Secondary Spectrum Trading Market – Auction-Based Approach to Spectrum Allocation and Profit Sharing

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Outline

- Background
- Motivation
- Problem formulation
- Efficient vs. optimal mechanism
  - Generalized Branco’s mechanism
- Incentive for cooperation among sellers
- Equitable profit sharing among sellers
  - Existence of nonempty core of cooperative game
  - Existence of equitable profit sharing scheme
- Conclusion
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Background (1)

- Inefficient spectrum allocation today
  - Conventional way
    - **Static** allocation by a government agency (e.g., Federal Communications Commission (FCC) in the U.S.)
  - Drawbacks
    - Hampers the entrance of a new service provider
      - Reduced competition
    - **Under-utilized** in many places
Background (2)

- Example of spectrum allocation (in the U.S.):
  - 614 ~ 806 MHz: Broadcasting (TV, channels 38-69)
  - 806 ~ 824 MHz: Pagers and public safety (uplink (e.g., T-GSM 810))
  - 824 ~ 849 MHz: Mobile phone (wireless comm. uplink)
  - 849 ~ 869 MHz: Pagers and public safety (downlink)
  - 869 ~ 894 MHz: Base station (wireless comm. downlink)

Source: http://en.wikipedia.org/wiki/Cellular_frequencies
Background (3)

- Limestone, Maine (2007)
Background (4)

- Limestone, Maine (2007)
Lessons from the measurements
- While spectrum is considered scarce (and expensive), allocated frequency bands are often under-utilized

Natural Question – In light of rapidly increasing demand for spectrum
- How can we increase frequency usage efficiency?
- Is there any way to allow other users (who need the frequency) to use under-utilized frequency bands?
Background (6)

- Proposed approaches
  - Pack more users in frequency spectrum
    - Mobile Virtual Network Operators (MVNOs), e.g., Virgin Mobile USA, 7-Eleven Speak Out Wireless, AirLink mobile, Credo Mobile
      - Share spectrum or infrastructure with Mobile Network Operators (MNOs), e.g., AT&T, Sprint, Verizon, T-Mobile
  - Allow dynamic frequency access to unlicensed users (secondary users)
    - e.g., Cognitive Radio (CR)
Background (7)

- Mobile Virtual Network Operator (MVNO)
  - Business agreement to use the spectrum and infrastructure of licensed Mobile Network Operators (MNOs)
    - Examples
      - Virgin Mobile USA (MVNO) with Sprint Nextel (MNO)
      - Credo Mobile (MVNO) with Spring Nextel (MNO)
      - Firefly Mobile (MVNO) with AT&T (MNO)
  - Runs own cellular mobile service business with its own brand, pricing scheme, numbering resources, and featured services
Background (8)

- Cognitive Radio (CR):
  - Underlying technology: Software-Defined Radio (SDR)
  - CR users (CRUs) can
    - switch its radio access technology based on the availability and/or performance of available networks
    - use any available frequency band
  - CRUs often called unlicensed users

- **Key constraint:**
  - Licensed users shall not be affected by CRUs’ use of frequency band
Background (9)

- Proposed methods for honoring the constraint include
  - **Frequency rental protocol**
    - Primary provider (i.e., licensed user) broadcasts available frequency bands
    - CRUs request (and use those bands granted for use)
    - When a licensed user needs the frequency bands, it sends a signal to stop CRUs
  - **Frequency sensing**
    - CRUs continuously monitor the usage on frequency bands
    - If no activity is detected, use the bands
    - When activity is detected, stop using the bands
  - **Interference temperature model**
    - Use frequency bands while total interference level at licensed user receivers remains below a predefined threshold
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Motivation (1)

- **Drawbacks of MVNOs**
  - Low flexibility for under-utilized frequency
    - Constrained to use the same radio technologies employed by MNOs
    - Can provide only (almost) the same set of services as MNOs

- **Research on CR**
  - Most of previous studies focus on resource allocation among CRUs
  - Often assume CRUs can use the spectrum free of charge
    - Private primary service providers may not be so generous
      - Likely to demand a payment
  - Individual CRUs responsible for finding and using under-utilized frequency spectrum (especially under frequency sensing and interference temperature model)
    - **Uncoordinated** access/use of under-utilized spectrum
Motivation (2)

- Secondary trading market for spectrum trading (to marry the previous two)
  - What if secondary service providers (acting as middle men)
    - Have own infrastructure with dynamic frequency access capability at both access point and user equipment (UE)
  - Lease the spectrum from primary service providers (licensees)
  - Collect the service/usage fee from their customers (CRUs)
  - Can use under-utilized spectrum in a more efficient and organized manner
  - Can provide more services
    - Not tied to the same radio technologies as MNOs
Motivation (3)

- Model:

  - Primary Service Providers
  - Secondary Service Providers
  - Spectrum Trading Market
Motivation (4)

- Need to design a spectrum sharing and pricing scheme between the primary service providers (PSPs) and secondary service providers (SSPs)
Motivation (5)

- Propose an auction-based framework for secondary spectrum trading market
  - Offers a natural tool for spectrum trading
    - Strategies of buyers
    - Methods for exchange of information
    - Allocation and payment schemes
  - Well designed auction mechanisms have desirable properties
    - Efficiency and/or optimality
    - Incentive compatibility
    - Individual rationality
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Problem formulation (1)

- In spectrum auction

- Frequency spectrum traded in a fixed unit
  - e.g., unit of 100 kHz
  - Total available spectrum from a primary service provider: 1 MHz
  - Primary service provider has 10 units of homogeneous good

- Frequency trading performed periodically or whenever needed

**Goods/Items:** Available frequency bands
**Sellers:** Primary service providers
**Buyers/Bidders:** Secondary service providers
Problem formulation (2)

- Sellers – primary service providers
  - Each seller interested in lending (a portion of) under-utilized spectrum it owns in different regions (i.e., operating markets)
  - Available spectrum divided according to a fixed unit (e.g., 100 kHz)
  - Sellers free to cooperate among themselves and form **coalitions** to sell their spectrum together
  - Each seller has a **value** associated with each unit of frequency band it wishes to lend
    - Determines its **reserve price**
  - **Risk neutral** – wish to maximize **expected profit** (i.e., revenue minus its values for sold frequency bands)
Problem formulation (3)

- **Buyers** – secondary service providers
  - Interested in purchasing frequency bands in different regions/markets
  - Have private information – type of buyer $j$ denoted by $T_j$
    - Has distribution $G_j$ with density $g_j$
    - Value of the $k$-th frequency band won by buyer $j$ given by $V_{k}^{j}(T_j)$
    - Independent and identically distributed (i.i.d.)
  - Interested in maximizing own expected payoffs
    - Payoff = total value from items won – price paid for the items
Problem formulation (4)

- Setup
  - Consider only a **single market**
  - $S = \text{set of primary service providers (sellers)}$
  - $B = \text{set of secondary service providers (buyers)}$
  - For each $s \in S$, $f_s$ denotes the number of frequency bands available for lease from seller $s$
Problem formulation (5)

- **Seller:**
  - Announces the list of frequency bands it wishes to lend and its reserve prices
  - May join other sellers to form a *coalition*
    - $\mathcal{P}$ - set of all possible partition of $S = \{1, 2, \ldots, S\}$
    - Each coalition of sellers holds a *separate* auction, sharing information among coalition members
Problem formulation (6)

**Buyer:**

- Each buyer first chooses one seller and participates in the auction of a coalition to which the chosen seller belongs.
  - Assume that the selection of a seller by a buyer does **not** depend on its type.
- Places a bid with the selected seller based on its private information.
Problem formulation (7)

- **Trading system:** For each auction,
  - Identifies winning bids and allocates goods *(allocation scheme)*
  - Computes the prices to charge winning bids *(pricing scheme)*
  - Distributes the revenue from the auction to the sellers according to a fixed and known revenue sharing scheme *(revenue sharing scheme)*
Problem formulation (8)

- **Goal:** Design a secondary spectrum trading market that will encourage and support trading between potential sellers and buyers
  - Should provide potential sellers with proper incentives to make their under-utilized frequency bands available to prospective buyers
    - Sellers likely to feel more compelled to put their under-utilized frequency bands up for sale when they anticipate higher revenue

- **Questions of interest**
  - How can the sellers maximize their revenue from auction?
  - Could they increase their revenue by cooperating with each other?
    - Cooperation would be “possible” only if (i) sellers feel that they can benefit from it and (ii) the revenue is shared fairly in sellers’ views
  - Is it possible to sustain cooperation among sellers?
  - If so, how should the revenue be shared among them to maintain such cooperation?
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Efficient vs. optimal mechanisms (1)

- **Efficient mechanism**
  - Maximizes social welfare
    - Assigns the item(s) to the buyer(s) who value the item(s) most
  - Suitable for auction of the public asset
  - Well studied - buyers’ strategies, allocation and payment rule
    - Well-known single item auctions
      - Dutch auction, English auction, first-price auction, second-price auction (Vickrey auction)
    - Well-known multiple item auctions
      - Discriminatory auction, uniform price auction, VCG mechanism
  - Designed for a single seller
Efficient vs. optimal mechanisms (1)

- **Optimal mechanism**
  - Maximizes seller’s expected revenue
  - Suitable for auction of a private asset
  - Much studied - buyers’ strategies, allocation, payment
    - Single item auction: Myerson’s mechanism
    - Multiple item auction: Branco’s mechanism
    - Mechanism given by a pair of functions \((p, c)\)
      - e.g., in Branco’s mechanism with \(m\) units of item
        \[
p : T \rightarrow \mathbb{R}^{(N \times m)}, \quad c : T \rightarrow \mathbb{R}^N
        \]
        \[
p_{jk}(t) : \text{probability that bidder } j \text{ will receive at least } k \text{ units}
        \]
        \[
c_j(t_j) : \text{bidder } j \text{'s expected payment}
        \]
  - Designed for a **single** seller
Generalized Branco’s mechanism (GBM) (1)

- $M$ buyers
  - $T_j \in T_j$ - type of buyer $j$ (private information)
  - Each buyer reports its type to seller(s) - $t^* = (t^*_j; j = 1, 2, \ldots, M)$
    - Not necessarily its true type

- Seller(s)
  - Have values for items for sale – $0 \leq V_0^{(1)} \leq V_0^{(2)} \leq \cdots \leq V_0^{(m)}$
  - Compute contributions: For each $j = 1, \ldots, M, k = 1, \ldots, m$
    \[
    \pi_{j,k}(t^*_j) = V_{j,k}(t^*_j) - \frac{\partial V_{j,k}(t_j)}{\partial t_j} \bigg|_{t_j=t^*_j} \frac{1 - g_j(t^*_j)}{g_j(t^*_j)}
    \]
  - Order the contributions by decreasing value
    - $\pi(\ell)(t^*)$ - $\ell$-th largest contribution among all buyers
Generalized Branco’s Mechanism (GBM) (2)

- **Regularity assumptions**

  - \((t_j - \tilde{t}_j)(\pi_{j,k}(t_j) - \pi_{j,k}(\tilde{t}_j)) \geq 0\) for all \(t_j, \tilde{t}_j \in \mathcal{T}_j\)

  - if \(\pi_{j,k+1}(t_j) \geq 0\), then \(\pi_{j,k}(t_j) \geq \pi_{j,k+1}(t_j)\) for all \(t_j \in \mathcal{T}_j\)
Generalized Branco’s Mechanism (GBM) (3)

- In a nutshell,
  - \( m^*(t^*) \) items are awarded to the buyers with the \( m^*(t^*) \) highest contributions, where

\[
m^*(t^*) := \max\{\ell \in \{1, 2, \ldots, m\} \mid \pi(\ell)(t^*) > V_0^{(\ell)}\}.
\]

- Buyer \( j \) pays \( V_{j,k}(c_{j,k}(t^*_{-j})) \) for the \( k \)-th item it wins, where

\[
c_{j,k}(t^*_{-j}) := \inf\{\hat{t}_j \in T_j \mid \pi_{j,k}(\hat{t}_j) \\
\geq \min\{\eta_\ell(\hat{t}_j, t^*_{-j}); \quad \ell = 1, 2, \ldots, m\}\}
\]

and \( \eta_\ell(t^*) = \max\{\pi(\ell+1)(t^*), V_0^{(\ell)}\} \)

- Smallest value for the \( k \)-th item that would win the item
Properties of GBM (1)

**Theorem:** The GBM satisfies following properties:

- **Incentive compatible**
  - Reporting true type is an optimal strategy for bidders
  - *We will assume buyers report their true types when GBM is employed by coalitions of sellers in our framework*

- **Individually rational**
  - No buyer will be worse off by participating in the auction

- **Optimal mechanism**
  - Maximizes the *expected profit* of the seller(s)
    - Profit = total revenue – total value of sold items
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Selfish buyers and non-cooperative game (1)

- Buyers assumed selfish
  - Interested in maximizing own expected payoffs
  - Interaction among selfish buyers modeled using a non-cooperative game
  - Only action is to select a seller

- **Seller selection probability vectors:**
  \[ p = (p_b; \ b \in \mathcal{B}) \]
  - \[ p_b = (p_{b,s}; \ s \in \mathcal{S}) \], where \( p_{b,s} \) is the probability that buyer \( b \) selects seller \( s \)
Non-cooperative game among buyers \((\mathcal{B} = \{1, 2, \ldots, B\})\)

- Payoff of buyer \(b\) given by \(p_b(A, T; P)\)
  - \(A = (a_1, a_2, \ldots, a_B)\) - sellers selected by buyers \((a_b \in \mathcal{S})\)
  - \(T = (T_b; b \in \mathcal{B})\) - vector of buyers’ (reported) types
  - \(P\) - partition of sellers, i.e., set of coalitions that emerge
    - \(P(s)\) - coalition to which seller \(s \in \mathcal{S}\) belongs
    - Each coalition \(C \in P\) holds a separate auction employing the generalized Branco’s mechanism (GBM)
  - \(p_b(A, T; P) = \) total value from items won – total price paid for the items won (according to the GBM)
Incentive for cooperation among sellers (1)

- Assume that buyers fix their seller selection probabilities
  - Any arbitrary probability vectors (mixed-strategy profile)

- For every $C \subset S$, let $v(C)$ denote the expected profit of coalition $C$ under the GBM

**Theorem:** For every $C_1, C_2 \subset S$ such that $C_1 \cap C_2 = \emptyset$

\[ v(C_1) + v(C_2) \leq v(C_1 \cup C_2) \]

Sellers are better off cooperating among themselves to maximize their expected profit
Source of difficulty (1)

- Calculation of prices to charge, hence total revenue from auction, difficult

- **Lack of monotonicity**
  - Profit/revenue does **NOT** necessarily increase with the set of items to be sold
    - Can easily find examples where introducing additional items to sell reduces the total revenue
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Cooperative game (1)

- How should sellers share the (expected) profit among themselves to promote and sustain cooperation?
  - Model the interaction as a **cooperative game**
  - **Characteristic function** $\nu$ defined through expected profit for different possible coalitions
    - $\nu(C^')$ - Expected payoff (i.e., expected profit) sellers in coalition $C'$ can guarantee themselves

**Definition:** An **imputation** is an expected payoff vector

$$x = (x_1, \ldots, x_S)$$ satisfying

- $\sum_{s \in S} x_s = \nu(S)$
- $x_s \geq \nu(\{s\})$ for all $s \in S$
**Cooperative game (2)**

**Definition:** Let \( x \) and \( y \) be two imputations. We say \( x \) dominates \( y \) through \( C \subset S \) if

- \( x_s > y_s \) for all \( s \in C \)
- \( \sum_{s \in C} x_s \leq v(C) \)

**Definition:** We say \( x \) dominates \( y \) if there is any coalition \( C \subset S \) such that \( x \) dominates \( y \) through \( C \)

**Definition:** The set of all undominated imputations is called the core.

- **Not** guaranteed to exist (i.e., non-empty)
Existence of non-empty core (1)

- **Theorem:** The core of the cooperative game among the sellers is always **non-empty**

- **Implication** – There always exists a way for sellers to share profit so that no subset of sellers will have an incentive or power to deviate from cooperation and increase their expected payoffs
Revenue sharing (1)

- Equitable sharing of revenue is possible
  - But, only in “expected” sense
  - Does not tell us how to share the revenue for each realization so as to achieve expected payoffs in the core

- Given an expected payoff vector $x^*$ in the core of cooperative game, how should the sellers distribute the profit for each realization so that their expected payoffs equal $x^*$?

- We would like to impose some additional natural constraints on the revenue sharing scheme we wish to design
Revenue sharing (2)

Revenue allocation scheme: \( \Theta : \Omega \rightarrow [0, \infty)^S \) with \( \sum_{s=1}^{S} \Theta_s(\omega) = 1 \)

C1. A seller that does not contribute anything to the auction (i.e., it brings neither winning contribution(s) nor allocated item(s)), called a non-contributing seller, receives nothing
   - Only contributing sellers receive positive payments

C2. Sellers shall have a nonnegative profit for every realization
   - Seller always receives a payment that is at least its total value of its items sold to the buyers

C3. \( \Theta(\omega) \) depends only on the set of contributing sellers
   - Can maintain the revenue allocation vectors in a finite table
Revenue sharing (2)

- Question: Is there a revenue allocation scheme, i.e., a mapping $\Theta^*$, that satisfies C1 through C3?

**Theorem:** Given any expected payoff vector in the core of cooperative game, there always exists a revenue allocation scheme that satisfies C1 through C3

- Recursive method for finding a mapping $\Theta^*$
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Conclusion

- Proposed an auction-based framework for designing a secondary spectrum trading market
  - Proposed an optimal auction mechanism (GBM) for allocating and pricing frequency bands
  - Showed the existence of an incentive for risk neutral sellers to cooperate in order to maximize their profits
  - By modeling the interaction among the sellers as a cooperative game, proved the existence of non-empty core of cooperative game
  - Designed a revenue sharing scheme that allows sellers to achieve any expected payoff vector in the non-empty core
Thank you....

Any questions?