REGEN3.3: USER MANUAL [1]

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Abstract

This report is the user manual for the REGEN3.3 computer program. This program models helium flow and heat transfer in a regenerator of the type used in cryocoolers. A finite difference approximation of the equations of motion in the gas and heat transfer in the matrix is used to advance the solution in time. Only the flow through the regenerator is modeled, the flow in the compressor or expansion component, such as the displacer or pulse tube, is not treated directly. Real gas properties of helium (He 4 or He 3) can be selected. The thermal heat capacity and conductivity of 33 different regenerator materials are included, or a user supplied table of the properties can be used. In addition to the semi-implicit numerical method used in the previous version of this model (REGEN3.2) another numerical method based on a conservation law formulation can be selected. The previous version required that the temperature and mass flow at the two ends be specified along with the initial pressure. If the new numerical method is selected the solution for a given pressure ratio, average pressure, and mass flow at the cold end may be obtained directly.

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1 Introduction.

This paper provides instructions for the REGEN3.3 model of a regenerative heat exchanger. This model has been developed at NIST over the past several years. The first version of this program was distributed by NIST as REGEN3.1 in 1989[1]. This was revised and distributed as REGEN3.2. The primary improvement in the latter is a graphical interface to simplify usage, more intuitive names for the input, and the addition of an iteration to achieve a given pressure ratio. This manual has been updated to describe a new version of this model, REGEN3.3. The major change is the addition of more material properties, an option to allow the use of user supplied material properties, an option to choose Helium-4 or Helium-3, and the addition of a second mathematical model of the regenerator based on conservation equations. The model used in the first two versions of REGEN is a semi-implicit model. The additional model available in REGEN3.3 is a full-implicit model based on the equations for conservation of mass, momentum and energy. The graphical user interface for REGEN3.3 will ask the user to choose between the two models.

In both versions the simplified physical model of a regenerator is a tube filled with a porous medium. An oscillatory flow of helium passes though the void space in the porous medium or matrix. The fluid is alternatively heated and cooled as the flow direction is reversed. The model is based on a numerical solution of the one dimensional equations for the flow of the helium gas through a porous matrix with an additional thermal conservation equation for the temperature of the matrix. In both versions the regenerator domain is subdivided into cells with the gas and matrix temperature computed for each cell along with the mass flux and pressure in the gas. A

finite difference approximation is used to convert the system of differential equations into discrete equations that are marched forward in time until a nearly time-periodic solution is obtained. A correlation is used to obtain the pressure drop in the gas due to viscous flow through the porous matrix and another correlation is used for the heat transfer between the gas and matrix. This report is a manual for usage of the model, therefore a complete description of the numerical method is not given.

1.1 The regen3.2 numerical approximation.

In the regen3.2 model the flow is driven by the sinusoidal mass flux at each end of the regenerator. The amplitude, frequency, and phase of the mass flux at the ends of the regenerator are input parameters. The initial pressure in the gas must also be specified. In addition the gas temperature at inflow must be given at each end. The initial temperature is generally taken to be a linear curve joining the end temperatures, although a quadratic profile can be selected. The average temperature profile (averaged in time over a cycle) will sometimes deviate greatly from the initial temperature profile which in turn will cause the average pressure to deviate from the pressure averaged over the first cycle. The differential equations used to describe the flow in this regen3.2 model are not the basic conservation equations. Instead the mass conservation equation is replaced by a predictive equation for pressure as described in section 5 below. The velocity is computed by solving a non-linear system for the velocity at all mesh cells in the regenerator. The velocity computation involves all mesh cells simultaneously, the remaining variables at each mesh cell are advanced in time using an explicit computation that involves only the one variable at the new time level. Hence the name semi-implicit for this numerical approximation. The pressure gradient is assumed to vanish so the pressure is dependent only on the time in the regen3.2 model. In the regen3.2 model the pressure drop in the regenerator is computed after the solution is advanced in time by use of the correlations described in section 5. An additional approximation in the regen3.2 model is the pressure correction at the end of each time step. This is done to insure mass conservation over the thousands of time steps that are usually necessary. This correction is not used in the regen3.3 model.

As an alternate input the user can specify three output values; the pressure ratio, the phase angle between mass flux and pressure at the cold end, and the average pressure. The initial pressure and the mass flux will then

be adjusted by means of a Newton iteration to achieve these three desired values. This is the more common way to use regen3.2 since the pressure ratio is more easily measured than the mass flux and is a more important design parameter. However, at least 5 integrations with different input data are required to obtain the desired pressure and mass flux.

1.2 The regen3.3 numerical approximation.

In this model the numerical approximation is a discretization of the differential equations for conservation of mass, momentum and energy in the gas and regenerator matrix. The resulting non-linear system of equations for the temperature, pressure and mass flux at all the mesh points simultaneously is solved by a Newton iteration. Hence, we refer to this as a fully-implicit model. In the regen3.3 model three sets of boundary conditions can be used. The first is identical to those used in the regen3.2 model. The mass flux must be given as a function of time at the two ends along with the temperature at inflow. The second requires the mass flux and pressure be given at the cold end as sinusoidal functions along with the phase angle of the cold end mass flux relative to the cold end pressure. The end temperatures at inflow are also required. The third choice for the boundary condition is similar to the second except that the pressure is given at the hot end as well as the phase of the mass flux at the cold end relative to the pressure at the hot end. In most cases the users of the regenerator model want to obtain results for a given mass flux and pressure at the cold end. This is provided directly by the second boundary condition. To obtain this result from the first or third boundary condition several integrations of the model are required with different input data as described in section 2.2. The regen3.3 model generally requires more computer time to perform an integration, but if the second boundary type is used no additional integrations are required to obtain the desired relation between the pressure and mass flux at the cold end. The regen3.3 computation does not assume a zero pressure drop across the regenerator nor does it require a pressure correction on each time step in order to conserve mass. In some cases the regen3.3 model converged when the regen3.2 model failed. These failures generally occurred in a layered regenerator where the layers had different values of parameters such as porosity, material, or geometry. The regen3.3 model seems to be more accurate. In comparisons that we have run the models generally agreed to within 5 or 10 percent depending on the resolution. However, the regen3.2 model has been

tested far more than the regen3.3 model.

1.3 The data obtained from the model.

The material of the matrix can be selected from a large menu of materials including various metals and rare earths. The correlation for heat transfer and pressure drop is determined by selection from a menu of geometries including spherical pellets and wire screens. The solution yields the enthalpy flux at the cold end of the regenerator. This flux is a major determinant for the effectiveness of the regenerator. The pressure ratio at both ends, the PV work, the ineffectiveness and many other parameters are displayed in the output file that is printed. The program will provide an estimate of the cooling power under the assumption of an isothermal expansion space. In addition data suitable for plotting is output in a file. The output is described in section 3.

2 The input variables.

Certain input parameters, for example those that define the geometry and material of the regenerator, are required. Other parameters are set by default or are used to select optional features. The former are described in the following section, the latter in a subsequent section. The input is split into these two parts to reduce the effort required to learn to use the model. In most cases the default values will suffice thus reducing the number of parameters that must be dealt with. All the parameters are input using Fortran90 NAMELIST format. A text editor can be used to create a file containing the NAMELIST input. On a Windows PC a graphical interface is available to create this input file. This file is then supplied as the input to execute the REGEN3.2 code. An example of such an input file is given in the appendix. The NAMELIST format requires that the name of the variable be given followed by its value. The names of the required variables are itemized in the following list. The context should make the type of these variables obvious. They are input as INTEGER or REAL variables. The hot end of the regenerator is always placed at the left end (x=0.0) and the cold end at the right (x=RG_LENGTH).

2.1 Required input parameters.

These input parameters do not have default values.

- FINAL_CYCLE (cycle).The number of cycles which will be run in the simulation. This parameter along with the frequency set by HERZ determine the time over which the model will be integrated. The value should be large enough to insure that a nearly periodic state is obtained. For a regenerator used in the first stage of cooler a value of 10000 or more may be required. This is a variable of integer type.

- GEOMETRY. The geometry of the regenerator. This integer variable is used to determine the correlation for heat transfer between gas and matrix. It also determines the correlation for the friction factor and thus the pressure drop. The following values are permitted.

  1. Flow between parallel plates.
  2. Axial flow through a set of parallel tubes. In this case the regenerator consists of one or more open tubes. The matrix consists of the walls of the tube.
  3. Flow transverse to tubes. Valid for Reynolds numbers between 300 and 1500.
  4. Flow through screens. Valid for porosity between 0.6 and 0.83.
  5. Flow through spherical pellets.
  6. Correlations determined by HTC1 and other input parameters.

  In regen3.2 these correlations can be overridden by setting the input parameter IHTPR. This option is not available in regen3.3. In this case the function routines USERHT and USERPR contained in the file userfn.f must be modified to supply the heat transfer and pressure gradient. We refer the reader to the internal comments in the userfn.f file for the usage of this option. Of course, use of this option requires the recompilation of the package.

- GAS_TEMP_COLD (K). The temperature of the incoming gas at the cold end. This is the temperature of the ideal heat exchanger and isothermal expansion space that the model assumes at the cold end.

- GAS_TEMP_HOT (K). The temperature of the incoming gas at the hot end. This is the temperature of the ideal heat exchanger and isothermal compressor the model assumes at the hot end.

- HERZ (s^{-1}). The frequency of the sinusoidal mass flow through the regenerator.

- HYDRA_DIAM (m). The hydraulic diameter of the matrix within the regenerator. Note that this is the diameter and not the radius.

- MASS_FLUX_COLD (kg/s). The amplitude A of the sinusoidal mass flux ($\dot{M} = A \sin(\omega t + \theta)$) at the cold end of the regenerator.

- MASS_FLUX_HOT (kg/s). *In most applications of regen3.3 this input parameter is not used, it is ignored. It is not used unless the value of the BDY_TYPE input parameter is 1. When BDY_TYPE=2 the mass flux at the hot end is obtained from the output and given by the output parameter MASFX0.* When BDY_TYPE=2 the value of the cold end pressure ratio is an input parameter. Other values of BDY_TYPE give a pressure ratio measured from the output pressure profile, the pressure ratio is not an input parameter. Giving the pressure ratio as an input parameter is usually more useful, therefore the value of BDY_TYPE is usually 2 and the value of MASS_FLUX_HOT is ignored.

  This mass flux is the amplitude A of the sinusoidal mass flux ($\dot{M} = A \sin(\omega t)$) at the hot end of the regenerator. This parameter is not used in the regen3.3 computation when BDY_TYPE > 1.

- MASS_PHASE (degree). *In most applications of regen3.3 this input parameter is not used, it is ignored. It is not used unless the value of the BDY_TYPE input parameter is 1 or 3. When BDY_TYPE=2 this phase angle is obtained from the output and is given by the value of the output parameter PHSMAS.* In regen3.2 and in regen3.3 when the value of BDY_TYPE=1 or 3 this is an input parameter. It is the phase angle θ of the mass flux at the cold end relative to the mass flux at the hot end. The mass flux at the cold end is ($\dot{M} = A \sin(\omega t + \theta)$). The phase is positive if the peak mass flux at the cold end occurs before that at the hot end. Note that the phase in this definition differs from most of the phase angles in this package which are taken relative to the cold end. In regen3.3 when BDY_TYPE = 1 this parameter has

the same meaning as in regen3.2. This parameter is not used in the regen3.3 model when BDY_TYPE = 2. In regen3.3 when BDY_TYPE = 3 this is the phase of the mass flow at the cold end relative to the pressure at the hot end.

- MATERIAL This integer parameter selects the material type. The following values are permitted. This parameter determines both the heat capacity and the thermal conductivity of the matrix. There is no default value. In regen3.2 only the first 34 values are allowed. The effect of materials 35 and 36 is obtained by setting MATERIAL_FORM to 3 or 4 in contrast to regen3.3 where MATERIAL_FORM is limited to values 1 and 2.

  1. Stainless steel.
  2. Epoxy glass.
  3. Nylon.
  4. Lead (95% Pb, 5% Sb).
  5. Brass.
  6. Nickel.
  7. Gd-Rh. Thermal conductivity same as material 8.
  8. Gd(0.6)-Er(0.4)-Rh.
  9. Er(3)-Ni.
  10. Er-Ni. Conductivity same as material 9.
  11. Er-Ni(2). Conductivity same as material 9.
  12. Er-Al(2). Conductivity same as material 9.
  13. Er-Dy(0.8)-Ni(2). Conductivity same as material 9.
  14. Kapton (NIST Report 3948).
  15. Neodymium. Conductivity same as material 8.
  16. Er(3)-Ni (NASA Ames). Conductivity same as material 9.
  17. Er(0.9)-Yb(0.1)-Ni. Conductivity same as material 9.
  18. Er(3)-Co. Conductivity same as material 9.
  19. Er(0.6)-Pr(0.4). Conductivity same as material 9.

20. A layered mixture of materials 1, 12, 13, 17, and 19.
21. A layered mixture of materials 1, 4, 10, and 18.
22. A layered mixture of materials 1, 18, and 19.
23. Ho-Cu(2) (uses same conductivity as material 1).
24. Er-Ni(0.9)-Co(0.1) (uses same conductivity as material 1).
25. Ho(2)-Al (uses same conductivity as material 1).
26. Er-Ag(0.9)-Al(0.1) (uses same conductivity as material 1).
27. Ho-Sb.
28. Dy-Sb.
29. Gd-Sb.
30. Commercially pure Er (Conductivity as material 1).
31. Er(0.5)-Pr(0.5) Conductivity as material 1.
32. From "GAP: A new Ceramic Magnetic Regenerator Material for 4 K Cryocoolers" T. Numazawa, T. Yanagitani, H. Nozawa, Y. Ikeya, R. Li, and T. Satoh 12th ICC
33. From "GOS: A new Ceramic Magnetic Regenerator Material for 4 K Cryocoolers" T. Numazawa, T. Yanagitani, H. Nozawa, Y. Ikeya, R. Li, and T. Satoh 12th ICC
34. Material properties taken from the file mattable. If this value is used then a file named "mattable" (must be lower case in linux) must be present in the directory where the program is executed. This file contains data that determines the material heat capacity and thermal conductivity as a function of temperature. The file contains a series of temperatures along with the heat capacity and conductivity at those temperatures. This is a text file. The first line must contain a integer value giving the number of temperature points. Each following line must contain 3 real numbers separated by a space or spaces. The first number is the temperature, the next the heat capacity, the last the thermal conductivity. The successive temperature values must form an increasing sequence. The lowest temperature must be no greater than the value of TABLE_TEMP_MIN and the highest (i.e. last) value must be no less than TABLE_TEMP_MAX. It is best to set these two input parameters explicitly if the mattable file is used in order to be sure

that this condition on the range of the temperatures in mattable is satisfied. The heat capacity is volumetric with units $(J/m^3 - K)$. The conductivity is also in SI units $(W/m - K)$.

35. A matrix made up of a mixture of the materials listed above. See the description of NUM_MATERIALS, MAT_FRACTION, and MATERIALS_LIST.

36. A hypothetical matrix choosen to give the maximal heat capacity over the list of materials contained in MATERIALS_LIST. See NUM_MATERIALS and MATERIALS_LIST.

- POROSITY. The porosity of the matrix.

- PRES_INITIAL (Pa). The initial pressure in the regenerator. This initial pressure combined with the initial temperature profile in the gas and the phase angle of the mass flow at the ends will determine the average pressure over the first flow cycle. The initial pressure and the first cycle average pressure will most likely differ. This parameter will be set to AVE_PRES in the regen3.3 model when BDY_TYPE > 1.

- RG_AREA (m^2). The cross-sectional area of the regenerator. This is the total area including the matrix. This value must be multiplied by the porosity to obtain the free-flow or void area.

- RG_LENGTH (m). The length of the regenerator.

The following input parameters are required in the regen3.3 model when the input parameter BDY_TYPE has value two. If FIND_PRATIO is greater than zero they are required when BDY_TYPE has value one or three as discussed in the next section. For further information see the discussion of the input parameter BDY_TYPE.

- AVE_PRES (Pa). The average pressure specified at the cold end. There is no default value.

- PRES_PHASE (degree). The phase angle of the mass flow at the cold end relative to the pressure at the cold end. This value is positive if the peak of the mass flow occurs before that of the pressure. There is no default value.

- PRES_RATIO. The pressure ratio specified at the cold end. This is the ratio of the maximum pressure over the cycle to the minimum over the same cycle. There is no default value.

2.2 Input to obtain a given pressure ratio.

In the regen3.2 model the required program input includes the mass flux at both ends as well as the phase angle between the two and the initial pressure. The pressure ratio and average pressure obtained after a periodic flow is achieved depends on this input. In most cases the desired output from the program is the behavior of the regenerator for a given average pressure and pressure ratio. The program can not yield this directly since the mass flow must be given at both ends. The program can be set up to perform a fixed number of Newton iterations to search for a given pressure ratio, average pressure and phase between the pressure oscillation and mass flow at the cold end. The input value of the mass flow at the cold end is used, then the mass flow at the hot end, the initial pressure, and the phase angle between the mass flow at the two ends is varied to achieve the desired output. There are three input parameters which can be varied to obtain the three output values. A fixed number of Newton iterations are used to obtain an improved estimate for the three values. Each iteration requires 5 integrations to compute the Jacobian matrix and then correct the initial guess. These 5 integrations can take several hours of computer time, so we prefer to do a fixed number of iterations rather than iterate until a convergence criterion is satisfied. The iteration requires the Jacobian matrix of the partial derivatives of the the three output values with respect to the three input values. This Jacobian is estimated by finite differences. That is, the model is first integrated over the interval given by FINAL_CYCLE at the given input values. Then each of the three input values are incremented in turn and the integration is repeated. With the data from these four integrations the Jacobian is estimated by a finite difference computation and a new guess for the input values is obtained from Newton's algorithm. A fifth integration is carried out with new input values to obtain improved output values. The desired values of the three output variables are given by the input variables AVE_PRES, PRES_RATIO, and PRES_PHASE. After the first Newton iteration each iteration uses the output from the preceding iteration as initial conditions.

In regen3.3, if the input parameter BDY_TYPE is set to one, then the boundary conditions are the same as in regen3.2 and the above discussion

applies to regen3.3. The following set of input parameters are not required for the regen3.2 model or for regen3.3 when BDY_TYPE = 1, since the FIND_PRATIO value is set to zero by default. However, in the regen3.2 simulations at NIST these parameters are almost always used. The main advantage of regen3.3 is obtained when BDY_TYPE = 2 which is the default value. We include the BDY_TYPE = 1 option in regen3.3 only for compatabilty.

- FIND_PRATIO. An integer parameter whose value is the number of Newton iterations used to search for a solution with a given pressure ratio, average pressure and phase between the pressure and mass flow at the cold end. The default value is 0.

- AVE_PRES (Pa). The average pressure at the cold end. If BDY_TYPE = 1 and the value of FIND_PRATIO is ≥ 1, or if regen3.2 is used and FIND_PRATIO is ≥ 1 then a value must be supplied for this variable. This input parameter must be set in the input to the regen3.3 model when BDY_TYPE > 1. There is no default value.

- PRES_PHASE (degree). The phase angle of the mass flow at the cold end relative to the pressure at the cold end. This value is positive if the peak of the mass flow occurs before that of the pressure. In regen3.2 this value must be supplied if FIND_PRATIO is ≥ 1. In regen3.3 if BDY_TYPE = 1 and FIND_PRATIO IS ≥ 1 or if BDY_TYPE > 1 it must be supplied. There is no default value.

- PRES_RATIO. The pressure ratio sought by the Newton iterations. This is the ratio of the maximum pressure to the minimum pressure over the cycle. In regen3.2 this value must be supplied if FIND_PRATIO is ≥ 1. In regen3.3 if BDY_TYPE = 1 and FIND_PRATIO IS ≥ 1 or if BDY_TYPE > 1 it must be supplied. There is no default value.

The following three variables are the increments used to compute a finite difference Jacobian for the Newton iterations when FIND_PRATIO > 0.

- MASS_FLUX_INC (kg/s). The increment for MASS_FLUX_HOT. The default value is 0.05*MASS_FLUX_HOT.

- MASS_PHASE_INC (degree). The increment for MASS_PHASE. The default value is 6.0.

- PRES_INC (Pa). The increment for PRES_INITIAL. The default value is 0.05*PRES_INITIAL.

2.3 Optional input parameters.

These are input parameters that may be used to alter the physical approximation or the numerical resolution. In most cases the default value for these parameters can be used.

- AREA_LAYERS. An array giving the values of the area in each layer of a multilayered regenerator. This is used only if MATERIAL_FORM=2. The default value is taken as RG_AREA.

- BDY_ORDER. An integer parameter to select the order of approximation used to estimate derivatives and extrapolations at the two ends of the domain. If the value is 1 a first order approximation is used. If the value is 2 the approximation is second order. Generally it is better to use the first order approximation. This parameter is not used in the regen3.2 model. The default value is 1.

- BDY_TYPE. An integer parameter to select the boundary condition used in the regen3.3 model. This parameter is not used in the regen3.2 model. The default value is 2. The following values are used.

    1. The boundary conditions are determined by the mass flow at the two ends and the initial pressure. This choice requires the following input variables: (PRES_INITIAL, MASS_FLUX_COLD, MASS_FLUX_HOT, and MASS_PHASE). As in regen3.2 the FIND_PRATIO parameter can be used to iterate for specified values of AVE_PRES, PRES_RATIO, and PRES_PHASE.
    2. The boundary conditions are determined by the pressure ratio and the average pressure at the cold end along with the mass flux at the cold end and the phase of this cold end mass flux relative to the pressure. This requires the following input: AVE_PRESS, PRES_PHASE, and PRES_RATIO. The value of FIND_PRATIO is not used when this option is selected.
    3. The boundary conditions are determined by the average pressure and pressure ratio at the hot end, the mass flux at the cold end,

and the phase of this cold end mass flux relative to the pressure at the hot end. This phase (MASS_PHASE) is positive if the cold end mass flux leads the hot end pressure. In this case the FIND_PRATIO parameter can be used to alter the value of MASS_PHASE to obtain the desired value of PRES_PHASE. This value of BDY_TYPE requires the input parameters AVE_PRES, PRES_RATIO. In addition the paramter PRES_PHASE must be input if FIND_PRATIO is > 0.

- COOLING_MULT. A factor used to reduce the gross refrigeration power. The cooling rate of a cryocooler based on the regenerator is estimated by assuming an isothermal expansion space. The isothermal cooling rate is reduced by this parameter to yield a more realistic cooling rate GRCADJ.

$$\text{GRCADJ} = \text{COOLING\_MULT} * \text{GRCOOL}.$$

  The default value is 1.0.

- DECAY_CYC (cycle). Used to return the temperature to its inflow value when the fluid velocity changes from outflow to inflow at either end. Since the outflow temperature is generally not equal to the temperature to which the inflow is set, a discontinuity is generated if the temperature is set to the inflow value on the time step when there is a reversal in the flow direction. The value of this parameter is the e-folding time in cycles over which the temperature is reduced to the inflow value. The temperature of the inflowing gas is given by the formula

$$\mathrm{T(t)} = \mathrm{T_0} + (\mathrm{T_r} - \mathrm{T_0})\mathrm{e}^{(\mathrm{t}-\mathrm{t_r})/(\tau \mathrm{D})}$$

  where $\mathrm{T_r}$ is the temperature of the gas when the flow changes from outflow to inflow, $\mathrm{T_0}$ is the inflow gas temperature, $\mathrm{t_r}$ is the time of the flow reversal, D is the value of the DECAY_CYC parameter, and τ is the period of the flow oscillation. The default value is 0.05 cycles.

- EPS_NEWTON. In regen3.3 this is the error tolerance in the Newton iteration used to solve the non-linear equations for each time-step. The error is measured in a normalized L_2 norm. In regen3.2 this is the relative error tolerance used in the Newton iteration for the velocity. The velocity is computed by using predicted values for the other variables in

the finite difference approximation of the energy equation for the gas. The velocity is then varied by a Newton iteration to solve the energy equation. When the maximum difference between the velocity on successive iterates is less than this parameter the iteration is successful. The default value is 10^{-6}.

- EPS_STEPS. In regen3.2 this is the error tolerance used for the predictor-corrector iteration for the gas and matrix temperature. If EPS_STEPS $> 0.$ then then the corrector iteration will continue until the difference between the temperatures on successive iterates relative to the average temperature is less than this parameter. If this parameter is set to zero, then a fixed number of iterations given by the value of NUM_ITT_STEP is used. This is used only in the regen3.2 model. The default value is 0.0.

- FULL_OUTPUT_INC (cycle). The increment in cycles over which full output is produced. See the section on output for a more complete discussion. This is an integer variable. The default value is set equal to the value of FINAL_CYCLE so that only one full output is produced on the last cycle of the integration.

- GAS_COND_COLD (W/(m $\cdot$ K)). Used to approximate the thermal conductivity of the gas by a linear function of temperature. If this linear approximation is selected by USE_GAS_COND the conductivity is determined by fitting the values of GAS_COND_COLD and GAS_COND_HOT at temperatures GAS_TEMP_COLD and GAS_TEMP_HOT. There is no default value.

- GAS_COND_HOT (W/(m $\cdot$ K)). See GAS_COND_COLD above.

- GEOM_LAYERS. An array whose elements select the geometry in each layer of a multilayered regenerator. This parameter is not used unless MATERIAL_FORM=2. The input data file is read using NAMELIST format in FORTRAN. The default values are taken from the GEOMETRY value.

- GRADED_CUTOFF. The distance over which the initial temperature drops to the value at the cold end (m). This parameter is only effective when USE_GRADED_MESH > 0. See the discussion

of USE_GRADED_MESH. This option is not available in the regen3.3 model. There is no default value.

- GRADED_RATIO. The ratio by which the length of successive mesh cells increase as the cells advance from the warm end to the GRADED_CUTOFF point. This parameter is only effective when USE_GRADED_MESH > 0. This option is not available in the regen3.3 model. The default value is 1.0 meaning that the cell size is constant across the regenerator.

- HELIUM. An integer parameter used to select the helium model. A value of 4 will select helium-4, a value of 3 selects helium-3. The default value is 4.

- HTALP. Used in the definition of the coefficient for heat transfer between the gas and matrix. The factor $R^{1-\beta}$ in this coefficient is smoothed in the vicinity of $R = 0$ by the factor $1 - e^{-\alpha R^2}$ for the four cases GEOMETRY=3,4,5,and 6. In these cases the heat transfer function given by Kays and London has the form $CR^{1-\beta}$ and the smoothed heat transfer used in this package is

$$H = C(\gamma e^{-\alpha R^2} + (1 - e^{-\alpha R^2})R^{1-\beta})$$

  where γ=HTGAM, α=HTALP, and R is the Reynolds number. The value of β is determined by GEOMETRY. The default value of HTALP is 0.04.

- HTBETA. Used in the definition of the heat transfer coefficient if GEOMETRY=6. See HTC1. The default value is 0.30.

- HTC1 ($W/(K \cdot m^2)$). Used to define the heat transfer between the gas and the matrix when GEOMETRY=6. The heat transfer coefficient in this case is given by

$$H = H_c(\gamma e^{-\alpha R^2} + (1 - e^{-\alpha R^2})R^{1-\beta})$$

  Here α=HTALP, β=HTBETA, H_c=HTC1, γ=HTGAM, and R is the Reynolds number of the flow. The default value is $30W/(K \cdot m^2)$.

- HT_FACTOR. The heat transfer coefficient that is determined by the geometry parameter will be multiplied by this factor. The default value is 1.0.

- HTGAM. Used to define the heat transfer between the gas and the matrix. See HTALP, above. The default value is 2.0.

- HIDIAM_LAYERS (m). An array of values giving the value of the hydraulic diameter in each layer of a multilayer regenerator. This array is not used unless MATERIAL_FORM=2. The default values are taken from the HYDRA_DIAM parameter.

- IHTPR. This parameter can be used to override the value of the GEOMETRY parameter so that the routines USERHT and USERPR are used to compute the heat transfer coefficient and the friction factor. These routines must be written and the package recompiled to link them with the REGEN3.2 code. See the comments in the file userfn.f where dummy versions of the USERHT and USERPR routines are included. The allowed values follow.

  0. Use the heat transfer and pressure drop determined by the GEOMETRY parameter.
  1. Use the USERHT and USERPR routines.

  This option is not available in the regen3.3 model. The default value is 0.

- IXHIST. An integer array containing the indices of the mesh points where time series output is selected. If I=IXHIST(K), then the temperature and other parameters will be graphed as a function of time at the mesh point X(I). The value of IXHIST(K) is restricted by $1 <= I <= NX$ and K must be in the range $1 <= K <= MUSHIS$ where MUSHIS is the input parameter that determines the number of these output points. If MUSHIS has the value zero, then this parameter is not used. There are no default values.

- LOCATE_HEAT (m). This parameter is used to specify the mesh cell to which heat is added when VOL_HEAT is non-zero (m). The cell chosen is the one which contains the point at a distance given by LOCATE_HEAT from the warm end. There is no default value.

- MATERIAL_FORM. This integer parameter determines the method used to compute the heat capacity and thermal conductivity of the solid matrix within the regenerator. It selects a single material or material

layers. There are 34 different materials selected by the MATERIAL parameter that can be formed into layers or mixtures. The default value is 1. In regen3.2 this parameter can take on values one through 4. See the regen3.2 documentation for more information. In regen3.3 this parameter can take on the following 2 values.

1. This value selects a matrix composed of a single material. The material is determined by the value of the MATERIAL parameter.
2. This value selects a matrix composed of layers of different materials. The number of layers is given by NUM_LAYERS. The break points that separate the layers is given by the array X_LAYERS. The number of elements in this array is one less than the value of NUM_LAYERS. The material in each layer is determined by the array MAT_LAYERS using the material numbers as listed below for MATERIAL. The porosity and hydraulic diameter of the layers are determined by POROS_LAYERS and HIDIAM_LAYERS. The other input arrays used for layers are AREA_LAYERS, GEOM_LAYERS, and MCDFACTOR_LAYERS. If MATERIAL_FORM=2 then at least one of these 6 arrays must be set in the input. If the remaining arrays are not included in the input they will be set by default using the values of RG_AREA, POROSITY. HYDRA_DIAM, MATERIAL, GEOMETRY, MAT_COND_FACTOR. These parameters are described in the section below that deals with optional input parameters.

- MAT_COND_COLD ($W/(m \cdot K)$). Used to approximate the thermal conductivity of the matrix by a linear function of temperature. If this linear approximation is selected by USE_MAT_COND the conductivity is determined by fitting the values of MAT_COND_COLD and MAT_COND_HOT at temperatures GAS_TEMP_COLD and GAS_TEMP_HOT. There is no default value.

- MAT_COND_FACTOR. A factor used to reduce the thermal conductivity of the matrix. The thermal conductivity used in the program is that for a solid material. However, the actual matrix may be screens or spherical pellets. The matrix conductivity will be multiplied by this factor to compensate for the structure of the matrix. The default value is 1.0.

- MAT_COND_HOT ($W/(m \cdot K)$). See MAT_COND_COLD above.
- MAT_CP_FACTOR. The heat capacity of the matrix that is determined by the MATERIAL parameter will be multiplied by this factor. The default value is 1.0.
- MAT_CPVOL_COLD ($J/m^3 \cdot K$). Used to approximate the heat capacity per unit volume of the matrix by a linear function of temperature. If this linear approximation is selected by USE_MAT_CPVOL the capacity is determined by fitting the values of MAT_CPVOL_COLD and MAT_CPVOL_HOT at temperatures GAS_TEMP_COLD and GAS_TEMP_HOT. There is no default value.
- MAT_CPVOL_HOT ($J/(m^3 \cdot K)$). See MAT_CPVOL_COLD above.
- MATERIALS_LIST. This is an array which must be defined when MATERIAL has the value 35 or 36. Otherwise it is not used. The number of elements in the array is given by the value of NUM_MATERIALS. Each entry is a number between 1 and 34 that identifies a material according to the definition of the MATERIAL parameter. If MATERIAL=35 this list defines the components of a uniform mixture of materials. The volumetric fraction occupied by each material is given by the array MAT_FRACTION. If MATERIAL=36 this list defines a set of materials to simulate a regenerator composed of layers of these materials chosen to maximize the heat capacity. The heat capacity of the matrix is, at each temperature, the maximum heat capacity over this list of materials. There is no default value.
- MAT_FRACTION. This array is used only if MATERIAL=35. It is an array whose elements contain the volumetric fraction of the material given in the corresponding element of the array MATERIAL_LIST. The number of elements in the array is given by the value of NUM_MATERIALS. The value of each element of this array must lie between 0.0 and 1.0 and the elements of the array must sum to 1.
- MAT_LAYERS. This array determines the material type in each layer of a multilayered regenerator. The index for each material type is given in the discussion of the MATERIAL parameter. The number of elements in the array is given by the value of NUM_LAYERS. This parameter

is used only if MATERIAL_FORM has the value 2. The default values are taken from the value of the MATERIAL parameter.

- MCDFACTOR_LAYERS. The values of this parameter determine the factor used to reduce the thermal conduction of the matrix for multilayer regenerators. This parameter is used only if MATERIAL_FORM=2. The default values are taken from MAT_COND_FACTOR.

- METHOD. This integer parameter selects the time discretization used in the regen3.3 model. If the value of METHOD is 1 then a first order backward difference approximation of the time derivatives is used. The value of 2 will result in use of a second order approximation. The second order approximation should be more accurate, but in rare cases results in an unstable computation. This parameter is not used in the regen3.2 model. The default value is 1.

- MFLUX_DC. A steady (DC) component of the mass flow at each end of the regenerator (kg/s). This value is added to MASS_FLUX_COLD and MASS_FLUX_HOT to provide a steady component of the mass flux. The default value is 0.0. If this input parameter is non-zero the output values INEFCT, RGLOSS, and PRLOSS will be corrected to remove the contribution made by this DC flux.

- MID_TEMP_RATIO. The normalized initial temperature at the midpoint of the regenerator. This value is used to set the initial temperature profile for the matrix and gas. The value of this parameter is the difference between the initial temperature at the midpoint of the regenerator and the inflow temperature GAS_TEMP_COLD at the cold end divided by the difference of two end point temperatures GAS_TEMP_COLD and GAS_TEMP_HOT. If the value of MID_TEMP_RATIO is 0.5 then the initial temperature is a linear function of distance along the regenerator. Otherwise a quadratic polynomial is passed through the temperature at the ends and the midpoint to determine the initial temperature as a function of distance along the regenerator. A good choice for this parameter can speed up the convergence to a periodic state if the final temperature profile is not linear. See the discussion of the GTPNRM output parameter. The default value is 0.5.

- MUSHIS. An integer parameter which is used along with the IXHIST input parameter to select points for time series graphs. The value is restricted by 1 <= MUSHIS <= NUMHIS. The value of NUMHIS is set to 11 in a PARAMETER statement in the main module. The default value is 0.

- NEWCAS. Selects the method used to initialize the integration. More than one NAMELIST input block may be read in. The input blocks that follow the first may start a new case or continue the integration resulting from the previous input block. The input values are not reset to the default values after the first case. Unless the input values are changed in the input for the following cases they will continue to have the values used in the first case. The default value is 1. The allowed values follow.

  0. Continue the integration changing only the input parameters that control the output. That is, parameters such as FINAL_CYCLE, NPLOT, etc.
  1. Start a new integration with the temperature profile determined by MID_TEMP_RATIO.
  2. Use the values of velocity, pressure, and temperature resulting from the previous case as initial conditions to start the next case. The initial time and cycle are set to 0. Here the previous integration must be in the same job step. This option is available only in regen3.2.
  3. Use the data from a file with prefix "RGSAV." and suffix a 4 digit run number. This file must have been written with the USE_SAVE option on an earlier run. In regen3.2 the the saved data from the previous run must be copied to a RGSAV.DAT file and initial values will be interpolated from the values in the this file. In REGEN3.2 the mesh resolution in the new run may differ from that in the run that produced the RGSAV file. In regen3.3 the mesh resolution must not change.

- NPR_PLOT. Controls output printed by the prtsol routine at the output intervals determined by PLOT_INC. If the value is 0 then no output is printed at these intervals. If the value is 1 then the solution arrays

(mass-flux, pressure, gas temperature, and matrix temperature) are printed across the mesh at the output intervals. The default value is 0.

- NPLOT. Selects graphics output as described in the graphics section. If the value is 0 no graphics are produced. The default value is 2.

- NRUN. The number used to label the current run. This number is used as the suffix for the output files that have the prefixes RGPR and RGWT. The value must not exceed 9999. This value must not be changed in cases following the first unless USE_CASE_RGPR = 1. There is no default value, a value must be supplied for this parameter.

- NRUN_RESTART. The run number $< nrun >$ used to select the file rgsav.$< nrun >$ used to set the initial values of the temperatures, pressure, and velocity when NEWCAS=3. The file rgsav.$< nrun >$ must have been written on an earlier run. See the discussion of NEWCAS, PRES_SAVE_MULT and USE_SAVE.

- NUM_ITT_STEP. The maximum number of predictor-corrector iterations used for the temperature calculation on each time step. If EPS_STEPS>0 then the difference between successive iterates for the temperatures relative to the temperature norm must be reduced below EPS_STEPS in no more than NUM_ITT_STEP iterations. If EPS_STEPS=0 then exactly NUM_ITT_STEP iterations are performed. This is not used in the regen3.3 model. The default value is 2.

- NUM_LAYERS. This parameter is used only when MATERIAL_FORM=2. Its value is the number of material layers in the regenerator. Each layer has a different value of porosity, hydraulic diameter, and material determined by the arrays POROS_LAYERS, HIDIAM_LAYERS, and MAT_LAYERS. The breakpoint between the layers is given by the values in the array X_LAYERS. For example, if the length of the regenerator is 0.05 m and the first half is composed of stainless steel and second half of Lead, then NUM_LAYERS=2, X_LAYERS=0.025, and MAT_LAYERS=[1,4]. The porosity and hydraulic diameter would be set in the arrays POROS_LAYERS and HIDIAM_LAYERS which would be of length 2 in this case. There is no default value.

- NUM_MATERIALS. The number of materials used to simulate an ideal regenerator whose matrix is a mixture of different materials, or an optimal material. This value must be less than 20. This parameter is used only if MATERIAL has value 35 or 36. There is no default value.

- NUM_POINTS_X. The number of mesh points used along the regenerator. A uniform mesh spacing is used unless the mesh is graded. This is a parameter of integer type. The default value is 21.

- NUM_STEPS_CYC. The number of time steps used in each cycle. The time step in seconds is therefore 1/(NUM_STEPS_CYC*HERZ). The default value is 80.

- OUTPUT_INC (cycle). The increment in cycles over which partial output is produced. See the section on output. This is an integer variable. The default value is set equal to the value of FINAL_CYCLE.

- P_GRAD_FACTOR. The correlation for the pressure gradient in the gas that is determined by the geometry parameter will be multiplied by this factor. The default value is 1.0.

- PLOT_INC (cycle). Diagnostic output is taken at intervals determined by the value of FULL_OUTPUT_INC. For example, if FULL_OUTPUT_INC = 1000 then detailed diagnostic information including the pressure ratio, the average enthalpy flux at the cold end and numerous other data will be printed at intervals of 1000 cycles. This information is taken over the last cycle in the interval. In addition "snapshot" information can be included in the RGWT file for later plotting. For example the temperature profile, that is the temperature as a function of distance along the regenerator, at a particular instant in the last cycle can be plotted. The frequency of these snapshots is determined by the value of PLOT_INC. If this value is 0.125 cycles then 8 of these snapshot outputs will be generated throughout the last cycle. The default value is 0.125 cycle.

- POROS_LAYERS. This array determines the porosity of each layer in a multilayered regenerator. It is only used when MATERIAL_FORM=2. The default values are taken from the POROSITY parameter.

- PRES_SAVE_MULT. When a case is started using initial conditions from the RGSAV file the pressure saved in that file from a previous run will be multiplied by this parameter to give the initial pressure used to start the case. This parameter is only used when NEWCAS has value 2 or 3. This is used to change the average pressure in the new case from that on the saved case. This is not used in the regen3.3 model. The default value is 1.0.

- TABLE_PRES_MAX (Pa). The upper limit in pressure used to construct the tables of thermodynamic properties when USE_PROPS_TABLE = 1. The pressure computed by the model must be below this limit. The table is computed once using the helium routines then table lookup is used throughout the computation. The default value is a multiple of PRES_INITIAL or it is determined from the pressure ratio and the average pressure input.

- TABLE_PRES_MIN (Pa). The lower limit in pressure used to construct the tables of thermodynamic properties when USE_PROPS_TABLE = 1. The default value is a multiple of PRES_INITIAL or it is determined from the pressure ratio and the average pressure input.

- TABLE_PRES_PTS. The number of points used to approximate the pressure in the helium properties table. This is a parameter of integer type. The default value is 200.

- TABLE_TEMP_MAX (K). The upper limit in temperature used to construct the tables of thermodynamic properties when USE_PROPS_TABLE = 1. The temperature computed by the model must be below this limit. The default value is 1.40*GAS_TEMP_HOT.

- TABLE_TEMP_MIN (K). The lower limit in temperature used to construct the tables of thermodynamic properties when USE_PROPS_TABLE = 1. The temperature computed by the model must be above this limit. The default value is 0.60*GAS_TEMP_COLD.

- TABLE_TEMP_PTS. The number of points used to approximate the temperature in the helium properties table. This is a parameter of integer type. The default value is 200.

- TUBE_H (m). This is the thickness of the tube containing the regenerator, that is the difference between the outer and inner radius. It is used to estimate the heat loss due to conduction along the tube. The default value is zero.

- USE_ADVEC. This integer parameter selects the momentum equation used in the regen3.3 model. If this parameter has value 0 then the correlation for the pressure gradient in a porous medium given in equation 2 is taken as the momentum equation. If the value is 1 then the momentum equation including the accelleration terms as well as the friction force given in equation 4 is used. The default value is 1.

- USE_CASE_RGPR. This option allows multiple NAMELIST input blocks within a single input data file to output separate RGPR output blocks each with a different run number NRUN. Each NAMELIST input block must define a different value of NRUN. These values will be used to name the RGPR output associated with the input block. If USE_CASE_RGPR = 1 then different RGPR files will be generated for each input block. If the value is 0, then this parameter has no effect. The default value is 0.

- USE_GRADED_MESH. This is a flag to select an initial temperature profile typical of devices with a cold end temperature below 10 K. Frequently the temperature drop in these devices occurs in the first small portion of the regenerator with the temperature in the remaining major portion nearly equal to the temperature at the cold end. This occurs when the heat capacity of the matrix is not large relative to that of the helium gas. If this parameter is greater than 0, then the GRADED_CUTOFF input parameter, denoted here by x_c, is used to define the point where the cold end temperature is assumed. Denoting the warm and cold end temperatures by T_h and T_c, the initial temperature as a function of distance x along the regenerator is $T(x) = T_h + x(T_c - T_h)/x_c$ with $T(x) = T_c$ for $x > x_c$. Generally the cold end temperature will be achieved at a point near x_c rather than exactly at x_c. This option is not available in the regen3.3 model. The default value is 0.

  If the GRADED_RATIO parameter is greater than 1.0 then a graded mesh is used for the mesh cells between the warm end and the cutoff

point x_c. This is the ratio between the length of successive mesh cells. If the value of GRADED_RATIO is denoted by λ and the cell endpoints are x_i then $(x_{i+2} - x_{i+1}) = \lambda(x_{i+1} - x_i)$. This ratio applies until the cutoff point x_c is reached, after that the cell size is constant.

- USE_GAS_COND. Selects the approximation for thermal conductivity. This parameter overrides the USE_PROPS_TABLE for the computation of the thermal conductivity of the gas. The default value is 0. The allowed values follow.

  0. If the value of USE_IDEAL_GAS is 0, use the HEPROPS routines to compute the thermal conductivity of the gas.
  1. Use a linear approximation. See the description of GAS_COND_HOT.

- USE_IDEAL_GAS. This is a flag to select the equation of state used for the helium gas. The default value is 0. The following values are permitted.

  0. The helium properties routine is used to obtain the properties of helium including viscosity and thermal conductivity, as well as the equation of state.
  1. The ideal gas law is used for the equation of state, the viscosity is obtained from the helium properties routine and the thermal conductivity of the gas is obtained from a fixed Prandtl number of 0.67.

- USE_MAT_COND. Selects the approximation used for the thermal conductivity of the matrix. The default value is 0. The allowed values follow.

  0. Use the value of the MATERIAL parameter to select a correlation for the specific material.
  1. Use a linear approximation. See the description of MAT_COND_HOT.

- USE_MAT_CPVOL. Selects the approximation used for the heat capacity of the matrix. The default value is 0. The allowed values follow.

0. Use the value of the MATERIAL parameter to select a correlation for the specific material.
1. Use a linear approximation. See the description of MAT_CPVOL_HOT.

- USE_PRES_CORR. Selects a correction for the pressure computation in the regen3.2 model. The difference equations in this model are not written in conservation form and thus mass is not conserved. If the value of this parameter is 1 then an increment will be added to the pressure at the end of each time step so that mass is conserved on each time step. Generally, this model will not give reasonable results unless the pressure correction is used. This parameter is not used in the regen3.3 model. The default value is 1. The allowed values follow.

  0. Do not correct the pressure.
  1. Correct the pressure.

- USE_PROPS_TABLE. Selects the method to approximate the thermodynamic properties of helium when the ideal gas equation of state is not used. The default value is 1. The allowed values follow.

  0. Use the helium routine directly to compute the thermodynamic properties of the helium as they are needed.
  1. Use the helium routine one time to compute tables for the properties as functions of temperature and pressure. Then table lookup is used to compute the properties as they are needed during the integration. Use of these tables can reduce the running time by a factor of 3 or 4 from that required by direct use of the helium properties routine.

- USE_SAVE. Selects an option to save the results of a run in a format which can be used to restart a subsequent run from the point of the save. A file with prefix "RGSAV." and suffix the value of the run number NRUN is written. The default value is 0. The allowed values follow.

  0. Do not save the results of the run.
  1. Save the results of the run.

- USE_STDOUT. Selects the file where the printed output is written. This parameter should only be set on the input for the first case. Default value is 0. The allowed values follow.

  0. The printed output is written to a file with prefix "RGPR" and suffix the run number. Thus, if the NRUN=1001, then a file named "RGPR1001" is opened and output is written to it.

  1. The printed output is written to standard output. The program is compiled with standard output set to logical device 6. If this is not the case the program should be recompiled with the global variable STDOUT reset to the proper value.

- VOL_HEAT. This parameter allows heat to be added to a single mesh cell along the regenerator. The value of VOL_HEAT is the rate (W) of heating added to the matrix cell containing the point given by the value of LOCATE_HEAT. This heat is add to the source term q in the thermal energy equation for the matrix (6). The default value is 0.0.

- X_LAYERS (m). The breakpoints which divide the layers in a multilayered regenerator. It is only used if MATERIAL_FORM=2. The number of elements in this array is one less than the value of NUM_LAYERS. See the description of the NUM_LAYERS parameter.

3 Printed output.

The output of the program is written to two files. The files have the prefix "RGPR" and "RGWT" with the four digit run number added as a suffix. The RGPR file contains printed output and the RGWT file contains data suitable for plotting. If the default values are used then output will be printed only on the last cycle. This means that integrals such as the heat transfer and enthalpy flux will be taken over the last cycle. Generally the value of the FINAL_CYCLE parameter is chosen so that the change in matrix energy over the last cycle is considerably less than the enthalpy flux. At this point the solution has achieved the nearly periodic state needed to evaluate the regenerator performance. In order to determine how rapidly this nearly periodic state is developed output can be taken at intermediate steps prior to reaching the final cycle.

The frequency with which the output is generated is determined by the input parameters FULL_OUTPUT_INC and OUTPUT_INC. The value of the OUTPUT_INC parameter is the interval over which this intermediate output is taken. Only a few variables are output at the intervals determined by OUTPUT_INC. These include parameters used to estimate the rate of convergence to a periodic solution such as the enthalpy flux at the cold end, the change in matrix thermal energy over the cycle, the heat transfer between gas and matrix, the average pressure, and the pressure ratio. The value of the FULL_OUTPUT_INC parameter is the interval over which more complete output is taken.

3.1 Output selected by OUTPUT_INC.

If the value of OUTPUT_INC is 1000, then the output described in this section will be printed every 1000 cycles. The first line of this output block in the regen3.3 model gives the value of 4 diagnostic parameters. These give insight into the difficulty that the model may be having with the convergence of the Newton iteration for the fully implicit equations. The average number of Newton iterations on the time-steps is ITTAVE. The total number of reductions in the time step required to gain convergence of the Newton iteration is ITFAIL. The average number of function evaluations on each time step is RESAVE. The average number of Jacobian recomputations on each time step is JACAVE. Usually this number is less than one.

- CFLMAX. The maximum value of the CFL ratio $c\Delta t/\Delta x$ where c is the maximum value of the velocity throughout the cycle. It is best to choose the value of NUM_STEPS_CYC so that this value does not exceed 1.
- EHTAVE (W). The average across the regenerator of the integral average over the cycle of the sum of the enthalpy flux in the gas and the heat flux in the matrix.
- EHTCOR (W). This is the difference EHTFLX-ENTCOR. That is, the enthalpy flux at the cold end reduced by the enthalpy flux due to the DC flow. This is printed only if MFLUX_DC is non-zero.
- EHTDIF (W). The difference between the maximum and minimum over the mesh of the integral average of the sum of the enthalpy flux and

matrix heat flux. When a periodic flow is achieved this value should be small in comparison to the value of EHTFLX.

- EHTFLX (W). The integral average of the sum of the enthalpy flux and the heat flux at the cold side of the regenerator

$$\int_{t-\tau}^{t} (\frac{\phi A \rho(L,t) v(L,t) h(P(t),T(L,t)) - (1-\phi) A k_m \frac{\partial T}{\partial x}(L))}{\tau} dt.$$

- ENGBAL (W). An estimate of the accuracy of energy conservation in the gas given by

$$\text{ENGDIF} + \text{enthalpyflux(L)} -$$

$$\text{enthalpyflux(0)} - \text{gasdiff} - \text{QINT}.$$

Here enthalpyflux(x) is the integral

$$\int_{t-\tau}^{t} \frac{\phi A \rho(x,t) v(x,t) h(P(t),T(x,t))}{\tau} dt.$$

and gasdiff is the integral of the thermal diffusion in the gas. Given a perfectly accurate method the value of ENGBAL would vanish. Therefore its value compared to the enthalpy flux gives some indication of the discretization error in the computation.

- ENGDIF (W). The rate of change of the energy in the gas over one cycle

$$\text{ENGDIF} = (E(t) - E(t-\tau))/\tau$$

where t is the time at the end of the cycle and τ the period of the oscillation.

- ENTCOR (W). If MFLUX_DC is non-zero this value is output. It is the enthalpy flux due to the DC flow. That is, the integral over the cycle of the product of MFLUX_DC with the enthalpy at the cold end.

- GTPNRM. The normed, average temperature at the midpoint of the regenerator. This is the difference between the average midpoint temperature and the cold end temperature divided by the difference between the inflow temperature at the ends. For a linear temperature profile this value is 0.5. The value may be useful in setting the input parameter MID_TEMP_RATIO on a subsequent run.

- MEGBAL (W). A measure of the accuracy of the computation of the energy balance in the matrix. This is computed from the relation

$$\text{MEGBAL} = \text{QINTW} + \text{MEGDIF}$$

  This value is not printed in the regen3.2 model.

- MEGDIF (w). The rate of change of the thermal energy of the matrix over the cycle. This is computed from the product of the difference between the matrix temperature at the end and beginning of the cycle with the matrix heat capacity at the final temperature integrated over the length of the regenerator.

- PAVE (Pa). The average pressure at the cold end over one cycle. The average is over time.

- PHSPRM (degree). The phase angle of the mass flow at the cold end relative to the pressure at the cold end.

- PRATIO. The ratio of maximum cold end pressure over the cycle to the minimum cold end pressure over the cycle.

- PRDIF (Pa). The change in the pressure at the cold end over the cycle.

- QINTW (W). The average rate at which the entire matrix is cooled by heat transfer from the gas and conduction through the ends of the regenerator. This is the integral of the heat transferred from the matrix over one cycle divided by the period of the cycle. A positive value means that heat is transferred from the matrix.

$$\text{QINTW} = -\frac{\int_{\text{t}-\tau}^{\text{t}}(\phi \text{A} \int_0^{\text{L}} \text{q}(\text{x},\text{t})\text{dx} + (1-\phi)\text{A}(\text{k}_\text{m}\frac{\partial \text{T}}{\partial \text{x}}(0,\text{t}) - \text{k}_\text{m}\frac{\partial \text{T}}{\partial \text{x}}(\text{L},\text{t})))\text{dt}}{\tau}$$

  Here t is taken at the end of a cycle, ϕ is the porosity, τ is the period, k_m the thermal conductivity of the matrix, and A the area of the regenerator. This value should approach zero as a periodic state is reached.

- QINT (W). The average rate of heat transfer from the entire matrix to the gas taken over the full cycle. This is similar to QINTW except that the heat transfer by conduction at the ends of the regenerator is not included.

3.2 Output selected by FULL_OUTPUT_INC.

The next set of parameters include the ineffectiveness, an estimate of the net cooling power, an estimate of the pressure drop generated by the velocity profile, the enthalpy flux at various points along the regenerator, as well as other parameters to help evaluate regenerator performance. These parameters are printed at intervals determined by the value of FULL_OUTPUT_INC.

- CAPF (J/K). The heat capacity of the gas which passes through the cold end of the regenerator during the outflow half cycle. This is defined by the following integral

$$\int_{v>0} \phi A \rho v c_p(L, t) dt.$$

- CAPR (J/K). The heat capacity of the matrix within the regenerator. This is defined by the following integral where τ is the period of the flow.

$$\frac{\int\int (1-\phi) A c_m(x, t) dx dt}{\tau}.$$

- CAPV (J/K). The heat capacity of the helium gas in the regenerator void volume. This is defined by the following integral where τ is the period of the flow.

$$\frac{\int\int \phi A \rho c_p(x, t) dx dt}{\tau}.$$

- CNTU. An estimate of the ratio between the heat transfer between the gas and the matrix over the cycle with the outflow heat at the cold end. This is the "N_{tu}" parameter. The heat transfer, HT, between the gas and the matrix over the cycle is computed from the following integral.

$$\int \frac{4H}{D_h} \phi A dx dt.$$

  where H is the heat transfer coefficient and D_h the hydraulic diameter. This integral is divided by the CAPF value to yield the CNTU value.

- DELPAV (Pa). This is the average pressure drop across the regenerator; that is, the absolute pressure drop averaged over one cycle. The solution is computed under the assumption that the pressure gradient

is 0. This computed pressure is a function of time but is constant along the length of the regenerator. The value of this pressure is assigned to the cold end of the regenerator. The pressure gradient at each point is obtained from the REGEN2 correlation. The estimated pressure gradient is integrated in x over the length of the regenerator, and then the absolute value of the pressure difference is integrated with respect to time over one cycle and then divided by the period to give DELPAV.

- DELPMX (Pa). The maximum pressure drop across the regenerator over the cycle.

- DLPMXN. The maximum pressure drop normalized by the average pressure.

- EHTPOS (W). The integral of the enthalpy flux at the cold end over the outflow portion of the cycle divided by the period of the cycle. Note that the total enthalpy flux, EHTFLX, is the difference between this outflow flux and the inflow flux at the cold end. The substraction of these two can amplify the error in the computation if the total flux is much less than the positive flux.

$$\text{ehtpos} = \frac{\int_{\text{v}>0} \phi\rho\text{Ahdt}}{\tau}$$

- EHXAVE (W). The energy flux averaged over the length of the regenerator. The energy flux is the enthalpy and matrix conduction flux with the addition of terms to account for kinetic energy and thermal diffusion in the gas. This flux at any point along the regenerator is the integral over time

$$\int (\phi A(E+p)\rho v - \phi A k_g \frac{\partial T}{\partial x} - (1-\phi) A k_m \frac{\partial T_m}{\partial x}) dt.$$

The value of EHXAVE is obtained by averaging this integral over the length of the regenerator. This flux will approach a constant value along the regenerator once a periodic state is achieved. This is the value that is conserved along the regenerator as inspection of equation (5) shows. This value is printed only when USE_ADVEC > 0 and is not printed in the regen3.2 model.

- EHXDIF (W). This is the difference between the maximum and minimum of this energy flux over the regenerator. This value is printed only when USE_ADVEC > 0 and is not printed in the regen3.2 model.

- EHXFLX (W). The energy flux defined above computed at the cold end of the regerator. The kinetic and gas thermal conduction are generally small so this value is close to EHTFLX. This value is printed only when USE_ADVEC > 0 and is not printed in the regen3.2 model.

- ENTFLX (W). The integral average of the enthalpy flux at the right side of the regenerator over one cycle

$$\int_{t-\tau}^{t} \frac{\phi A \rho(L,t) v(L,t) h(P(t),T(L,t))}{\tau} dt$$

- GASAR (m^2). The cross-sectional area of the regenerator open to gas flow. This the product of the area and the porosity.

- GASARS ($m^2 s/kg$). Specific gas cross-sectional area. This is the cross-sectional area GASAR divided by the mass flow rate at the cold end.

- GASVO (m^3). Gas volume in the regenerator. This is the product of the gas cross-sectional area GASAR and the length ZLEN.

- GASVOR. The gas volume ratio. This is the ratio of the gas volume in the regenerator to the peak expansion volume, that is the quotient $GASVO/PTVMX2$.

- EFFIC. The second law efficiency. This is the product of the coeficient of performance with the ratio of the temperature at the ends of the regenerator, that is $NTACOP * GAS\_TEMP\_HOT/GAS\_TEMP\_COLD$.

- GRCADJ (W). The gross refrigeration power adjusted for losses in the expansion space. It is the product of the gross refrigeration GRCOOL with the user supplied adjustment factor COOLING_MULT.

$$\text{GRCADJ} = \text{GRCOOL} * \text{COOLING\_MULT}.$$

  If MFLUX_DC is non-zero the enthalpy flux in this definition is reduced by enthalpy flux due to the DC flow.

- GRCOOL (W). This is the gross refrigeration power reduced by the pressurization loss. An isothermal expansion space is assumed.

$$\mathrm{GRCOOL} = \mathrm{PVWK1} - \mathrm{PRLOSS}.$$

  If MFLUX_DC is non-zero the enthalpy flux in this definition is reduced by enthalpy flux due to the DC flow.

- GTPDIF (K). The maximum, taken over all the mesh points, of the absolute difference between the gas temperature at the beginning and the end of the cycle. This value should approach zero as a periodic state is reached.

- HTFLUX (W). The heat flux due to thermal conduction through the cold side of the matrix.

- HTFLX0 (W). The heat flux due to thermal conduction through the hot side of the matrix.

- HTFMAX ($\mathrm{W}/(\mathrm{m}^2 \cdot \mathrm{s})$). The maximum of the heat transfer coefficient, H, taken over both space and time.

- HTIAVE ($\mathrm{W}/(\mathrm{m}^2 \cdot \mathrm{s})$). The average value of the heat transfer coefficient, H. The coefficient H is averaged over both the length of the regenerator and the cycle.

- INEFCT. The regenerator ineffectiveness, defined as follows

$$\mathrm{INEFCT} = \frac{\mathrm{ENTFLX} - \mathrm{PRLOSS}}{\mathrm{INEFNM}}.$$

  If MFLUX_DC is non-zero the enthalpy flux in this definition is reduced by enthalpy flux due to the DC flow.

- INEFNM (W). The normalization factor for the ineffectiveness.

$$\mathrm{INEFNM} = \max\left[\int_{\mathrm{v}\le 0} \phi \mathrm{A} \frac{\rho(\mathrm{L},\mathrm{t})\mathrm{v}(\mathrm{L},\mathrm{t})(\mathrm{h}(\mathrm{p}(\mathrm{t}),\mathrm{T}_0) - \mathrm{h}(\mathrm{p}(\mathrm{t}),\mathrm{T}_1))}{\tau}\mathrm{dt},\right.$$

$$\left.\int_{\mathrm{v}\ge 0} \phi \mathrm{A} \frac{\rho(\mathrm{L},\mathrm{t})\mathrm{v}(\mathrm{L},\mathrm{t})(\mathrm{h}(\mathrm{p}(\mathrm{t}),\mathrm{T}_0) - \mathrm{h}(\mathrm{p}(\mathrm{t}),\mathrm{T}_1))}{\tau}\mathrm{dt}\right].$$

  Here T_1 is the inflow temperature at the cold (right) end, and T_0 is the inflow temperature at the hot (left) end.

- MASS (kg). The mass of the gas in the regenerator void space at the end of the cycle.
- MASFX0 (kg/s). The peak mass flow rate at the warm (left) end of the regenerator over the cycle.
- MASFX1 (kg/s). The peak mass flow rate at the cold (right) end of the regenerator over the cycle. For most boundary conditions this should equal MASS_FLUX_COLD.
- ME2DIF (W). The rate of change of the thermal energy of the matrix over the cycle. This is computed from the difference of the heat capacity of the matrix at the end and beginning of the cycle.

$$\mathrm{ME2DIF} = \frac{\int (\mathrm{D(x, T_m(x, t_1))} - \mathrm{D(x, T_m(x, t_0))})\mathrm{dx}}{\tau}$$

  where the heat capacity $D(x,T)$ is defined in equation (7), $T_m(x,t)$ denotes the matrix temperature, t_1 the temperature at the end of the cycle, t_0 the temperature at the start, and τ the period of the oscillation. When the solution is nearly periodic this value should be close to that of -QINTW. This value is not printed in the regen3.2 model.
- MG2BAL (W). A measure of the accuracy of the computation of the energy balance in the matrix. This term should be zero if the matrix conservation of energy equation is solved exactly. This is computed from the relation

$$\mathrm{MG2BAL} = \mathrm{QINTW} - \mathrm{ME2DIF}$$

  This value is not printed in the regen3.2 model.
- MHTCAP (J/K). The heat capacity of the matrix computed at the end of the cycle.

$$\mathrm{MHTCAP} = \int (1-\phi)\mathrm{Ac_m(x, T)dx}.$$

  This value is not printed in the regen3.2 model.
- MTPDIF (K). The maximum absolute difference between the matrix temperature at the start and end of the cycle. The maximum is taken over the length of the regenerator.

- NTACOP. The coefficient of performance. This is the adjusted net refrigeration power normalized by the PV work at the hot end.

$$\text{NTACOP} = \text{NTCADJ}/\left|\text{PVWK0T}\right|$$

- NTCADJ (W). The adjusted net refrigeration power

$$\text{NTCADJ} = \text{GRCADJ} - \text{RGLOSS} - \text{HTFLUX} - \text{TUBECD}.$$

- PDPPHS (degree). The phase angle of the oscillation in the pressure drop relative to the pressure at the cold end.
- PHSCV (degree). The phase angle of the compression volume relative to the pressure at the cold end.
- PHSEV (degree). The phase angle of the expansion volume relative to the pressure at the cold end.
- PHSMAS (degree). The phase angle of the mass flow at the cold end relative to the mass flow at the hot end. Note that this is the phase relative to the hot end which is opposite from that used to define the other phase angles in this section. This is computed from the flow and may differ slightly from the MASS_PHASE parameter when the boundary conditions specify the mass flow at both ends.
- PHSPLM degree). The phase angle of the mass flow at the hot end relative to the cold end pressure.
- PHSTPM (degree). The phase angle of the temperature at the regenerator midpoint relative to the cold end pressure.
- PLRPHS (degree). The phase angle of the pressure at the hot end relative to the cold end pressure.
- PMAX (Pa). The maximum value of the pressure throughout the cycle.
- PMIN (Pa). The minimum value of the pressure throughout the cycle.
- PNORM. The amplitude of the pressure oscillation

$$\text{PNORM} = (\text{PMAX} - \text{PMIN})/(2.0 * \text{PAVE}).$$

- PRLOSS (W). A correction term used to estimate the effect of pressurization on the enthalpy flux of the gas.

$$\max\left[\int_{t-\tau}^{t}\frac{\phi A\rho(0,t)v(0,t)h(p(t),T_0)}{\tau}dt\ ,\ \int_{t-\tau}^{t}\frac{\phi A\rho(L,t)v(L,t)h(p(t),T_1)}{\tau}dt\right]$$

  If MFLUX_DC is non-zero the enthalpy flux in this definition is reduced by enthalpy flux due to the DC flow.

- PRTLFT. The ratio of maximum to minimum pressure at the hot end.

- PTVMX1 (m^3). The peak value of the compression volume. This is obtained by the maximum of the integral during the cycle

$$-\int\frac{\phi Av(0,t)\rho(0,t)}{\rho_0}dt$$

  where $\rho_0 = \rho(T_0, p(t))$ is the density at the hot side input temperature T_0. The integral is taken from the time the flow first becomes outflow at the hot end until it reverses to inflow. This computation does not require that the start of outflow and the reversal occur in the same cycle.

- PTVMX2 (m^3). The peak value of the expansion volume.

- PVLOSS (W). The hot end pV work due to nonzero delta T. This is given by

$$PVWK0 + PVWK1\frac{\rho_c}{\rho_h}$$

  where ρ_c is the density computed at the cold end inflow temperature T_1 and average pressure PAVE and ρ_h is the corresponding density at the hot end. Note that PVWK0 is generally negative and PVWK1 positive.

- PVWKPR (W). The pV work due to pressure drop across the regenerator. This is the integral $\int \delta PdV$ over the cycle where δP is the pressure drop and dV refers to the compression volume at the hot end obtained from the assumption of an isothermal compression space.

- PVWK0T (W). The hot end pV work. In regen3.2 the pressure at the hot end is obtained by adding the pressure drop to the pressure at the

cold end. The pressure drop is obtained by integration of the pressure gradient. The latter is based on fitting [2] the data of Kays and London [3]. See the discussion of PVWK1.

- PVWK0 (W). The pV work term at the hot (left) end of the regenerator based on the cold end pressure. This is the $\int \mathrm{pdV}$ where V is the compression volume. See the discussion of PVWK1.

- PVWK1 (W). The pV work term at the cold (right) end of the regenerator. This is computed from the integral $\int_{\mathrm{t}-\tau}^{\mathrm{t}} \mathrm{pdV}$ where V is the volume of gas in the isothermal expansion chamber at the cold end. This is equivlent to the integral

$$\int_{\mathrm{t}-\tau}^{\mathrm{t}} \frac{\phi \mathrm{Av(L,t)p(t)}\rho\mathrm{(L,t)}}{\rho_1 \tau}\mathrm{dt}$$

  where $\rho_1 = \rho(\mathrm{T}_1, \mathrm{p(t)})$ is the density at the cold side input temperature T_1.

- RELLFT. This is maximum penetration into the tube of a gas particle moving in from the hot side. It is measured relative to the length of the tube, thus a value of one implies that the particle has reached the cold end of the tube.

- RELRHT. This is maximum penetration into the tube of a gas particle moving in from the cold side. It is measured relative to the length of the tube, thus a value of one implies that the particle has reached the hot end of the tube.

- RENMAX. The maximum of the Reynolds number taken over space and time.

- RGLOSS (W). The loss due to regenerator ineffectiveness.

$$\mathrm{RGLOSS} = \mathrm{ENTFLX} - \mathrm{PRLOSS}$$

  . This excludes the loss due to pressurization and conduction.

- SFLUX0 (W/K). The entropy flux at the hot end. This is computed by integration of the product of the entropy with the mass flux over the cycle. The entropy as a function of GAS_TEMP_HOT and pressure is obtained from the HEPROPS package. The cold end pressure is used to compute the entropy.

- SFLUX1 (W/K). The entropy flux at the cold end. This is computed by integration of the product of the entropy with the mass flux over the cycle. The entropy as a function of GAS_TEMP_COLD and cold end pressure is obtained from the HEPROPS package.

- SFLXP0 (W/K). The entropy flux at the hot end computed using the pressure at the hot end. This is computed by integration of the product of the entropy with the mass flux over the cycle. The entropy as a function of GAS_TEMP_HOT and pressure is obtained from the HEPROPS package.

- SF0ALT (W/K). An alternate approximation of the entropy flux at the hot end

$$\mathrm{SF0ALT} = (\mathrm{PVWK0} + \mathrm{ENTHT0})/\mathrm{T0})$$

  where PVWK0 is the pV work at the hot end based on cold end pressure, ENTHT0 is the sum of enthalpy flux at the constant temperature T_0 and thermal conduction flux at the hot end. T_0 is the inflow temperature at the hot end (T_0 = GAS_TEMP_HOT).

- SF1ALT (W/K). An alternate approximation of the entropy flux at the cold end

$$\mathrm{SF1ALT} = (-\mathrm{PVWK1} + \mathrm{ENTHT1})/\mathrm{T_1})$$

  where PVWK1 is the pV work at the cold end, ENTHT1 is the sum of enthalpy flux at the constant temperature T_1 and thermal conduction flux at the cold end, and T_1 is the inflow temperature at the cold end (T_1 = GAS_TEMP_COLD).

- TNORM. The difference between the maximum gas temperature and minimum gas temperature at the regenerator midpoint divided by the average temperature at the midpoint. The maximum and minimum are taken over the cycle.

- TUBECD (W). Thermal conduction through the tube containing the regenerator matrix. The tube is assumed to be stainless steel with a steady state temperature profile and the thermal exchange between the tube and the matrix is ignored. The following relation is used

$$\mathrm{TUBECD} = -\frac{\sqrt{4\pi \mathrm{A_r}}\mathrm{h}}{\mathrm{L}} \int_{\mathrm{T_0}}^{\mathrm{T_1}} \sigma(\mathrm{T})\mathrm{dT}$$

where A_r = RG_AREA, h = TUBE_H, L = RG_LENGTH, T_1 = GAS_TEMP_HOT, T_0 = GAS_TEMP_COLD, and σ is the thermal conductivity of stainless steel. In a steady state with no heat transfer from tube to matrix the thermal conduction along the tube, $c = -\sigma(T)\partial T/\partial x = -\sigma(T)T_x$ must be constant. This leads to

$$\int_{x_0}^{x_1} c\,dx = cL = -\int_{x_0}^{x_1} \sigma(T)T_x\,dx = -\int_{T_0}^{T_1} \sigma(T)\,dT$$

and the above relation for TUBECD. The last integral is computed using the temperature profile along the regenerator at the end of the cycle.

- UABMAX (m/s). The maximum gas velocity with the maximum taken over both space and time.

A table showing integrals over time at 6 points along the regenerator is given at the end of the printed output that is selected by the FULL_OUTPUT_INC parameter. The integral of the mass flux as well as the peak mass flux is included. The conductive heat flux in the matrix at each point is given along with the integral of the enthalpy flux in the gas and the sum of the enthalpy and heat fluxes.

4 Graphic output.

This is a description of the data for plotting that is written out to the RGWT files. Variables such as the pressure or the temperature at a specified mesh point can be plotted as a function of time. These plots are taken over a single cycle determined by the FULL_OUTPUT_INC parameter. Variables are also plotted as a function of the distance along the regenerator. These plots are taken at a time determined by a combination of the FULL_OUTPUT_INC and PLOT_INC parameters. The data for a single plot frame consists of a header containing labels and the number of items in the graph. This is followed by two or more columns of data containing the x and y coordinates of the points to be graphed. This data is written to the RGWT file in ASCII text format. A script (plotwt.m) that will display the file using the MATLAB [4] analysis routine is included with the source code. An executable code (VIEWRGWT.EXE) to display and print the data contained in the RGWT

files on a PC is also included. Up to four curves may be displayed on a graph. The title and axis labels for the graph use the four markers (a,b,c,d) to label the curves on the MATLAB display. The PC display uses a solid line, short dash line, long dash line, and a short-long dash line to distinguish the curves whose variables are labeled (a,b,c,d) respectively. In some cases the plotted variables are normalized within the range -1 to 1 in order to fit several variables on the same graph. The title and axis labels will indicate this normalization and will also give the maximum absolute value of the variables that have been normalized. If the plotted variable is dimensioned the units will be given in the label.

4.1 Matrix heat capacity and the heat transfer rate.

The first graph in the output shows the heat capacity of the matrix (CPVOL MATRIX) (J/K) as a function of initial temperature profile as well as the heat capacity of the helium gas (CPVOL GAS) at the initial pressure and temperature. REGEN3.2 will generate a second graph that shows the heat transfer coefficient H ($W/m^2\dot{K}$) as a function of the Reynolds number.

4.2 Graphs selected by FULL_OUTPUT_INC.

The following three graphs are generated at the end of the cycle. The first graph will be output if nplot has value greater than 0, the second will be output if the value of NPLOT is 2 or 3, the third if NPLOT has the value 3.

1. The first graph shows the average temperature (K) in the gas and also in the matrix (K) plotted as a function of distance along the regenerator. The value at each mesh point is averaged over the cycle.

2. The second graph shows three curves plotted as a function of distance along the regenerator. These are the integral average over time, taken at each point, of the enthalpy flux (ENT-FLUX) (W), the conductive heat flux (COND-FLUX) (W) in the matrix, and the sum (ENTH+COND) of the two.

3. The third graph shows the net heat (QINTGL) (W) passed from the matrix to the gas integrated over the cycle. This is given as a function of distance along the regenerator. QINTGL is the heat transfer rate per

unit length at each point multiplied by the length of the regenerator and averaged over the cycle.

The following graphs are selected if the NPLOT value is 2 or 3. These will be output at intervals determined by the FULL_OUTPUT_INC parameter. All are plotted as a function of time over a period of one cycle.

1. A frame showing the gas (TGAS) and matrix (TMAT) temperature (K) at the cold end. They are plotted as a function of time over the cycle.

2. A frame showing the gas (TGAS) and matrix (TMAT) temperature (K) at the hot end. They are plotted as a function of time over the cycle.

3. This graph shows three curves of pressure (Pa) versus volume (cm^3). The first curve, (HOT-V) marked with "a", is the cold end pressure plotted against the compression volume. The second, (COLD-V) marked with "b", is the pressure at the cold end plotted against the expansion volume. The last, (HOT-P-COLD-V) marked with "c", is the pressure at the hot end including the pressure drop plotted against the compression volume.

The following graphs will be produced if the value of NPLOT is 3.

1. This graph contains two curves (PHOT) and (PCOLD) that show the pressure (Pa) as a function of time at the hot and cold ends of the regenerator. In REGEN3.2 the pressure at the hot end is corrected to include the pressure drop as computed from the correlation.

2. This frame shows the cold end pressure (P) (Pa), the mass flux (MFX0) (kg/s) at hot end, and the mass flux (MFX1) (kg/s) at the cold end plotted against the time. In order to show the disparate units on the same plot all three variables are normalized to lie between -1 and 1. The normalization factors are shown on the graph.

3. This frame shows the pressure drop (Pa) across the regenerator as a function of time over the cycle. The value is positive if the cold end pressure is less than the hot end pressure.

4. This graph plots the compression (HOT END PTVOL) and expansion (COLD END PTVOL) volumes (cm^3) as a function of time over the cycle.

A sequence of graphs can be selected by the MUSHIS parameter. The value of MUSHIS determines the number of points where output will be taken. The value of IXHIST(K) is the mesh index of the point. (See the discussion of MUSHIS given previously.) For each point the following frames are output to the RGWT file if NPLOT is greater than 1.

1. The first graph shows the gas (TGAS) and matrix (TMATRIX) temperatures (K) at the point as a function of time over the cycle.

2. The first curve on the second graph shows the heat transfer rate (QHIS) from the matrix to the gas (W). If q represents the heat transfer per unit volume, ϕ the porosity, L the length, and A the area then the heat transfer on the graph is ϕAqL. The second curve shows the mass flux rate (MFX) (kg/s), and the the third the temperature difference (TDIF) between the gas and matrix (K) at the point as a function of time. The three functions are normalized to the interval [-1,1] for plotting.

4.3 Graphs selected by PLOT_INC.

These graphs show various fields plotted as a function of distance along the regenerator. If the value of PLOT_INC is 0.10 and the value of FULL_OUTPUT_INC is 1000, then 10 of the following set of plots will be generated every 1000 cycles. If the value of HERZ is 10, then each set of plots will be snapshots of the solution on a single time step taken at intervals of 0.01 s throughout the cycle. If PLOT_INC is greater than 1.0 then these plots will not be generated. The next frame is output if the value of NPLOT is 2 or 3.

1. This frame shows the gas (TGAS) and matrix (TMAT) temperature (K) as a function of distance along the regenerator.

The following frames will be generated if the value of NPLOT is 3.

1. This frame shows the instantaneous mass flux (MFX), that is ϕAρv (kg/s), the enthalpy flux (MFX*h), that is $\rho\phi$Avh (W), and the heat

transfer rate (Q), that is q, (W/m^3) as a function of distance along the regenerator. The values are normalized to lie in the $[-1, 1]$ interval.

2. This frame shows the gas velocity (VEL) (m/s) as a function of distance along the regenerator.

3. This frame shows the result of the cell energy balance in the gas. This states that the rate of change of energy of the gas in the cell is determined by the energy flux through the sides of the cell and the heating between the gas and the matrix. Four functions that determine the energy balance in the cell containing each mesh point are plotted. The first is the rate of change (DT(ENG)) over the time step of the energy of the gas contained in the cell (W). The second is net enthalpy and conductive flux (DX(ENT)) (W) through the ends of the cell. The third is the average heating rate (Q) (W) from the gas to the matrix over the time step. The last curve (BAL) is the energy balance over the cell (W); this would vanish if the computation was perfectly accurate.

4. This frame contains three curves. The first is the value of the heat transfer coefficient (H) ($W/(m^2-s)$) at the time of the plot. The second is the Reynolds number (REN). The third (PHI) is the parameter Φ that appears in the expression for the heat transfer coefficient. These curves are all normalized to lie in the [-1,1] interval.

5. The next curve is a plot of the enthalpy flux (W) across the regenerator. This curve is the value of $\phi A \rho v h$ at each mesh point.

6. The last curve is the conductive heat flux (W) in the matrix.

5 The differential equations.

The conservation equations for mass, momentum, and energy for one dimensional flow of a gas in an open tube are the following:

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} &= 0 \\ \frac{\partial \rho v}{\partial t} + \frac{\partial(\rho v^2 + p)}{\partial x} &= 0 \\ \frac{\partial e}{\partial t} + \frac{\partial((e + p)v)}{\partial x} &= 0. \end{aligned} \tag{1}$$

These equations might be adequate for flow in an open tube, but they are not reasonable for flow in a regenerator. Such flow is better described as flow in a porous medium. The derivation of the flow equations for a porous media is discussed in many sources, such as the book by Kaviany [5]. Here, we simply assume that the effect of the porous media on the momentum equation can be modeled by the addition of a "friction" term in the momentum equation and a heat transfer term in the energy equation. These terms are taken to be functions of pressure, temperature, and velocity obtained by fitting data in Kays and London [3] [2]. This data was obtained from steady flow, and may not be as accurate for oscillating flow in a regenerator. The modified momentum equation is then

$$\frac{\partial \mathrm{v}}{\partial \mathrm{t}} + \mathrm{v}\frac{\partial \mathrm{v}}{\partial \mathrm{x}} + \frac{1}{\rho}(\frac{\partial \mathrm{p}}{\partial \mathrm{x}} - \mathrm{f}(\mathrm{p}, \mathrm{T}, \mathrm{v})) = 0$$

where $f(p, T, v)$ represents the friction term.

5.1 Reduced momentum equation.

If the velocity is rendered non-dimensional by division by the peak velocity, the time is divided by the period of the oscillatory flow, the pressure and density are scaled by their mean or peak values, then the following non-dimensional form of the momentum equation is obtained

$$\frac{\partial \mathrm{v}}{\partial \mathrm{t}} + \mathrm{v}\frac{\partial \mathrm{v}}{\partial \mathrm{x}} + (\frac{1}{\gamma \mathrm{M}_0^2})\frac{1}{\rho}(\frac{\partial \mathrm{p}}{\partial \mathrm{x}} - \alpha \mathrm{f}) = 0.$$

Here γ is the ratio of specific heats, M_0 the Mach number, and the variables $\mathrm{v}, \mathrm{p}, \rho$ are dimensionless. The friction term f has been scaled by $\alpha \mathrm{P}_0/\mathrm{L}$ with α chosen so the non-dimensional f is order 1. If the pressure gradient is no more than 5 or 10 percent of the mean pressure P_0, then α would be less than 0.1. For flow in a regenerator the Mach number is generally very small, perhaps 0.01 to 0.001. The reduction to dimensionless form has been defined so that the dimensionless time derivative should be of order one, and the term involving the spatial derivative of the velocity should be no larger than order one. Therefore the difference between the pressure gradient and the friction would have to be of the order of the square of the Mach number. This leads to a "reduced system", in which the first two terms are removed from the momentum equation, so that the momentum equation is replaced

by an equation which simply equates the pressure gradient to the friction term, namely

$$\frac{1}{\rho}(\frac{\partial \mathrm{p}}{\partial \mathrm{x}} - \mathrm{f}(\mathrm{p}, \mathrm{T}, \mathrm{v})) = 0. \tag{2}$$

This reduction eliminates the sound waves from the solution of the system.

5.2 The conservation equations for flow through a regenerator.

In addition to the friction term, the conservation equations must be extended to account for the thermal capacity of the matrix and the heat transfer between the matrix and the gas. This leads to the following conservation equations for the regenerator

$$\frac{\partial \rho}{\partial \mathrm{t}} + \frac{\partial(\rho \mathrm{v})}{\partial \mathrm{x}} = 0 \tag{3}$$

$$\frac{\partial \rho v}{\partial t} + \frac{\partial(\rho v^2 + p)}{\partial x} - f(\rho, T, v) = 0 \tag{4}$$

$$\frac{\partial \phi A E}{\partial t} + \frac{\partial(\phi A(E+p)v)}{\partial x} - \frac{\partial(\phi A k_g \partial T/\partial x)}{\partial x} - \phi A q(p, T, T_m, v) = 0 \tag{5}$$

$$\frac{\partial D}{\partial t} + \phi A q(p, T, T_m, v) - \frac{(1-\phi) A k_m \partial T_m/\partial x}{\partial x} = 0. \tag{6}$$

where the matrix thermal content term $D(x, T)$ is defined by

$$D(x, T) = \int_{T_{min}}^{T} (1-\phi) A c_m(x, T) dT. \tag{7}$$

Note that the area and porosity can depend on the position x along the regenerator. The heat transfer term has the form

$$\mathrm{q}(\mathrm{p}, \mathrm{T}, \mathrm{T_m}, \mathrm{v}) = 4\mathrm{H}(\mathrm{p}, \mathrm{T}, \mathrm{v})(\mathrm{T_m} - \mathrm{T})/\mathrm{D_h}.$$

The friction term f(p, v) and the heat transfer coefficient H(p, v) are obtained by fitting data for steady flow. These functions are described in a subsequent section. A discrete form of the above conservation equations (3-6) is the basis for the REGEN3.3 model. If the input parameter USE_ADVEC has value zero, then equation (4) is replaced by equation (2) in the REGEN3.3 model.

5.3 Boundary conditions for regenerator flow.

The next question concerns the proper boundary conditions for these equations. We assume that only the regenerator portion of a cryocooler is modeled, so that the system is not closed, which makes the boundary conditions more difficult to define. The first set of equations (1) are the Euler equations for one dimensional inviscid flow in open tube. These are hyperbolic so that the boundary conditions can be obtained by determining the characteristics for the equations. Inspection of the characteristic directions for the Euler equations indicates that two variables must be specified when the flow is into the regenerator and exactly one value must be specified at an outflow boundary. This argument is valid for a linear system with constant coefficients; however it should provide good guidance for the boundary conditions of the nonlinear system. The assumption that these equations have a hyperbolic nature that we can use to determine boundary conditions may not be valid because the time derivative and velocity gradient terms are small compared to the friction force and we have added the matrix thermal energy equation to the system. In fact, we have two different boundary conditions for the REGEN3.3 model that have given good results. The first selected by BDY_TYPE=1 has given good agreement with REGEN3.2 and also with the second selected by BDY_TYPE=2. The third boundary condition selected by BDY_TYPE=3 has not been tested as much as the first two.

To complete the finite difference approximation of the matrix temperature equation the matrix temperature is extrapolated at the boundary. This is equivalent to equating a centered difference approximation for the temperature derivative in the boundary cell to a centered approximation for the derivative in the cell adjacent to the boundary. To specify the matrix temperature at the boundary, or specify zero heat flux at the boundary would not be consistent with the geometry of a typical regenerator. Our extrapolated boundary would not be proper for a simple heat equation, but has been satisfactory for this model.

5.4 The reduced equations for the REGEN3.2 model.

The REGEN3.2 model does not use the conservation form (3-6) of the differential equations. Instead the equations are modified in order to obtain a numerical method which reduces the computational cost. This has the disadvantage that the pressure gradient is assumed to vanish. After the solution is

computed, then the pressure drop is estimated from the correlation of Kays and London [3]. Also, a rather ad hoc method must be used to conserve mass. However, this method has given good results and is sometimes more efficient than a fully implicit method.

Instead of using the mass conservation equation, we use a variant which involves the time derivative of the pressure. The energy conservation equation is replaced by a variant which involves the time derivative of the temperature. This results in an implicit scheme only for the velocity variable; the other three variables can be advanced by an explicit scheme.

The mass conservation equation can be combined with various thermodynamic relations involving the entropy and heat flux to yield the following equation involving the time derivative of the pressure [6]

$$\frac{\partial \mathrm{p}}{\partial \mathrm{t}} + \mathrm{v}\frac{\partial \mathrm{p}}{\partial \mathrm{x}} + \rho \mathrm{c}^2 \frac{\partial \mathrm{v}}{\partial \mathrm{x}} - \mathrm{G_r q} - \mathrm{G_r k_g}\frac{\partial^2 \mathrm{T}}{\partial \mathrm{x}^2} = 0. \tag{8}$$

For convenience, we have replaced $\partial(\mathrm{k_g}\partial \mathrm{T}/\partial \mathrm{x})/\partial \mathrm{x}$ in the equation by $\mathrm{k_g}\partial^2 \mathrm{T}/\partial \mathrm{x}^2$. This term seems generally to be small, and could probably be eliminated altogether. This equation is valid for a real gas. If the first equation for the pressure is differentiated with respect to x then an equation for v is obtained. Differentiation with respect to x yields

$$\frac{\partial}{\partial \mathrm{x}}\frac{\partial \mathrm{p}}{\partial \mathrm{t}} + \frac{\partial}{\partial \mathrm{x}}\left(\mathrm{v}\frac{\partial \mathrm{p}}{\partial \mathrm{x}} + \rho \mathrm{c}^2 \frac{\partial \mathrm{v}}{\partial \mathrm{x}} - \mathrm{G_r q} - \mathrm{G_r k_g}\frac{\partial^2 \mathrm{T}}{\partial \mathrm{x}^2}\right) = 0.$$

Since $\partial \mathrm{p}/\partial \mathrm{x} = \mathrm{f}(\mathrm{p}, \mathrm{T}, \mathrm{v})$, the following equation for the velocity is obtained

$$\frac{\partial \mathrm{f}(\mathrm{p}, \mathrm{T}, \mathrm{v})}{\partial \mathrm{t}} + \frac{\partial}{\partial \mathrm{x}}\left(\mathrm{v}\mathrm{f}(\mathrm{p}, \mathrm{T}, \mathrm{v}) + \rho \mathrm{c}^2 \frac{\partial \mathrm{v}}{\partial \mathrm{x}} - \mathrm{G_r q}(\mathrm{p}, \mathrm{T}, \mathrm{v}) - \mathrm{G_r k_g}\frac{\partial^2 \mathrm{T}}{\partial \mathrm{x}^2}\right) = 0. \tag{9}$$

In order to further simplify this nonlinear equation for the velocity, the pressure gradient is assumed to vanish. Thus the friction term, f, is replaced by zero in the above equation. This provides a two-point boundary value problem for the velocity once the other variables are known and the value of the velocity is given at the ends of the regenerator. The solution of this equation requires the solution of a tridiagonal system of equations. The remaining three equations can be solved by a scheme which is essentially explicit. This is much less expensive than a scheme which is implicit in all four equations. However, it has the disadvantage of differentiating the heat transfer term.

Also, this system is not in conservation form, which can lead to a failure to conserve mass.

The differential equations, which form the basis for the REGEN3.2 method include the porosity and the cross sectional area of the regenerator which may be functions of the distance x along the regenerator. If the area and porosity vary but are constant within each segment of the regenerator then a solution is still possible in either model. The equations for the REGEN3.2 method with the porosity and area assumed constant for simplicity are the following:

$$\frac{\partial}{\partial \mathrm{x}}\left(\rho \mathrm{c}^2 \frac{\partial \mathrm{v}}{\partial \mathrm{x}}\right) - \frac{\partial}{\partial \mathrm{x}}\left(\frac{4\mathrm{G_r H(p,T,v)}}{\mathrm{D_h}}(\mathrm{T_m}-\mathrm{T}) + \mathrm{G_r k_g}\frac{\partial^2 \mathrm{T}}{\partial \mathrm{x}^2}\right) = 0 \quad (10)$$

$$\frac{\partial \mathrm{p}}{\partial \mathrm{t}} + \rho \mathrm{c}^2 \frac{\partial \mathrm{v}}{\partial \mathrm{x}} - \frac{4\mathrm{G_r H(p,T,v)}}{\mathrm{D_h}}(\mathrm{T_m}-\mathrm{T}) - \mathrm{G_r k_g}\frac{\partial^2 \mathrm{T}}{\partial \mathrm{x}^2} = 0 \quad (11)$$

$$\frac{\partial \mathrm{T}}{\partial \mathrm{t}} + \mathrm{v}\frac{\partial \mathrm{T}}{\partial \mathrm{x}} + \mathrm{G_r T}\frac{\partial \mathrm{v}}{\partial \mathrm{x}} - \frac{4\mathrm{H(p,T,v)}}{\rho \mathrm{c_v D_h}}(\mathrm{T_m}-\mathrm{T}) - \frac{\mathrm{k_g}}{\rho \mathrm{c_v}}\frac{\partial^2 \mathrm{T}}{\partial \mathrm{x}^2} = 0 \quad (12)$$

$$\frac{\partial \mathrm{T_m}}{\partial \mathrm{t}} + \frac{4\phi \mathrm{H(p,T,v)}}{(1-\phi)\mathrm{c_m D_h}}(\mathrm{T_m}-\mathrm{T}) - \frac{1}{\mathrm{c_m}}\frac{\partial(\mathrm{k_m}\partial \mathrm{T_m}/\partial \mathrm{x})}{\partial \mathrm{x}} = 0. \quad (13)$$

Note that the pressure is assumed to be independent of the spatial coordinate x, thus $\mathrm{p} = \mathrm{p(t)}$.

5.5 The correlation for the heat transfer coefficient.

The correlation expresses the heat transfer as a function of the geometry and the properties of the flowing gas. This allows the flow in the porous media to be modeled in terms of the averaged flow variables instead of looking at the flow on the scale of the porous media. The coefficient which determines the heat transfer between the gas and the matrix within the regenerator is taken from the REGEN report by Arp and Radebaugh [2]. These coefficients for different geometries are determined from fitting the data of Kays and London [3]. The heat transfer, from the matrix to the gas, is given by $\mathrm{q} = \mathrm{HA_w}\theta$ where $\theta = \mathrm{T_m} - \mathrm{T}$ is the difference of the matrix and gas temperatures, $\mathrm{A_w}$ is the "wetted area", and H is the heat transfer coefficient. The hydraulic diameter is defined by $\mathrm{D_h} = 4\phi \mathrm{V}/\mathrm{A_w}$ where ϕV is the void volume. The heat transfer rate per unit volume of the gas is thus $\mathrm{q} = 4\mathrm{H}\theta/\mathrm{D_h}$. The units of H are $(\mathrm{W/m^2 \cdot K})$. The heat transfer coefficient is computed in terms of

the Stanton number, N_{st} defined by $N_{st} = H/(\rho|v|c_p)$ where ρ is density, v velocity, and c_p specific heat. The Stanton number is expressed as a function of the Reynolds number $R = D_h\rho|v|/\mu$ where μ is viscosity. An empirical fit of the product $Y = N_{st}\sigma^{2/3}$ is made in terms of the Reynolds number. Then the heat transfer function can be expressed in terms of the Reynolds number as $H = \Phi R Y(R)$ where

$$\Phi = \mu c_p/(D_h\sigma^{2/3})$$

where σ is the Prandtl number. For a matrix formed from screens or packed spheres the function Y is approximated by the following two parameter function of the Reynolds number, $Y(R) = aR^{-\beta}$. The values of a and β are chosen to approximate the data in Kays and London [3]. In the case of parallel plate or axial tube flow, four parameters are used for Y(R); namely, $Y = a_1R^{-\beta_1} + a_2R^{-\beta_2}$. In these cases, in order to account for a transition to turbulence, two such four parameter approximations are pieced together to give a smooth approximation which has different parameters in the regions below and above R=2000. Note that R, Y, and the coefficients a, a_1, and a_2 are dimensionless.

The approximations for the regenerator geometries included in the package are given next. The parallel plate geometry is described by

$$\begin{aligned} Y(R) &= 8.5R^{-1} \quad \text{for} \quad R \le 2000 \\ Y(R) &= 0.0034 + 2.72 \times 10^{13}\ R^{-5} \quad \text{for} \quad R > 2000. \end{aligned}$$

The tube geometry has

$$\begin{aligned} Y(R) &= 4.20R^{-1} \quad \text{for} \quad R \le 2000 \\ Y(R) &= 0.00168 + 1.344 \times 10^{13}\ R^{-5} \quad \text{for} \quad R > 2000. \end{aligned}$$

For flow transverse to tubes

$$Y(R) = 0.343R^{-0.4}.$$

For a regenerator matrix composed of screens

$$\begin{aligned} Y(R) &= aR^{-0.43} \quad \text{where} \\ a &= 0.715(5.6 + \phi(-16.363 + \phi 13.928)). \end{aligned}$$

For flow through packed spheres

$$\begin{aligned} Y(R) &= aR^{-0.30} \quad \text{where} \\ a &= 0.23(1.0 + 0.7772(0.38 - \phi)). \end{aligned}$$

The approximations described above are those given in the report by Arp and Radebaugh [2]. They did not always work well in this regenerator model. The parameter β used in the definition of Y(R) is generally between zero and one. Therefore RY(R) is a continuous function but has a singularity in its derivative at R = 0. This singularity would sometimes cause the iteration for the velocity to fail. Note that equation (7), which is used to determine the velocity, is a nonlinear equation for the velocity. In the marching scheme, once new values are computed for the temperature and pressure, then this nonlinear equation is solved for the velocity. The Newton iteration used to solve this equation would sometimes fail, apparently because of the singularity in the function RY(R) at R = 0. Therefore, for geometry values of 3,4 and 5 the term $\mathrm{RY(R)} = \mathrm{aR}^{1-\beta}$ is replaced by the following function which is smoothed in the vicinity of R=0

$$\mathrm{RY(R)} = \mathrm{a}\gamma_0 \mathrm{e}^{-\alpha \mathrm{R}^2} + \mathrm{a}(1 - \mathrm{e}^{-\alpha \mathrm{R}^2})\mathrm{R}^{1-\beta}.$$

For small values of R, for packed spheres a value less than 4, there are no measurements of H to approximate. Therefore it is not possible to evaluate the accuracy of this approximation. We did vary the values from $\gamma_0 = 0.25$ with $\alpha = 0.1$ to $\gamma_0 = 2.0$ with $\alpha = 0.04$ and found little effect on the computed results. We looked at three cases, two were running at low temperatures between 10 and 30 K, at pressure ratios of 1.3 and 2.0, the third between 65 and 340K with a pressure ratio of 1.5. There was less than a 2 percent change in the enthalpy flux or ineffectiveness with the changes in these parameters.

5.6 The correlation for the friction factor.

The pressure gradient along the regenerator can be estimated by fitting data given by Kays and London. In this reference the friction factor is defined in terms of two functions, A(R) and Y(R) where R is the Reynolds number, Y(R) is defined in the previous section, and the function A(R) is determined empirically for different geometries. Then the pressure gradient is, as given in the report by Arp and Radebaugh,

$$\frac{\partial \mathrm{p}}{\partial \mathrm{x}} = \mathrm{sign(v)} \frac{2\mathrm{Y(R)R}^2 \mu^2}{\mathrm{A(R)D}_\mathrm{h}^3 \rho}.$$

For the parallel plate geometry

$$\mathrm{A(R)} = 0.33 + \frac{0.09}{1.0 + \exp^{(3500-\mathrm{R})/1500}}.$$

For axial flow through a tube

$$\mathrm{A(R)} = 0.27 + \frac{0.16}{1.0 + \exp^{(5500-\mathrm{R})/2000}}.$$

The parameter for flow transverse to tubes is

$$\mathrm{A(R)} = 3.59 - 2.37/\max(1.0, \mathrm{R}^{0.2}).$$

The screen geometry is described by

$$\begin{aligned}
\mathrm{A(R)} &= 0.0074\mathrm{R} \quad \text{for} \quad \mathrm{R} \leq 10 \\
\mathrm{A(R)} &= 0.129 - 0.0058(\log(\mathrm{R}/200))^2 \quad \text{for} \quad 10 < \mathrm{R} \leq 3000 \\
\mathrm{A(R)} &= 0.149 - 0.0239\log(\mathrm{R}/200) \quad \text{otherwise.}
\end{aligned}$$

The parameter for flow through packed spheres is

$$\begin{aligned}
\mathrm{A(R)} &= 0.0022\mathrm{R} \quad \text{for} \quad \mathrm{R} \leq 10 \\
\mathrm{A(R)} &= 0.1032 - 0.00695\log\mathrm{R} \quad \text{for} \quad \mathrm{R} \geq 330 \\
\mathrm{A(R)} &= -0.007 + 0.0126\log\mathrm{R} \quad \text{otherwise} \\
\mathrm{A(R)} &= \max(0.62, \mathrm{A(R)}).
\end{aligned}$$

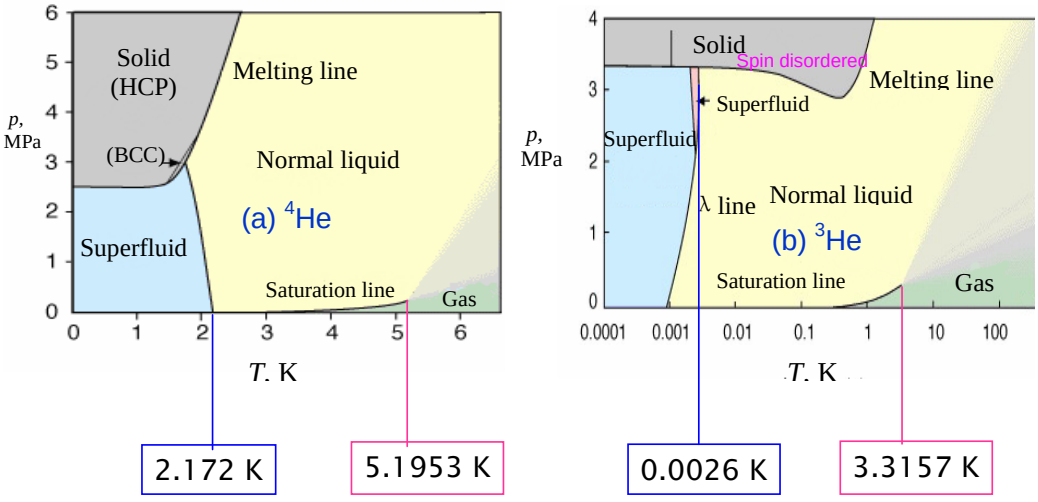
5.7 Thermal properties of the regenerator matrix.

The data for the volumetric heat capacity and the thermal conductivity of the regenerator matrix are taken from the REGEN report by Arp and Radebaugh [2] and from data obtained from different sources. This data is used to generate an approximation for each of these parameters. Some of these have a piecewise linear dependence on the temperature, others are fitted by an analytic function. At present there are approximations for 33 materials or mixtures included in the code. In addition any linear function of temperature can easily be used for these thermal properties.

The user of the package is required to provide a correction (MAT_COND_FACTOR) to the thermal conductivity of the matrix to account for a matrix which is not solid material, but may instead be porous. This correction factor is multiplied by the thermal conductivity for solid material to give the conductivity used in the model.

6 Modeling the Helium properties.

^{4}He and ^{3}He, the two stable helium isotopes in nature are the best working substances for cryocoolers at very low temperatures. The normal boiling temperature of ^{3}He is 3.197 K, and it becomes superfluid below 0.0026 K, in marked contrast to (the ordinary) ^{4}He with a normal boiling temperature of 4.23 K and which transits to a superfluid phase below 2.1768 K. The thermophysical properties of liquid ^{4}He and ^{3}He show dramatic differences at low temperatures. ^{4}He, with an even number of nucleons, obeys Bose-Einstein statistics while ^{3}He, with an odd number, obeys Fermi-Dirac statistics. The following two graphs show the phase diagrams of both these two isotopes.



The thermophysical properties of the working substances in the regenerator are required by REGEN. These properties include density, specific heat at constant pressure, enthalpy, entropy, internal energy, velocity of sound, thermal conductivity and viscosity as functions of temperature and pressure. In the previous versions of REGEN, only the HE4PROPS subroutine was available to compute the real gas properties of ^{4}He at temperatures from 2 to 1500 K and pressures up to 2000 MPa. This routine was developed at NIST[7][8].

By using ^{3}He as the working substance, rather than ^{4}He, inspection of the thermodynamic properties shows that better performance of low temperature cryocoolers could be achieved. Starting from REGEN 3.3, a model for the thermophysical properties of ^{3}He was added as the HE3PROPS subroutine. The thermodynamic part of the properties of ^{3}He were the work by Dr.

Yonghua Huang [9], [10], [11], [12] in Zhejiang university, China. An equation of state using the Helmholtz potential function based on Debye theory was used to calculate almost all the thermodynamic properties of real fluid ^{3}He in a wide range of temperature from 0.01 K to 1500 K and pressure up to 15 MPa. The transport properties of ^{3}He, i.e., thermal conductivity and viscosity, were developed also by Yonghua Huang at NIST in 2006 (unpublished). The equations cover a range of temperature from 2.6 mK to 450 K and pressure up to 20 MPa.

Since two working fluids are available in the current version of REGEN, the HELIUM input parameter is added to select the working fluid. Setting HELIUM=4 selects ^{4}He and HELIUM=3 selects ^{3}He.

6.1 Interpolation for helium properties.

Computing the helium properties using these routines can be the most time consuming part of the computation. To reduce this computation time an option is provided that computes these properties as a function of pressure and temperature and stores the results for the 8 properties in a large table, generally (200x200) or (400x400). The size of the table can be set by the input data. The table lookup is much faster than direct computation using the properties routines. Generally the table lookup does not have a significant effect on accuracy, but checking the accuracy of the lower temperature problems is advisable. Selection of the table lookup option is determined by the USE-PROPS-TABLE input parameter.

7 Program usage.

This section describes the format of the input data required to run a simulation. There is also a discussion of the parameters that determine the accuracy of a simulation, the interpretation of the output from a simulation, and the difficulties inherent in cases which have the cold end temperature near 4K. The model can be executed in 2 modes. The first requires typing commands into a command window in Linux, Unix, or Windows. In this case the data file must be in NAMELIST format and a text editor may be used to create or modify the input data file. In the second mode the model can be executed in a "point-and-click" environment within Windows Explorer. At present, only the REGEN3.2 model can be executed within this graphical interface.

However, a graphical interface is available that can create or modify a input data file. This file is created in the NAMELIST format. This file can then be used as input to the REGEN3.3 model so the user does not have to use a text editor or deal directly with the NAMELIST input.

7.1 Input data format.

The input data for the REGEN3.2 and REGEN3.3 programs must be in Fortran NAMELIST format. The group name for this input is "INP". The input for each case must be preceded by an ampersand followed by the group name, that is "&INP". The data for the case must be terminated by a forward slash "/". Note that there can be data for more than one case in the input. The NEWCAS parameter will determine if the new input data block will cause the computation from the previous block to continue to a larger value of FINAL_CYCLE (NEWCAS=0) or start a new computation (NEWCAS=1). If the group name (INP) is misspelled most systems will read the data and ignore it without giving an error message and the code will treat this as an empty input data file. Each input item or list of values must be preceded by the variable name with an "=" as the separator. The use of NAMELIST input makes default values for input parameters convenient since these parameters can be omitted from the input. In this case the default values will be assumed. In the both programs the input is passed to the program as standard input. If the data is in the file DATA1000, then the code is executed by the command

./RG33.EXE < DATA1000.

The version of the model used by the graphical interface on the Windows PC is RG32_GUI.EXE. This will take the same data format, but the data must be copied into a file named DATA.DAT that must be in the same directory as the RG32_GUI.EXE program file. The program will read the data from this file. This version is used with the graphical interface.

7.2 Operation using the graphical interface.

The graphical interface program RUNREGEN.EXE can be used on a Windows PC to create the data file, execute the program and view the graphics output. This interface can also be used to modify the data from a previous

run to create a new data file. In this way the model can be run without the use of a text editor. A separate graphical interface program, GUI.EXE, is available that will create the data file needed by the two models. Another program is available to view the graphical output from a run. This is the VIEWRGWT.EXE program. The RUNREGEN.EXE program is a combination of these two programs with the computational programs RG32_GUI.EXE and RG33_GUI.EXE.

7.3 Terminatation criteria.

The integration should continue until the solution has reached a periodic state. Then the variables that determine the solution will have the same value at the end of a cycle that they had at the start of the cycle. This means that the temperature of the matrix and gas at each point and the total thermal energy within the matrix should return to the starting values. Also when the solution has converged to the periodic state the integral over the cycle of the sum of the enthalpy flux in the gas and the heat flux in the matrix should be the same along the length of the regenerator. The value of QINTW measures the change in the total thermal energy within the matrix over the cycle. The value of EHTDIF is the maximum difference in the enthalpy plus heat flux integral over the length of the regenerator. The convergence of a simulation to a periodic state is best judged by the value of the QINTW and EHTDIF output parameters. This value should be relatively small compared to the value of the enthalpy flux EHTFLX. If one end of the regenerator is at room temperature, then convergence may require 10000 to 15000 time steps which may mean several hours of computing depending on the mesh resolution. The large difference between the heat capacity of the gas and matrix means that only a small change in temperature occurs over a single cycle. If the periodic solution has a matrix temperature that deviates from the initial temperature profile then the convergence may be slow. To judge the convergence the QINTW and EHTDIF values should always be checked.

7.4 Initial temperature adjustment.

The convergence will be accelerated if the initial temperature profile of the matrix is more nearly equal to the final temperature when the periodic state is achieved. The MID_TEMP_RATIO parameter controls the initial matrix

temperature profile. The value of this parameter is the difference between the initial temperature at the midpoint of the regenerator and the inflow temperature GAS_TEMP_COLD at the cold end divided by the difference of two end point temperatures GAS_TEMP_COLD and GAS_TEMP_HOT. If the input value of MID_TEMP_RATIO is 0.5 then the initial temperature is a linear function of distance along the regenerator. If this value is not 0.5, then the initial temperature is a quadratic which passes through the two end temperatures and the midpoint temperature determined by MID_TEMP_RATIO. If the value of MID_TEMP_RATIO is greater than 0.5 the initial temperature will lie above the midpoint of the linear profile. If MID_TEMP_RATIO is set to the value of the output parameter GTPNRM attained by the final periodic solution of a run, then the convergence of a subsequent run with similar input data is likely to be improved. We have found that a value of 0.2 is best for cases where the cold end temperature is below 10 K.

7.5 Problems with low temperature cases.

Low temperature cases generally show a rapid drop to the cold end temperature close to the warm end. A regenerator 10 cm in length may have the temperature drop to the cold end in the first 2 cm with the remaining 8 cm close to the cold end temperature. If the initial temperature is a linear drop over the full 10 cm then the major portion of the gas will, on average, be at a much cooler temperature after a periodic state is achieved than it was initially. In the case of the REGEN3.2 model this means that the average pressure over the initial cycle will be much higher than the average pressure when the periodic state is reached at the end of the run. Note that the average mass of the gas in the regenerator does not change from cycle to cycle. In this case the input parameter PRES_INITIAL may be much larger than the average pressure at the end. The steep drop near the warm end may require a smaller cell size than a more uniform temperature drop. For a stage one model whose cold end is at 80 K 20 mesh cells across the regenerator may be adequate, but low temperature case whose cold end is at 4 K may need 200 cells. In the REGEN3.2 model the USE_GRADED_MESH option allows a non-uniform mesh with the smaller mesh cells concentrated near the warm end. The use of this option along with the GRADED_CUTOFF parameter may also change the initial temperature profile so that the temperature drops more rapidly down to the cold end temperature at this cutoff point. The rapid drop near the warm end can cause the temperature to undershoot

the cold end temperature, that can cause failure due to the appearance of a liquid or solid phase. Sometimes this undershoot can be reduced by using more mesh points, but it is caused by the use of central differences in the numerical approximation combined with the steep temperature profile and may be difficult to eliminate. The REGEN3.3 model has similar difficulty with this undeshoot.

7.6 Adjustment for convergence in the REGEN3.3 case

. Each time-step in regen3.3 requires the solution of a non-linear system of equations for the new values of mass flux, gas temperature, pressure, and matrix temperature. This solution is obtained by means of a Newton iteration for all these variables simultaneously. If the discretized conservation of energy equation (5) for the gas is summed over all the mesh cells and over a complete cycle the interior flux terms cancel leaving an energy balance value that is stored in the ENGBAL variable. The energy balance for the matrix thermal energy equation is given by the MEGBAL variable. If the Newton iteration converged perfectly both these variables would have value zero even if the solution has not converged to a periodic state. The size of these balance variables can give an estimate of the accuracy of the integration.

There are two parameters used in the regen3.3 model that may require adjustment if the Newton iteration fails. If the EPS_NEWTON value is too small it may cause the Newton iteration to fail. If the parameter is too large the accuracy may be reduced. This reduced accuracy may be indicated by larger values of the gas energy balance ENGBAL or the matrix energy balance MEGBAL relative to the enthalpy flux. Usually values of EPS_NEWTON between 10^{-6} and 10^{-7} give good results, sometimes values of 10^{-8} or 10^{-9} will cause the Newton iteration to fail. Another parameter that may need adjustment is METHOD. The default value of METHOD is 1 which selects a first order approximation of the time derivatives in conservation equations. Using the second order approximation (METHOD=2) yields more accurate results in most cases. However, we observed a subtle failure in one case using the second order method. The enthalpy flux would jump to a higher value and the temperature would oscillate over a range of cycles and then revert to an average value. In another case use of METHOD=2 caused the Newton iteration to fail. Using the first order method eliminated this problem.

Another problem can occur if the values of TABLE_PRES_MIN, TABLE_PRES_MAX, TABLE_TEMP_MIN, or TABLE_TEMP_MAX do not provide a large enough range (a table is always used to compute the total matrix thermal content even if USE_PROPS_TABLE=0). In this case the run will terminate with an error message. These values are set by default and some values may need to be reset.

7.7 Evaluating the accuracy of the solution.

If the energy equation for the gas (5) is added to the energy equation for the matrix (6) and then integrated over one cycle the result relates the difference in the energy at the start and end of the cycle with the sum of the enthalpy flux in the gas and the conductive flux in the gas and matrix. If the solution is periodic the energy difference will cancel out and thus the sum of the enthalpy and conductive flux must be zero. This sum evaluated at the cold end of the regenerator is the value of the EHTFLX variable. The maximum variation in the sum of the enthalpy and conductive flux across the length of the regenerator is EHTDIF. If the value of this enthalpy/conduction variation (EHTDIF) is large compared to the sum of the enthalpy and matrix thermal flux (EHTFLX) and is not reduced by running more cycles, this is evidence of a poor solution. If these values are 5 % or 10 % of the enthalpy flux the solution is probably acceptable, a value of 20-30 % is probably not acceptable. If the energy balance ENGBAL is comparable to the EHTFLX term and if ENGBAL is not reduced by use of a smaller value of EPS_NEWTON this may also indicate an inaccurate solution. Another check, perhaps the best check, is to rerun the case with double the number of mesh points and double the time steps per cycle and compare with the lower resolution run.

8 Program installation.

An executable binary version of the program is available for an IBM compatible PC running Microsoft Windows. For linux or Unix systems the source code and a makefile script file is available from NIST so that an executable version can be built. The source code is written in the Fortran90 language. The instructions for using the package on a PC are contained in the QUICKSTART file on the CD. Input for several runs is contained in the DATA1500, DATA1501,..., DATA1519 files. These are intended to illustrate different

ways to operate the model, but most will only the run the model for a few cycles so that they will execute quickly. Thus they will not achieve a useful periodic solution. This user guide is contained in the PDF file MANUAL.PDF and also in postscript form in MANUAL.PS.

The executable file is RG33. This executable always takes its input from the a file "DATA.DAT". It is intended to work in a " point-and-click" environment in Windows Explorer rather than in a command window.

For Windows systems there is a graphical interface, RUNGUI.EXE, that can be used to generate a data file, execute the model, and view the graphical output from the package. For more details on the usage of the package see the QUICKSTART file.

The program will generate two output files one with prefix RGPR and the other with RGWT. If the USE_SAVE option is selected then a file with prefix RGSAV file will be written.

9 Error messages.

If the REGEN3.3 code detects an error condition such as a failure of the Newton iteration to converge, or negative temperature, or some other out-of-range condition the case will abort and an error message will be written to both standard output (.i.e. the terminal) and the 'rgpr' file. The REGEN3.2 code will also generate error messages, but we have not prepared a list of those messages. They are similar to those listed here. In REGEN3.3 the first line of the error message will have the following appearance:
' ***** REGEN3.3 ERROR, error number= <num> '.
The messages corresponding to each error number are listed below following the error number.

- (1) This error number reprsents errors found in the input data. This could be inconsistent data or required data that is missing. There are 50 error conditions that can be reported under this number. These messages are listed below, one per line. If the code finds more than one error in the input all will be reported under error number one. That is, unlike the other error numbers, multiple errors may be detected and reported in the same error message.
  ' ***** nrun must be input '
  ' ***** graded mesh not used in rgfull'
  ' ***** newcas out of range'

‘ ***** nrun_restart must be input if newcas=3’
‘ ***** bdy_type must be given’
‘ ***** bdy_type out of range’
‘ ***** if bdy_type=2,3 ave_pres must be input’
‘ ***** if bdy_type=2,3 pres_ratio must be input’
‘ ***** orifice must be given when bdy_type=4’
‘ ***** geometry out of range geometry=’
‘ ***** error input mushis out range ’
‘ ***** material_form out of range’
‘ ***** num_layers out of range’
‘ ***** error : material not set, material_form=1’
‘ ***** method out of range’
‘ ***** use_mat_cond out of range’
‘ ***** use_props_table out of range’
‘ ***** tbmin too small, < 1.0 K ’
‘ ***** tbmin larger than gas_temp_cold’
‘ ***** tbmax smaller than gas_temp_hot’
‘ ***** final_cycle < 0’
‘ ***** gas_temp_cold < 0.’
‘ ***** gas_temp_hot < 0.’
‘ ***** herz < 0’
‘ ***** hydra_diam < 0.’
‘ ***** mass_flux_cold < 0.’
‘ ***** mass_flux_hot < 0.’
‘ ***** mass_phase < -360.’
‘ ***** porosity < 0.’
‘ ***** pres_initial < 0.’
‘ ***** rg_area < 0.’
‘ ***** rg_length < 0.’
‘ ***** material out of range’
‘ ***** num_materials out of range’
‘ ***** materials_list out of range’
‘ ***** mat_fraction(i) out of range’
‘ ***** num_layers out of range’
‘ ***** x_layers(i) out of range’
‘ ***** hidiam_layers < 0.’
‘ ***** poros_layers < 0.’
‘ ***** x_layers(1)<0’

‘ ***** x_layers()>xlenrg’
‘ ***** x_layers(i-1)>x_layers(i)’
‘ ***** num_materials out of range’
‘ ***** materials_list() out of range’
‘ ***** table_temp_min or table_temp_max must be input’
‘ ***** find_pratio > 0 pres_ratio < 0.’
‘ ***** find_pratio > 0 ave_pres < 0.’
‘ ***** find_pratio > 0 pres_phase < -180’
‘ ***** x_layers out of bounds’
‘ ***** geom_layers out of bounds’
‘ ***** The graded mesh option is not available ’
‘ ***** nrun must not be changed if use_case_rgpr=0’

- (101, 102) ‘ Error in heprops called from gprops: ’
- (103) ‘ Error in heprops called from settab: pres=, temp=’
- (104) ‘ Error in settab nbt or nbp out of range must have nbt≤ , nbp≤’
- (105) ‘ Error in cpvol_m called from settab: negative temp=’
- (106) ‘ Error in cpvol_m: negative temp=’
- (107) ‘ Error in cpvol called from gprops: negative temp=’
- (108) ‘ Error in matcnd called from gprops: negative temp=’
- (109) ‘ Error in dmint called from gprops: negative temp=’
  ‘ ..... Increase of limits TABLE_TEMP_MIN or MAX may be needed’
- (110) ‘ Error in cpvol_m: negative temp=’
- (111) ‘ Error in cpvol: negative temp=’
- (112) ‘ Error in matcnd: negative temp=’
- (113) ‘ Error in dmint: negative temp=’
  ‘ ..... Increase of limits TABLE_TEMP_MIN or MAX may be needed’
- (114) ‘ Error in heprops, called from full_output’

- (115) ‘ Error in cpvol_m, called from full_output: negative temp=’
- (116) ‘ Error in matcnd_m called from tube_loss: negative temp=’
- (117) ‘ Pressure out of range in table gprops_tab, pres=’
  ‘ ..... A change of TABLE_PRES_MIN or MAX may be needed’
- (118) ‘ Temperature out of range in table gprops_tab, temp=’
  ‘ ..... A change of TABLE_TEMP_MIN or MAX may be needed’
- (119) ‘ Pressure out of range in gprops_tab, pres=’
  ‘ ..... A change of TABLE_PRES_MIN or MAX may be needed’
- (120) ‘ Temperature out of range in gprops_tab, temp=’
  ‘ ..... A change of TABLE_TEMP_MIN or MAX may be needed’
- (121) ‘ Failure reading input file in routine rdinp iostatus=’
- (122) ‘ Failure in routine set_optimal: num_materials is out of range’
- (123) ‘ Failure in routine set_mixture: num_materials is out of range’
- (124) ‘ Failure in routine initial: mid-point of last cell must be in last layer’
- (125) ‘ Failure in routine set_mat_tab: Could not read file, iostatus=’
- (126) ‘ Failure in routine set_mat_tab:’
  ‘..... In mattable data temp(i) $\geq$ temp(i-1), temp=’
- (127) ‘ Failure in routine set_mat_tab: ’
  ‘ ..... In mattable data temp(1)>table_temp_min’
- (128) ‘ Failure in routine set_mat_tab: ’
  ‘ ..... In mattable data temp(npts)<table_temp_max’
- (129) ‘ Failure advan: dt reduction failed, dt=’
- (130) ‘ Failure advan: fdjac failed to set Jacobian matrix’
- (131) ‘ Failure in advan caused by gprops call in resfun’
  ‘ .....The error number from gprops is ierr=’
  ‘ ..... Additional information from resfun is dt=’

- (132) ‘ Failure in fdjac, zero on diagonal, i=’
- (133) ‘ Failure in getsav, nx .ne. saved nx saved nx=nx_lc=’
- (134) ‘ Failure in getsav, final_cycle must be > restart’
- (135) ‘ Failure in getsav, error opening file, iostatus=’
- (136) ‘ Failure in routine jacslv, singular matrix’
- (137) ‘ Failure in routine rename_input’
  ‘ num_layers > max_layer’
- (138) ‘ Failure in routine rename_input’ ‘ At least one of layers parameters must be set’
- (139) ‘ Error in heprops, called from setden in set_soln’
- (140) ‘ Error in heprops, called from setden ’
- (141) ‘ Error in heprops, called from setden in output_mod’

The following are error numbers and messages associated with the helium 3 and helium 4 properties routines.

- (-101) ‘ !He4props failure: pressure = 0 or negative, idid=’
- (-102) ‘ !He4props failure: pressure too high, idid=’
- (-103) ‘ !He4props failure: temperature < 0.8 K, idid=’
- (-104) ‘ !He4props failure: temperature > 1500 K, idid=’
- (-105) ‘ !He4props failure: density = 0 or negative, idid=’
- (-106) ‘ !He4props failure: density too high, idid=’
- (-107) ‘ !He4props failure: solid phase, idid=’
- (-201) ‘ He3props failure: Input Pressure ≤ zero idid=’
- (-202) ‘ He3props failure: ’
  ‘ Input Pressure too high; out of range, idid=’

- (-203) ‘ He3props failure: ’
  ‘ Input Temperature < 0.0026 K; out of range, idid=’
- (-204) ‘ He3props failure: ’
  ‘ Input Temperature > 1500 K; out of range, idid=’
- (-205) ‘ He3props failure: Input Density $\leq$ zero idid=’
- (-206) ‘ He3props failure: ’
  ‘ Input Density is outside of valid range, idid=’
- (-207) ‘ He3props failure: Iteration ’
  ‘ failure with (P,T) input. Out of range?, idid=’
- (-208) ‘ He3props failure: Unexpected iteration failure near the critial point’
  ‘ idid=’

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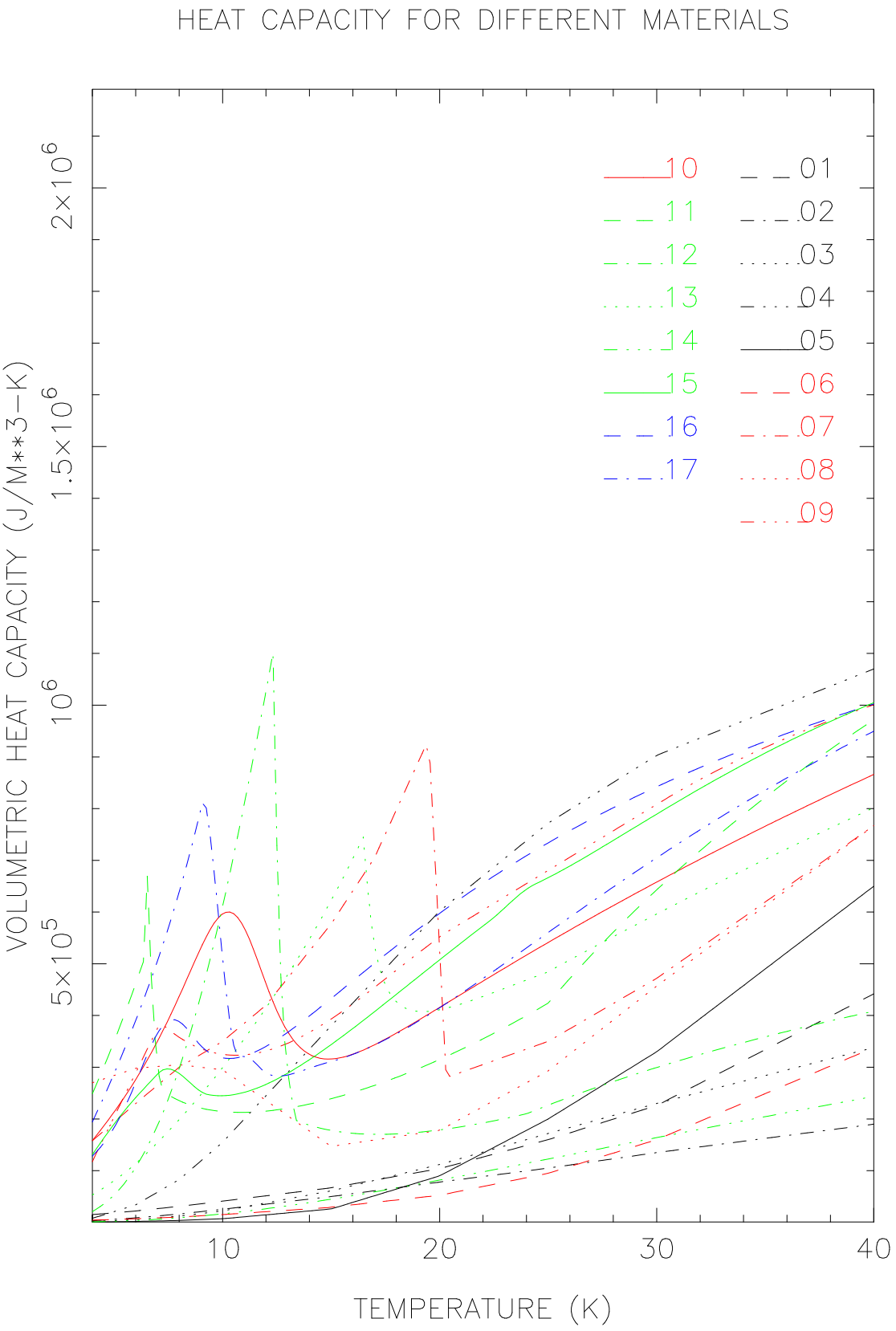

Figure 1: Heat capacity of various materials.

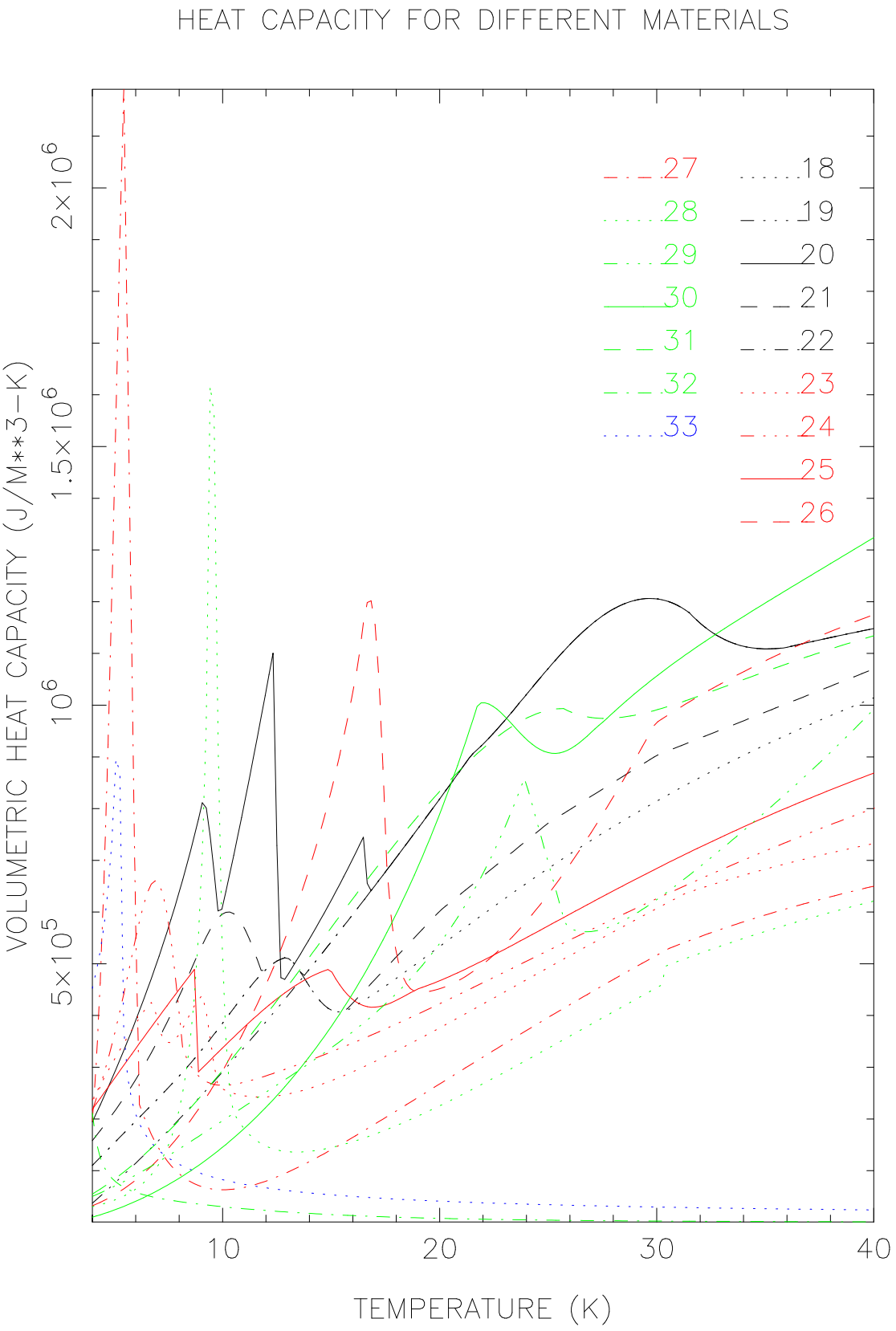

Figure 2: Heat capacity of various materials.

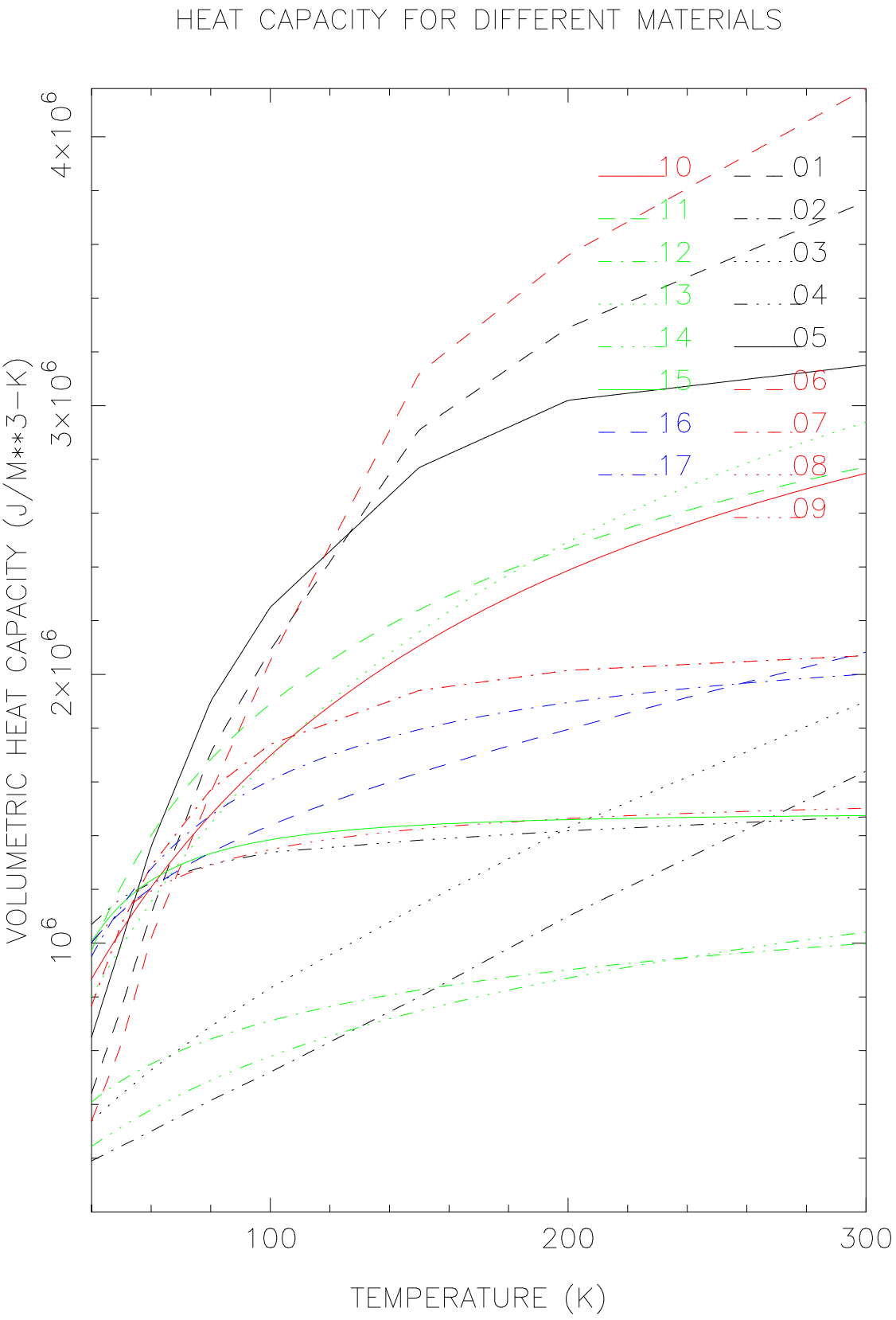

Figure 3: Heat capacity of various materials.

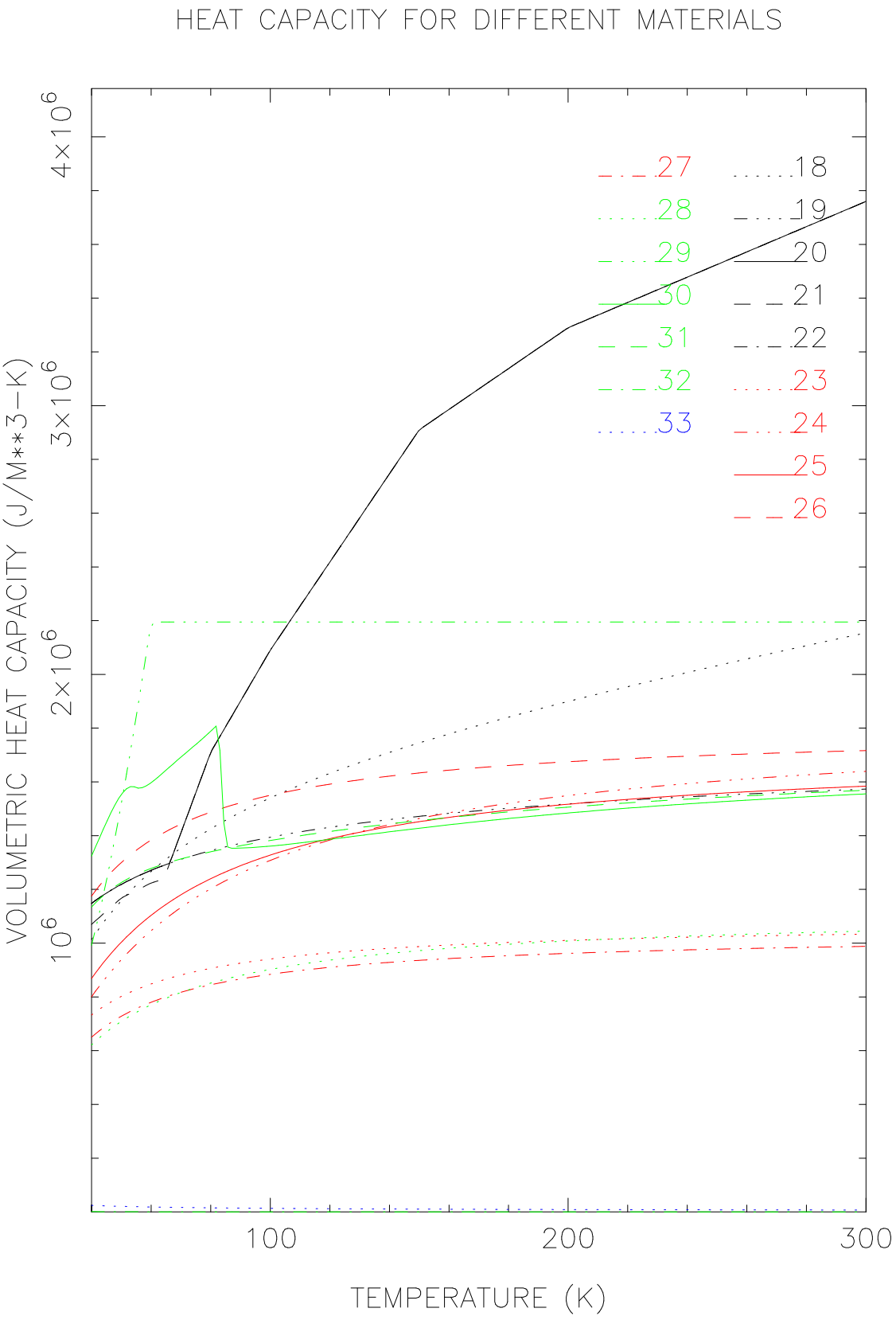

Figure 4: Heat capacity of various materials.

10 Appendix A: notation.

The variables are the following (SI units are used throughout):
A - The cross-sectional area of the regenerator (m^2).
$c(p, T)$ - sound speed (m/s)
$c_m(T)$ - heat capacity per unit volume of the matrix $(J/(m^3 \cdot K))$
$c_v(p, T)$ - specific heat of the gas $(J/(kg \cdot K))$
D_h - hydraulic diameter for the matrix (m)
D - Matrix thermal content (see 7).
e - ρu energy (per unit volume) of the gas (J/m^3)
E - $e + \rho v^2$ total energy (per unit volume) of the gas (J/m^3)
$f(p, T, v)$ - friction factor for flow in the regenerator (Pa/m)
$G_r(p, T)$ - Gruneisen parameter
$h(p, T)$ - enthalpy $(J/(kg \cdot K)$
$H(p, T, v)$ - heat transfer rate between the gas and matrix $(W/(m^2 \cdot K))$
L - length of the regenerator (m)
$k_g(p, T)$ - thermal conductivity of the gas $(W/(m \cdot K))$
$k_m(T)$ - thermal conductivity of the matrix $(W/(m \cdot K))$
$p(t)$ - pressure in the gas (Pa)
$q(x, t)$ - heat transfer rate per unit gas volume from matrix to gas (W/m^3)
$q = 4H(p, T, v)(T_m - T)/D_h$
R - Reynolds Number $D_h \rho |v|/\mu$
t - time (s)
$T(x, t)$ temperature of the gas (K)
T_c - inflow temperature of gas at cold (right) side (K)
T_h - inflow temperature of gas at warm (left) side (K)
$T_m(x, t)$ - temperature of the matrix (K)
Δt - time step (s)
$u(p, T)$ - internal energy of the gas (J/kg).
$v(x, t)$ - velocity of the gas (m/s)
x - spatial coordinate, $0 \leq x \leq L$ (m)
Δx - mesh increment (m)
$\rho(p, T)$ - density of the gas (kg/m^3)
ϕ - porosity of the matrix
ψ - volumetric velocity, $\psi = \phi Av$ (m^3/s)
μ - viscosity $(Pa \cdot s)$
σ - Prandtl number
γ - ratio of specific heats, c_p/c_v

τ - period of the flow=1/HERZ (s)

11 Appendix B: input sample.

```
&inp
  nrun=4002
  cooling_mult=0.5d0      final_cycle=10
  geometry = 4            gas_temp_cold=80.
  gas_temp_hot = 330.     herz = 13.
  hydra_diam = 4.14e-5    use_ideal_gas = 0          mass_flux_cold = 4.32e-3
  mass_flux_hot =6.76e-3  mass_phase = -28.8         material = 1
  mat_cond_factor = 0.3
  porosity = 0.62         pres_initial = 1.91e6
  rg_area = 7.92e-4       rg_length = 0.11
  find_pratio = 1
  ave_pres=2.03e6         pres_phase=25.4            pres_ratio=1.242
/
```

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