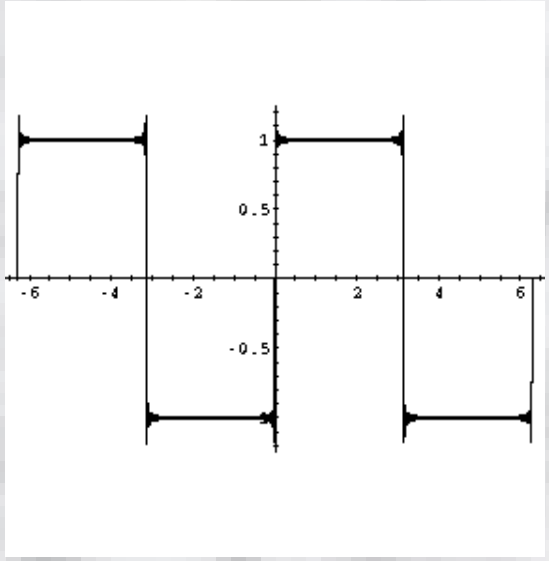
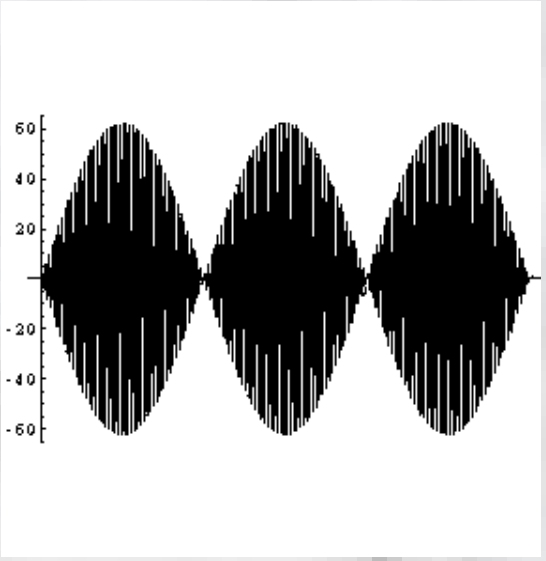
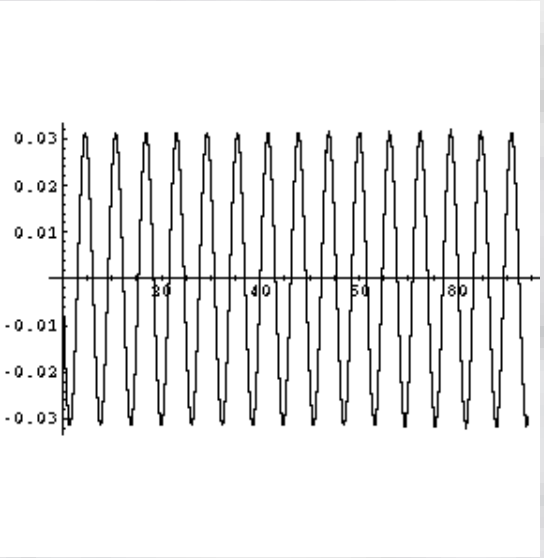
The Gibbs Phenomenon in solutions to the Forced Harmonic Oscillator





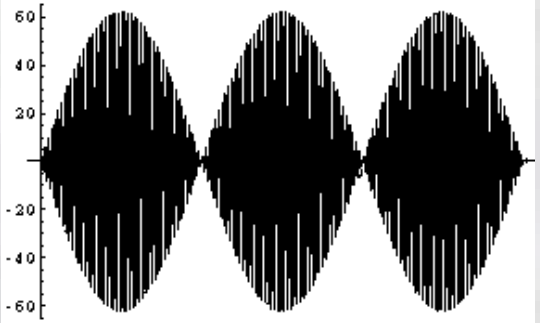


Initial value probem:

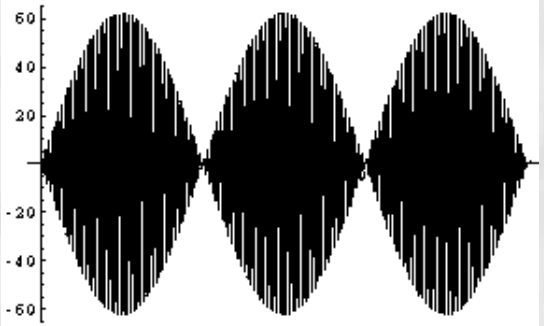
$$y'' + L^2 y = f(t)$$

$$y(0) = y'(0) = 0$$

- 1. Laplace Transform



- 2. Fourier Series



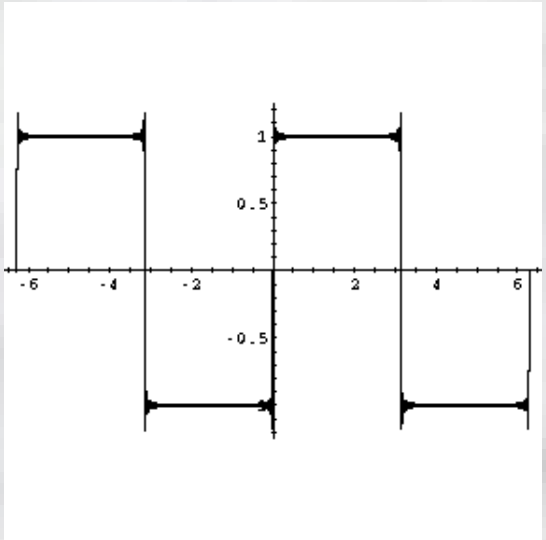
The Forcing Function and approximations

- u(t) = 0 for t <= 0
- 1 for t > 0
-

- h(t) = -1 -Pi < t < 0
- 0 t = -PI , 0 , Pi

- g(t) = 0 for t <= 0
- 1 for 0 < t < Pi
- 0 for t= Pi
- -1 for Pi < t < 2Pi
- 0 for t >= 2Pi

- f(t,n) = g(t-2nPi) u(t-2nPi)

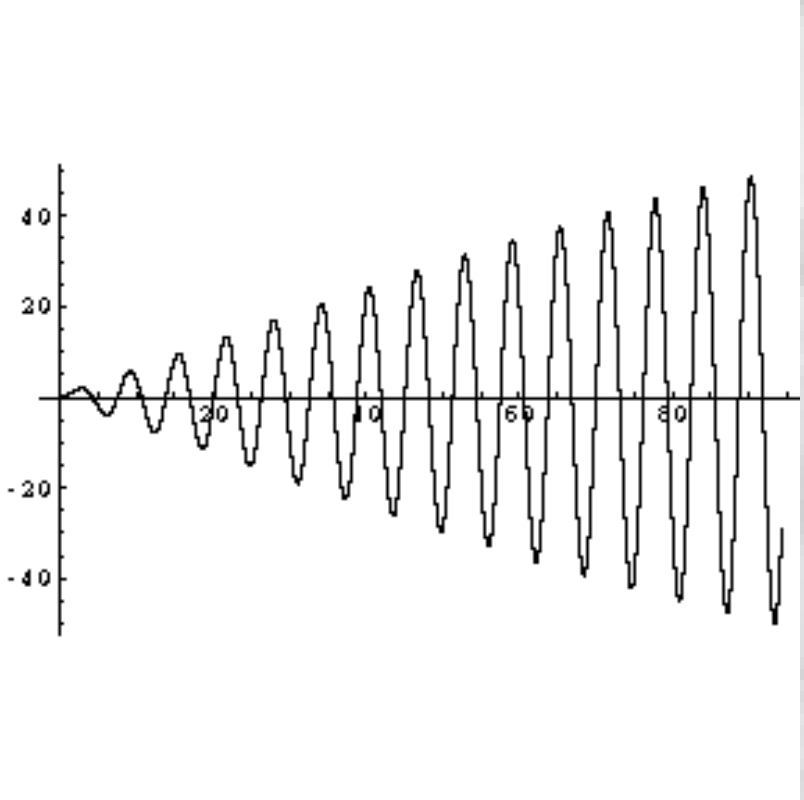


Laplace Transform

- S^2 Y(s) + L^2 Y(s) = $\mathcal{L}$ (g(t))

- Y(s) = 1/(s^2 + L^2) $\mathcal{L}$ (g(t))
- = 1/(L^2) (1/s - s/(s^2 + L^2))
- * (1 - 2e^(-Pi*s) + e^(- 2Pi*s))

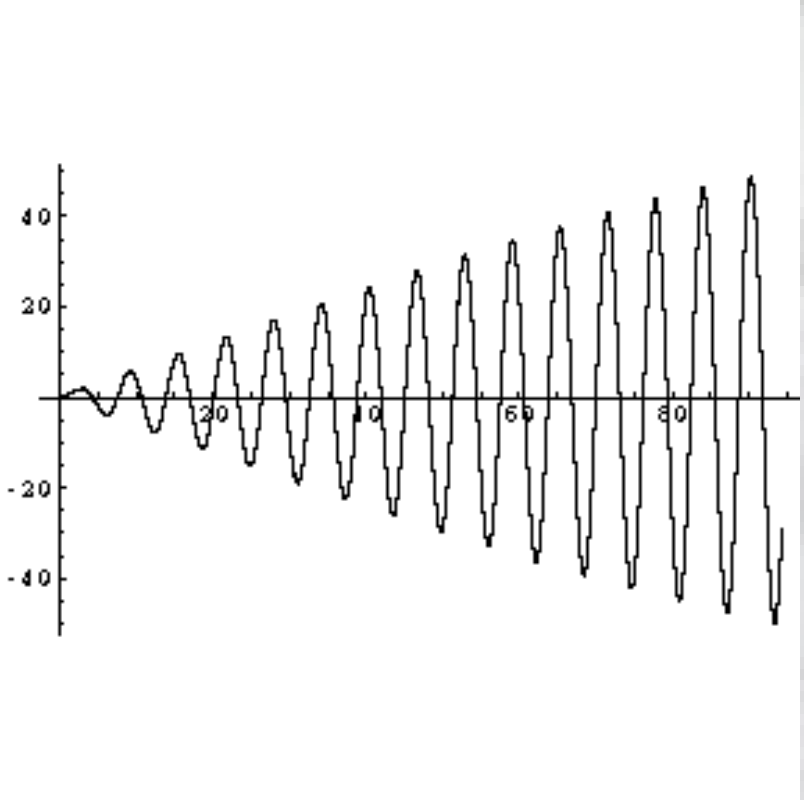
- y(t,n) = 1/L^2 [1- Cos(L(t -2Pi))u(t-2nPi)]
- - 2/L^2 [1- Cos(L(t-(2n+1)Pi)) u(t -(2n+1)Pi)]
- + 1/L^2 [1- Cos(L(t -2(n+1)Pi))u(t-2(n+1)Pi)]



Fourier Series

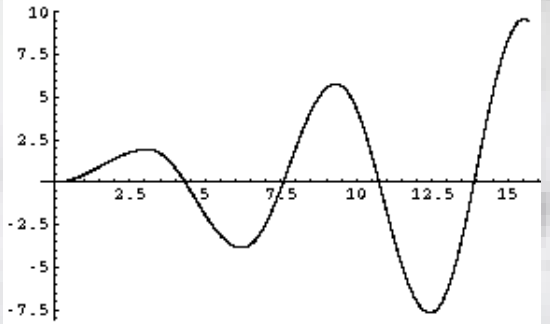
- y_r ”+L^2 y_r = 4/(Pi *(2r-1)) * Sin((2$_r$ -1)t)

- y(t,n)= 4/(Pi *L)*Sum[(L Sin((2$_n$ -1)t) - (2$_n$ -1)Sin(Lt))
- (L^2 - (2$_n$ -1)^2)(2$_n$ - 1)

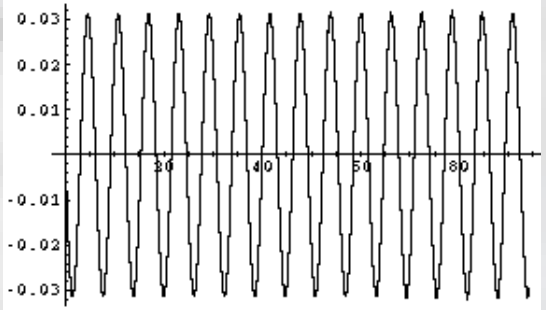


How close is the approximation ?

- a: Laplace Transform solution,
- and Fourier Series solution

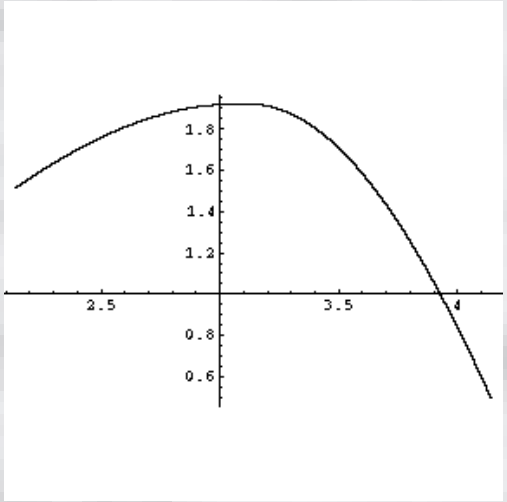


- b: Fourier Series solution minus
- the Laplace Trasform solution

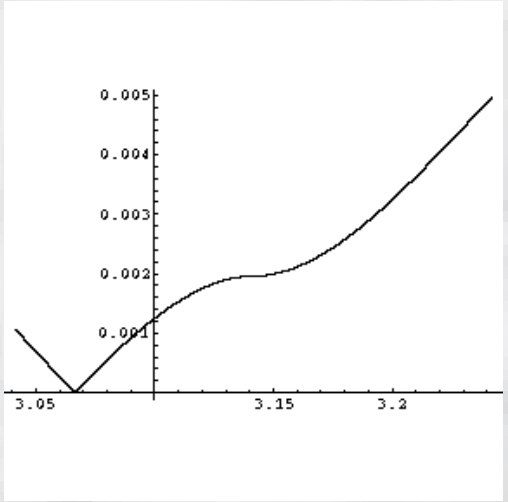


What is the behavior at the jumps ?

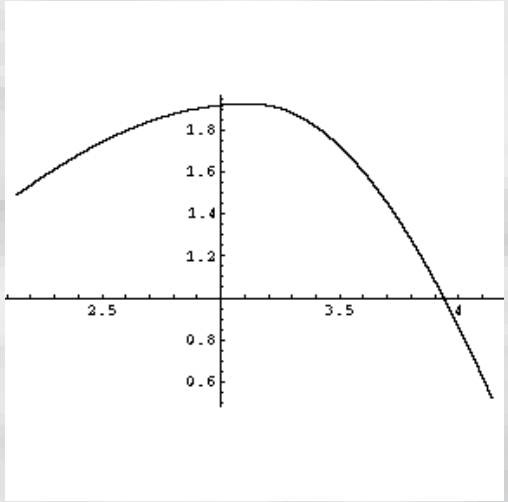
- a



b



c



- a: Laplace Trasform solution
- b: Absolute value of the Fourier Series solution - the Laplace Transform solution
- c: Fourier Series solution