Revolutionaries and Spies on Graphs

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Slides available on my webpage

Joint with Jane Butterfield, Greg Puleo,
Doug West, and Reza Zamani

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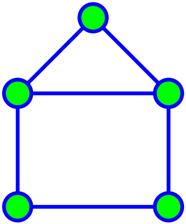

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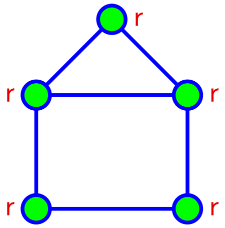

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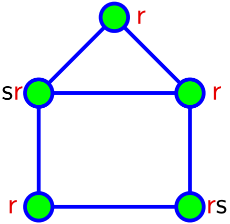

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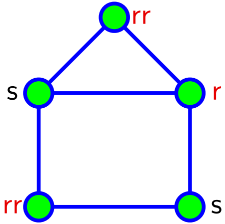

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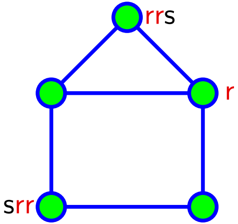

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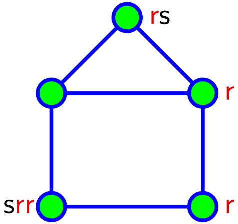

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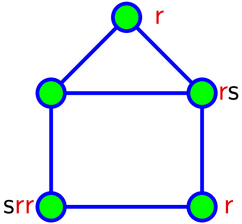

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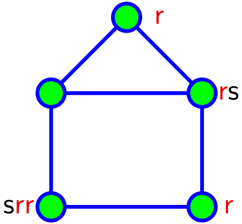

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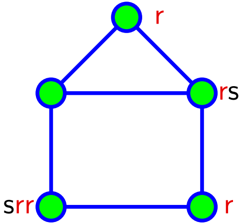

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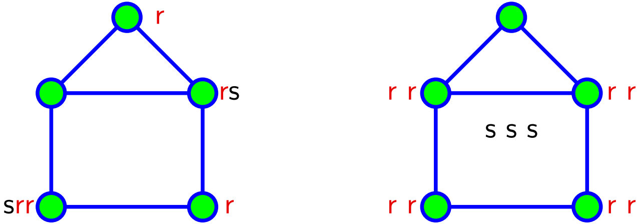

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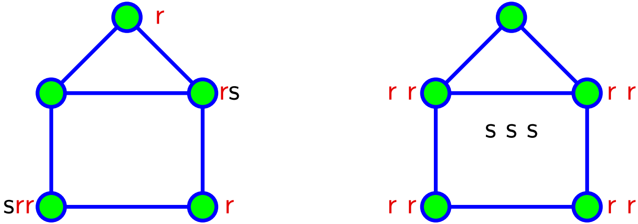

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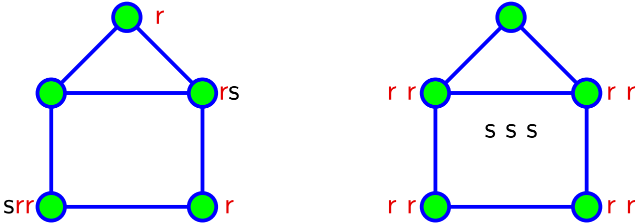

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Def: $\sigma(G, m, r)$ is minimum number of spies needed to win on G.

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Conj: As m grows: $\sigma(G,m,r) \sim \frac{3}{2}\frac{r}{m}$

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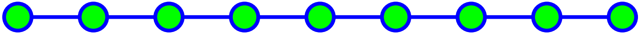

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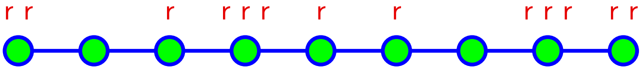

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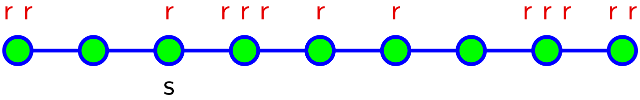

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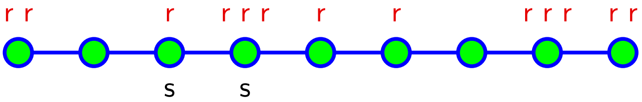

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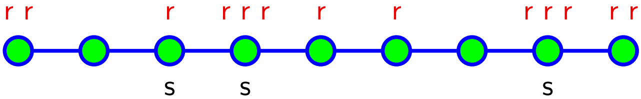

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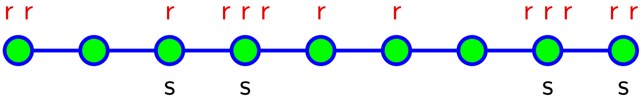

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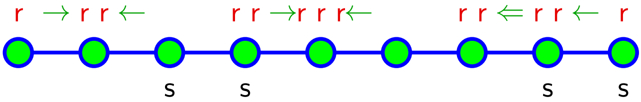

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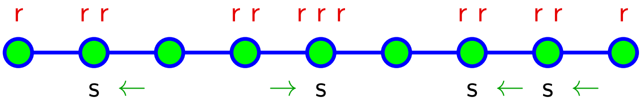

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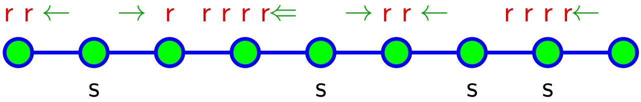

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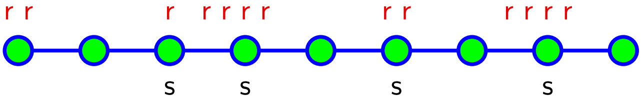

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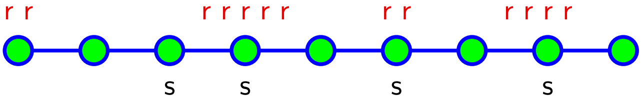

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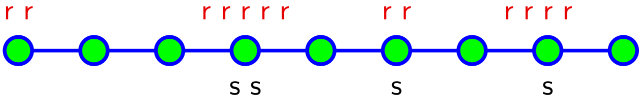

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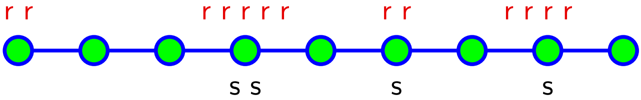

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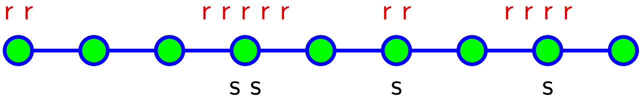

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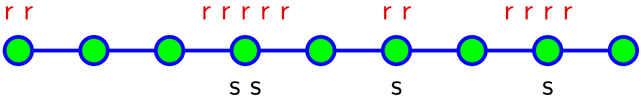

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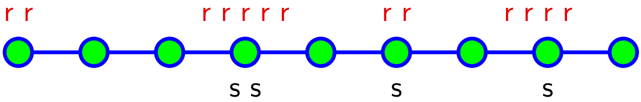

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2. If $r(v) \geq m$, then $s(v) \geq \left\lfloor \frac{w(v)}{m} \right\rfloor - \left\lfloor \frac{w(v)-r(v)}{m} \right\rfloor \geq 1$

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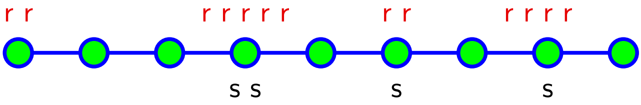

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Pf Sketch: Write $r(v)$ and $s(v)$ for num. of rev's and spies at v; $C(v)$ is children of v; and $w(v)$ is num. of rev's at descendants.

$$s(v) = \left\lfloor \frac{w(v)}{m} \right\rfloor - \sum_{x \in C(v)} \left\lfloor \frac{w(x)}{m} \right\rfloor$$

1. Since $\lfloor a + b \rfloor \geq \lfloor a \rfloor + \lfloor b \rfloor$, $s(v)$ is nonnegative
2. If $r(v) \geq m$, then $s(v) \geq \left\lfloor \frac{w(v)}{m} \right\rfloor - \left\lfloor \frac{w(v) - r(v)}{m} \right\rfloor \geq 1$
3. $\sum_{v \in T} s(v) = \left\lfloor \frac{w(u)}{m} \right\rfloor = \left\lfloor \frac{r}{m} \right\rfloor$

Spy-good graphs: Dominated Graphs and Webbed Trees

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Partition $E(G)$ into subgraphs $G(v) = G[v \cup C(v)]$. Simulate a game in each $G(v)$; use those moves in the actual game. Each $G(v)$ is a dominated graph, so we can use that result.

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Thm: For a large complete bipartite graph G

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- To win, on each round the spies maintain an invariant; the proof goes by induction on the number of rounds.

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   Conj: As m grows: $\sigma(G,m,r) \sim \frac{3}{2}\frac{r}{m}$