Clustering Algorithms for Streaming and Online Settings

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Big Data Challenges for ML

We face an explosion in data!

Internet transactions
DNA sequencing
Satellite imagery
Environmental sensors
…

Real-world data can be:

Vast
High-dimensional
Noisy, raw
Sparse
Streaming, time-varying
Sensitive/private

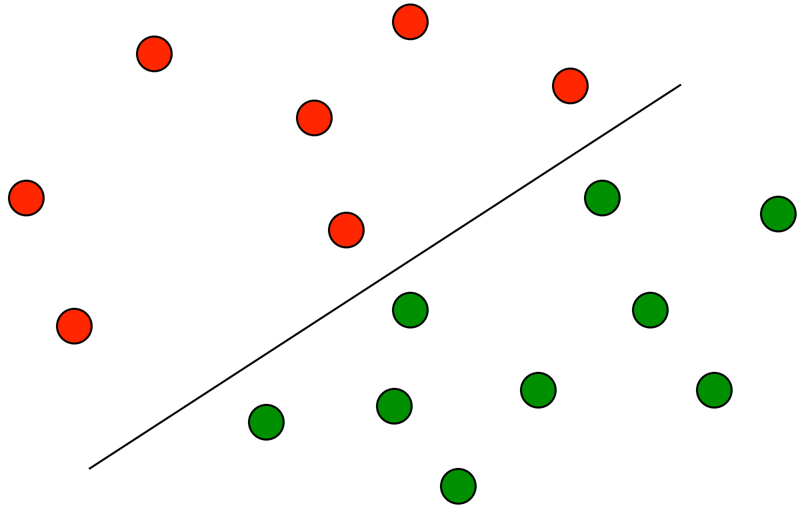
Machine Learning

Given labeled data points, find a good classification rule.

Describes the data

Generalizes well

E.g. linear classifiers:



Machine Learning algorithms for real data sources

Goal: design algorithms to detect patterns in real data sources.

Want efficient algorithms, with performance guarantees.

- Data streams
- Raw (unlabeled or partially-labeled) data
  - Active learning
  - Clustering
- Sensitive/private data
  - Privacy-preserving machine learning
- New applications of Machine Learning
  - Climate Informatics

Machine Learning algorithms for real data sources

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  - Clustering

Scaling-up unsupervised learning to the velocity and volume of big data.

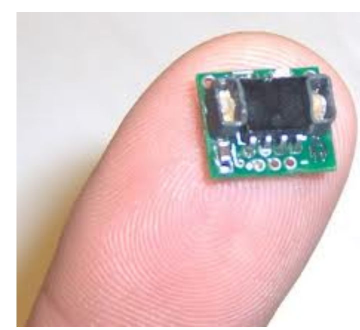
Data stream motivations

Data velocity: data arrives in a stream over time.

e.g. forecasting, real-time decision making, streaming data applications.

Data volume: data is large compared to memory or computation resources.



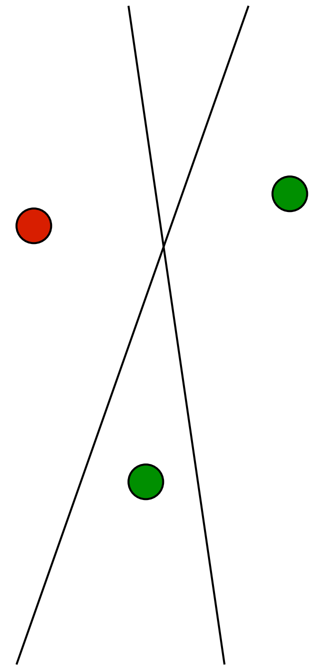


e.g. resource-constrained learning.

Learning from data streams

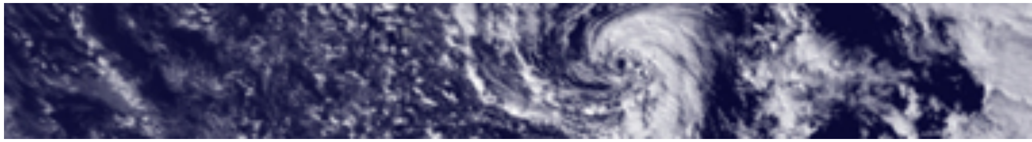
Data arrives in a stream over time.

E.g. linear classifiers:

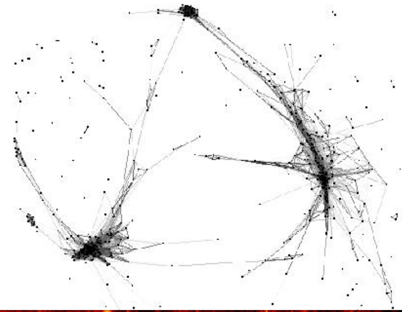


Clustering data streams: Motivations

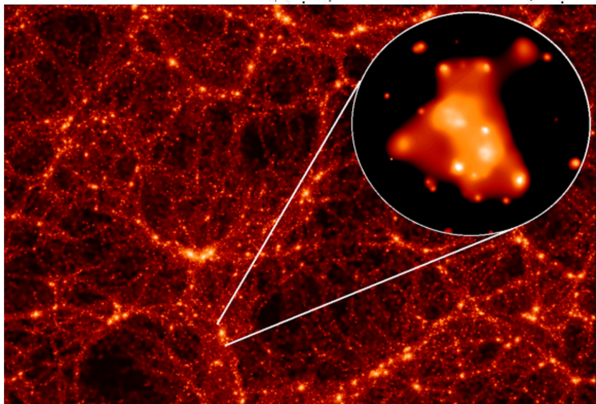
- Multimedia:
  - Aggregating and detecting topics in streaming media
    - e.g. clustering video, music, news stories



- Climate / weather:
  - Grouping / detecting spatiotemporal patterns
    - e.g. droughts, storms



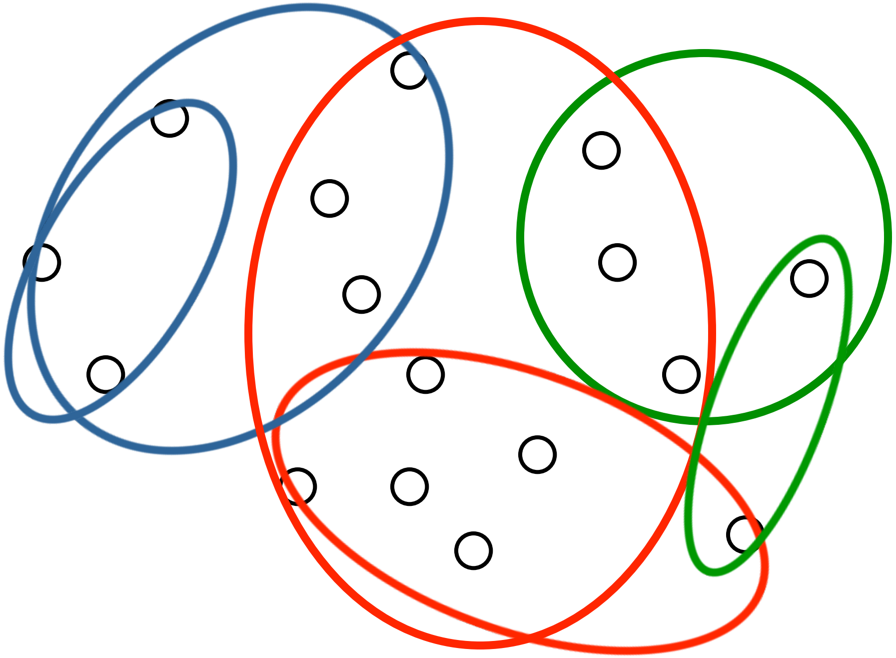
- Exploratory data analysis:
  - e.g. Neuroscience:
    - online spike classification
    - pattern detection in networks of neurons
    - network monitoring
  - Astronomy



Clustering

What can be done without any labels?

Unsupervised learning, Clustering.



How to evaluate a clustering algorithm?

k-means clustering objective

Clustering algorithms can be hard to evaluate without prior information or assumptions on the data.

With *no* assumptions on the data, one evaluation technique is w.r.t. some objective function.

A widely-cited and studied objective is the k-means clustering objective: Given set, $X \subset R^d$, choose $C \subset R^d$, $|C| = k$, to minimize:

$$\phi_C = \sum_{x \in X} \min_{c \in C} \|x - c\|^2$$

k-means approximation

Optimizing k-means is NP-hard, even for k=2.
[Dasgupta '08; Deshpande & Popat '08].

Very few algorithms approximate the k-means objective.

Definition: b-approximation: $\phi_C \leq b \cdot \phi_{OPT}$

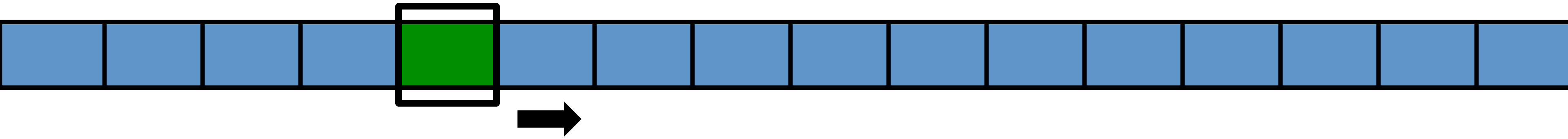
Definition: Bi-criteria (a,b)-approximation guarantee: a·k centers,

b-approximation.

Even "the k-means algorithm" [Lloyd 1957] does not have an approximation guarantee. Can suffer from bad initialization.

Goal: approximate the k-means clustering objective with streaming or online clustering algorithms [Open problems, Dasgupta '08]

Learning from data streams



“Streaming” model:

- Stream of of known length n.
- Memory available is o(n)
- Tested only at the end
- A (small) constant number of passes allowed

“Online” model:

- Endless stream of data
- Fixed amount of memory
- Tested at every time step
- Each point in stream is seen only once

Outline

Streaming clustering

[Ailon, Jaiswal & M, NIPS 2009]

Online clustering

[Choromanska & M, AISTATS 2012]

Streaming k-means approximation

[Ailon, Jaiswal & M, NIPS 2009]:

Goal: approximate the k-means objective with a one-pass streaming clustering algorithm

Related work:

[Arthur & Vassilvitskii, SODA '07]: *k*-means++, a batch clustering algorithm with O(log *k*)-approx. of *k*-means.

[Guha, Meyerson, Mishra, Motwani, & O'Callaghan, TKDE '03]: Divide and conquer streaming (a,b)-approximate *k*-medoid clustering.

Contributions to streaming clustering

Extend *k*-means++ to *k*-means#, an (O(log *k*), O(1))-approximation to k-means, in batch setting.

Analyze Guha *et al.* divide and conquer algorithm, using (a,b)-approximate *k*-means clustering.

Use Guha *et al.* with k-means# and then k-means++ to yield a one-pass O(log k)-approximation algorithm to *k*-means objective.

Analyze multi-level hierarchy version for improved memory vs. approximation tradeoff.

Experiments on real and simulated data.

k-means++

Algorithm:

```
Choose first center c₁ uniformly at random from X,
and let C = {c₁}.
Repeat (k-1) times:
   Choose next center cᵢ = x' ∈ X with prob. D(x',C)² / Σ_{x∈X} D(x,C)²
   C ← C ∪ {cᵢ}
```

Choose first center c_1 uniformly at random from X, and let $C = \{c_1\}$.

Repeat $(k-1)$ times:

- Choose next center $c_i = x' \in X$ with prob. $\frac{D(x', C)^2}{\sum_{x \in \mathcal{X}} D(x, C)^2}$
- $C \leftarrow C \cup \{c_i\}$

where $D(x, C) = \min_{c \in C} \|x - c\|$

Theorem (Arthur & Vassilvitskii '07): Returns an O(log *k*)-approximation, in expectation.

k-means#

Idea: *k*-means++ returns *k* centers, with O(log *k*)-approximation. Can we design a variant that returns O(*k* log *k*) centers, but constant approximation?

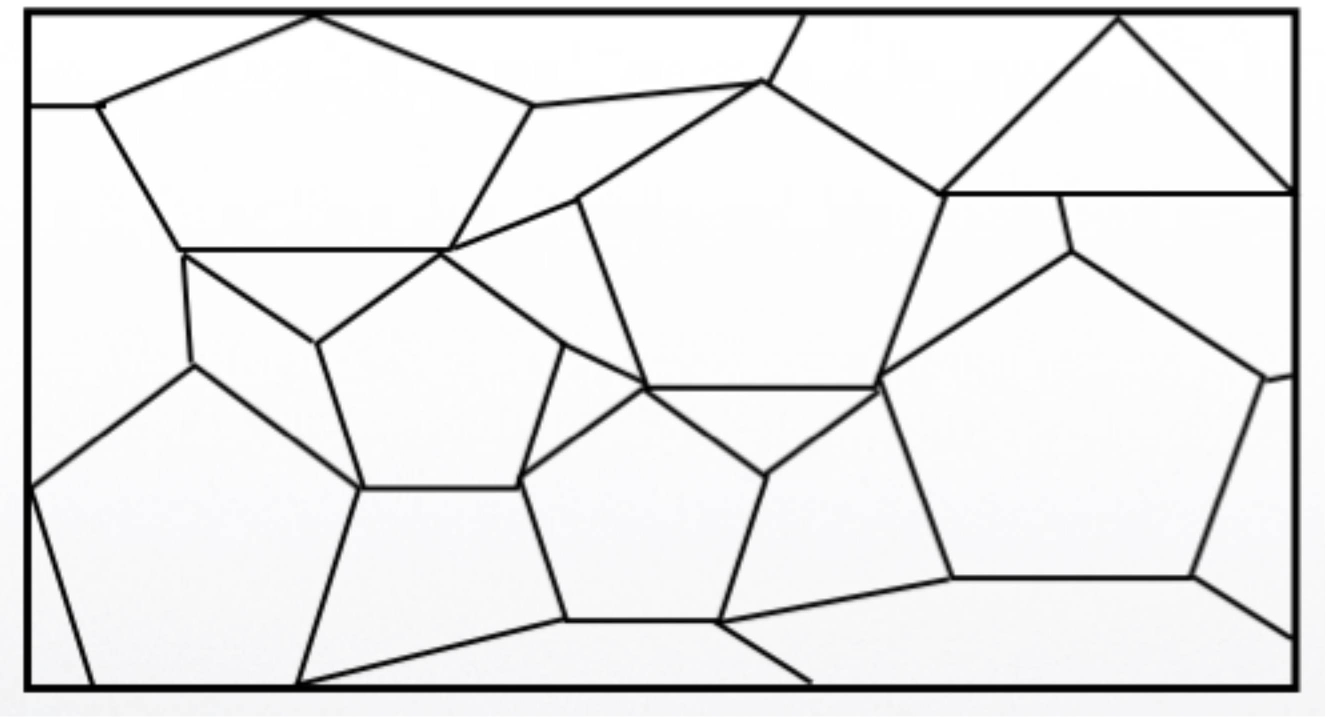
Algorithm:

```
Initialize C={}.
Choose 3·log(k) centers independently and uniformly
at random from X, and add them to C.
Repeat (k-1) times:
   Choose 3·log(k) centers indep. with prob. D(x',C)^2 / Σ_{x∈X} D(x,C)^2
   and add them to C.
```

$$\frac{D(x',C)^2}{\sum_{x\in\mathcal{X}} D(x,C)^2}$$

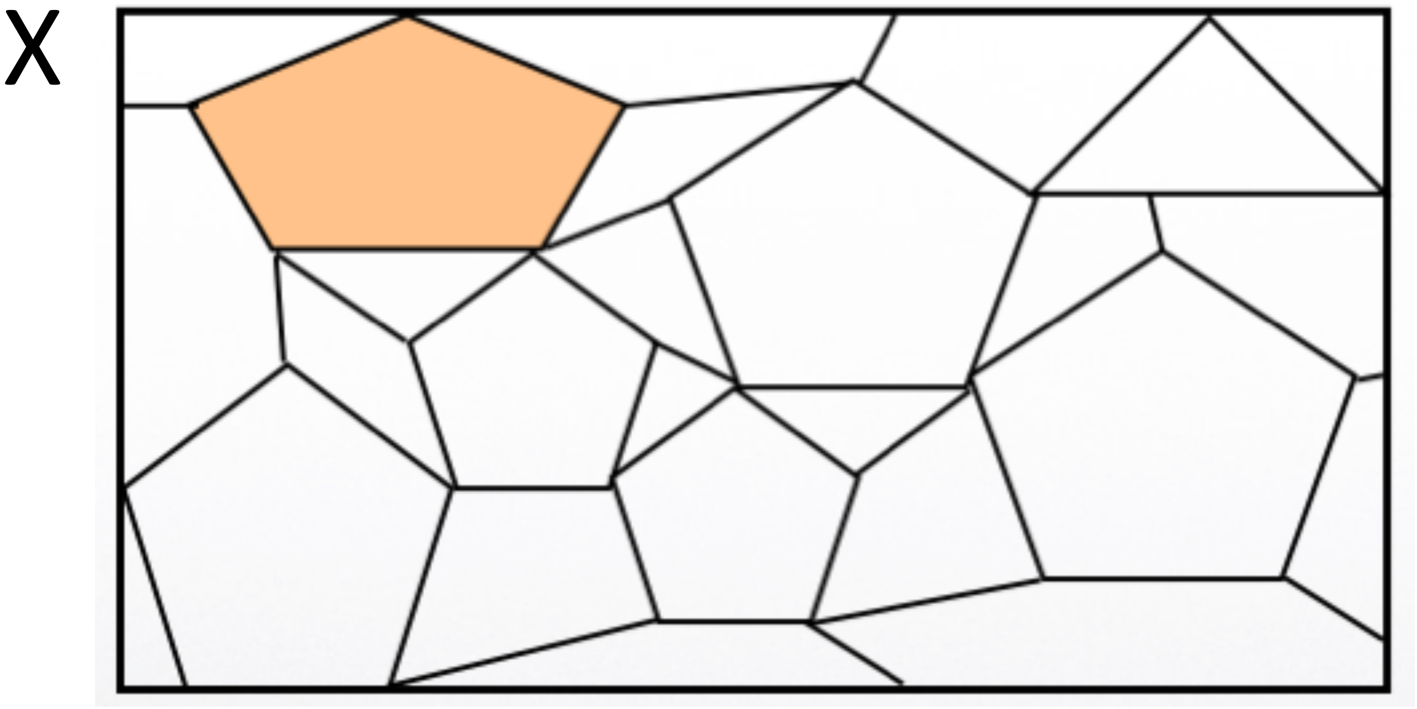
k-means# proof idea

X



The clustering (partition) induced by OPT.

k-means# proof idea



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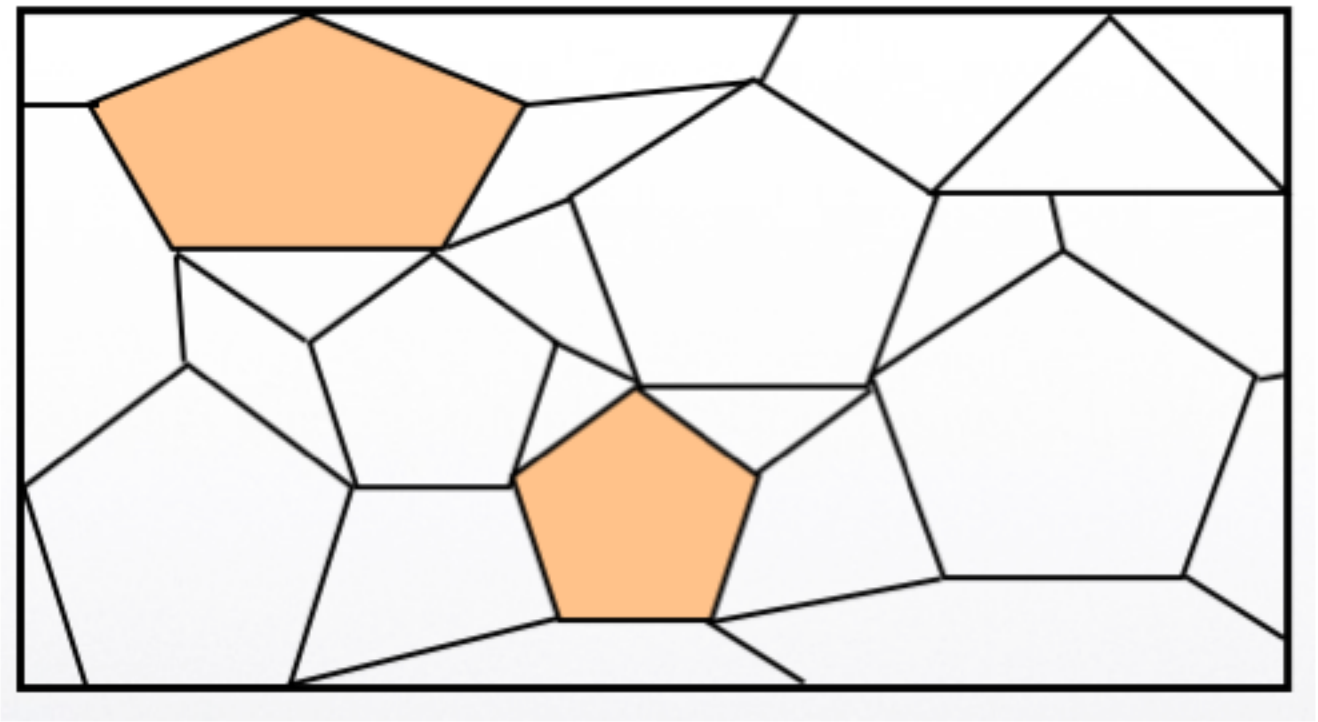

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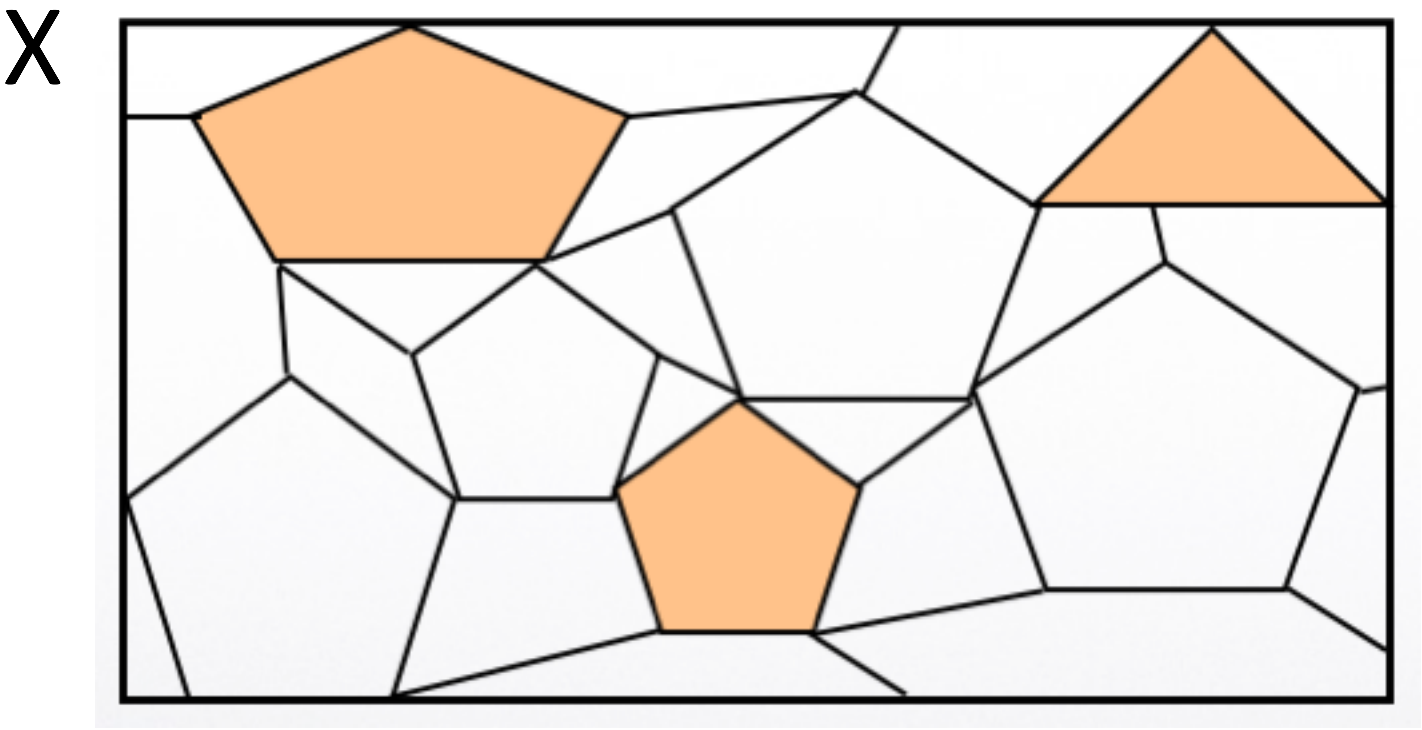
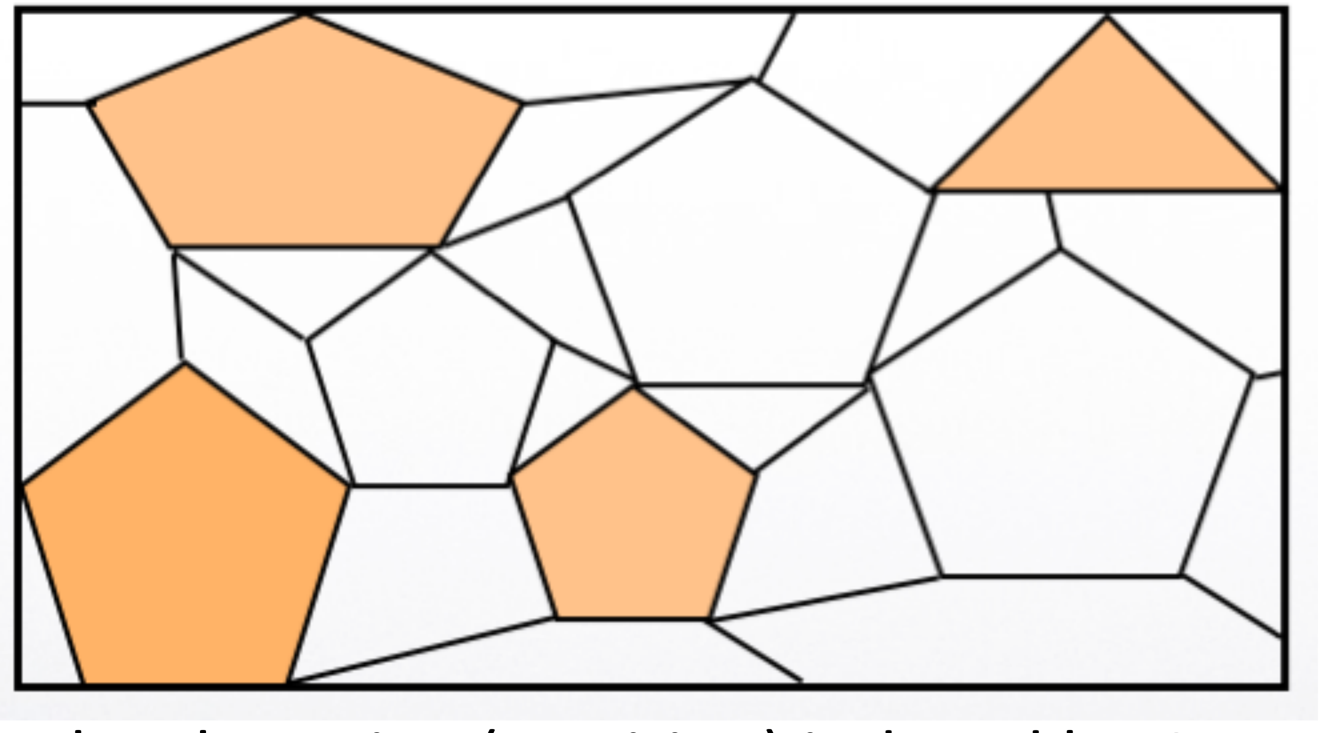
k-means# proof idea



The clustering (partition) induced by OPT.

k-means# proof idea

X



The clustering (partition) induced by OPT.

→ We cover the *k* clusters in OPT, after choosing *O(k log k)* centers.

k-means#

Theorem: With probability at least 1/4, *k*-means# yields an O(1)-approximation, on O(*k* log *k*) centers.

Proof outline: Definition "covered": cluster A ∈ OPT is covered if:

$\phi_C(A) < 32 \cdot \phi_{OPT}(A)$, where $\phi_C(A) = \sum_{x \in A} D(x, C)^2$.

Define $\{X_c, X_u\}$: the partition of X into covered, uncovered.

- In first round we cover one cluster in OPT.
- In any later round, either:

Case 1: $\phi_C(X_c) > \phi_C(X_u)$: We are done. (Reached 64-approx.)

Case 2 : $\phi_C(X_c) \leq \phi_C(X_u)$: We are likely to hit and cover another uncovered cluster in OPT.

We show *k*-means# is a (3 · log(*k*), 64)-approximation to k-means.

k-means# proof: First round

Fix any point x chosen in the first step. Define A as the unique cluster in OPT, s.t. x $\in$ A.

Lemma (AV '07): Fix A $\in$ OPT, and let C be the 1-clustering with the center chosen uniformly at random from A. Then $E[\phi_C(A)] = 2 \cdot \phi_{OPT}(A)$.

Corollary: $Pr[\phi_C(A) < 8 \cdot \phi_{OPT}(A)] \geq 3/4$. Pf. Apply Markov's inequality.

After 3 · log(*k*) random points, probability of hitting a cluster A with a point that is good for A is at least $1 - (1/4)^{3 \log k} \geq 1 - 1/k$.

So after first step, w.p. at least (1-1/k), at least 1 cluster is covered.

k-means# proof: Case 1

Case 1: $\phi_C(X_c) > \phi_C(X_u)$:

Since X= $X_c \cup X_u$ and by definition of ϕ,

$$\phi_C(X) = \phi_C(X_c) + \phi_C(X_u) \leq 2 \cdot \phi_C(X_c) \leq 64 \cdot \phi_{OPT}(X_c) \leq 64 \cdot \phi_{OPT}(X)$$

by definition of Case 1, and definition of covered.

Last inequality is by $X_c \subseteq X$, and definition of ϕ (each term in sum is nonnegative).

k-means# proof: Case 2

Case 2: $\phi_C(X_c) \leq \phi_C(X_u)$:

The probability of picking a point in X_u at the next round is:

$$\frac{\sum_{x \in X_u} D(x,C)^2}{\sum_{x \in X} D(x,C)^2} = \frac{\phi_C(X_u)}{\phi_C(X_u) + \phi_C(X_c)} \geq \frac{1}{2}$$

Lemma (AV '07): Fix A ∈ OPT, and let C be any clustering. If we add a center to C, sampled randomly from the D^2 weighting over A, yielding C' then: $E[\phi_{C'}(A)] \leq 8 \cdot \phi_{OPT}(A)$.

Corollary: $Pr[\phi_{C'}(A) < 32 \cdot \phi_{OPT}(A)] \geq 3/4$. By Markov's inequality.

So, w.p. $\geq \frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$ we pick a point in X_u that covers a new cluster in OPT.

After $3 \cdot \log(k)$ picks, prob. of covering a new cluster is at least (1-1/k).

k-means# proof summary

For the first round, prob. of covering a cluster in OPT is at least (1-1/k).

For the k-1 remaining rounds, either Case 1 holds, and we have achieved a 64-approximation, or Case 2 holds, and the probability of covering a new cluster in OPT, in the next round, is at least (1-1/k).

So the probability that after k rounds there exists an uncovered cluster in OPT is $\leq 1-(1-1/k)^k \leq 3/4$.

Thus the algorithm achieves a 64-approximation on $3k \cdot \log(k)$ centers, with probability at least 1/4.

k-means#

Theorem: With probability at least 1/4, k-means# yields an O(1)-approximation, on O(k log k) centers.

Corollary: With probability at least 1-1/n, running k-means# for $3 \cdot \log n$ independent runs yields an O(1)-approximation (on O(k log k) centers).

Proof: Call it repeatedly, $3 \cdot \log n$ times, independently, and choose the clustering that yields the minimum cost. Corollary follows, since

$$\left(1-(3/4)^{3\log n}\right) \geq \left(1-\frac{1}{n}\right) .$$

Divide and conquer clustering

S - stream

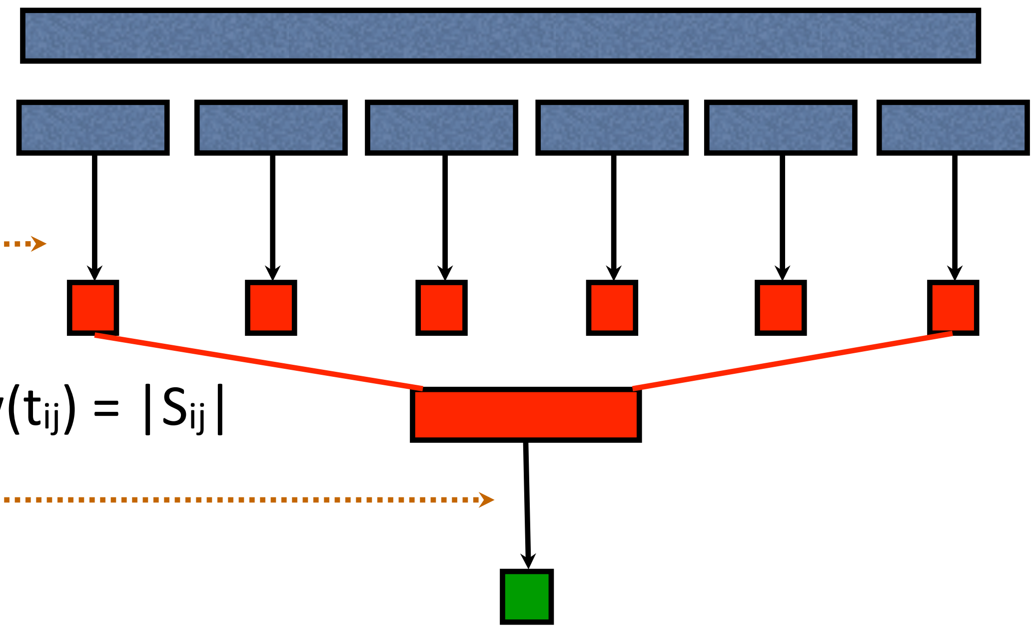
$\{S_i\}$ – partition

(a,b)-clustering

$\{T_i\}$ - sets of 'centers'

S_w - $\cup_i T_i$ with weights $w(t_{ij}) = |S_{ij}|$

(a',b')-clustering



T – final "centers"

[Guha *et al.* '03] analyzed this template for *k*-medoid clustering:

(a', O(bb'))-approximation.

One-pass *k*-means approximation

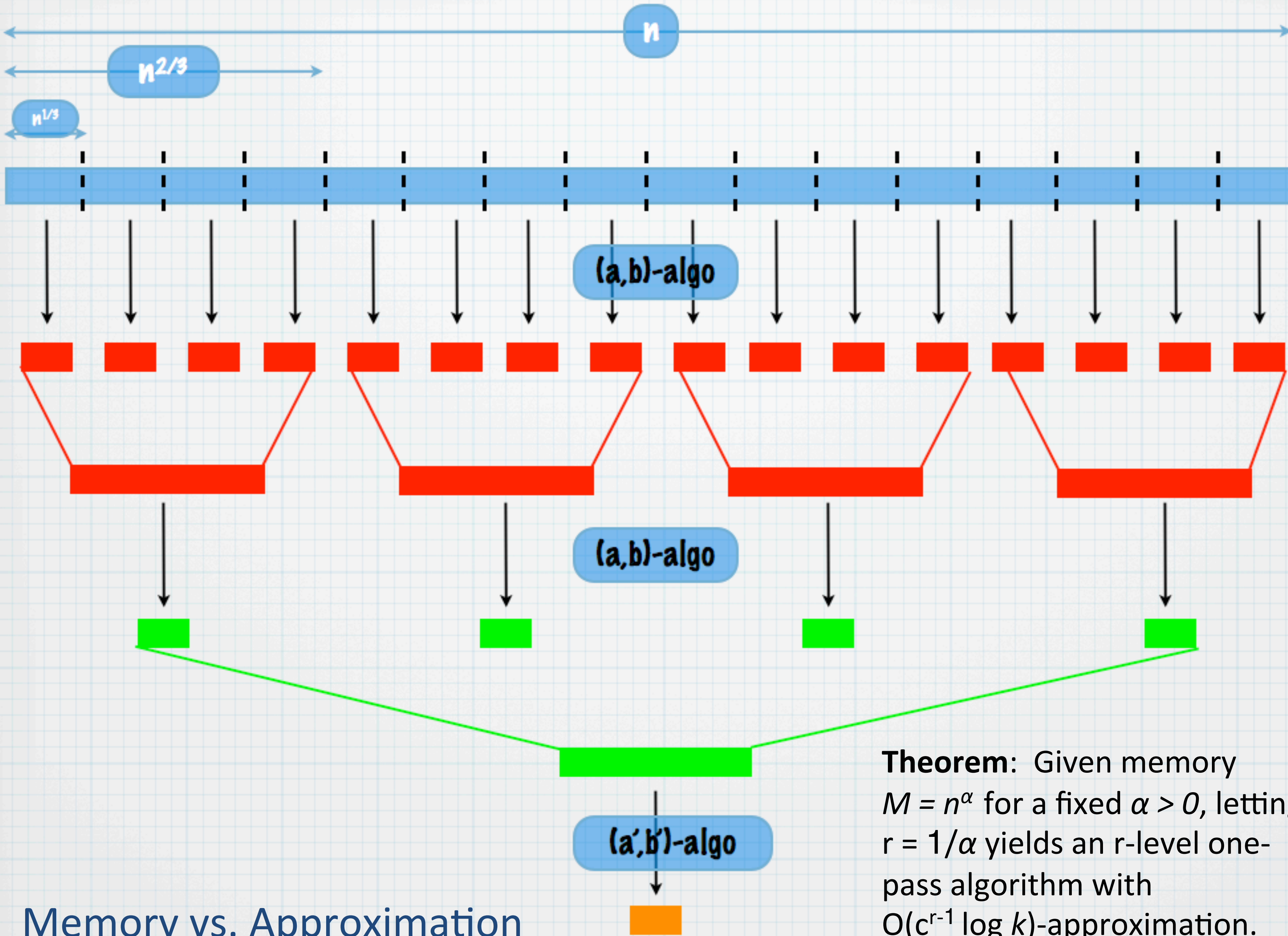
We analyze the Guha *et al.* scheme for (a,b)-approximation algorithms w.r.t. *k*-means: yields a one-pass (a', O(bb'))-approximation algorithm.

Our algorithm:

For the (a,b) algorithm, use (repeated) k-means#: a = O(log *k*), b = O(1).

For the (a',b') algorithm, use k-means++: a' = 1, b' = O(log k)

So the combined algorithm is a (1, O(log k))-approximation to *k*-means.



Memory vs. Approximation

Theorem: Given memory $M = n^{\alpha}$ for a fixed $\alpha > 0$, letting $r = 1/\alpha$ yields an r-level one-pass algorithm with $O(c^{r-1} \log k)$-approximation.

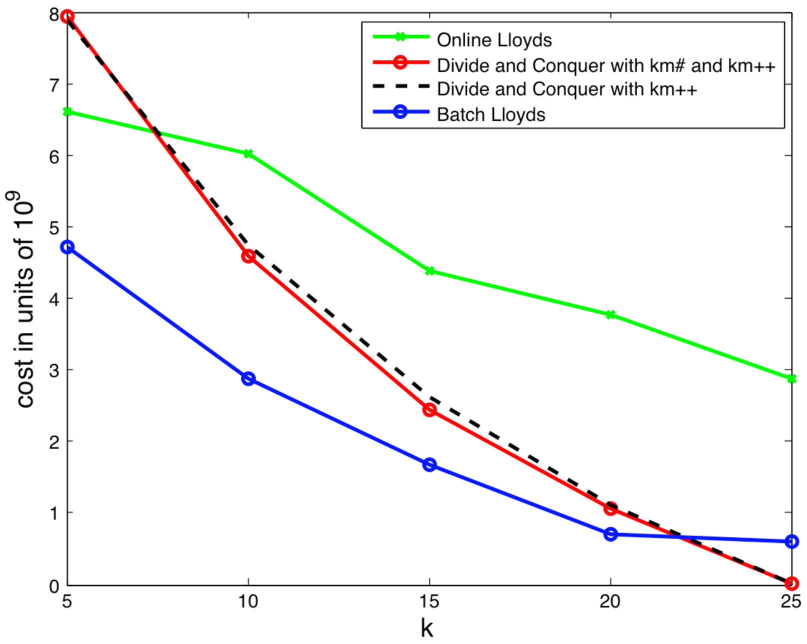
Experiments

k	BL	OL	DC-1	DC-2	BL	OL	DC-1	DC-2
5	$4.7254 \cdot 10^9$	$6.5967 \cdot 10^9$	$7.9336 \cdot 109$	$7.8752 \cdot 109$	5.80	1.44	16.95	12.22
10	$2.8738 \cdot 10^9$	$6.0146 \cdot 10^9$	$4.5968 \cdot 10^9$	$4.7288 \cdot 10^9$	7.33	2.76	53.10	24.74
15	$1.6753 \cdot 10^9$	$4.3743 \cdot 10^9$	$2.4338 \cdot 10^9$	$2.6280 \cdot 10^9$	8.85	4.00	112.68	36.86
20	$7.0016 \cdot 10^8$	$3.7794 \cdot 10^9$	$1.0661 \cdot 10^9$	$1.1017 \cdot 10^9$	11.75	6.04	250.21	48.57
25	$6.0011 \cdot 10^8$	$2.8859 \cdot 10^9$	$2.7493 \cdot 10^5$	$2.7906 \cdot 10^5$	13.83	7.00	403.81	60.96

Table 1: norm25 dataset. (columns 2-5 has the clustering cost and columns 6-9 has time in sec.)

Mixture of 25 Gaussians:

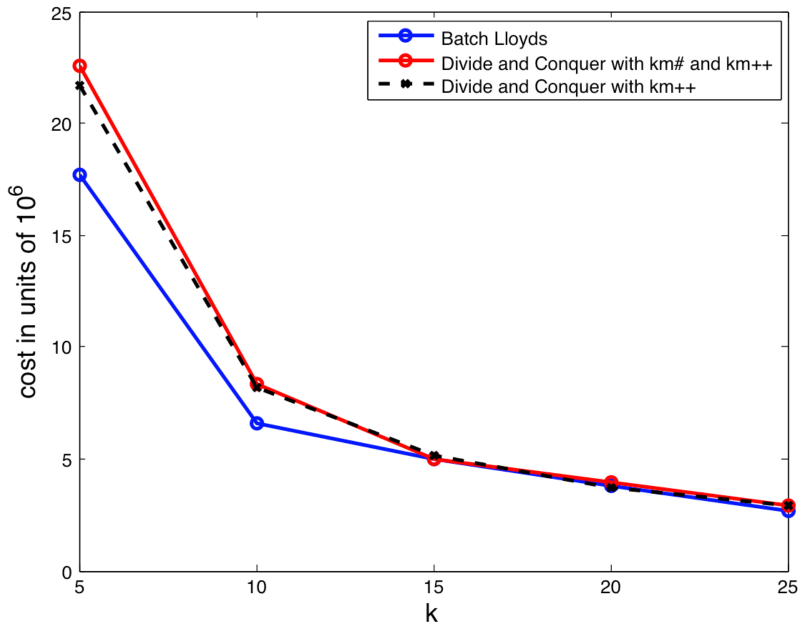
10K points sampled from a mixture of 25 Gaussians chosen at random from 15 dimensional hypercube (side 500).

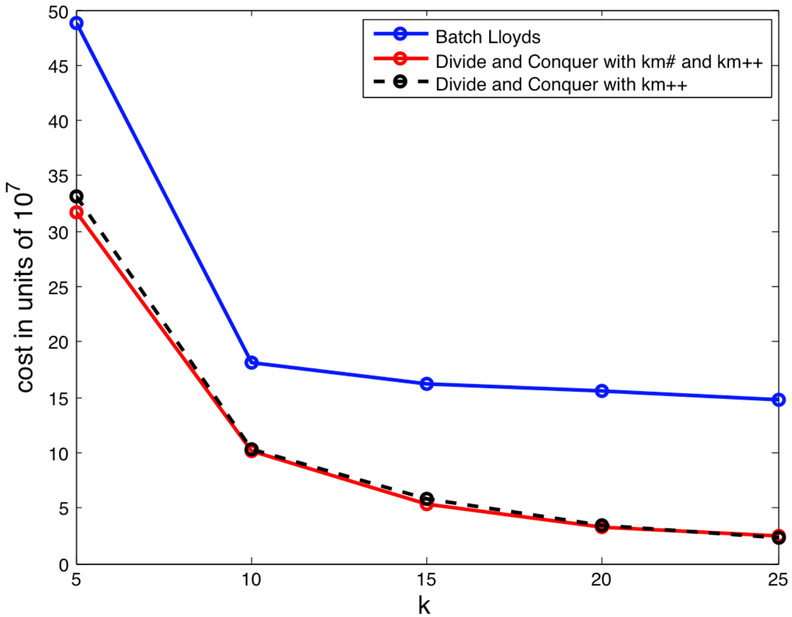


Experiments

k	BL	OL	DC-1	DC-2	BL	OL	DC-1	DC-2
5	1.7713 · 107	1.2401 · 108	2.2582 · 107	2.1683 · 107	1.78	0.15	2.30	1.10
10	$6.5871 \cdot 10^6$	$8.5684 \cdot 10^7$	$8.3452 \cdot 10^6$	$8.2037 \cdot 10^6$	2.27	0.31	7.45	2.40
15	$4.9851 \cdot 10^6$	$8.4633 \cdot 10^7$	$4.9935 \cdot 10^6$	$5.1391 \cdot 10^6$	3.42	0.45	13.34	3.32
20	$3.7836 \cdot 10^6$	$6.5110 \cdot 10^7$	$3.9289 \cdot 10^6$	$3.7279 \cdot 10^6$	3.38	0.59	32.42	5.00
25	$2.6363 \cdot 10^6$	$6.3758 \cdot 10^7$	$2.8899 \cdot 10^6$	$2.9470 \cdot 10^6$	4.54	0.62	46.45	5.89

Table 2: Cloud dataset. (columns 2-5 has the clustering cost and columns 6-9 has time in sec.)





k	BL	OL	DC-1	DC-2	BL	OL	DC-1	DC-2
5	$4.8769 \cdot 10^8$	$1.7001 \cdot 10^9$	$3.1770 \cdot 10^8$	$3.3191 \cdot 10^8$	3.74	0.87	14.60	6.53
10	$1.8169 \cdot 10^8$	$1.6930 \cdot 10^9$	$1.0104 \cdot 10^8$	$1.0271 \cdot 10^8$	5.59	1.66	47.92	12.17
15	$1.6227 \cdot 10^8$	$1.4762 \cdot 10^9$	$5.3517 \cdot 10^7$	$5.7865 \cdot 10^7$	7.04	2.19	86.54	17.53
20	$1.5580 \cdot 10^8$	$1.4766 \cdot 10^9$	$3.2577 \cdot 10^7$	$3.4155 \cdot 10^7$	9.87	2.83	218.95	25.70
25	$1.4704 \cdot 10^8$	$1.4754 \cdot 10^9$	$2.3981 \cdot 10^8$	$2.2735 \cdot 10^8$	13.26	4.41	331.77	40.64

Table 3: Spambase dataset. (columns 2-5 has the clustering cost and columns 6-9 has time in sec.)

UCI data: Clouds and Spambase.

Outline

Streaming clustering

[Ailon, Jaiswal & M, NIPS 2009]

Online clustering

[Choromanska & M, AISTATS 2012]

Open problems posed by Dasgupta

Provide an online algorithm for k-means clustering endless streams in either framework [Dasgupta, Spring '08, Lecture 6]:

1. At time t, algorithm sees data point x_t, and outputs the set of k centers C_t. For some constant $\alpha \geq 1$ and for all t:

$$\texttt{cost}(C_t) \leq \alpha \cdot OPT_t.$$

where $OPT_t = \texttt{cost(best } k \texttt{ centers for } x_1, \ldots, x_t\texttt{)}.$

2. At time t, algorithm announces set of k centers C_t, then sees x_t and incurs loss equal to cost of x_t under C_t: the squared distance from x_t to closest center in C_t. Goal: bound the regret G, between cumulative loss at time T, and OPT for the stream seen so far:

$$L_T(\texttt{alg}) = \sum_{t \leq T} \min_{c \in C_t} \|x_t - c\|^2 \leq OPT_T + G$$

Online clustering with experts

[Choromanska & M, AISTATS 2012]

Goal: approximate the k-means clustering objective with an online clustering algorithm

- A new evaluation framework, extending Dasgupta's
  - Bound variant of 2 w.r.t. performance of a set of experts: clustering algorithms
- A new family of online clustering algorithms
  - Extend algorithms for online learning with experts
- Performance guarantees with no data assumptions
  - Regret bounds
  - Novel form of online clustering approximation guarantees, w.r.t. OPT!
- Encouraging experimental performance

Contributions to online clustering

- Extend online learning algorithms from [Herbster & Warmuth '98] and [M & Jaakkola '03] to clustering setting.
  - Instead of using prediction errors to update weights over experts, use a proxy for k-means cost obtained so far.

- Prove (c,η)-realizability of our clustering and loss function.
  - Allows us to extend regret bounds from [HW98] and [MJ03].

- Add assumptions that experts are b-approximation algorithms w.r.t. k-means objective, to extend regret bounds

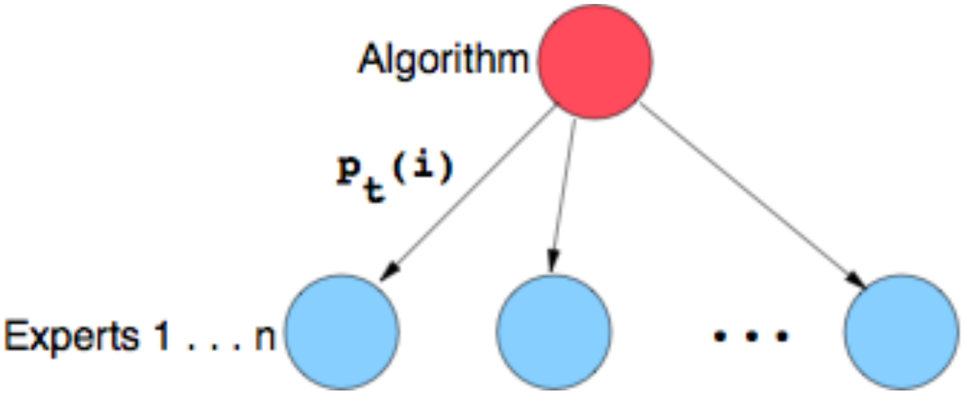
→ Novel online approximation bounds w.r.t. OPT for the entire stream!

Online learning (supervised setting)

- Learning proceeds in stages.
  - Algorithm first predicts a label for the current data point.
  - Loss is then computed: function of predicted and observed label.
  - Learner can update its hypothesis (usually taking into account loss).
- Framework models regression, or classification
  - By varying choice of loss function:
    - Many hypothesis classes
    - Problem need not be separable
- Non-stochastic setting: no statistical assumptions.
  - No assumptions on observation sequence.
  - Observations can even be generated online by an adaptive adversary.
- Analyze regret: difference in cumulative loss from that of the optimal comparator algorithm for the observed sequence (computed in hindsight) .

Online learning with experts

Learner maintains distribution over *n* “experts.”



Experts are black boxes: need not be good algorithms, can vary with time, and depend on one another.

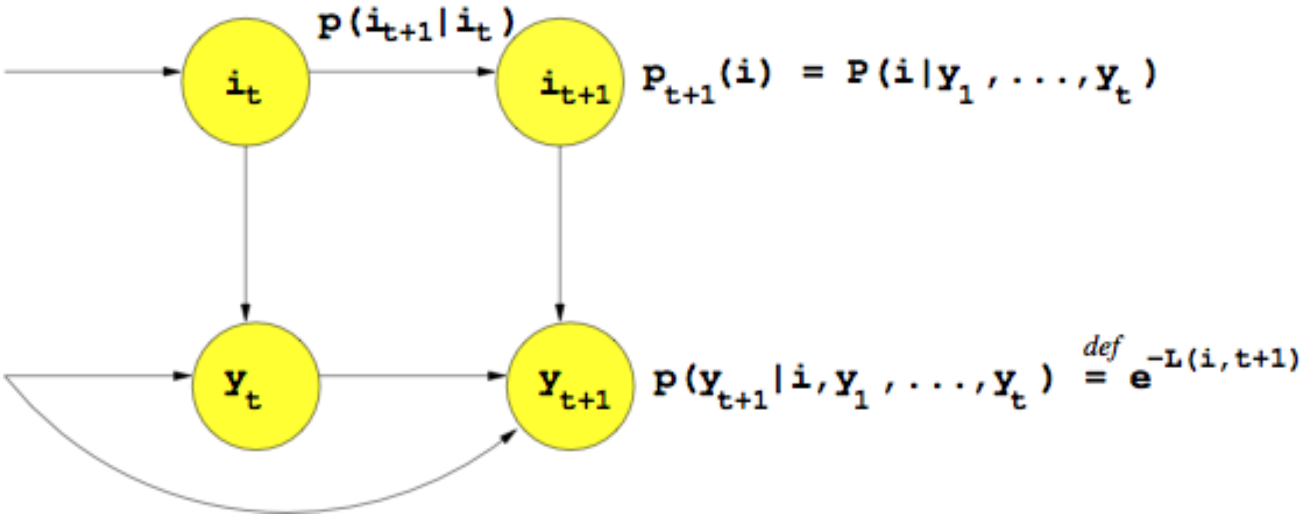
Learner informs prediction using a probability distribution $p_t(i)$ over experts, i, depending on L(i,t), loss of expert i’s output w.r.t. observation – defined per problem.

Different algorithms to update p_t (i) - based on the model of time-varying data.

Shifting algorithms

To handle changing observations, maintain $p_t(i)$ via an HMM.

Hidden state: identity of the current best expert.



[M&Jaakkola'03]: Performing Bayesian updates on this HMM yields existing online learning algorithms.

$$p_{t+1}(i) \propto \sum_j p_t(j) e^{-L(j,t)} p(i|j)$$

Static update, $P(i \mid j) = \delta(i,j)$ gives [Littlestone&Warmuth'89] algorithm: The Weighted Majority Algorithm, a.k.a. Static-Expert.

$$p_{t+1}(i) \propto p_t(i) e^{-L(i,t)}$$

Shifting algorithms

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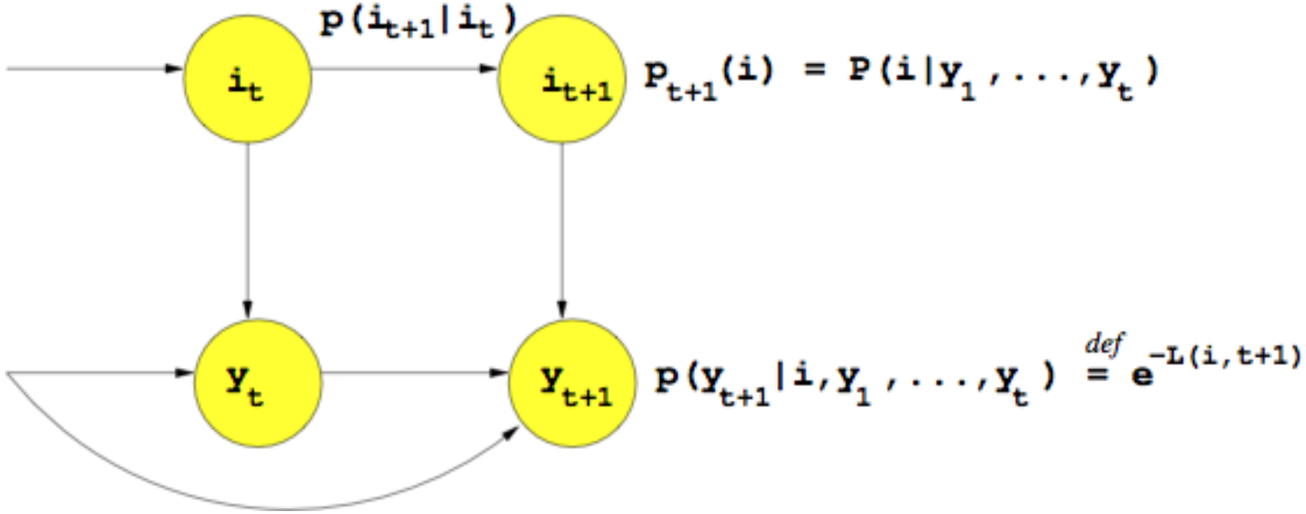


Performing Bayesian updates on this HMM yields existing OL algorithms.

$$p_{t+1}(i) \propto \sum_j p_t(j) e^{-L(j,t)} p(i|j)$$

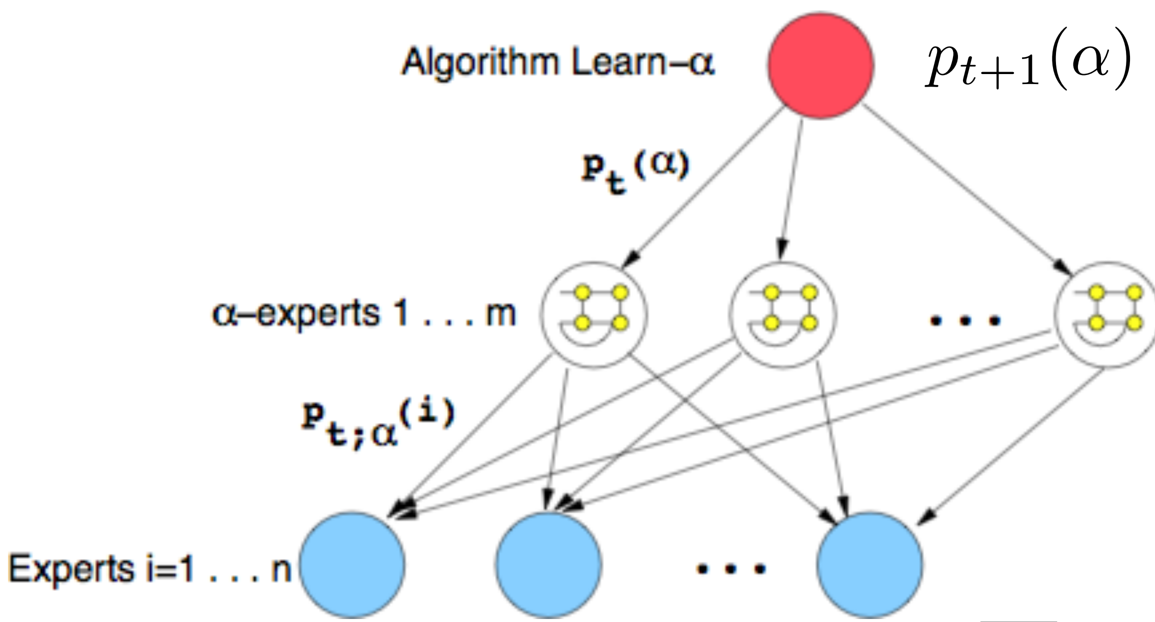
[Herbster&Warmuth'98]
Model shifting concepts via:

$$P(i|j;\alpha) = \begin{cases} (1-\alpha) & i = j \\ \frac{\alpha}{n-1} & i \neq j \end{cases}$$

Learn-α algorithm

[M, 2003] [M & Jaakkola, NIPS 2003]

Learn-α algorithm: Learn the α parameter using α–experts, each updating with different value of α. Use Bayesian updates to track the best α.



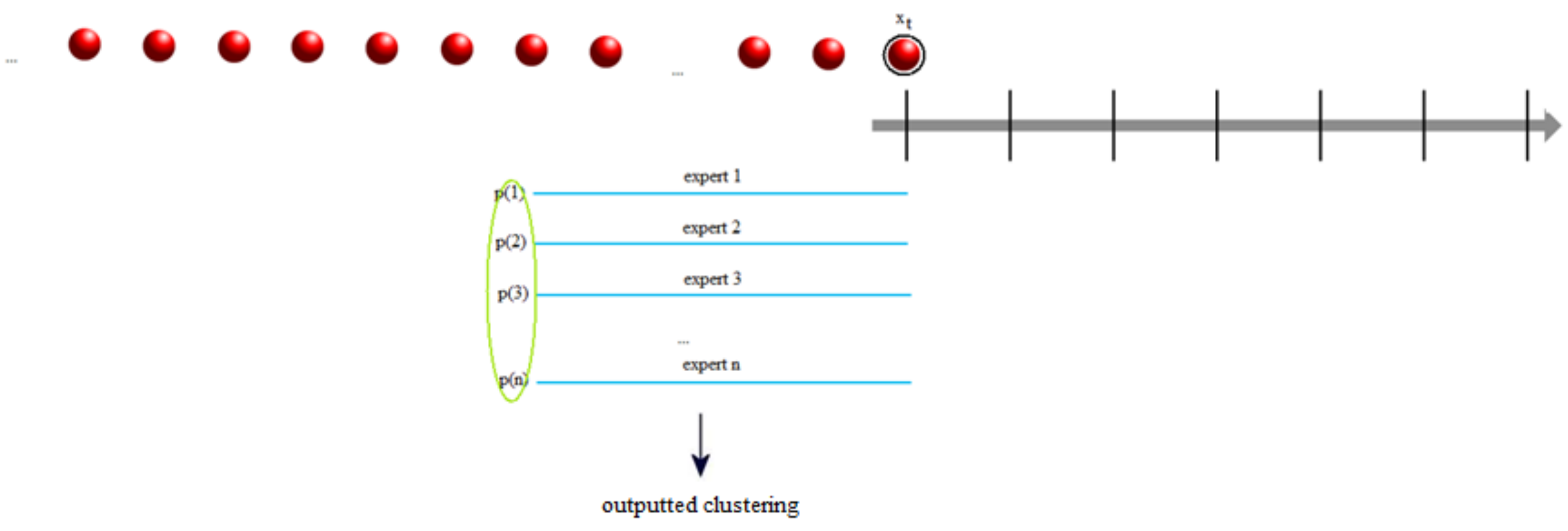
$$p_{t+1}(\alpha) \propto p_t(\alpha) e^{-L(\alpha,t)}$$

$$p_{t+1;\alpha}(i) \propto \sum_j p_t(j) e^{-L(j,t)} p(i|j;\alpha)$$

Online clustering with experts

Algorithm produces clustering informed by experts' clusterings:

- Clustering "experts" output centers at each time t.
- Approximation assumptions on the batch clustering algorithms used as experts yields novel online approximation guarantees.
- At time t, algorithm receives experts' clusterings and outputs a clustering informed by experts.



Analysis ideas

- Prove clustering analogs of regret bounds.
  - Define clustering and loss functions. $L(x_t, c_t) = \left\| \frac{x_t - c_t}{2R} \right\|^2$

    $clust(t) = \sum_{i=1}^{n} p_t(i) c_t^i.$
  - Prove (c,η)-realizability to relate our loss to log-loss.

- Instantiate experts as (batch) clustering algorithms with b-approximation assumptions, run on sliding window
  - Starting from regret bounds, extend with approximation assumptions to yield novel online approximation guarantees.

Performance Guarantees

- Static expert: $L_T(\boldsymbol{alg}) \leq L_T(a_i*) + 2\log n$

$$L_T(alg) \leq \frac{bW}{4R^2} OPT_T + 2\log n$$

- Fixed-share: $L_T^{\log}(\alpha) \leq L_T^{\log}(\alpha^*) + (T-1)D(\alpha^* \| \alpha)$

$$L_T(\alpha) \leq \frac{bW}{4R^2} OPT_T + 2(T-1)D(\alpha^* \| \alpha)$$

- Learn-α:

$$L_T^{\log}(\boldsymbol{alg}) \leq L_T^{\log}(\alpha^*) + (T-1) \min_{\{\alpha_j\}} D(\alpha^* \| \alpha_j) + \log m$$

$$L_T(\boldsymbol{alg}) \leq \frac{bW}{4R^2} OPT_T + 2(T-1) \min_{\{\alpha_j\}} D(\alpha^* \| \alpha_j) + \log m$$

Results: final k-means cost

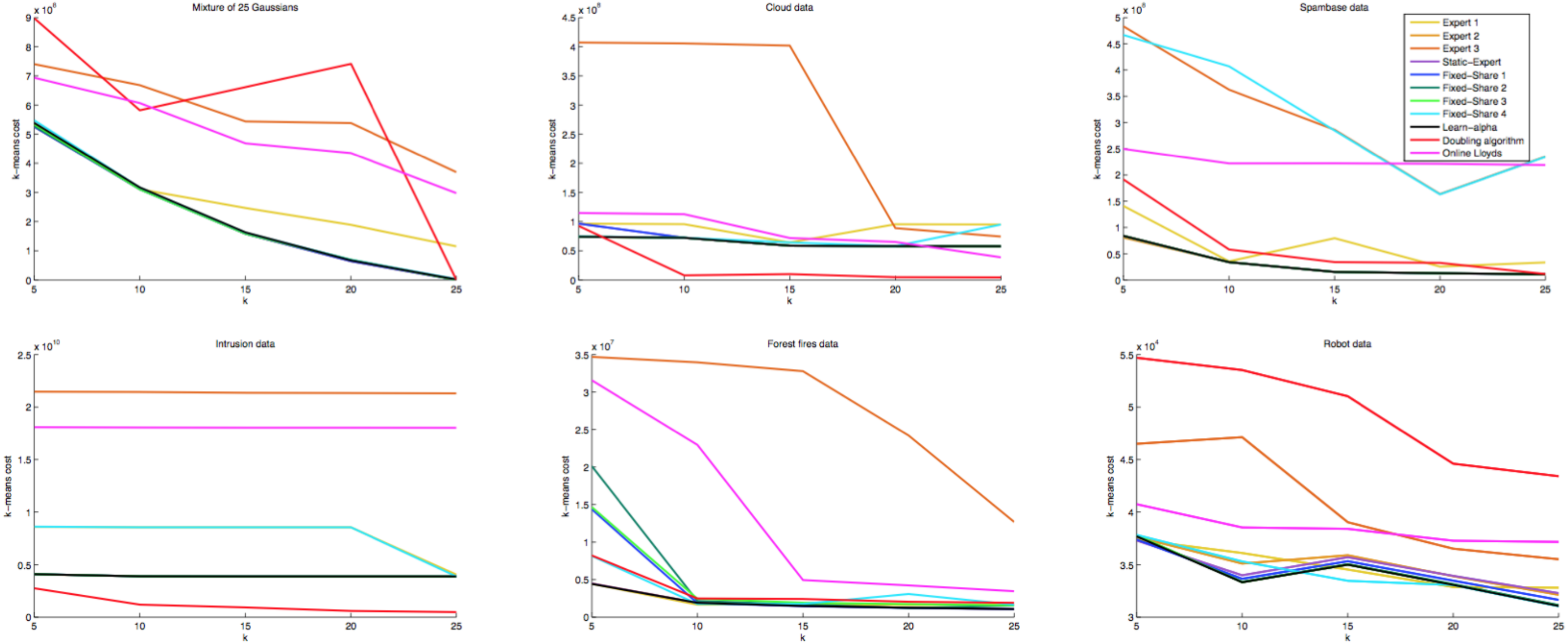


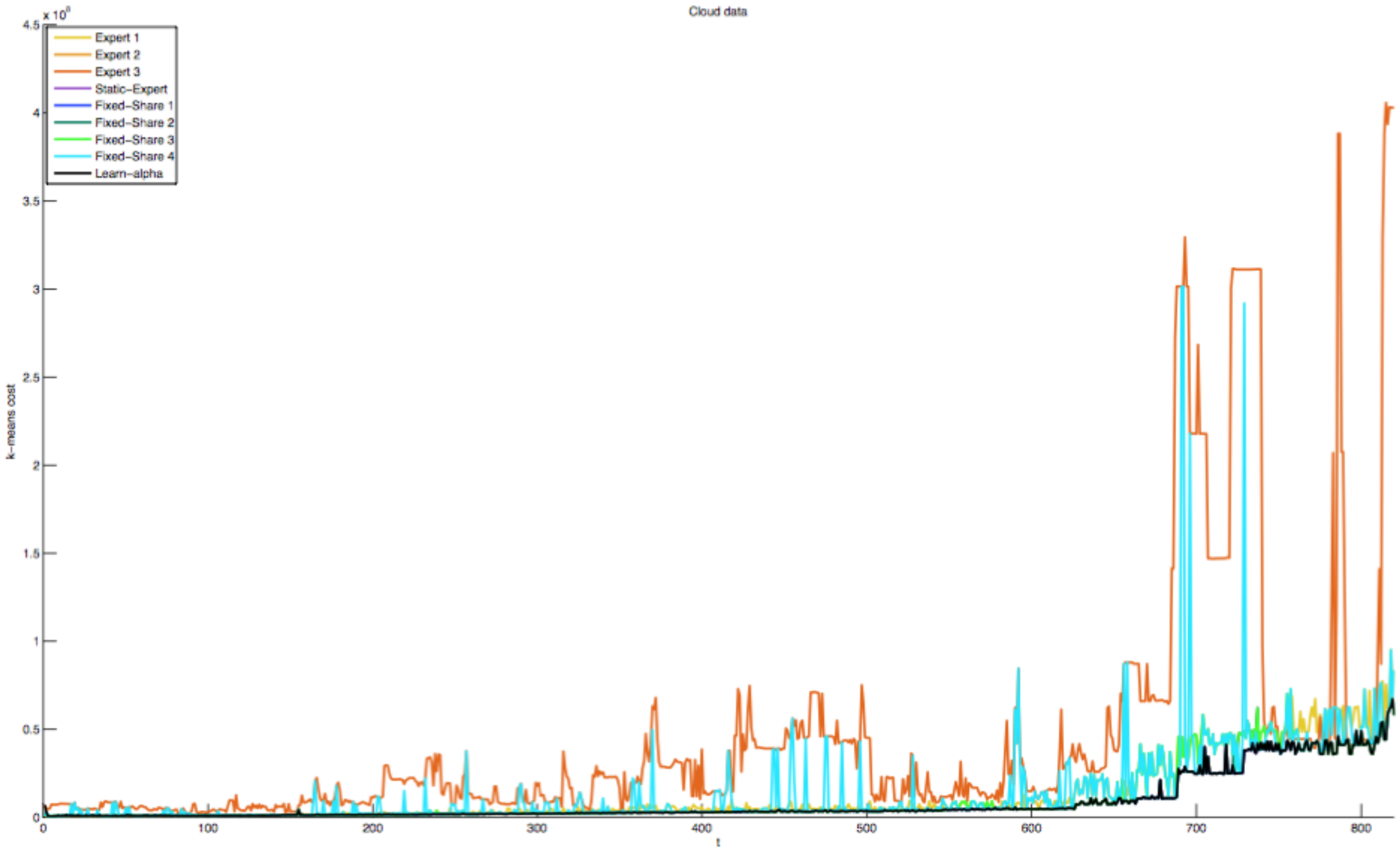
Figure 1: k-means cost on the entire sequence, versus k, per experiment. Legend in upper right.

Results: mean cost over sequence

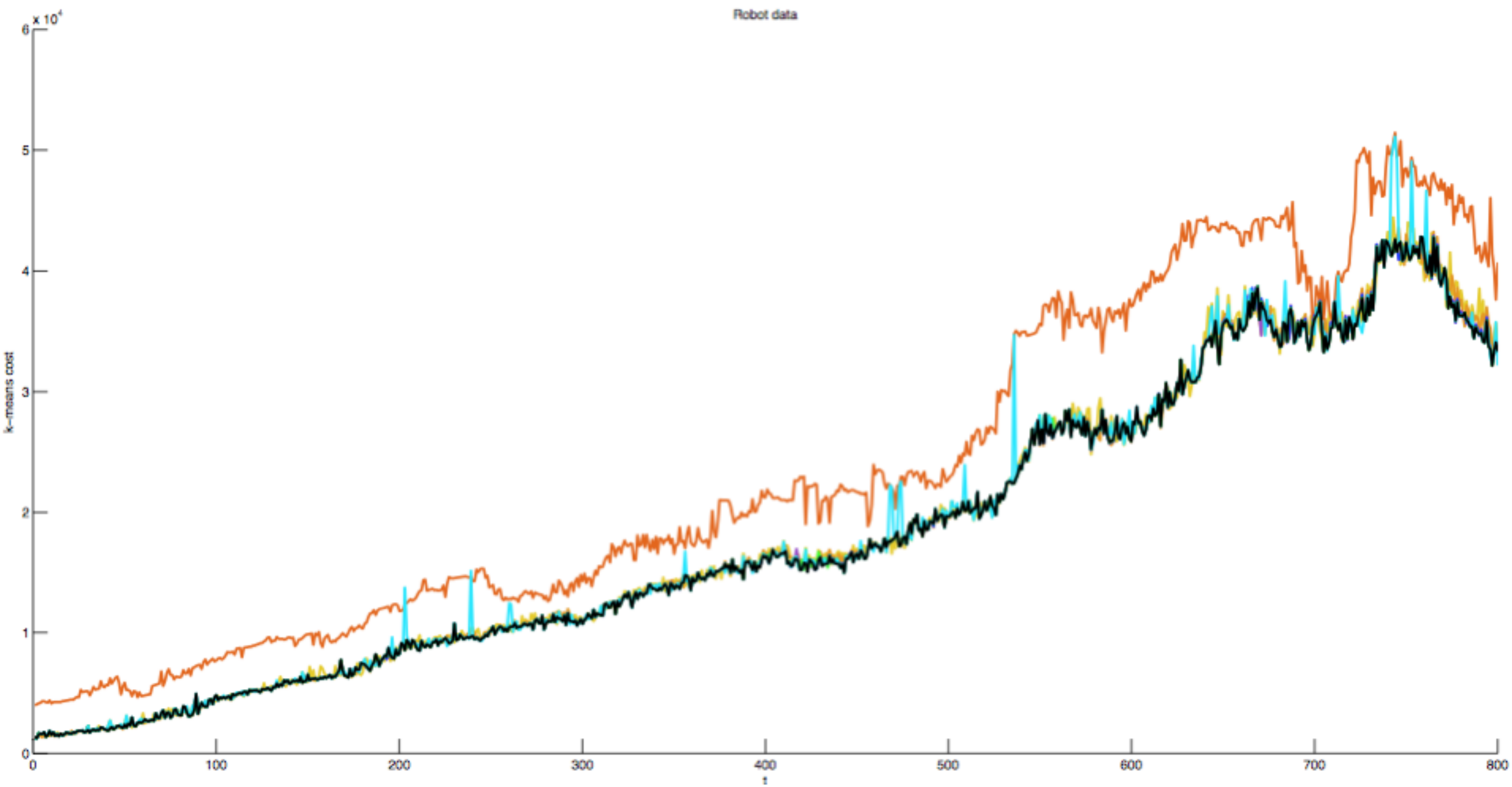
	25 Gaussians	Cloud $\times 10^7$	Spam $\times 10^8$	Intrus. $\times 10^{10}$	For. fire $\times 10^6$	Robot $\times 10^4$
e_1	0.6193±0.3195×10^8	1.3180±1.9395	0.2706±0.2793	0.1988±0.2104	0.7766±0.6413	1.8362±1.2172
e_2	**0.0036**±0.0290×10^7	**0.8837**±1.3834	**0.1042**±0.1463	**0.0743**±0.1041	**0.6616**±0.4832	**1.8199**±1.2102
e_3	2.0859±0.9204×10^8	4.6601±7.8013	1.6291±1.3292	0.7145±0.5376	7.1172±7.6576	2.3590±1.4070
da	0.0179±0.0723×10^8	**0.5285**±0.2959	0.1971±0.0826	**0.0050**±0.0529	1.4496±0.6484	2.5514±1.4239
ol	1.7714±0.6888×10^8	4.2322±2.4965	0.8222±0.7619	1.3518±0.3827	2.9617±1.3006	1.9806±1.0160
se	**0.0014**±0.0143×10^8	**0.8855**±1.3824	**0.1059**±0.1469	**0.0778**±0.1094	0.6620±0.4831	1.8139±1.2032
f_1	**0.0014**±0.0143×10^8	**0.8855**±1.3823	**0.1059**±0.1469	0.0779±0.1100	**0.6614**±0.4819	1.8137±1.2032
f_2	**0.0014**±0.0143×10^8	0.9114±1.4381	**0.1059**±0.1470	**0.0778**±0.1099	0.7008±0.5382	**1.8134**±1.2031
f_3	**0.0014**±0.0143×10^8	1.0715±1.6511	**0.1059**±0.1470	0.0779±0.1099	0.6996±0.5361	1.8145±1.2031
f_4	0.0124±0.0193×10^8	1.4806±2.6257	0.3723±0.7351	0.1803±0.2358	1.0489±1.4817	1.8334±1.2212
f_5	1.3811±1.0881×10^8	3.0837±6.3553	0.8212±1.1583	0.4126±0.5040	4.4481±6.2816	2.2576±1.3849
la	**0.0012**±0.0136×10^8	0.8862±1.3920	**0.1076**±0.1483	0.0785±0.1108	**0.6616**±0.4805	**1.8130**±1.2026
e_4	**7.3703**±4.2635×10^3	**0.6742**±1.2301	**0.0687**±0.1355	**0.0704**±0.1042	**0.2316**±0.2573	**1.3667**±1.0176
e_5	8.2289±4.4386×10^3	0.6833±1.2278	0.0692±0.1356	**0.0704**±0.1042	0.2625±0.2685	1.4385±1.0495
e_6	9.8080±4.7863×10^3	0.7079±1.2364	0.0710±0.1360	0.0705±0.1042	0.3256±0.2889	1.5713±1.1011
se	**0.1360**±1.4323×10^6	**0.6743**±1.2300	**0.0687**±0.1355	**0.0705**±0.1045	0.2322±0.2571	1.3642±1.0138
f_1	**0.1360**±1.4323×10^6	**0.6743**±1.2300	**0.0687**±0.1355	**0.0705**±0.1045	0.2322±0.2571	1.3640±1.0135
f_2	0.1361±1.4322×10^6	0.6746±1.2298	**0.0687**±0.1355	**0.0705**±0.1045	0.2322±0.2572	1.3636±1.0130
f_3	0.1364±1.4322×10^6	**0.6743**±1.2300	**0.0687**±0.1355	0.0711±0.1055	**0.2321** ±0.2570	**1.3634**±1.0127
f_4	0.0027±0.0144×10^8	0.7207±1.3025	**0.0707**±0.1357	0.0773±0.1203	0.2776±0.4917	1.3963±1.0339
f_5	1.4039±1.0790×10^8	3.0786±6.4109	0.7155±1.0650	0.4227±0.5179	4.6103±6.3019	2.3142±1.4127
la	**0.0012**±0.0134×10^8	**0.6742**±1.2300	**0.0687**±0.1355	**0.0708**±0.1046	**0.2318**±0.2573	**1.3632**±1.0128

Table 1: Mean and standard deviation, over the sequence, of k-means cost on points seen so far. $k = 25$ for Gaussians, $k = 15$ otherwise. The best expert and the best 2 scores of the algorithms, per experiment, are bold. Below the triple lines, 3 more experts are added to the ensemble.

Clustering analogs to learning curves



Clustering analogs to learning curves



Future work on clustering data streams

- Online clustering with experts, where experts need not be clustering algorithms

- A negative result for Dasgupta's conjecture (framework 1).

- Other open problems in online clustering
  - Online spectral clustering
  - Hierarchical clustering with k-means approximation guarantees for all k simultaneously
  - How to allow k to vary with time-varying data
  - Your suggestions?

Thank you!

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"Streaming k-means approximation"

Nir Ailon, Technion

Ragesh Jaiswal, IIT Delhi

"Online Clustering with Experts"

Anna Choromanska, Columbia