ declined non-monotonically from ≈ 0.6 in 1850 to ≈ 0.4 in 2000.*

1 Atmospheric CO_2 and Global Temperatures

The upper plot in Figure 1 shows an optimal regression spline [11] fit $c(t)$ to the record of atmospheric CO_2 concentrations obtained by combining atmospheric measurements at the South Pole [5] with reconstructions from Antarctic ice cores [1, 7]. Although the latter display larger random variations than the former, the two records are consistent in the years where they overlap. The spline $c(t)$ was used to model the Climatic Research Unit's record [4] of annual average global surface temperature anomalies shown in the lower plot. The solid curve was obtained by fitting the model

$$T(t) = T_0 + \eta\,[c(t) - 277.04] + A \sin\left[\frac{2\pi}{\tau}(t+\phi)\right] \ ,$$

with free parameters T_0, η, A, τ, and ϕ. The constant 277.04 ppmv is the preindustrial CO_2 concentration estimated by averaging ice-core measurements for 1647-1764. The corresponding temperature anomaly, estimated by the fit, was $\hat{T}_0 = (-0.507 \pm .016)$°C. The sinusoid, with $\hat{\tau} = (71.5 \pm 2.2)$ yr and $\hat{A} = (0.099 \pm .012)$°C, represents the oscillation discovered by Schlesinger and Ramankutty [10]. It accounts for ≈ 8 % of the variance in the record. The baseline $T_0 + \eta\,[c(t) - 277.04]$, with $\hat{\eta} = (0.01039 \pm .00042)$ °C/ppmv, accounts for ≈ 77 % of the variance. It indicates *a linear relationship between global warming and increasing atmospheric CO_2*. The total warming since 1856 has been ≈ 0.9°C, and *that warming is accelerating*.

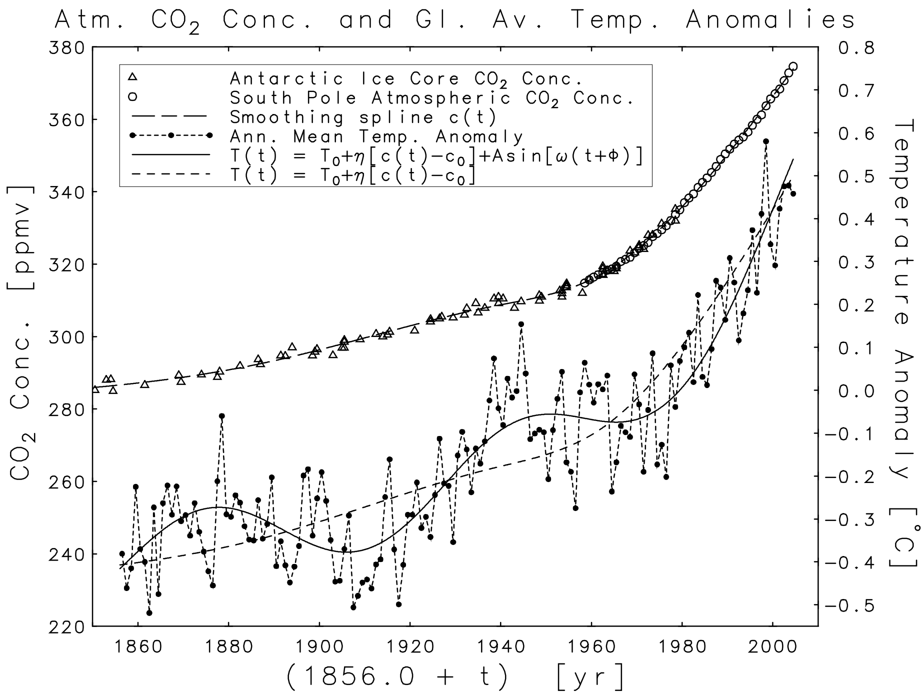

Figure 1: The relationship between atmospheric carbon dioxide and global temperatures.

2 Man-made Emissions and Atmospheric CO_2 Concentrations

Annual total man-made CO_2 emissions $F(t_i)$, for the years 1850-2000, are shown in the lower plot of Figure 2. These totals are the sums of annual fossil fuel emissions [6] and emissions due to changes in land use [3]. Taking $t_0 = 1850.0$ gives, for any later time t_i,

$$c(t_i) = c_0 + \int_{t_0}^{t_i} \gamma(\tau) F(\tau) d\tau + \delta S(t_i) \ ,$$

where $c_0 = c(t_0)$, $\gamma(\tau)$ is the fraction remaining in the atmosphere, and $S(t)$ is a ramp function representing the Mt. Pinatubo eruption on June 15, 1991. $S(t)$ is 0 on $[1850.0, 1991.54]$, increases linearly to 1 on $[1991.54, 1993.54]$, and remains 1 thereafter. The amplitude constant δ turns out to be negative [2].

One way to estimate $\gamma(\tau)$ is to assume a harmonic expansion of the form

$$\gamma(t) = A_0 + B_0 t + \sum_{k=1}^{n_h} \left[A_k \cos\left(2\pi k t/150\right) + B_k \sin\left(2\pi k t/150\right) \right] \ , \quad 1850 \leq \ t \ \leq 2000 \ ,$$

with n_h chosen so that $2n_h + 4$ is less than the number of observed $c(t_i)$. Substituting the expansion into the above integral leads to linear least squares estimates for c_0, δ, and the A_k and B_k. Choosing n_h too large produces implausibly oscillating estimates which violate the constraints $0 \leq \gamma(t) \leq 1$. The estimate for $n_h = 2$ is plotted as a smooth curve in Figure 3. The corresponding estimate for $c(t)$ is shown as a dashed curve in the upper plot of Figure 2.

Another approach, which seeks a vector approximation $\boldsymbol{\gamma}(\tau_j)$, is to approximate the integral using a rectangular quadrature rule with $\Delta\tau = 1$ year. This gives a linear regression model

$$\mathbf{c}(t_i) = [\ \mathbf{1}\ ,\ \mathbf{F}(t_i, \tau_j)\ ,\ \boldsymbol{s}\ ] \begin{bmatrix} c_0 \\ \boldsymbol{\gamma}(\tau_j) \\ \delta \end{bmatrix} + \boldsymbol{\epsilon}(t_i) \ , \quad \boldsymbol{\epsilon}(t_i) \sim N\left(\mathbf{0}, \boldsymbol{\Sigma}^2\right) \ ,$$

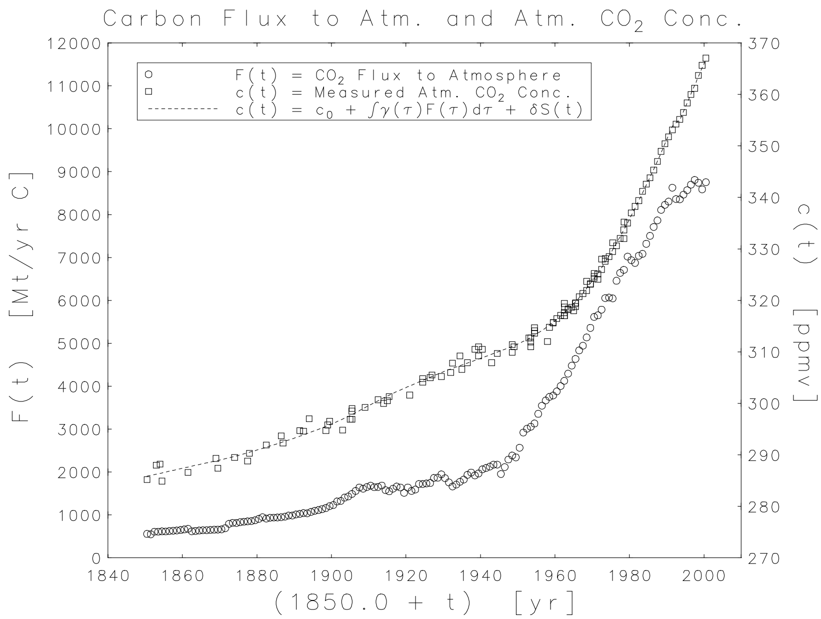

Figure 2: Man-made CO_2 emissions and CO_2 concentration in the atmosphere.

where $\mathbf{F}(t_i, \tau_j)$ is a matrix of columns formed from zeroes and the values $F(\tau_j)$, $\boldsymbol{s}$ is a vector representation of $S(t_i)$, and $\boldsymbol{\epsilon}(t_i)$ is a vector of measurement errors. The covariance matrix $\boldsymbol{\Sigma}^2$ was estimated by assuming constant variance for the ice core records and a different constant variance for the atmospheric measurements. These two constants were estimated from deviations of the measurements from optimal regression spline fits [11] to the two sets of data.

Because of gaps in the $c(t_i)$ record, the matrix $[\mathbf{1}, \mathbf{F}(t_i, \tau_j), \boldsymbol{s}]$ has more columns than rows, but estimation is possible because $\mathbf{0} \leq \boldsymbol{\gamma}(\tau_j) \leq \mathbf{1}$. Even so, the estimates $\hat{\boldsymbol{\gamma}}(\tau_j)$ oscillate wildly between those bounds, so it was necessary to estimate average values of $\gamma(\tau)$ on various time subintervals of $[1850, 2000]$. O'Leary's BRAKET-LS algorithm [8] was used to compute 95 % confidence intervals for 6 nonoverlapping 25-year subintervals shown in Figure 3. The pre-1925 uncertainties are large, but the bounds give good agreement with the $\hat{\gamma}(t)$ estimated from the harmonic expansion. Combining the fit of the corresponding $\hat{c}(t)$ to the measurements in Figure 2 with the results in Figure 1 suggests that *man-made CO_2 emissions are a major contributor to global warming*.

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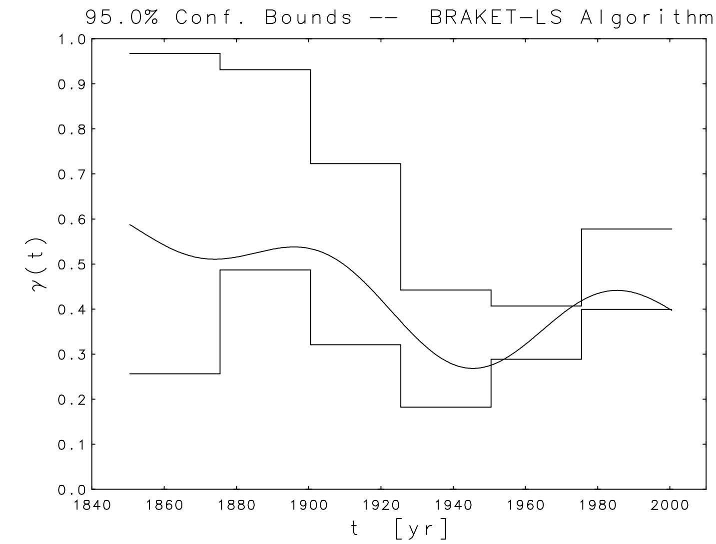

Figure 3: Fraction of CO_2 emissions remaining in the atmosphere.

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