Inferences Drawn From Atmospheric CO_2 Data[1]

BERT W. RUST, RALPH M. ROTTY, AND GREGG MARLAND

Institute for Energy Analysis, Oak Ridge Associated Universities, Oak Ridge, Tennessee 37830

Detailed analyses of the atmospheric carbon dioxide concentration measurements from Mauna Loa Observatory have produced a model which describes most of the variability in the data. The final model is $y(t) = a + de^{\alpha t} + \sum_{i=1}^{2} A_i \sin[(2\pi/T_i)(t + \phi_i)] + \sum_{j=1}^{2} F_j \sin[(2\pi/\tau_j)(t + \Theta_j)]$, where a is the preindustrial value, α is the exponential long-term growth rate, i represents the annual cycle and its harmonics, and j represents longer-term periodicities. The exponential growth rate is nearly identical to that calculated for the production of CO_2 from fossil fuel burning. A long-period component of about 44 months correlates with the southern oscillation index. Another long-period oscillation—about 142 months—has tentatively been identified with the solar activity cycle and complicates the determination of α because the two sources of curvature interact in the fitting procedure, and the data record is not long enough to specify both of them independently. The generality of the model has been demonstrated by testing it against shorter data sequences from the South Pole and Australia.

INTRODUCTION

There can no longer be any doubt that our industrial society is introducing significant perturbations on the global carbon cycle. Monitoring records show clearly that the concentration of CO_2 in the atmosphere is increasing, and current climate models suggest that this increase is likely to cause major changes in the global climate [*Manabe and Wetherald,* 1975]. If the predictions are correct, a fundamental remodeling of our industrial system may be required, and the basic changes which this suggests include serious implications for the aspirations of the developing nations of the world. The challenge facing us now is to determine unambiguously whether or not drastic climate changes are imminent and to do so sufficiently far in advance that we can have some options in dealing with the associated problems.

This suggests two fundamental questions: How is atmospheric CO_2 changing? and, What climate changes are likely as a function of atmospheric CO_2 changes? This paper is concerned with only the first question.

Acknowledging that our goal is to achieve the best possible understanding of atmospheric CO_2 changes at the earliest possible date, we are attempting to extract as much information as possible from the data available now. A thorough understanding of the CO_2 fluxes to and from the atmosphere and their changes with time is required. There are a variety of data that bear on the question of CO_2 fluxes, but among the most useful for understanding the atmospheric fluxes are the actual measurements of the atmospheric CO_2 concentration—particularly the 18-year record of monitoring at Mauna Loa Observatory. This paper is devoted to extracting from that data, and to a lesser extent from South Pole and Australian data, as much insight as possible from a statistical and mathematical analysis. Our principal starting point is the monthly averaged Mauna Loa data shown in Figure 1, drawn from *Keeling et al.* [1976*a*]. It is difficult to describe the extraordinary care and scientific acumen that went into the effort that finally produced the time series shown here. The measurement precision level is estimated to be 0.1 ppm [*Keeling et al.*, 1976*a*].

The most obvious properties of the Mauna Loa data set are an annual cycle and a long-term increasing trend [*Machta,* 1973]. The purpose here is to determine the functional form of the long-term trend, identify any other components that may be interacting with the annual cycle and the general trend, and, if possible, relate all these components to physical factors and forcing functions. For example, *Bacastow* [1976] has suggested a relationship between the southern oscillation and the atmospheric carbon dioxide observations.

Most of the carbon dioxide literature of recent years has emphasized modeling of the CO_2 fluxes between the atmosphere and other reservoirs. *Olson* [1970], *Baes et al.* [1976], *Lieth and Whittaker* [1975], and *Bolin* [1977] have all addressed the terrestrial ecosystem-atmosphere fluxes. The ocean fluxes and oceanic CO_2 have been modeled by *Bacastow and Keeling* [1973], *Broecker et al.* [1971], *Stuiver* [1975], and *Oeschger et al.* [1975], among others. Our approach here is significantly different. We are attempting to learn from mathematical analyses of the carefully obtained data of Figure 1 what types of physical processes might be influencing the observations.

ANALYSIS

In our initial effort we used the method of linear least squares to fit the Mauna Loa data with a model consisting of a linear trend and sinusoidal components for the annual cycle and its first four harmonics. That is,

$$y(t) = a + bt + \sum_{i=1}^{5} A_i \sin\left[\frac{2\pi}{T_i}(t + \phi_i)\right] \qquad (1)$$

where $T_1 = 12$, $T_2 = 6$, $T_3 = 4$, $T_4 = 3$, and $T_5 = 2.4$ months. The time t was measured in months with the zero point taken to be the beginning of 1958. The constants a, b, A_1, ϕ_1, A_2, ϕ_2, $\cdots$, A_5, and ϕ_5 were determined by linear least squares. Only the first two of the sinusoidal terms, i.e., $T_1 = 12$ and $T_2 = 6$, contributed significantly to the fit. A plot of the residuals (data points minus best fit) versus time (Figure 2) shows that this model fails to capture the long-term trend adequately and that some fundamental form with concave upward curvature is required.

The principal anthropogenic perturbation of the natural CO_2 cycle (the probable forcing function for the long-term increasing trend) is generally acknowledged to be the combustion of fossil fuels with a contribution likely from forest clearing, particularly in the tropics. *Rotty* [1977] has shown that the annual CO_2 production from fossil fuels and cement

[1] Editor's note: This paper was presented at the International Association of Meteorology and Atmospheric Physics Symposium on the Carbon Dioxide Cycle, Seattle, Washington, August 31, 1977.

Paper number 8C1164.

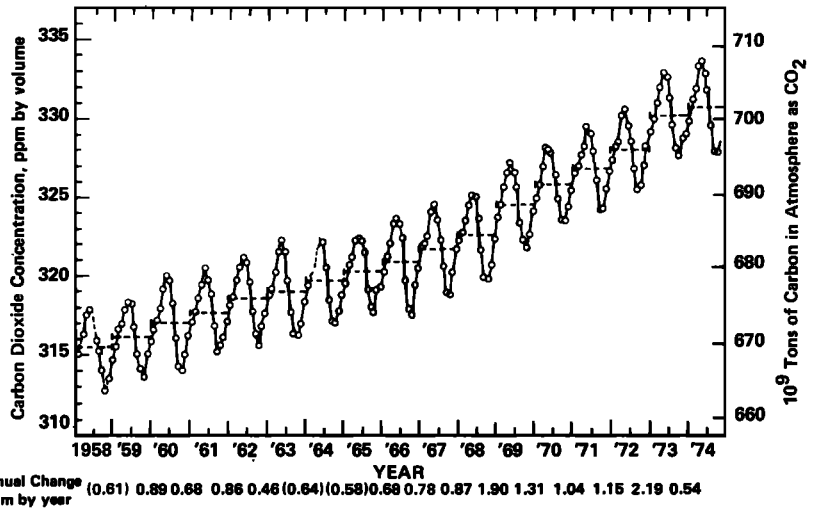

Fig. 1. Atmospheric carbon dioxide concentration at Mauna Loa Observatory (1958–1971 data from *Keeling et al.* [1976*a*]; 1972–1974 data from C. D. Keeling, private communication, 1976].

closely follows a straight line relation when plotted on a log scale (Figure 3), and we find by a least squares fit that the carbon in fossil fuel burned annually since 1950 is closely approximated by the expression

$$X(t) = d'e^{\alpha' t} \tag{2}$$

where $\alpha' = 0.0443 \pm 0.008$ yr^{-1} or 0.00369 month$^{-1} \pm 0.0007$ (Figure 4). The zero point for t was taken to be 1950.0. The value of α' is independent of this choice and in fact has remained almost constant since 1860 except for the three short periods of major global upheaval—two world wars and the great depression (see Figure 3). Consequently, to provide for the concave curvature suggested in Figure 2, we selected an exponential base line and postulated for the Mauna Loa data a model of the form

$$y(t) = a + de^{\alpha t} + \sum_{i=1}^{2} A_i \sin\left[\frac{2\pi}{T_i}(t + \phi_i)\right] \tag{3}$$

where the zero point for t was again taken to be 1958.0 and the constants a, d, α, A_1, ϕ_1, A_2, and ϕ_2 are parameters determined by nonlinear least squares fitting. If atmospheric CO_2 growth is largely a consequence of fossil fuel burning and an exponential forest clearing, we would expect a to be the preindustrial concentration and α to be the growth rate, hopefully near the 0.00369 month^{-1} suggested by the fossil fuel data. Also, we have observed that the harmonics of the annual cycle

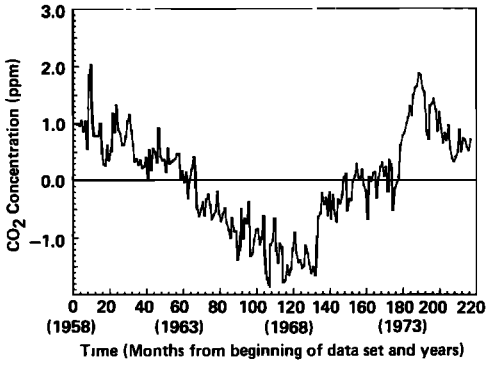

Fig. 2. Residual variability when Mauna Loa CO_2 data are fitted with the model defined by (1).

beyond the first are negligible, so only the 12- and 6-month period components were retained in the analysis. The unconstrained best fit to the data was achieved with $a = 306.4 \pm 0.9$ and $\alpha = 0.0050 \pm 0.0003$, but a plot of the residuals (Figure 5) suggests that there is a long-term periodicity in the data which is not accounted for in the model, and this could affect the value of α. Others [*Keeling et al.*, 1976*b*] have suggested that the low values of CO_2 concentration centered on 1968 are a singularity related perhaps to the March 1963 eruption of Mount Agung. The resulting abnormally low solar radiation reaching the earth could have resulted in cooler surface ocean water being a stronger than usual sink for CO_2. However, spectral analysis shows that the perturbation also can be treated mathematically as a long-term periodicity, and we suggest the possibility that there might be long-period physical phenomena which could have impacts on the atmospheric CO_2 concentration.

Both Fourier and maximum entropy spectral analysis were applied to the detrended data of Figure 5 to identify regularities remaining in the residuals. Maximum entropy spectral analysis (Mesa) is in many ways superior to the classical Fourier techniques because (1) it uses all the measured data without alterations (e.g., no tapering windows are used) and (2) it minimizes the implicit assumptions about the unavailable data (i.e., the assumptions about the time series outside the data window). The reward for this more scrupulous treatment of the data is a much enhanced frequency resolution. This improvement is especially noticeable at lower frequencies. Mesa is capable of resolving cycles with periods comparable to the length of the time series record [*Ulrych*, 1972]. *Kirk et al.* [1979] describe a Fortran program for computing Mesa spectra and also provide an extensive bibliography on the method. One disadvantage of Mesa is that peak heights in a spectrum do not give accurate estimates of the amount of variation or power associated with that frequency. The operating philosophy that we have adopted for isolating cyclic components in time series data is to use Mesa to estimate frequencies and the Fourier spectrum to estimate the associated amplitudes.

Figure 6 is the Mesa spectrum of the residual data from Figure 5. It is dominated by two peaks with periods of 44.4 months and approximately 109 months. The 44.4-month pe-

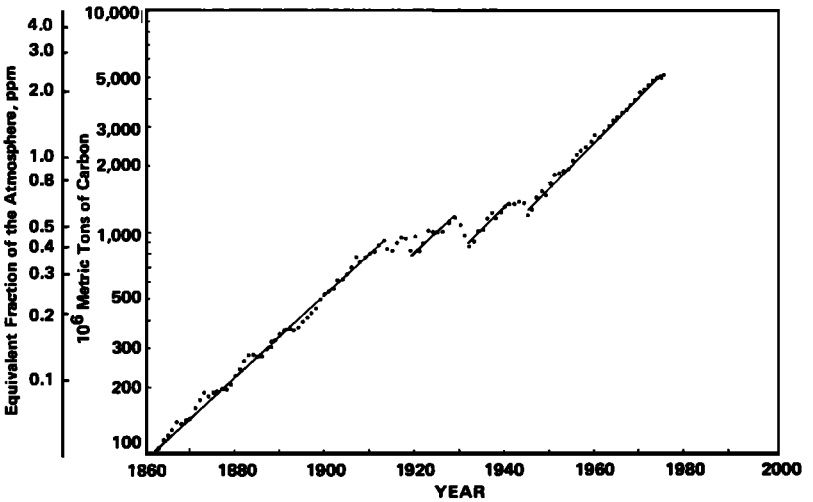

Fig. 3. Carbon dioxide production from fossil fuels and cement [from *Rotty*, 1977].

riod corresponds closely to a periodicity in the southern oscillation index identified by *Bacastow* [1976]. The longer-period peak, which appears in the Fourier spectrum with about 3 times the power of the 44-month peak, must somehow be taken into account in order to determine accurately the exponential rate constant α. To demonstrate this fact, we selected a series of reasonable estimates of the preindustrial concentration a and for each one fitted the model of (3) to the data with a held fixed. For each fit we calculated Fourier and Mesa spectra of the residuals. The results of this exercise are shown graphically in Figure 7. The top graph gives the variation of α with a. The solid circles represent the individual fits with a held fixed. The open circle represents the unconstrained fit, i.e., the fit in which a is allowed to vary also. The bottom graph demonstrates why higher than expected values were obtained for a and α in the unconstrained fit. The plot gives the square of the amplitude of the long-term cycle (estimated from the Fourier spectra) as a function of the value of a. In the unconstrained fit the values of a and α are adjusted to minimize the power or variance remaining in the data and hence in the long-term cycle. Thus it seems likely that in the unconstrained fit some of the variation that properly belongs in the long-term cycle, which was not included in the model of (3), was transferred by the fitting procedure into the exponential term in an effort to minimize the total squared variation. This could produce anomalously high values of both a and α. The middle graph of Figure 7 gives the Mesa-determined period of the long-term cycle as a function of a. We note that $a = 300$ gives a period of about 124 months.

The model suggested by the analysis thus far is

$$y(t) = a + de^{\alpha t} + \sum_{i=1}^{2} A_i \sin\left[\frac{2\pi}{T_i}(t + \phi_i)\right] + \sum_{j=1}^{2} F_j \sin\left[\frac{2\pi}{\tau_j}(t + \Theta_j)\right] \quad (4)$$

where the two new harmonic terms have periods τ_j which must be determined by the fit. This model has only three nonlinear parameters: $\alpha \cong 0.0036$ month^{-1}, $\tau_1 \cong 124$ months, and $\tau_2 \cong 44$ months, but attempting to fit it to the data as it stands yields a very ill conditioned problem, i.e., a problem in which the estimated parameter values are extremely sensitive to very small perturbations in the data. As a result of this ill conditioning, it is possible to find an infinite number of sets of parameter estimates (with each parameter spanning a wide range), each of which 'solves' the fitting problem equally well, i.e., which minimizes the sum of squares of the residuals (Table 1). The situation is not hopeless, however, because the parameters are all highly correlated with one another; e.g., the

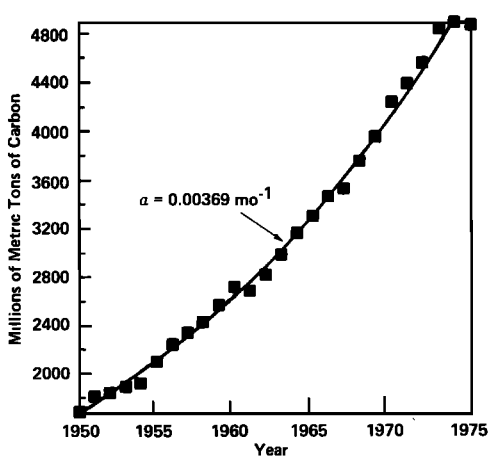

Fig. 4. Total carbon in fossil fuel burned plus carbon released from cement manufacture, by year. Data are fitted by the model given in (2).

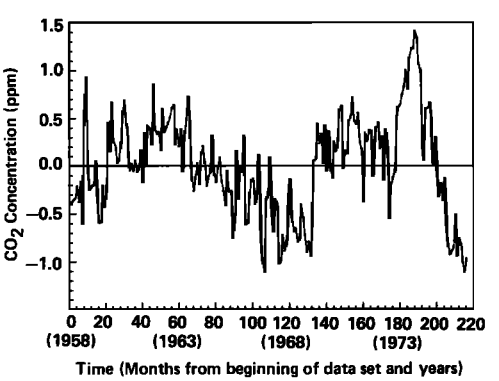

Fig. 5. Residual variability when Mauna Loa CO_2 data are fitted with the model defined by (3) with $a = 306.4$ ppm and $\alpha = 0.0050$ month^{-1}.

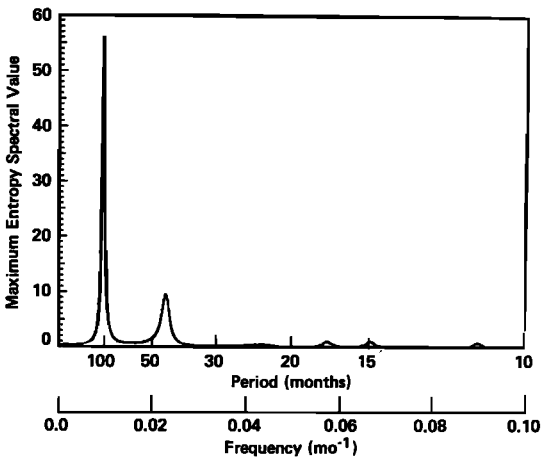

Fig. 6. Maximum entropy spectrum of the residual variability shown in Figure 5.

coefficient of correlation between α and τ is 0.976, and that between α and a is 0.9998. This means that if we can independently determine the value of one of the parameters and hold it fixed, the fitting procedure will determine the others.

Since a and α are the two parameters for which we have the best physical models and since α is one of the critical values for which we would like to have some conclusive indication, we decided to see if it were possible to estimate a independently. There is a great volume of early data, of widely varying credibility, on atmospheric CO_2 concentration.

Both *Callendar* [1958] and *Bray* [1959] concluded that no significant change in atmospheric CO_2 occurred prior to 1900, and Callendar concluded that 290 ppm ± 1% was the best choice for the North Atlantic region at the end of the last century. Bray selected among the data only slightly differently and obtained 293 ppm for a pre-1906 weighted average for the western Europe–Scandinavian region. Having now some idea of the magnitude of the diurnal cycle for a rural continental area [*Verma and Rosenberg*, 1976] and noting from *Bolin and Bischof* [1970] that CO_2 in the lower troposphere over northern Europe varies by 15 ppm over the annual cycle, we can get additional insight from the compilations of Callendar and Bray.

It appears that Bray's slightly higher average is a consequence of more conscious considerations of the annual cycle, and it is therefore to be preferred. The obvious bias which remains in both studies arises from the diurnal cycle. Callendar did present a daytime mean for his 'preferred' studies, plus enough data to see clearly that his overall mean was dominated by daytime measurements. If one distinguishes between his daytime and nighttime values and weights them so as to correct for the preponderance of daytime measurements, Callendar's average is increased by about 5 ppm. Adding about 5 ppm to Bray's average provides a number that recognizes both the annual and diurnal variations. The error suggested by Callendar, the diurnal and annual variations, and the variations among the various estimates are all constrained within ±2 or 3%. Thus while we should resist putting too much credence in the early data, the CO_2 content of the atmosphere over northern Europe was approximately 298 ppm (+4, −6)—indicating a wider error band on the low side than on the high side.

We might further inquire if the preindustrial value at Mauna Loa was different from that in northern Europe. Probably the best that can be done is to select the earliest of the modern Swedish aircraft measurements [*Bolin and Bischof*, 1970] and compare them with the earliest Mauna Loa data [*Keeling et al.*, 1976*a*]. Hence we observe that in 1960 the Mauna Loa average was less than the Swedish aircraft average by 0.26 ppm, i.e., no significant difference.

Table 1 shows the variation of the other parameters over this range of a. In the bottom row the expected small variation of the sum of squares of the residuals is given. Note that the amplitudes of the shorter-period harmonic terms A_1, A_2, and F_2 are quite insensitive to changes in a. The period of the long-period term τ_1, its amplitude F_1, and the amplitude of the exponential rate factor α are strongly dependent on a. At a = 300 the exponential rate is very close to the growth rate derived for fossil fuel production of CO_2.

Figure 8 illustrates the quality of the fit when a is specified to be 298, and Figure 9*a* presents the residuals after the exponential, seasonal, and 45- and 142-month terms are removed. Less than 3% of the 216 values are different from the predicted value by more than ±0.6 ppm. Table 2 gives an analysis of the variance. The sums of squares for the terms in the model were calculated from the reductions of the residual sum of squares as the terms were successively added to the model, with the order of inclusion being the same as the ordering of the terms in the table. The terms are not completely independent, but the final results do not vary much when the ordering is perturbed. The order chosen for the table reflects the relative strengths of the corresponding effects in the data. The important point is that the model accounts for 99.6% of the variance. This does not mean that the remaining 0.4% corresponds to the true noise level. Inspection of the residuals in Figure 9*a* reveals systematic runs of positive and negative values. A cumulative periodogram for these residuals is given in Figure 9*b*. If the residuals were truly white noise, this function should remain inside the 95% confidence band described by the two diagonal lines. A Fourier amplitude spectrum of the residuals (Figure 9*a*) is given in Figure 9*c*. There are six low-frequency peaks whose amplitudes exceed the estimated precision level of the measurements (i.e., whose peak-

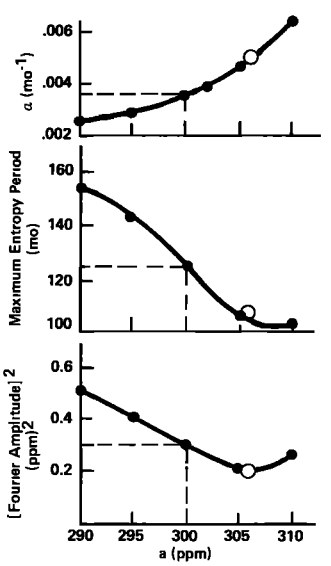

Fig. 7. Parametric study to illustrate interrelationships among exponential growth rate, period of an apparent long-term sinusoid, the amplitude of this sinusoid, and base line CO_2 concentration. The unconstrained best fit, i.e., when the base line CO_2 concentration a is allowed to vary also, is shown by open circles.

TABLE 1. Solutions for the Fit of Mauna Loa CO_2 Data by the Model Given by (4) as a Function of the Preindustrial CO_2 Concentration

Parameter	a = 292	a = 295	[a = 298]*	a = 300	a = 302
α	0.00264	0.00293	[0.00329]	0.00358	0.00394
d	22.6	19.7	16.8	14.9	13.0
τ_1	150	147	[142]	139	134
F_1	0.709	0.678	0.641	0.614	0.585
τ_2	45.0	44.9	[44.9]	44.8	44.7
F_2	0.261	0.262	0.263	0.265	0.266
A_1	2.55	2.55	2.55	2.55	2.55
A_2	0.691	0.691	0.691	0.690	0.690
$\sum$(resid.)^2	22.72	22.77	22.85	22.96	23.14

Preindustrial CO_2 concentration a was held fixed in each fit. The annual components are T_1 = 12 months and T_2 = 6 months.
*The preferred value of a is shown to be 298 ppm.

to-peak variation is more than twice the precision level). These peaks occur at frequencies which could be associated with harmonics of the 142-month cycle.

The weaker of the two long-period sinusoids discovered in the data, i.e., the one at 44.9 ± 0.7 months (a = 298), seems to represent the correlation with the southern oscillation index suggested by *Bacastow* [1976]. Analysis of the data for atmospheric pressure difference between Easter Island and Darwin, Australia (as suggested by *Quinn and Burt* [1972]), from 1962 to 1975 yielded an oscillation with a period of 44.7 ± 1.1 months. An examination of the phases of the two cycles confirmed that the maxima in the CO_2 oscillation lag that in the ΔP record by about 2½ months, as previously found by Bacastow. This periodicity in the CO_2 data has a peak-to-peak range of approximately 0.53 ppm, well above the measurement error; thus the correlation appears to be real.

We do not wish to speculate very far at this time about the apparent long-period oscillation, but we offer Figure 10, the detrended Mauna Loa data when all components other than the 142-month periodicity have been removed. Superimposed is the 142-month cycle with peak-to-peak range of 1.3 ppm, as obtained from the analysis when a = 298. This amplitude is about one fourth that of the annual cycle and about 3 times that of the 44-month cycle. In Figure 11 we have plotted the rate of change for the 142-month cycle part of the CO_2 concentration, obtained by differentiating the curve of Figure 10, superimposed on a plot of monthly sunspot numbers, obtained from World Data Center A for Solar-Terrestrial Physics, NOAA, Boulder, Colorado.

We recognize that a 216-month record is not long enough to attempt a definitive analysis of periods as long as 142-months—scarcely 1½ cycles. However, the correlations are so striking that we feel these observations should be noted. We do not suggest that the atmospheric CO_2 concentration is directly coupled to sunspot activity, although an indirect relationship is not beyond possibility. The possibility remains that this long periodicity is an artifact produced by a single (or few) random occurrence with an effect decaying over a 5- or 6-year time

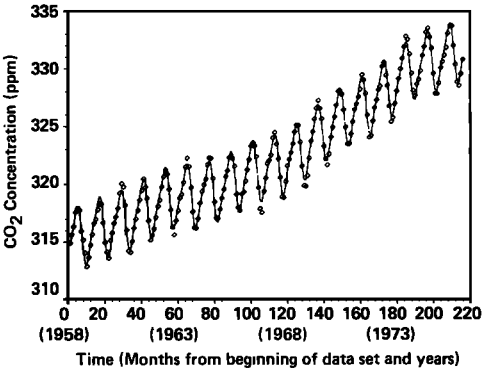

Fig. 8. Observed CO_2 concentrations at Mauna Loa (points) and results for fit of the model given by (4) (curve) when a = 298.

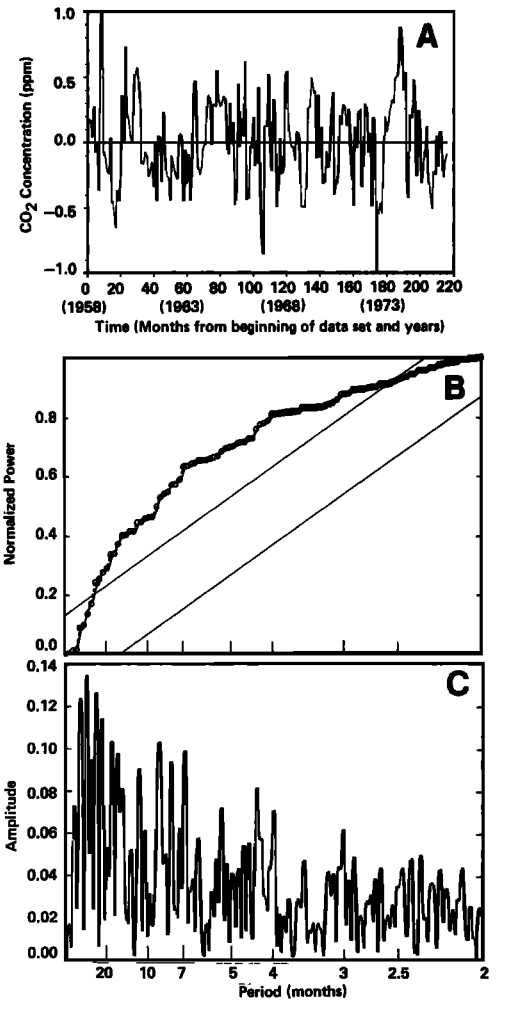

Fig. 9. Residual variability for the model described in Figure 8. (*a*) Residual variability, (*b*) Cumulative periodogram, and (*c*) Fourier amplitude spectrum of the residuals.

TABLE 2. Analysis of Variance for the Model Given in (4) When a = 298 ppm

Source of Variation	Degrees of Freedom	Sum of Squares	Percent Variance Explained
Model component			
$de^{\alpha t}$	1	5240	86.4
Annual cycle			
$A_1 \sin [(2\pi/12)(t + \phi_1)]$	1	704	11.6
$A_2 \sin [(2\pi/6)(t + \phi_2)]$	1	51	0.8
Total	2	755	12.4
142-month cycle	1	40	0.7
44.9-month cycle	1	8	0.1
Model (total)	5	6043	99.6
Error (residuals)	210	23	0.4
Corrected total	215	6066	100

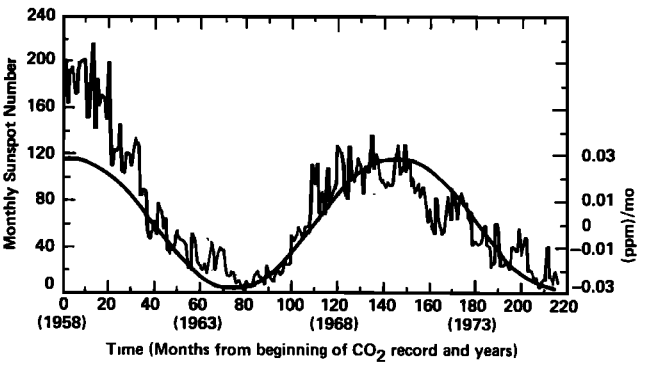

Fig. 11. Rate of change of 142-month cycle part of CO_2 concentration at Mauna Loa (a = 298) superimposed on a plot of monthly sunspot numbers.

span, e.g., the Mount Agung eruption, and consequent cooling of atmosphere and ocean surface water. Neither do we offer this example as an explanation for the data, only as a possibility.

To demonstrate the technique further and to test the model developed from the Mauna Loa data, we analyzed a time series consisting of 58 monthly average atmospheric CO_2 measurements obtained by *Pearman* [1977] and his colleagues at the Council for Scientific and Industrial Research. The measurements were taken from aircraft at altitudes of 3–5 km over southeastern Australia. They cover the months March 1972 to December 1976, overlapping the last quarter cycle shown in Figure 10. We fit the data with two models:

$$y(t) = a + de^{\alpha t} + \sum_{i=1}^{2} A_i \sin\left[\frac{2\pi}{T_i}(t + \phi_i)\right] \quad (5)$$

$$y(t) = a + de^{\alpha t} + \sum_{i=1}^{2} A_i \sin\left[\frac{2\pi}{T_i}(t + \phi_i)\right] + F_1 \sin\left[\frac{2\pi}{\tau_1}(t + \theta_1)\right] \quad (6)$$

In both fits we kept the values a = 298 ppm and α = 0.00329 month^{-1} fixed—the latter value obtained from the Mauna Loa data. In (6) we also kept the Mauna Loa values for τ_1 and θ_1 and determined only the amplitude F_1 from the fit. The residuals for the two models are shown in Figure 12. Residuals using (5) clearly display a systematic trend which is eliminated when the 142-month sine wave is added. Furthermore, the value for F_1 (0.76 ± 0.09) is comparable to the amplitude obtained at Mauna Loa.

The second longest continuous record of carefully determined monthly average values of atmospheric CO_2 is that from the South Pole [*Keeling et al.*, 1976*b*]. When the South Pole data were fitted by an exponential growth curve with yearly harmonics but no long-term periodicities (equation (3)), the unconstrained best fit was obtained with a = 301.9 and α = 0.0038. Holding a fixed at 300, the model gave α = 0.00348 ± 0.00008 (Figure 13). It has been obvious for some time that the amplitude of the annual cycle in the South Pole data is much less than that at Mauna Loa, and this is evident in the relative amplitudes A_i. The amplitude of the 12-month cycle is 2.55 at Mauna Loa and 0.453 at the South Pole; the 6-month cycle has an amplitude of 0.69 at Mauna Loa and only 0.033 at the South Pole. (Actually, all harmonics other than the 12-month period itself are nearly negligible at the South Pole.) A plot of the residuals (Figure 14) clearly reveals a periodic variation with about the 44-month wavelength as identified in the Mauna Loa data, but the continuous record from the South Pole is too short to define a cycle of 140 ± 1 month periodicity.

An important question raised by comparison of the analyses is whether the Mauna Loa and the South Pole CO_2 concentra-

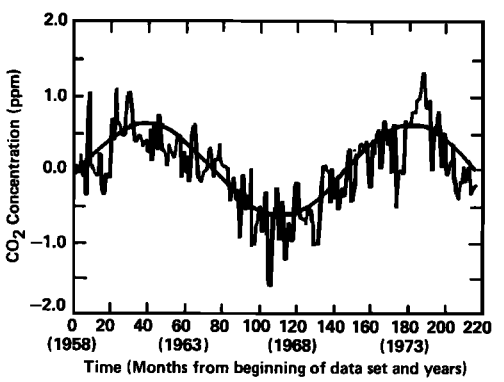

Fig. 10. Residual variability when all elements of the model in (4), except the approximately 142-month period, are subtracted from the data. The 142-month cycle (a = 298 ppm) is superimposed.

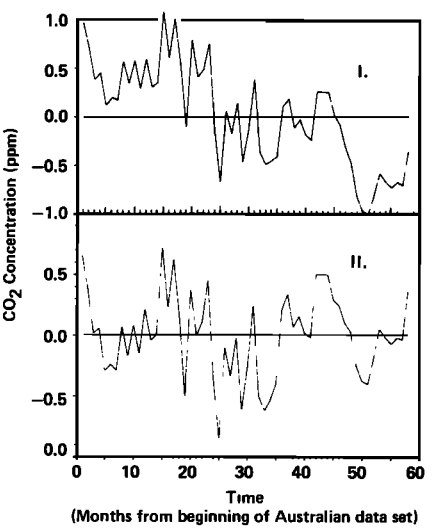

Fig. 12. Residual variability when Australian CO_2 data [*Pearman*, 1977] are fitted by two models (equations (5) (6)) with a = 298 ppm and α = 0.00329 month^{-1}.

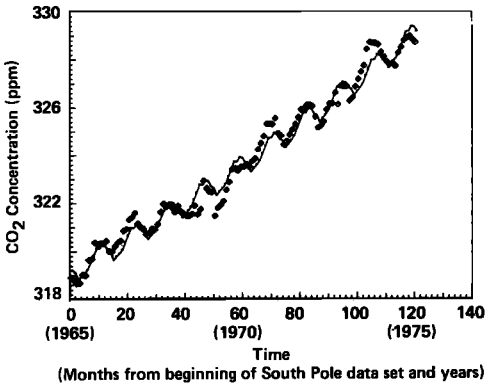

Fig. 13. Observed concentrations and model results for atmospheric CO_2 concentration at the South Pole. The model is $y(t) = 300 + de^{\alpha t} + \sum_{i=1}^{2} A_i \sin[(2\pi/T_i)(t + \phi_i)]$. Data for 1965–1971 are from *Keeling et al.* [1976*b*] and for 1972–1975 are from Keeling (private communication, 1976).

tions are experiencing exponential growth with different rate constants. A plot of the difference between the Mauna Loa and the South Pole values (Figure 15) does suggest that the Mauna Loa observations are increasing more rapidly than those at the South Pole. Interruptions in the record at the South Pole make it difficult to compare with the 18-year record at Mauna Loa, and this led us to examine the Mauna Loa data in parts. That is, since the South Pole data cover a shorter time, how might the Mauna Loa data look if it had been restricted to a shorter period? It turns out that the Mauna Loa data, fitted by (3) (i.e., without the long-period cyclic components), does give different values of α for different data periods. Holding a at 300 for the early interval 1958–1968, we obtain $\alpha = 0.0030$; for a middle interval of 1961–1971, $\alpha = 0.0036$; and for 1965–1975, $\alpha = 0.0039$. Figure 10 shows that this is the kind of variation in α that should be expected when the long-period cycle is omitted from the model. The conclusion is that only over a long data period are we able to discern and separate long-period components from the general exponential growth trend. For the South Pole data this is not yet possible and even for the Mauna Loa data the separation must be considered somewhat tentative and our specification of α correspondingly uncertain. If there is embedded in the South Pole data a 140 ± 1 month periodicity (or alternatively, some mid-1960 singularity) having a smaller amplitude in the CO_2 observations than at Mauna Loa, the apparently different growth rates could perhaps be explained.

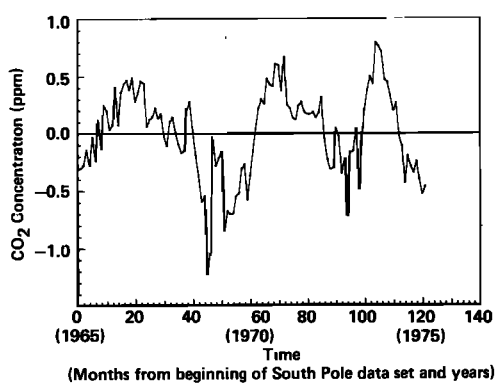

Fig. 14. Residual variability for the model described in Figure 13. Only the continuous data set, beginning in 1965, is used.

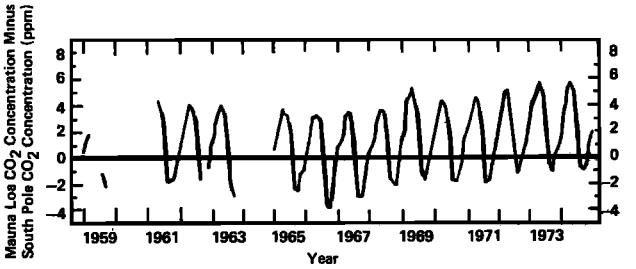

Fig. 15. Difference between atmospheric CO_2 concentration at Mauna Loa and atmospheric CO_2 concentration at the South Pole.

CONCLUSION

A mathematical model to describe temporal changes in the atmospheric concentration of CO_2 at Mauna Loa has been developed, and the generality of the model has been verified by testing it against shorter data sequences from the South Pole and Australia. The most compelling conclusions from the model are that the growth of atmospheric CO_2 is accurately represented by an exponential term and that the exponential growth rate is approximately the same as the exponential growth rate for the generation of CO_2 from fossil fuel burning and cement production. In contemplating the ultimate consequences of changes in atmospheric CO_2 it is important to realize that recent growth has been exponential.

Because of the similarity in rate constants, it is tempting to conclude that the atmospheric CO_2 growth is largely a consequence of fossil fuel burning. We note, however, that atmospheric CO_2 change is a function both of input rates (fossil fuel burning and forest clearing) and of sequestering rates in the ocean and in the terrestrial biota. It is likely that forest clearing has expanded in a manner approaching exponential growth over the period of CO_2 records and that fluxes to both the ocean and the terrestrial biota are proportional to the atmospheric concentration. Thus an exponential growth in the amount being stored in the atmosphere is not surprising. The analysis does not permit separation of effects but suggests that all of the input processes affecting atmospheric CO_2 concentration have grown at rates over the last 20 years similar to the rate of fossil fuel combustion.

We have been able to verify a component of the fluctuations in the CO_2 concentrations with a periodicity similar to that of the southern oscillation index and to identify at least one long-period component of the variability which compounds the difficulty of establishing long-term growth rates or of comparing growth rates from different monitoring sites.

REFERENCES

Bacastow, R. B., Modulation of atmospheric carbon dioxide by the southern oscillation, *Nature, 261*, 116–118, 1976.

Bacastow, R., and D. C. Keeling, atmospheric carbon dioxide and radiocarbon in the natural carbon cycle, II, Changes from A.D. 1700 to 2070 as deduced from a geochemical model, in *Carbon and the Biosphere*, edited by G. M. Woodwell and E. V. Pecan, U.S. Atomic Energy Commission, Washington, D.C., 1973.

Baes, C. F., Jr., H. E. Goeller, J. S. Olson, and R. M. Rotty, The global carbon dioxide problem, *Rep. ORNL-5194*, Oak Ridge Nat. Lab., Oak Ridge, Tenn., 1976.

Bolin, B., Changes of land biota and their importance for the carbon cycle, *Science, 196*, 613–615, 1977.

Bolin, B., and W. Bischof, Variations of the carbon dioxide content of the atmosphere in the Northern Hemisphere, *Tellus, 22*, 431–442, 1970.

Bray, J. R., An analysis of the possible recent change in atmospheric carbon dioxide concentration, *Tellus, 11*, 220–230, 1959.

Broecker, W. S., Y.-H. Li, and T.-H. Peng, Carbon dioxide—Man's unseen artifact, in *Impingement of Man on the Ocean*, edited by D. H. Hood, pp. 287–324, Interscience, New York, 1971.

Callendar, G. S., On the amount of carbon dioxide in the atmosphere, *Tellus, 10*, 243–248, 1958.

Keeling, C. D., R. B. Bacastow, A. E. Bainbridge, C. A. Ekdahl, Jr., P. R. Guenther, and L. S. Waterman, Atmospheric carbon dioxide variations at Mauna Loa Observatory, Hawaii, *Tellus, 28*, 538–551, 1976*a*.

Keeling, C. D., J. A. Adams, Jr., C. A. Ekdahl, Jr., and P. R. Guenther, Atmospheric carbon dioxide variations at the South Pole, *Tellus, 28*, 552–564, 1976*b*.

Kirk, B. L., B. W. Rust, and W. Van Winkle, Time series analysis by the maximum entropy method, *Rep. ORNL-5332*, Oak Ridge Nat. Lab., Oak Ridge, Tenn., 1979.

Lieth, H., and R. H. Whittaker, *Primary Productivity of the Biosphere, Ecol. Stud.*, vol. 14, Springer, New York, 1975.

Machta, L., Prediction of CO_2 in the atmosphere, in *Carbon and the Biosphere*, edited by G. M. Woodwell and E. V. Pecan, pp. 21–31, U.S. Atomic Energy Commission, Washington, D.C., 1973.

Manabe, S., and R. T. Wetherald, The effects of doubling the CO_2 concentration on the climate of a general circulation model, *J. Atmos. Sci., 32*, 3–15, 1975.

Oeschger, H., U. Siegenthaler, U. Schotterer, and A. Fogelmann, A box diffusion model to study the carbon dioxide exchange in nature, *Tellus, 27*, 168–192, 1975.

Olson, J. S., Carbon cycles and temperate woodlands, in *Analysis of Temperate Forest Ecosystems*, edited by D. E. Reichle, Springer, New York, 1970.

Pearman, G. I., The carbon dioxide-climate problem: Recent developments, *Clean Air, 11*, 21–26, 1977.

Quinn, W. H., and W. V. Burt, Use of the southern oscillation in weather prediction, *J. Appl. Meteorol., 11*, 616–628, 1972.

Rotty, R. M., Global carbon dioxide production from fossil fuels and cement A.D. 1950–A.D. 2000, in *The Fate of Fossil Fuel CO_2 in the Oceans*, edited by N. R. Andersen and A. Malahoff, Plenum, New York, 1977.

Stuiver, M., The ^{14}C cycle and its implications for mixing rates in the ocean-atmosphere system, in *Carbon and the Biosphere*, edited by G. M. Woodwell and E. V. Pecan, pp. 6–20, U.S. Atomic Energy Commission, Washington, D.C., 1975.

Ulrych, T. J., Maximum entropy power spectrum of truncated sinusoids, *J. Geophys. Res., 77*, 1396–1404, 1972.

Verma, S. B., and N. J. Rosenberg, Carbon dioxide concentration and flux in a large agricultural region of the Great Plains of North America, *J. Geophys. Res., 81*, 399–405, 1976.

(Received February 6, 1978;
revised July 3, 1978;
accepted July 18, 1978.)