Calibrating a Standard Candle for Extragalactic Distance Measurements

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Astronomical Measurements and Units

$B \equiv$ astronomical **apparent magnitude**

$L_B \equiv$ **relative luminosity** (rel. to B_{ref})

$$\frac{L_B(t_i)}{L_{B_{ref}}} = 10^{-0.4[B(t_i)-B_{ref}]}$$

$$B(t_i) = B_{ref} - 2.5\log_{10}[L_B(t_i)]$$

$M_B \equiv$ astronomical **absolute magnitude**

$\mu \equiv B - M_B \equiv$ **distance modulus**

$\mu \equiv 5\log D + 25 \quad (D \equiv$ **distance** [Mpc])

Radioactive Decay Model for Light Curve

$$W(t) \longrightarrow \boxed{^{56}\mathrm{Ni}} \xrightarrow{k_1} \boxed{^{56}\mathrm{Co}} \xrightarrow{k_2} \boxed{^{56}\mathrm{Fe}}$$

$$W(t) \equiv W(t\,;\alpha_1,\alpha_2,\alpha_3) = (\text{Weibull pdf})$$

$$k_1 = \frac{1}{8.764\alpha_4}\,, \quad k_2 = \frac{1}{111.42\alpha_4}\,, \quad 0 < \alpha_4 \le 1$$

$$\frac{dN_1}{dt} = W(t;\alpha_1,\alpha_2,\alpha_3) - \frac{1}{8.764\alpha_4}N_1\,, \qquad N_1(\alpha_1) = 0$$

$$\frac{dN_2}{dt} = \frac{1}{8.764\alpha_4}N_1 - \frac{1}{111.42\alpha_4}N_2\,, \qquad N_2(\alpha_1) = 0$$

$$\frac{dN_3}{dt} = \frac{1}{111.42\alpha_4}N_2\,, \qquad N_3(\alpha_1) = 0$$

$$L(t) = C_1 W(t) + C_2\left[\frac{1}{8.764\alpha_4}N_1(t) + \frac{2.146}{111.42\alpha_4}N_2(t)\right]$$

Radioactive Decay Model for Light Curve

$$W(t\,;\alpha_1,\alpha_2,\alpha_3) = \frac{\alpha_2}{\alpha_3}\left(\frac{t-\alpha_1}{\alpha_3}\right)^{(\alpha_2-1)} \exp\left[-\left(\frac{t-\alpha_1}{\alpha_3}\right)^{\alpha_2}\right]$$

At $t = \alpha_1 : W(t\,;\alpha_1,\alpha_2,\alpha_3) = 0$

$\alpha_2 =$ shape parameter $\quad \alpha_3 =$ scale parameter

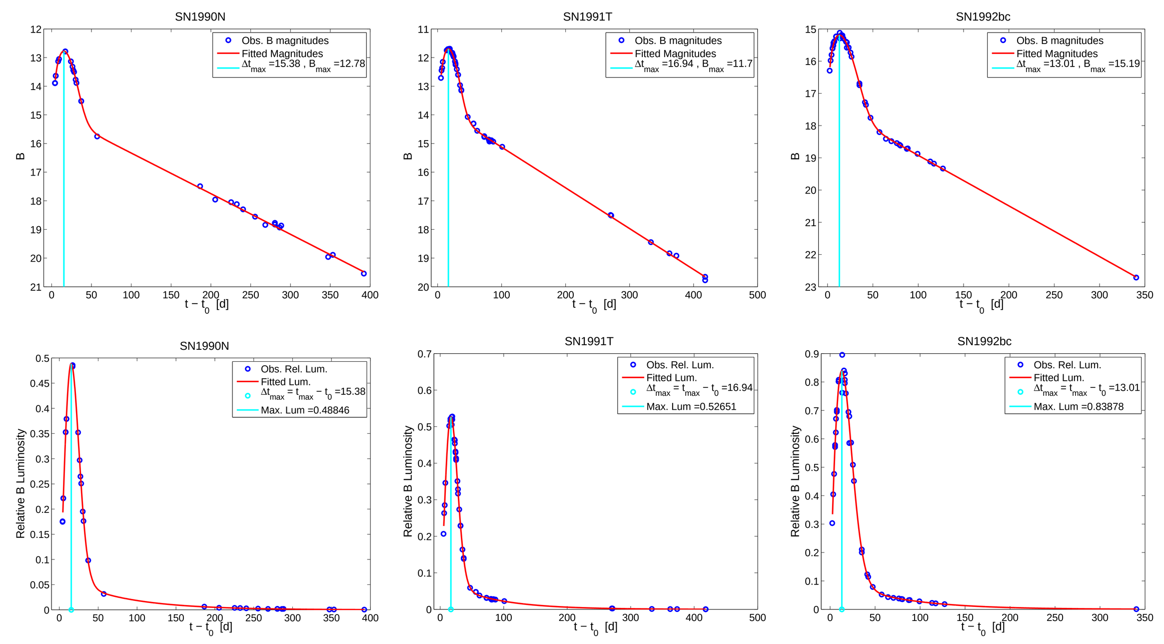
$$E(\text{Ni depos.}) = \int_{\alpha_1}^{\infty} C_1 W(t)dt = C_1$$

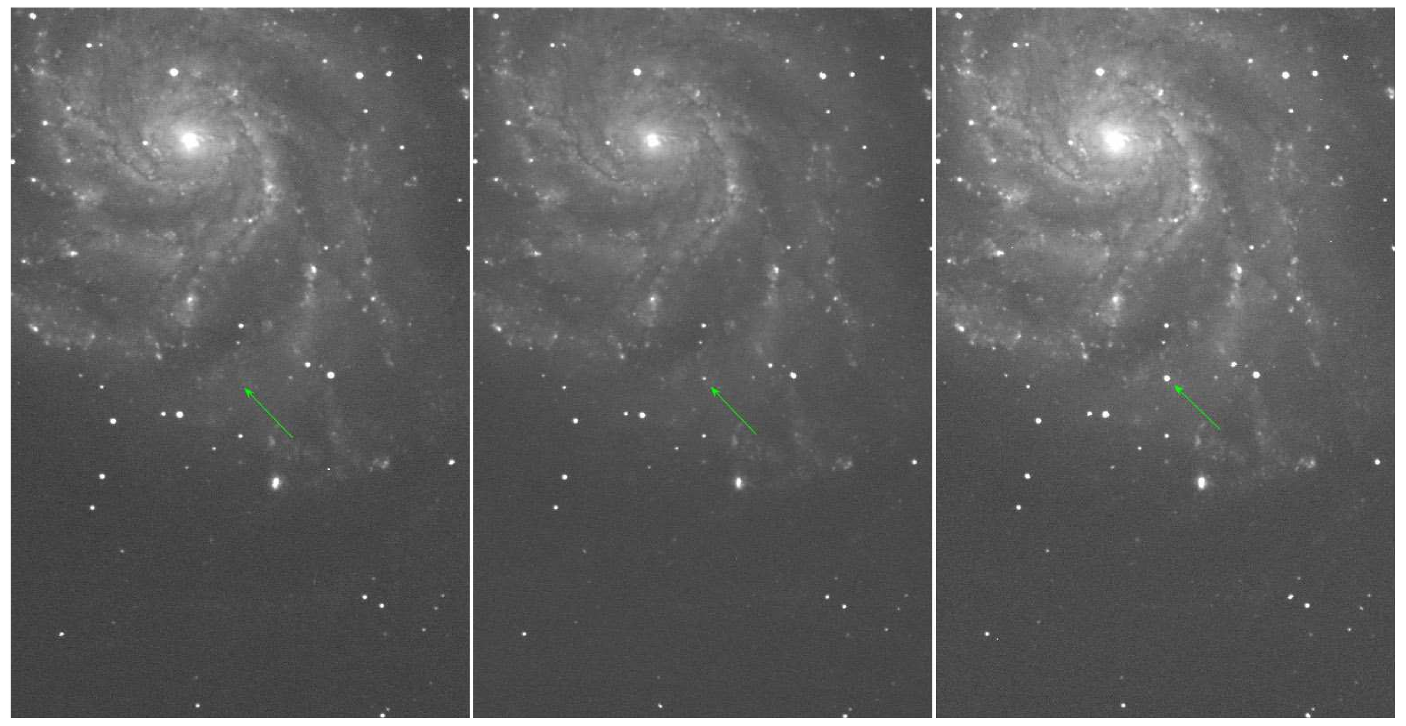
$$E(\text{Ni decay}) = \int_{\alpha_1}^{\infty} \frac{C_2}{8.764\alpha_4}N_1(t)dt = C_2$$

$$E(\text{Co decay}) = \int_{\alpha_1}^{\infty} \frac{2.146C_2}{111.42\alpha_4}N_2(t)dt = 2.146C_2$$

$$\frac{E(\text{Ni depos.})}{\text{nucleon}} = \frac{C_1}{C_2}[1.71\ \text{MeV}]$$

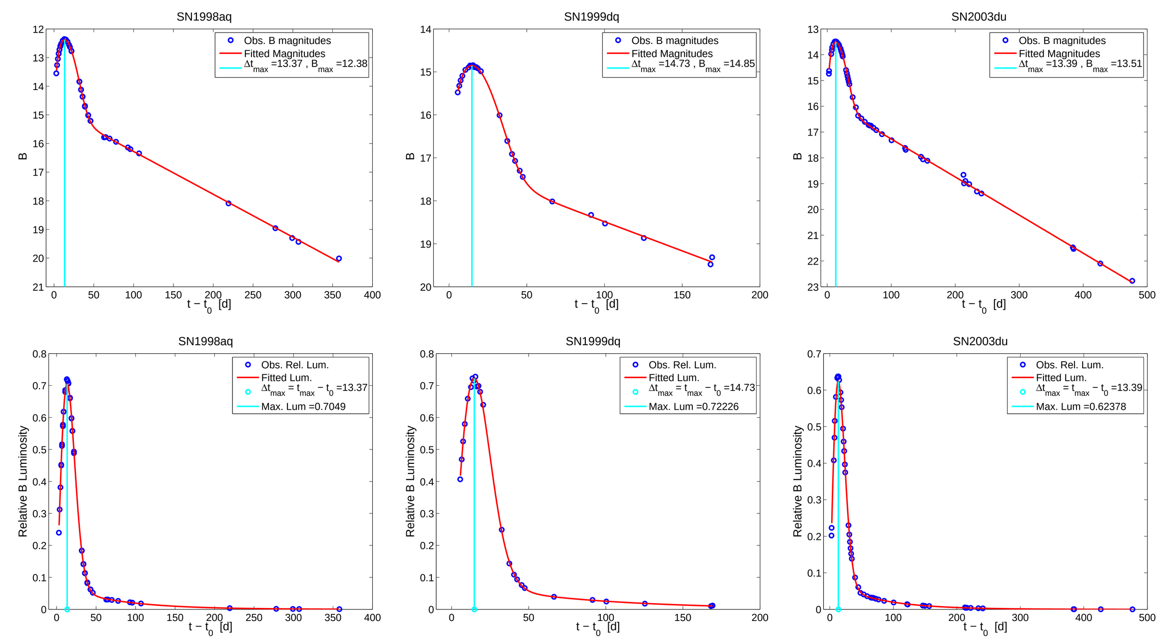
Light Curve Data and Fits



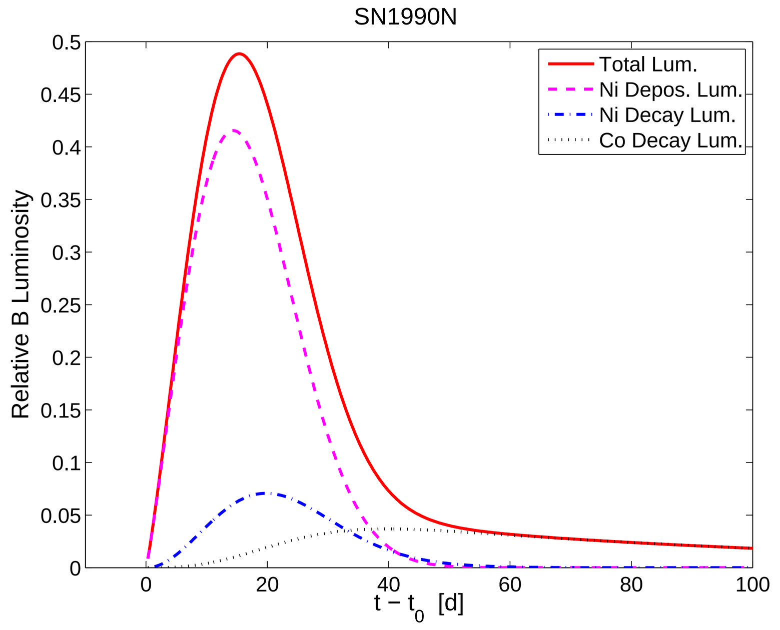


Snapshots of the luminosity of a recent supernova, indicated by the green arrow.
The luminosity appears suddenly, peaks, and then decays over a period of months.

Light Curve Data and Fits



Components of the Luminosity



Supernova	E(Ni depos.) / nucleon
SN1990N	8.577 MeV
SN1991T	8.174 MeV
SN1992bc	10.832 MeV
SN1998aq	11.548 MeV
SN1999dq	9.929 MeV
SN2003du	10.432 MeV

Binding Energy $\left(^{56}\mathrm{Fe}\right) = 8.790$ MeV

Distance Calibration

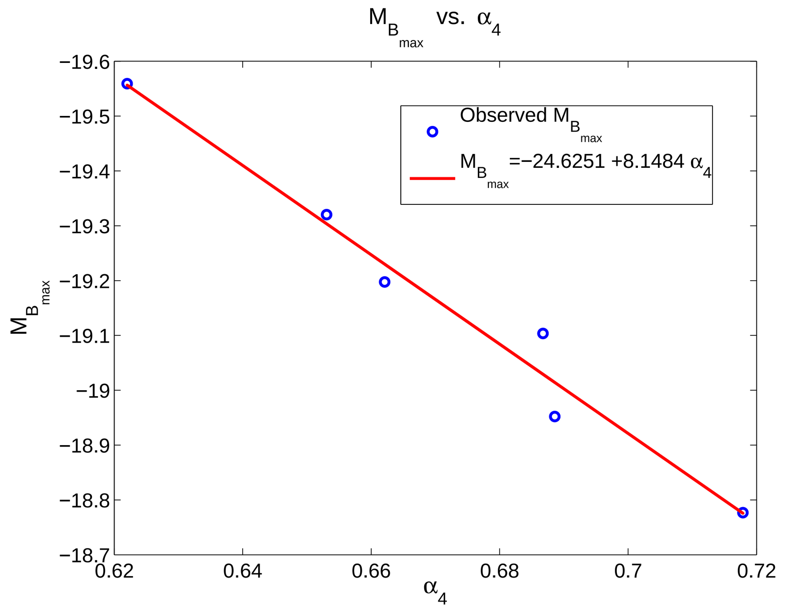
Supernova	$D[Mpc]$*	μ	B_{max}	$M_{B_{max}}$
SN1990N	22.283	31.73	12.78	−18.95
SN1991T	14.750	30.80	11.70	−19.10
SN1992bc	87.747	34.75	15.19	−19.56
SN1998aq	21.952	31.70	12.38	−19.32
SN1999dq	52.656	33.63	14.85	−18.78
SN2003du	34.760	32.71	13.51	−19.20

* D from NASA/IPAC Extragalactic Database

$$\mu \;=\; 5\log D + 25 \;=\; B_{max} - M_{B_{max}}$$

$$M_{B_{max}} \;=\; B_{max} - \mu$$

Distance Calibration



To estimate distance to a faraway supernova:

1. measure its light curve (more than 150 days)
2. fit the model to determine α_4 and B_{max}
3. $M_{B_{max}} = -24.63 + 8.148\alpha_4$
4. $\mu = B_{max} - M_{B_{max}}$
5. $D = \log^{-1}\left[\frac{\mu - 25}{5}\right][Mpc]$