A Benes Packet Network

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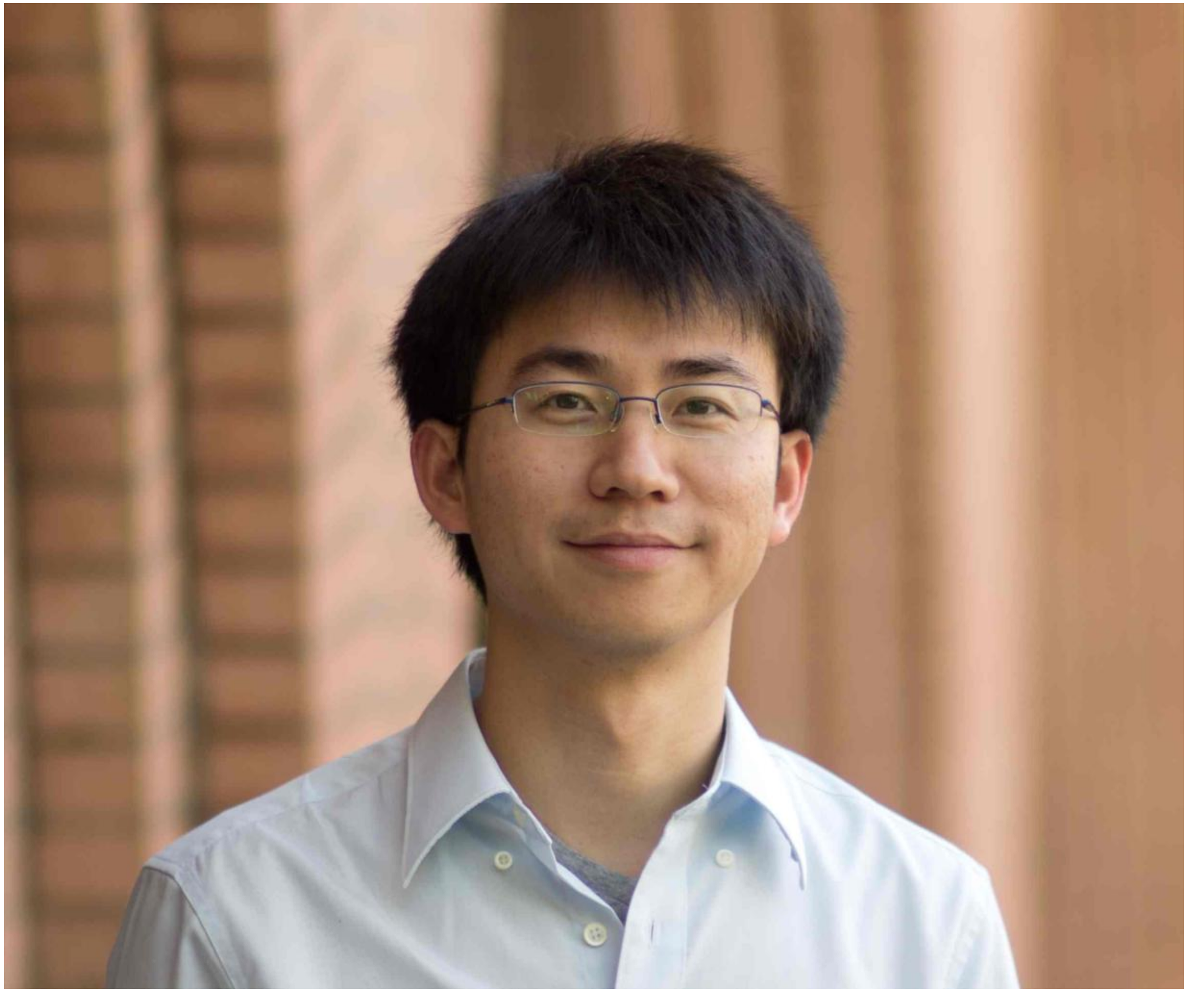

Longbo Huang
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Tsinghua University

Data centers are important computing resources

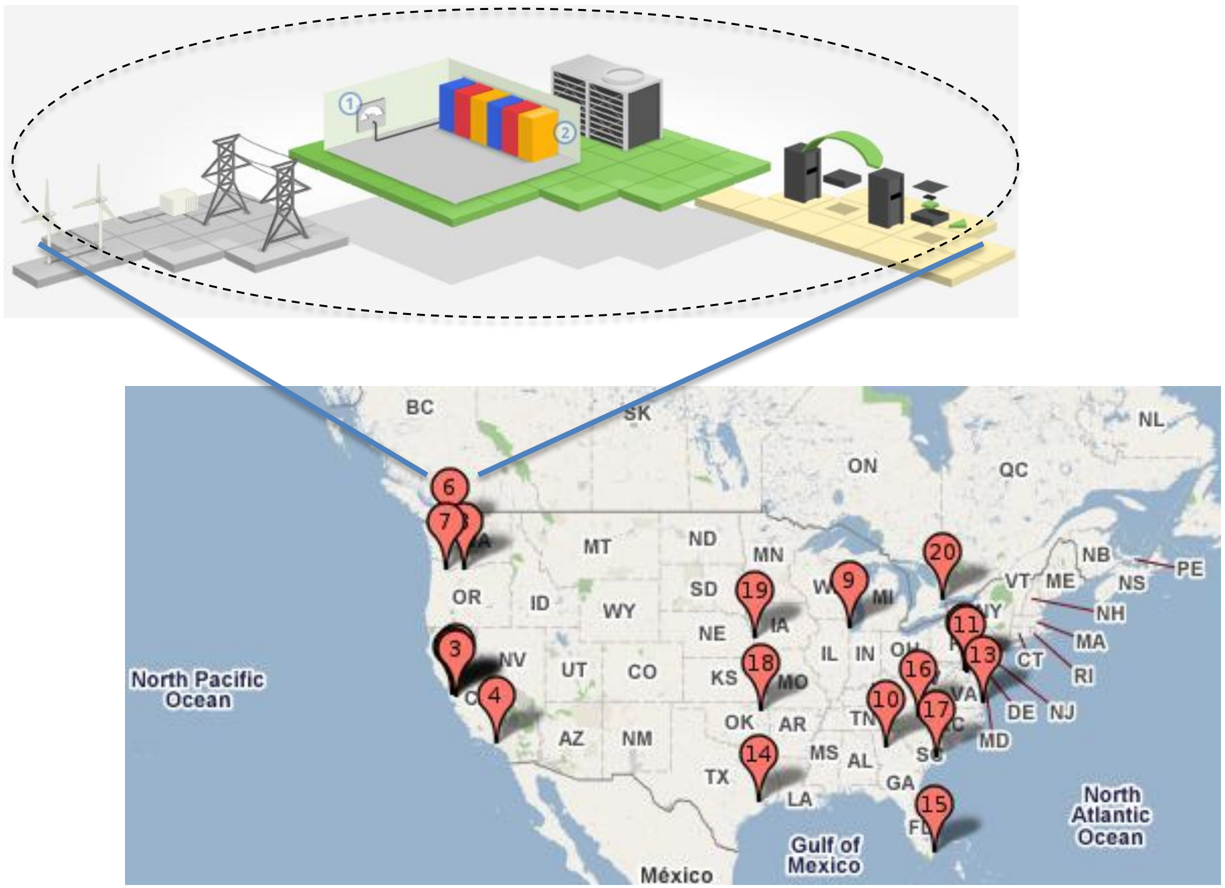
Provide most of our computing services

- Web service: Facebook, Email
- Information processing: MapReduce
- Data storage: Flickr, Google Drive

Google data centers within US

Src: http://royal.pingdom.com/2008/04/11/map-of-all-google-data-center-locations/

Data centers are important computing resources



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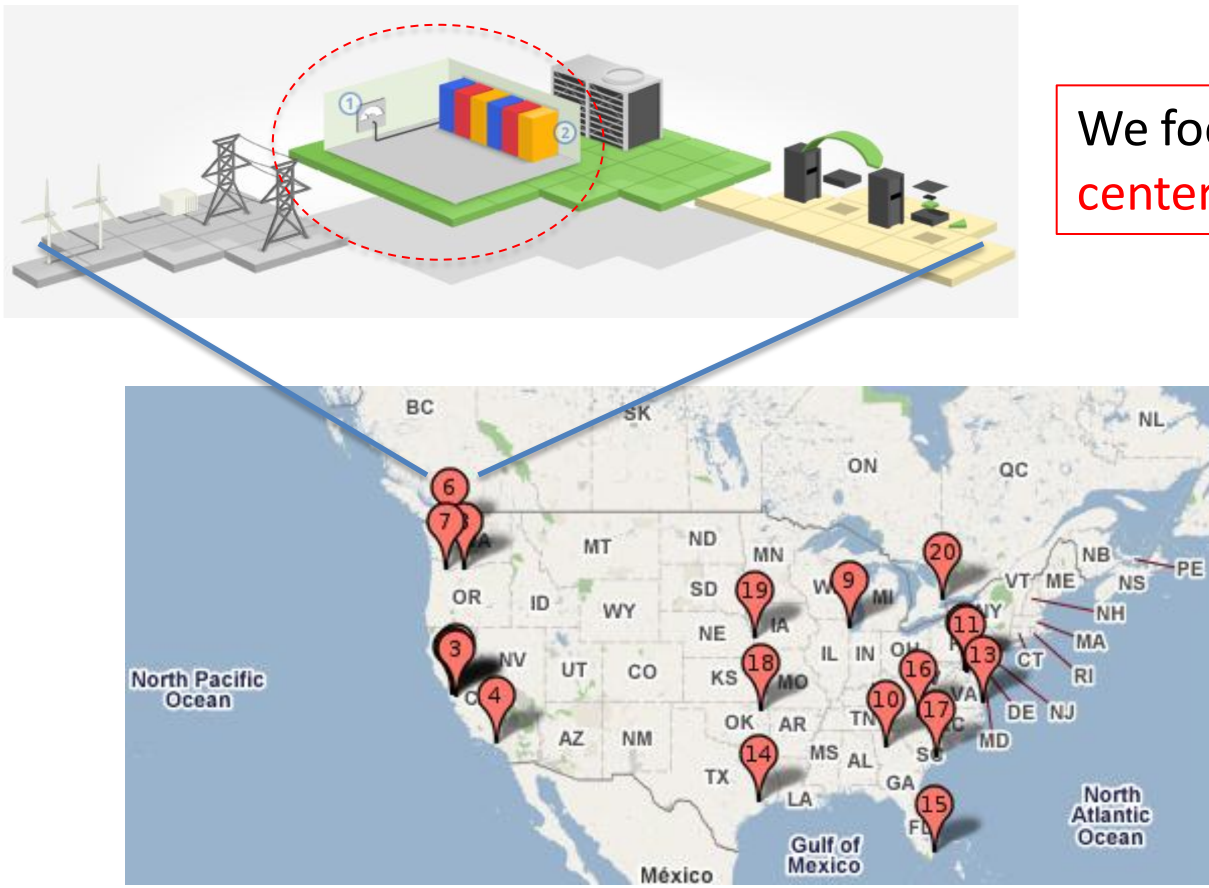

We focus on data center networking!

Google data centers within US

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The data center networking problem

Networking is the foundation of data centers' functionality

- Hundreds of thousands of interconnected servers
- Dynamic traffic flowing among servers
- Large volume of data requiring small latency
- Traffic statistical info may be hard to obtain

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Questions:

- How to connect the servers?
- How to route traffic to achieve best rate allocation?
- How to ensure small delay?
- How to adapt to traffic changes?

Benes Network + Utility Optimization + Backpressure

Benes Network:

- High throughput
- Small delay (logarithmic in network size)
- Connecting 2N servers with O(NlogN) switch modules

Flow Utility Maximization

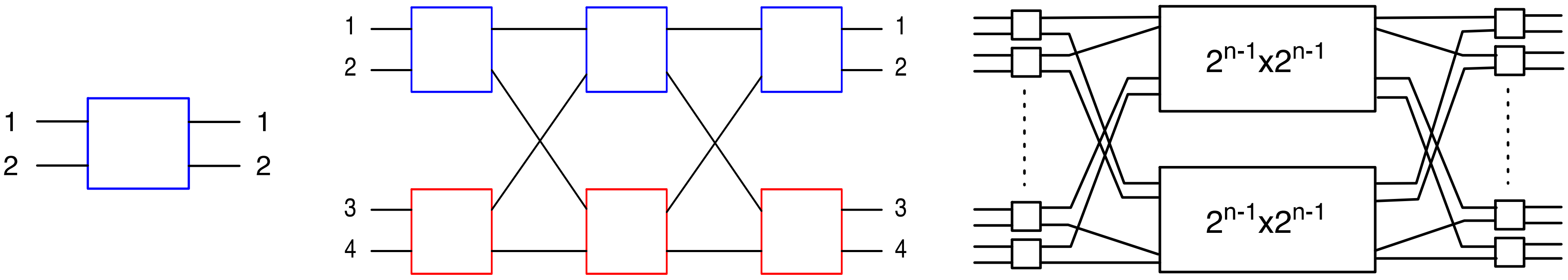
- Ensure best allocation of resources

Backpressure:

- Throughput optimal
- Robust to system dynamics
- Require no statistical info

Benes Network

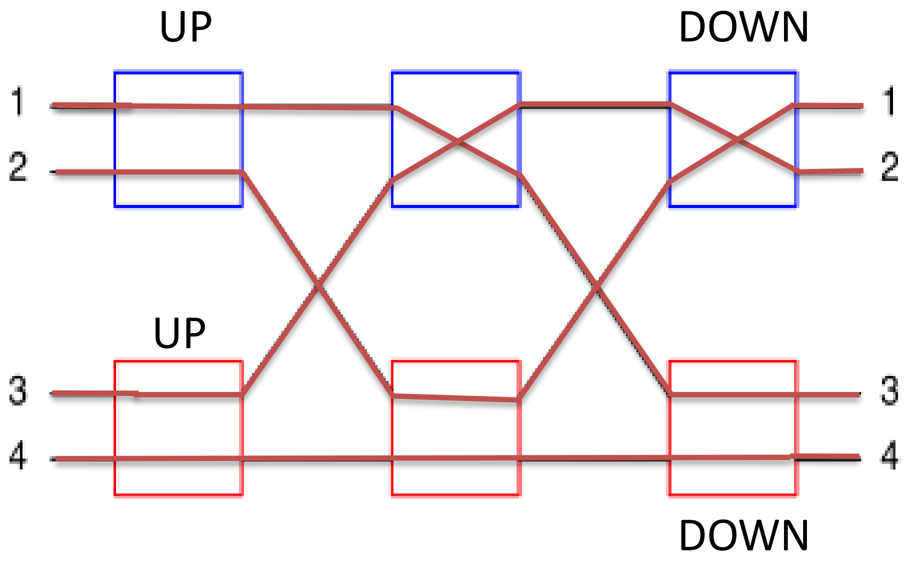
Building a $2^n x 2^n$ Benes network



Benes Network

Routing circuits:

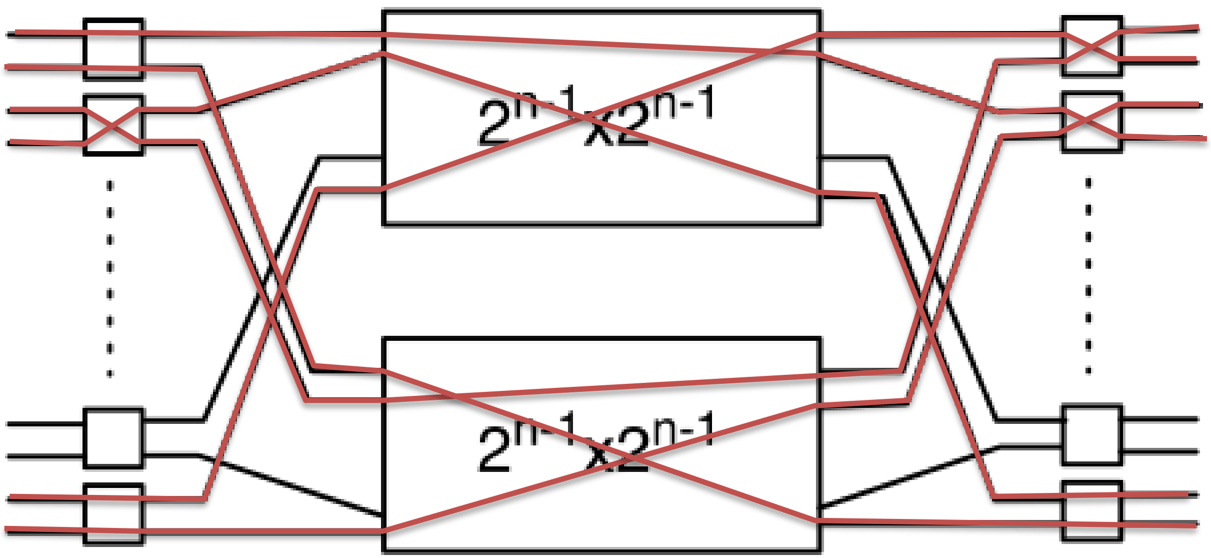
1 → 3
2 → 1
3 → 2
4 → 4



Benes Network

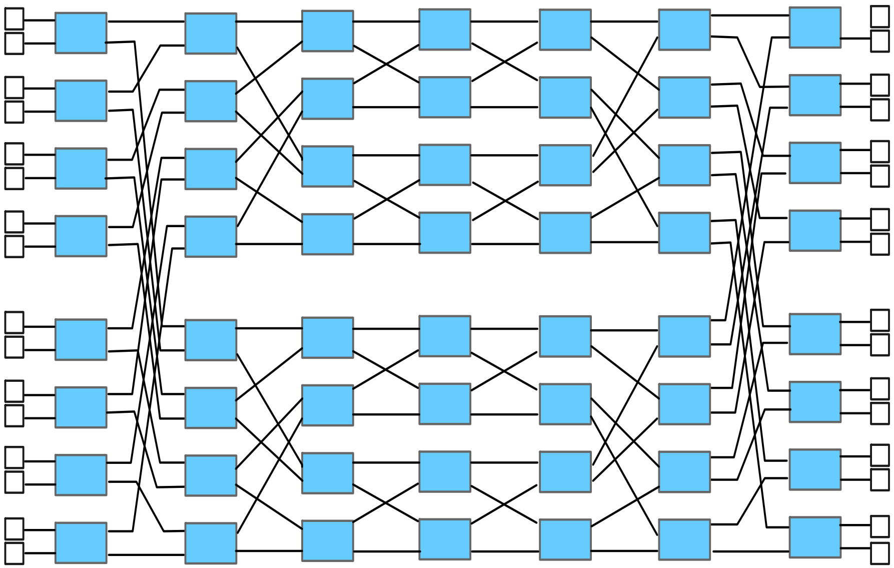
Routing circuits:

1 → 4
2 → 2^n
3 → 1
4 → $2^n - 1$
…….
$2^n - 1$ → 2
2^n → 3



➔ non-blocking for circuits
➔ full-throughput for packets

Benes Network Flow Utility Maximization



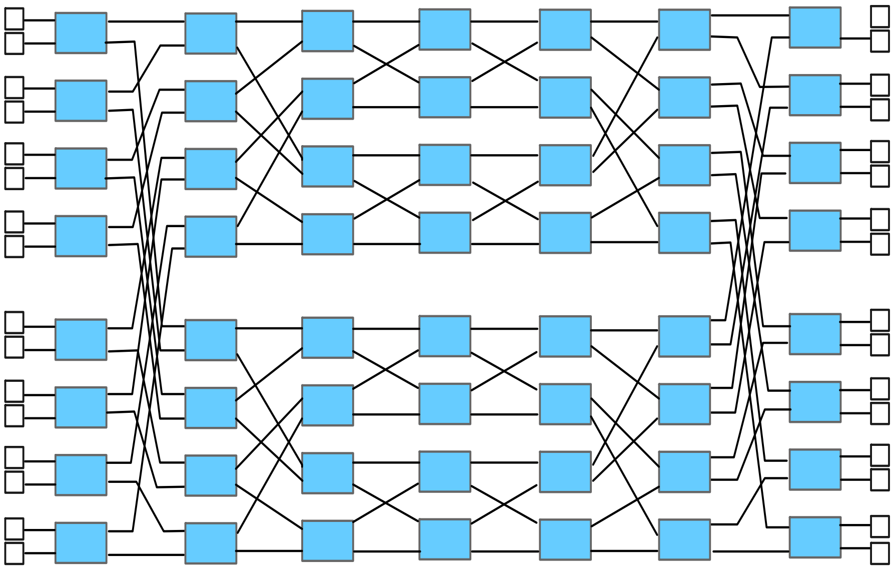
- Random arrival $A_{sd}(t)$
- Flow control, admit $R_{sd}(t)$ in $[0, A_{sd}(t)]$
- Each (s, d) flow has utility $U_{sd}(r_{sd})$
- Each link has capacity 1pk/s

The flow utility maximization problem:

$$\max: \quad \sum_{sd} U_{sd}(r_{sd})$$

$$\text{s.t.} \quad \text{Stability}$$

Benes Network Flow Utility Maximization



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- Each (s, d) flow has utility $U_{sd}(r_{sd})$
- Each link has capacity 1pk/s

Backpressure can be directly applied. However, each node needs 2^n queues, one for each destination

Grouped-Backpressure (G-BP)

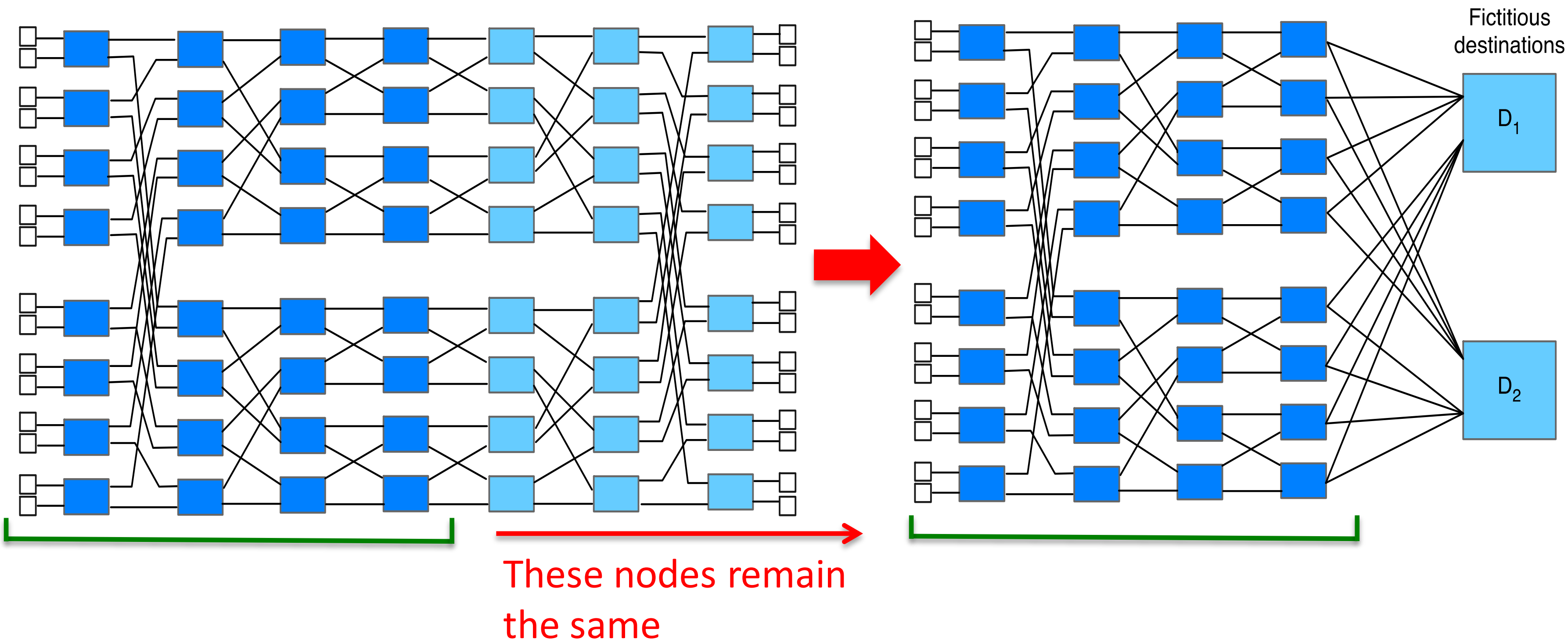
The idea:

- Divide traffic into two groups
- Perform routing & scheduling on the mixed traffic
- Rely on Backpressure & symmetry for stability

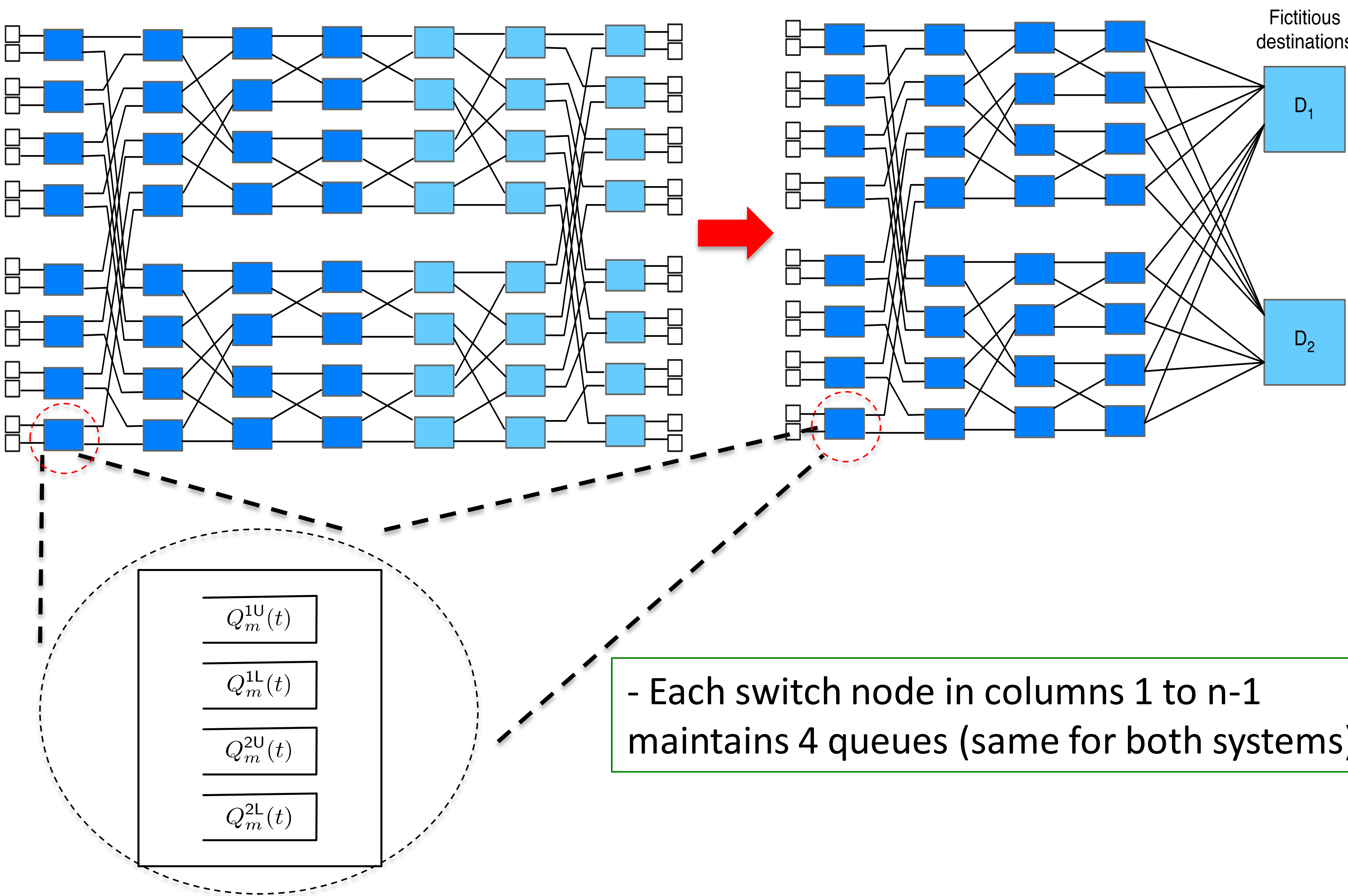
Key components

1. A fictitious reference system for control
2. A special queueing structure
3. An admission & regulation mechanism
4. Dynamic scheduling

G-BP Component 1 - Reference System

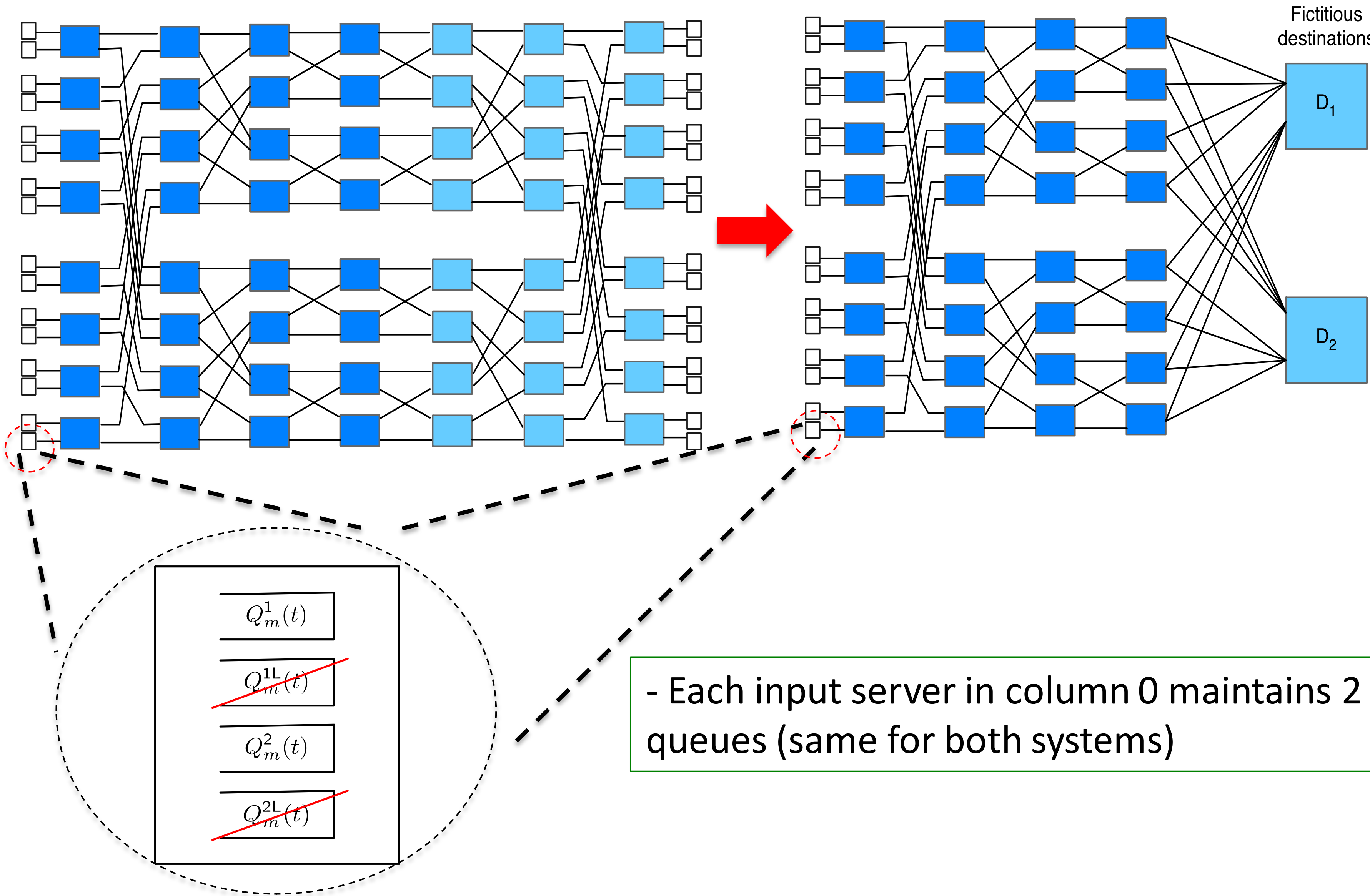


G-BP Component 2 – Queueing Structure



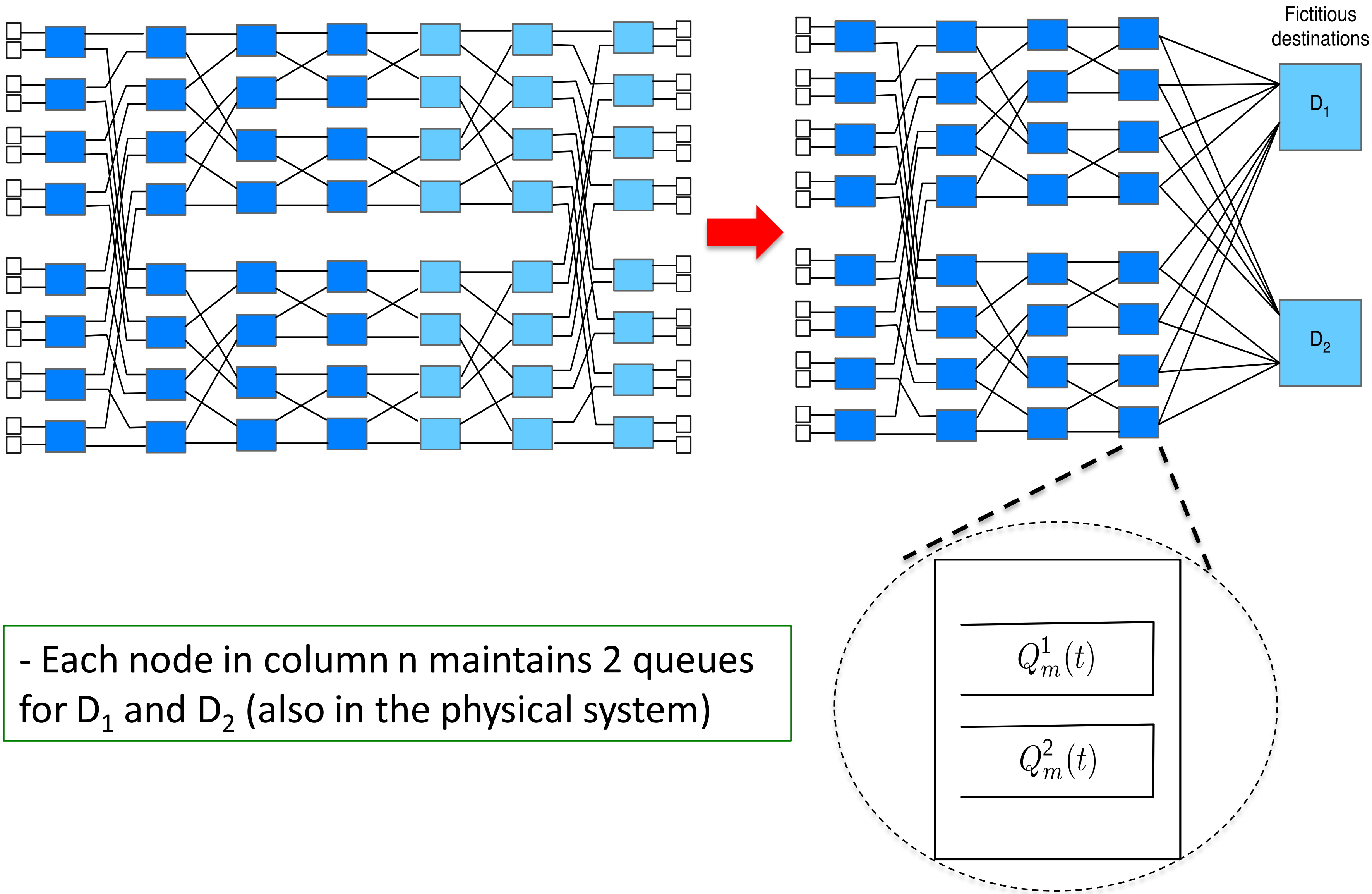
- Each switch node in columns 1 to n-1 maintains 4 queues (same for both systems)

G-BP Component 2 – Queueing Structure



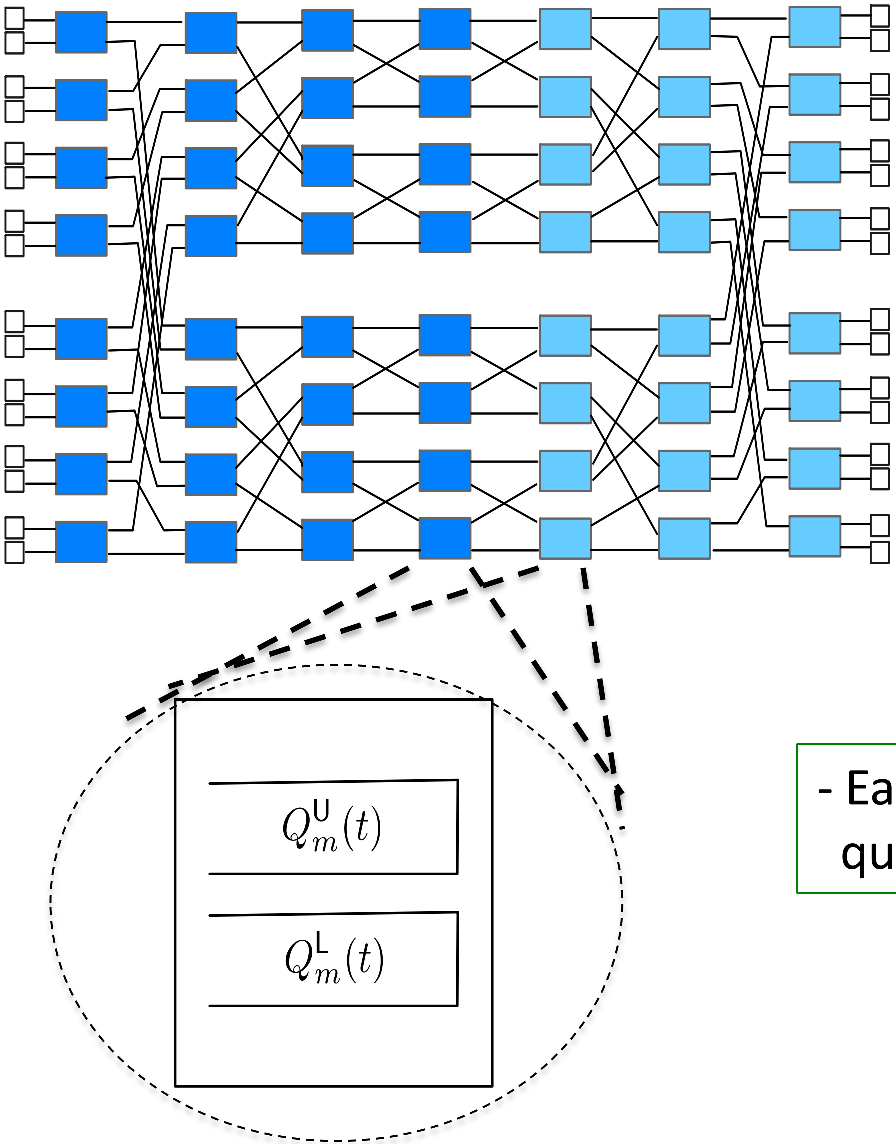
- Each input server in column 0 maintains 2 queues (same for both systems)

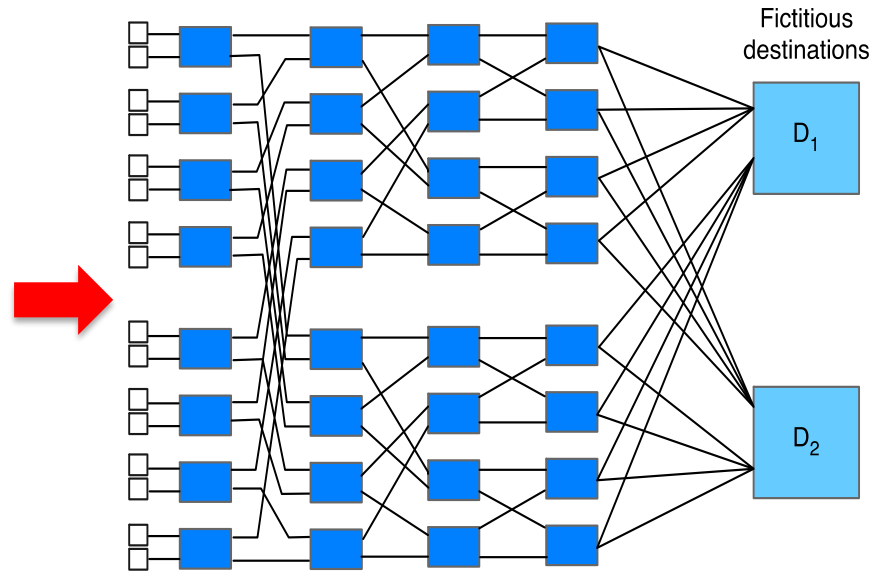
G-BP Component 2 – Queueing Structure



- Each node in column n maintains 2 queues for D_1 and D_2 (also in the physical system)

G-BP Component 2 – Queueing Structure

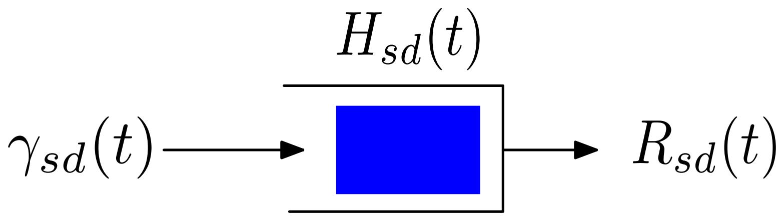




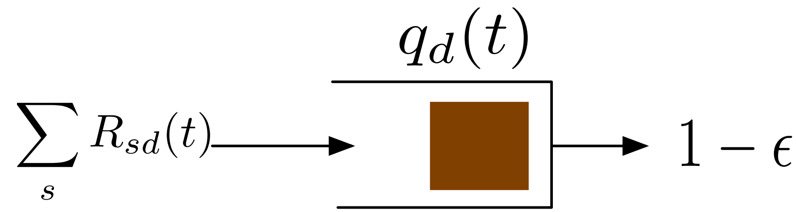
- Each node in columns n to 2n-1 maintains 2 queues (only the physical system)

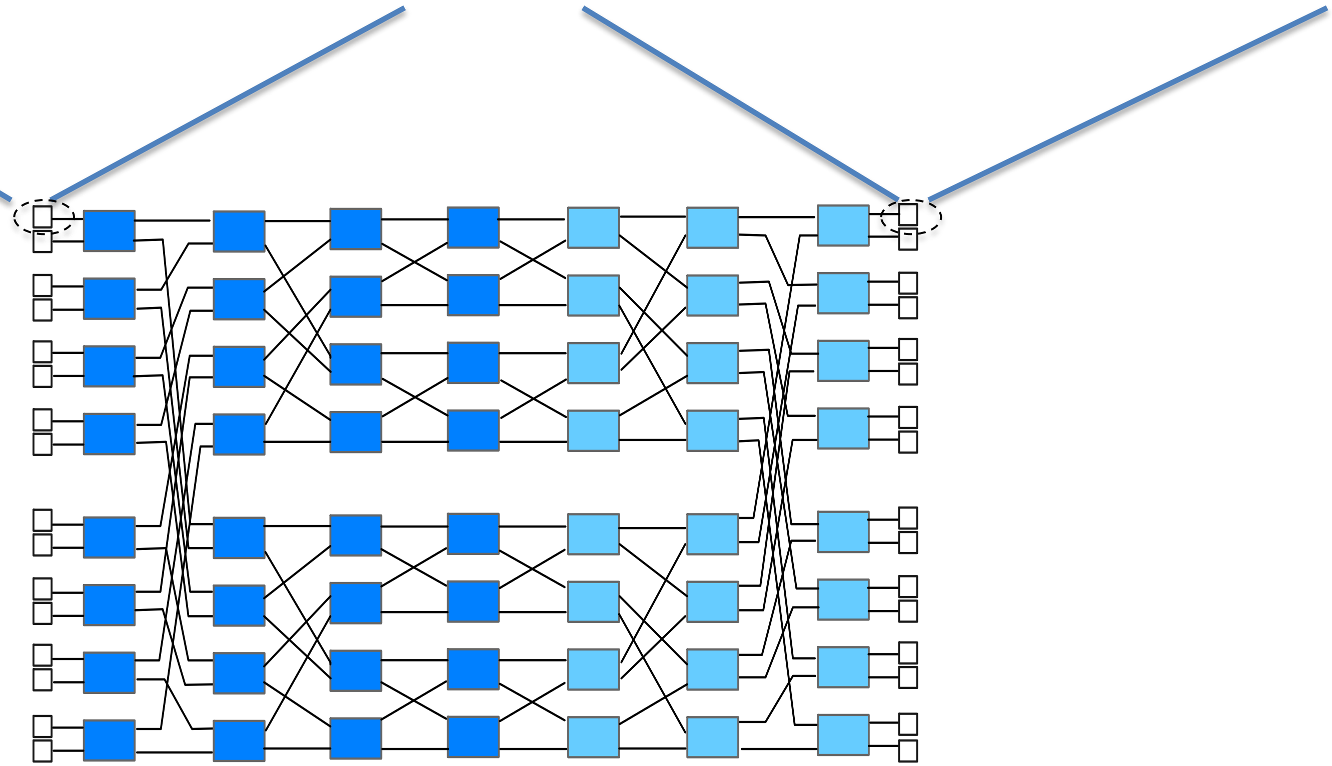
G-BP Component 3 – Admission & Regulation

Admission queue at input:



Regulation queue at output:





G-BP Component 3 – Admission & Regulation

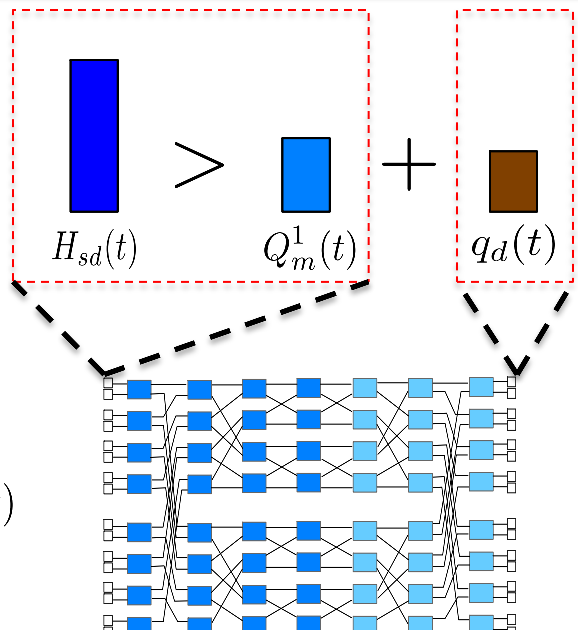
Admission decisions at input:

- Update $\gamma_{sd}(t)$:

$$\max : \quad VU_{sd}(\gamma_{sd}(t)) - H_{sd}(t)\gamma_{sd}(t),$$
$$\text{s.t.} \quad 0 \leq \gamma_{sd}(t) \leq A_{\max}$$

- Admit packets:
(up flow to d in D_1)

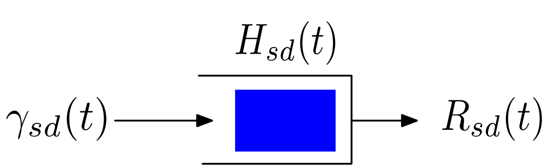
$$R_{sd}(t) = \begin{cases} A_{sd}(t) & \text{if } H_{sd}(t) > Q_m^1(t) + q_d(t), \\ 0 & \text{else.} \end{cases}$$

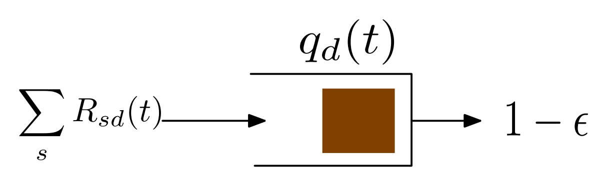


Note: $q_d(t)$ is "idealized"

In practice:
- delayed arrivals at d
- delayed feedback to s

Input server admits pkts





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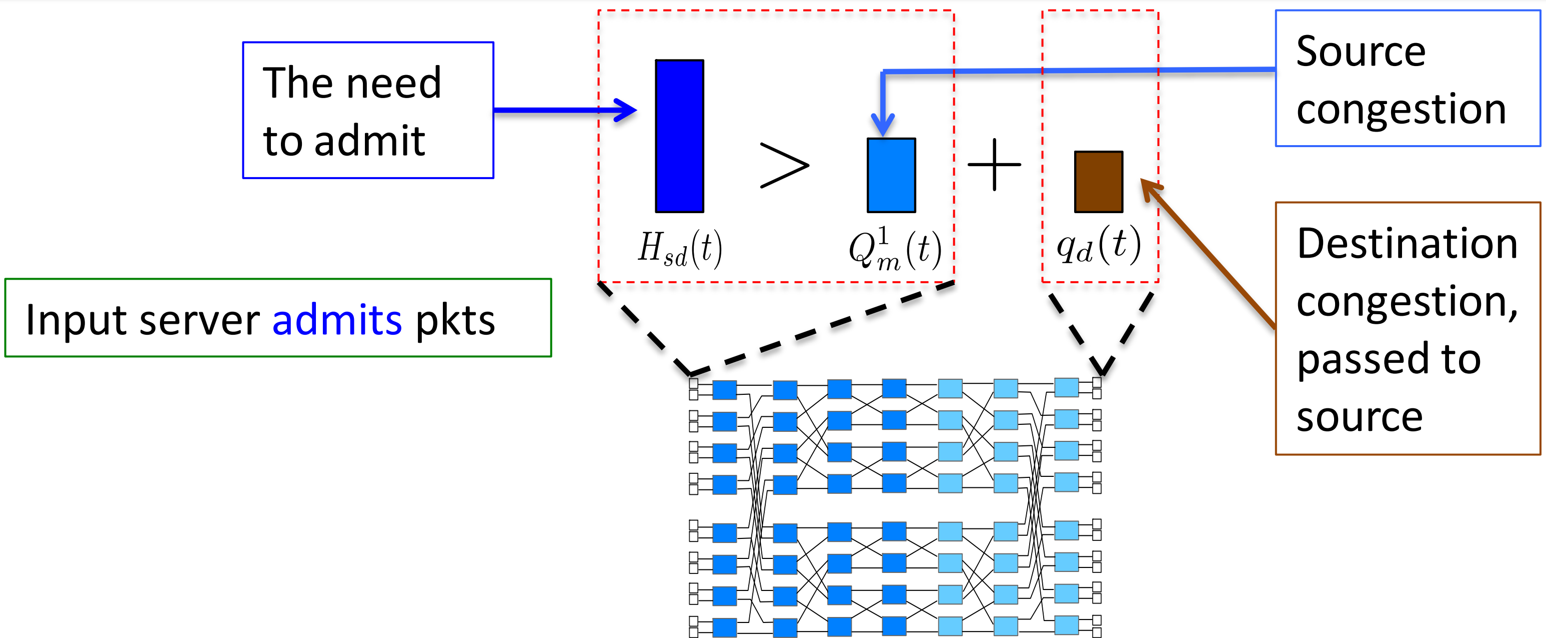

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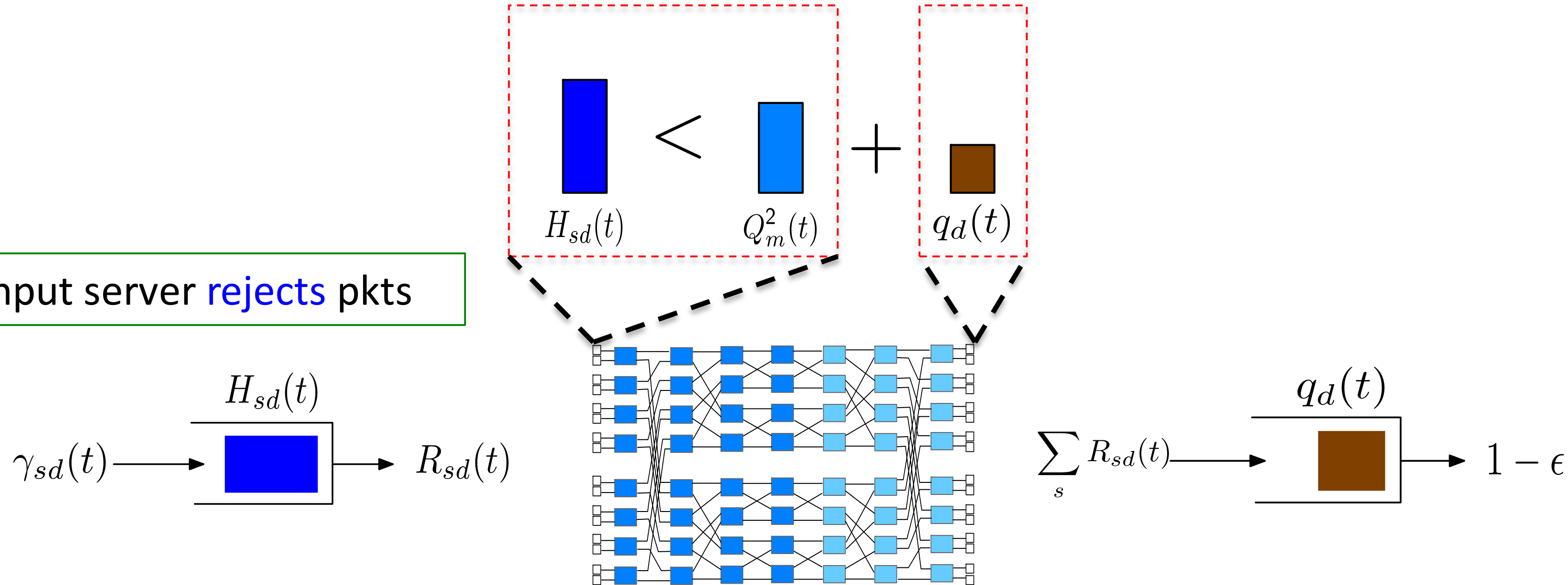
$$\begin{aligned} \max: \quad & VU_{sd}(\gamma_{sd}(t)) - H_{sd}(t)\gamma_{sd}(t), \\ \text{s.t.} \quad & 0 \leq \gamma_{sd}(t) \leq A_{\max} \end{aligned}$$

- Admit packets:
(low flow to d in D_2)

$$R_{sd}(t) = \begin{cases} A_{sd}(t) & \text{if } H_{sd}(t) > Q^2_m(t) + q_d(t), \\ 0 & \text{else.} \end{cases}$$

Input server rejects pkts



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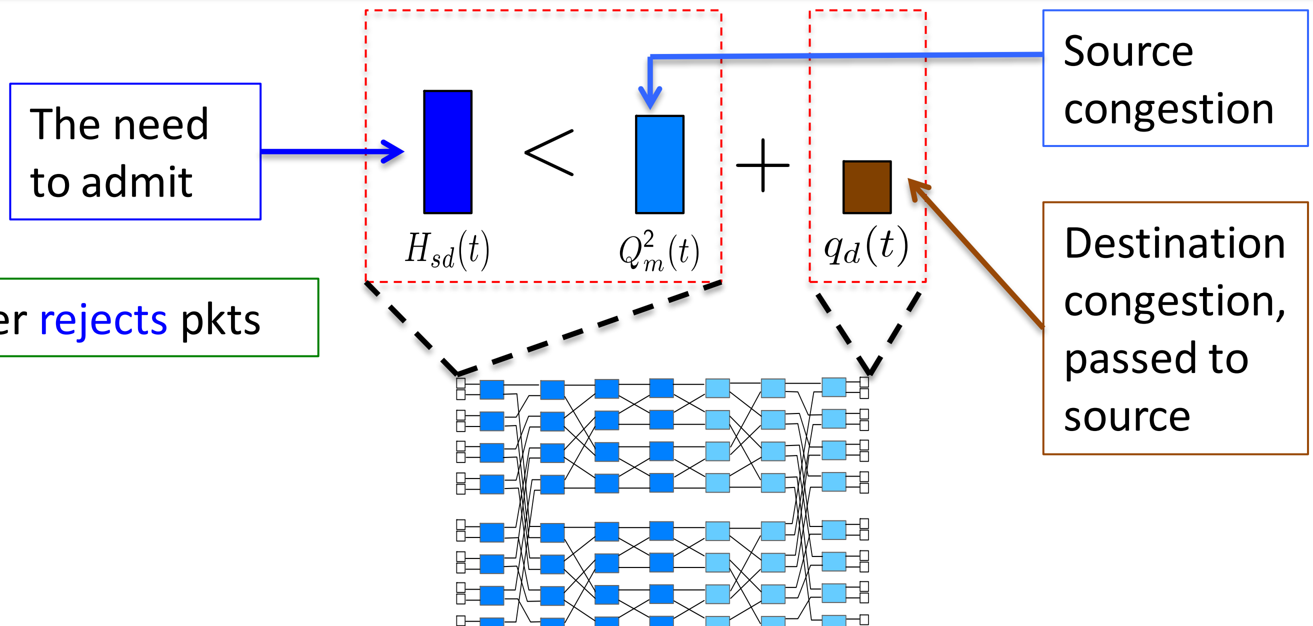
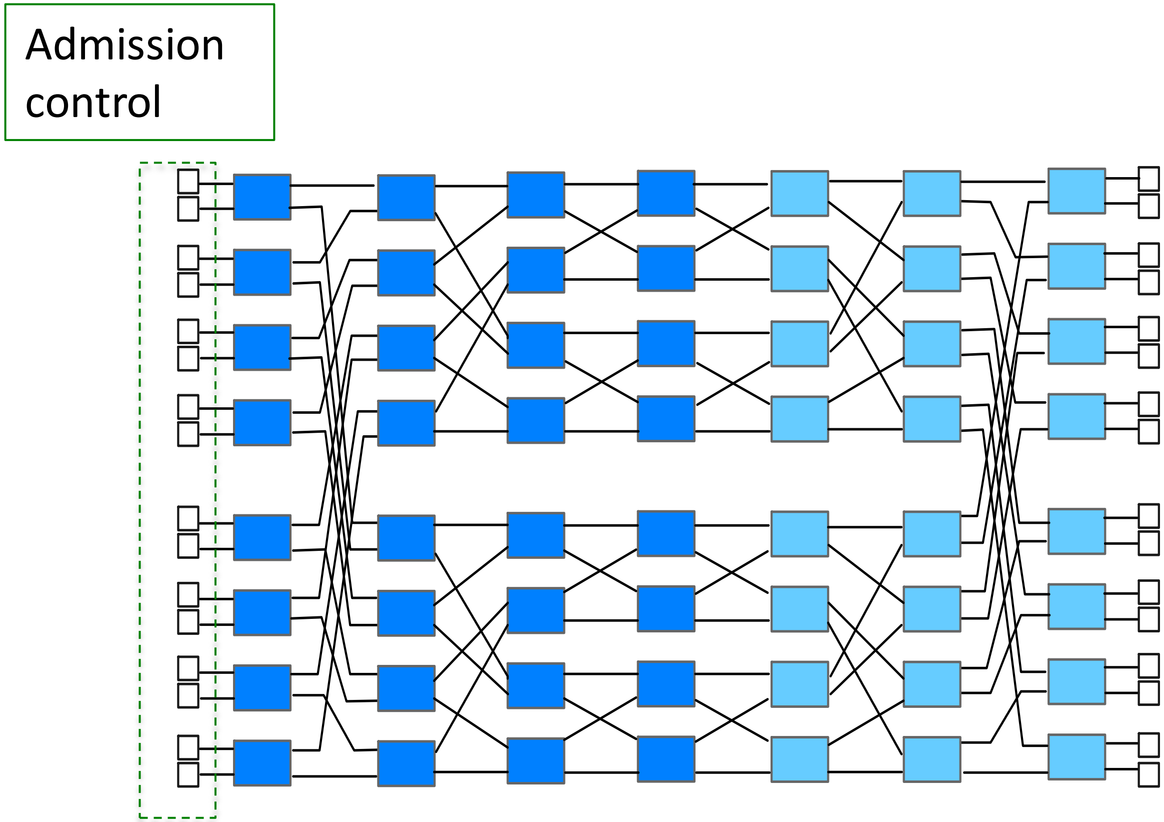
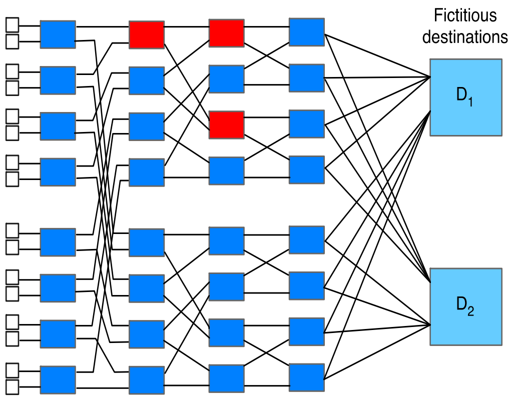


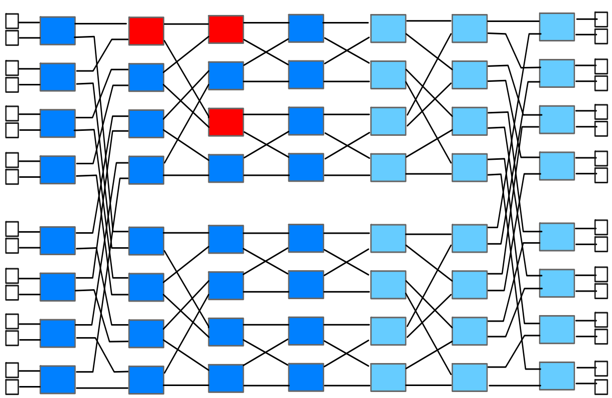
Input server rejects pkts

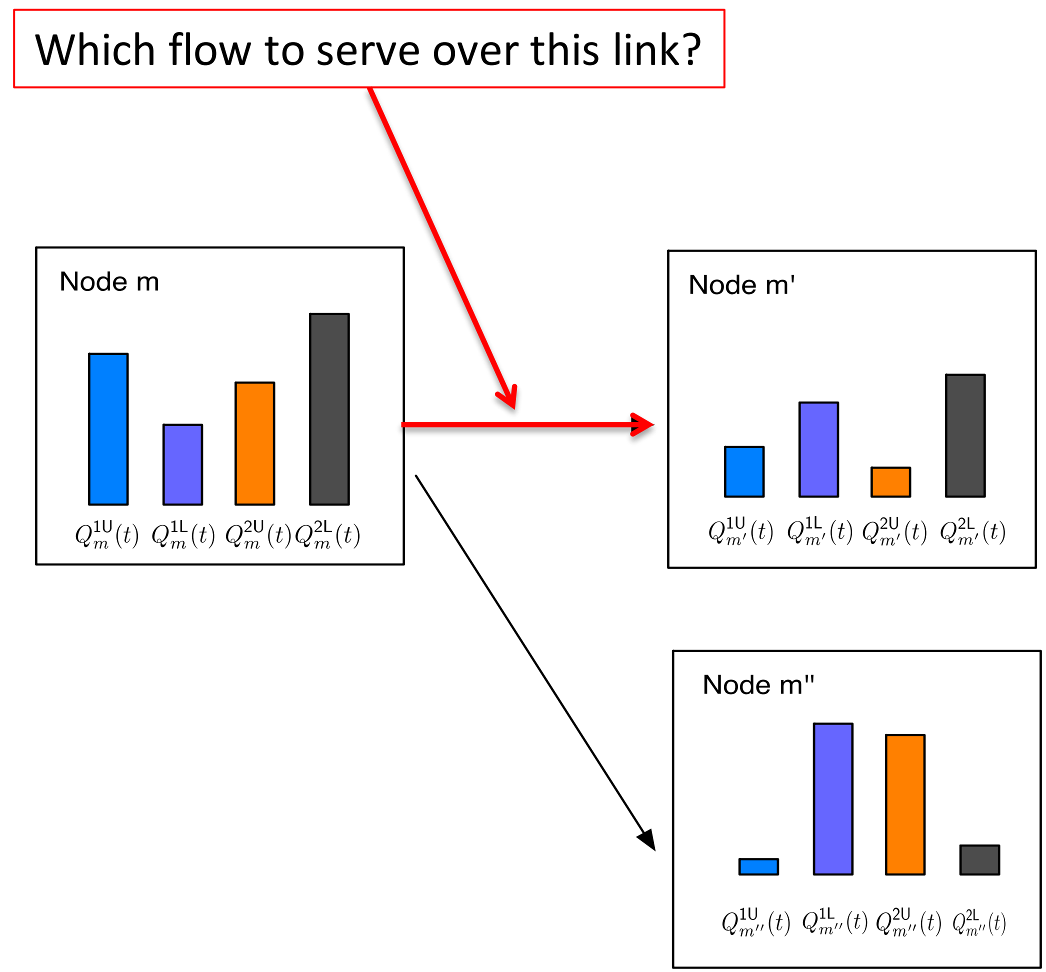
Grouped-Backpressure



G-BP Component 4 – Dynamic Scheduling







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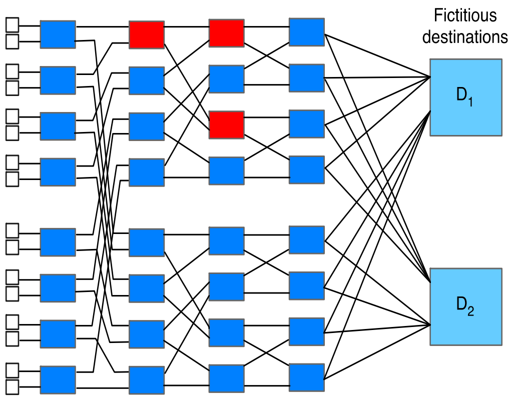
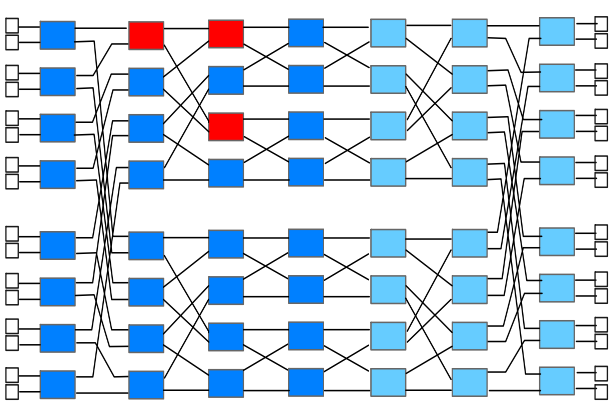
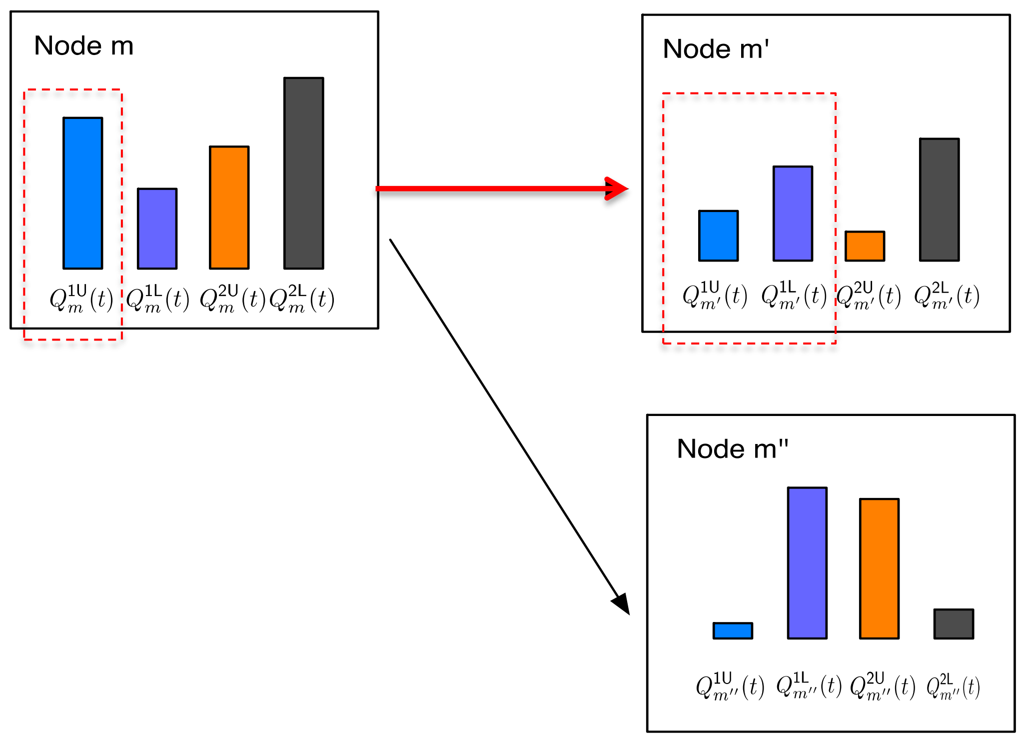


Define flow weights:

$$W^{1\mathrm{U}} = \left[2Q_m^{1\mathrm{U}}(t) - Q_{m'}^{1\mathrm{U}}(t) - Q_{m'}^{1\mathrm{L}}(t)\right]^+$$





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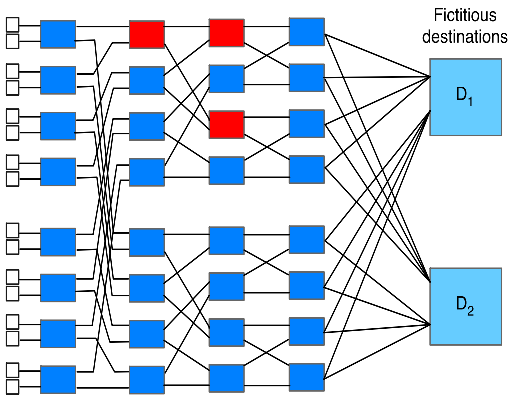

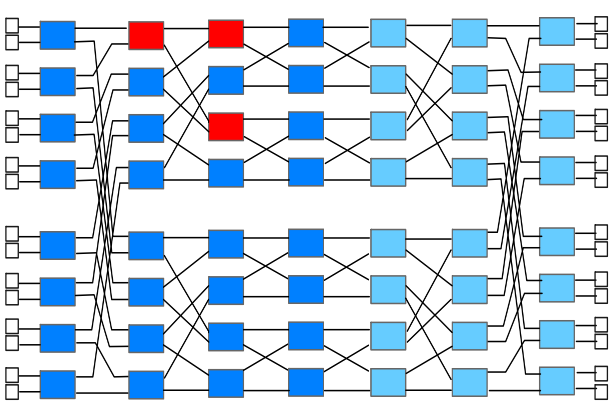

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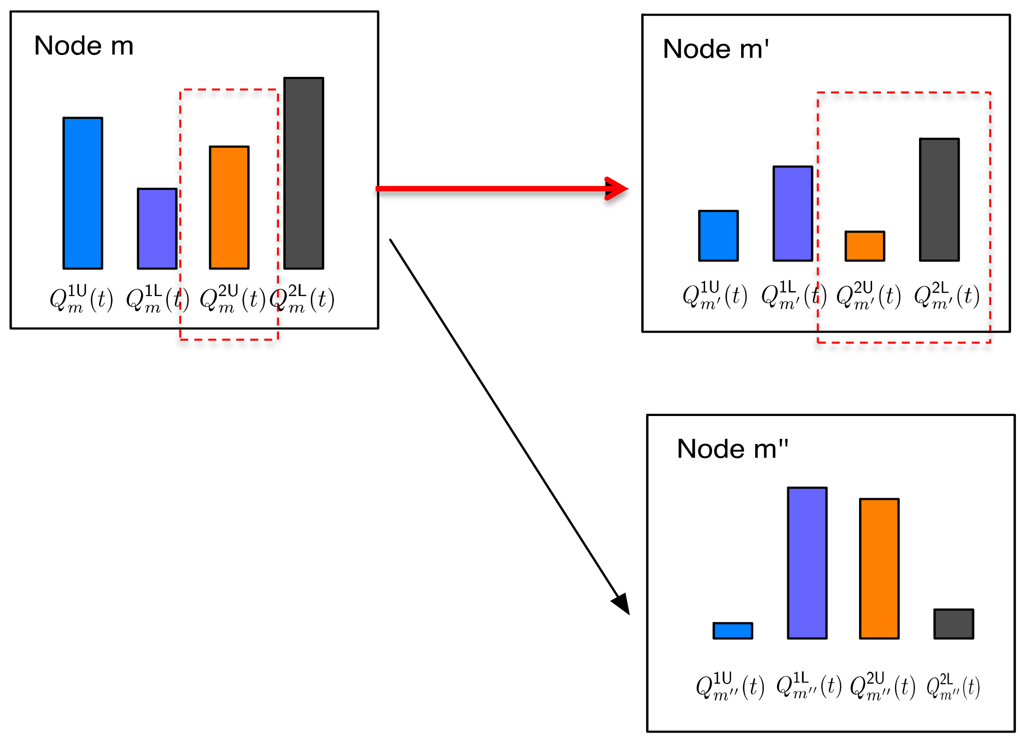

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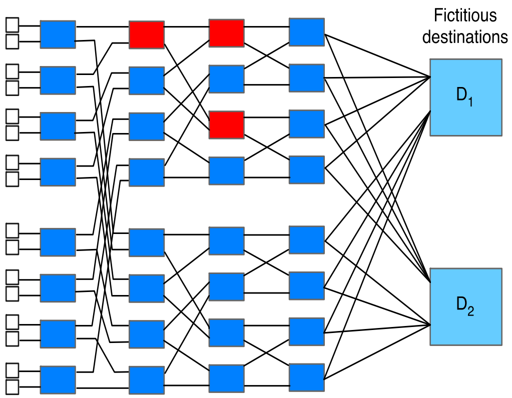
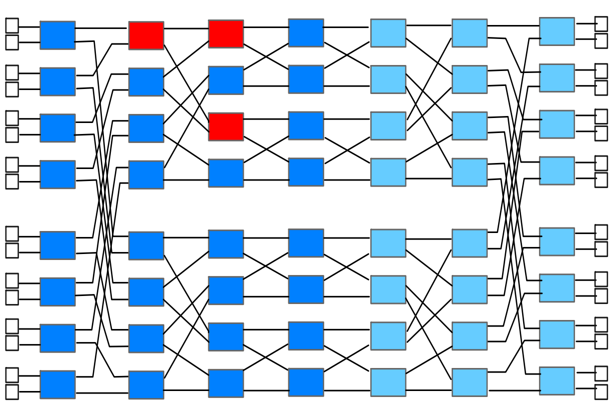
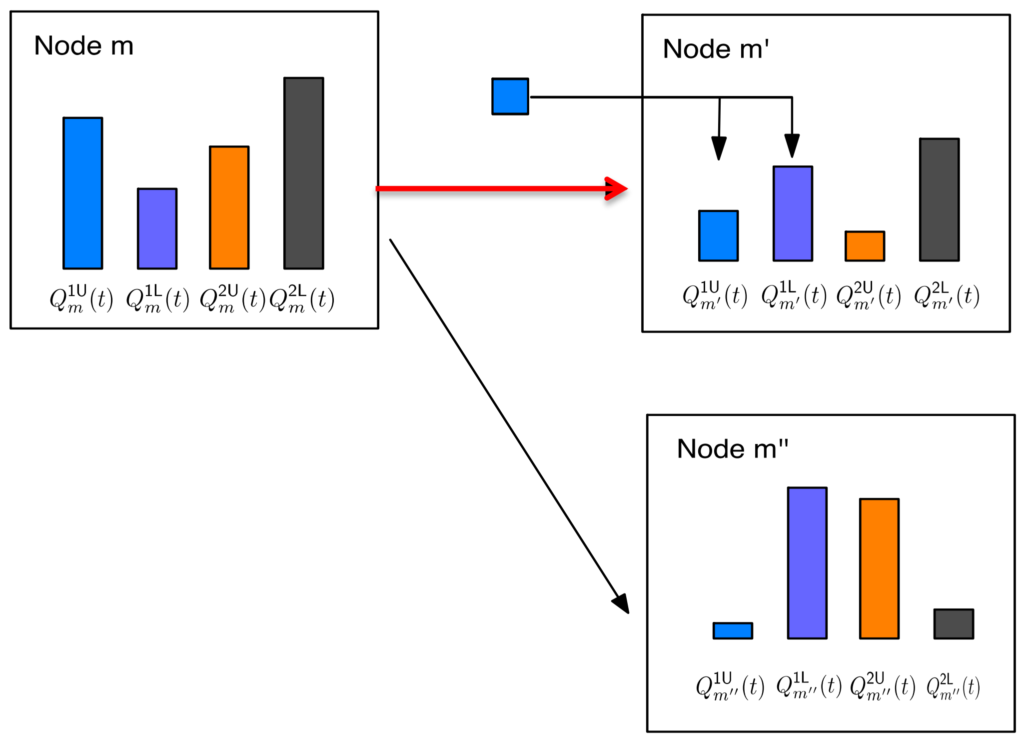
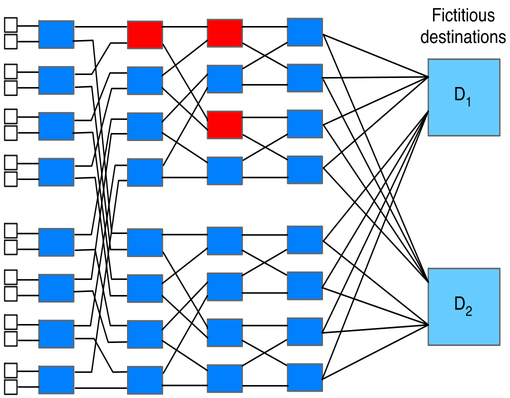




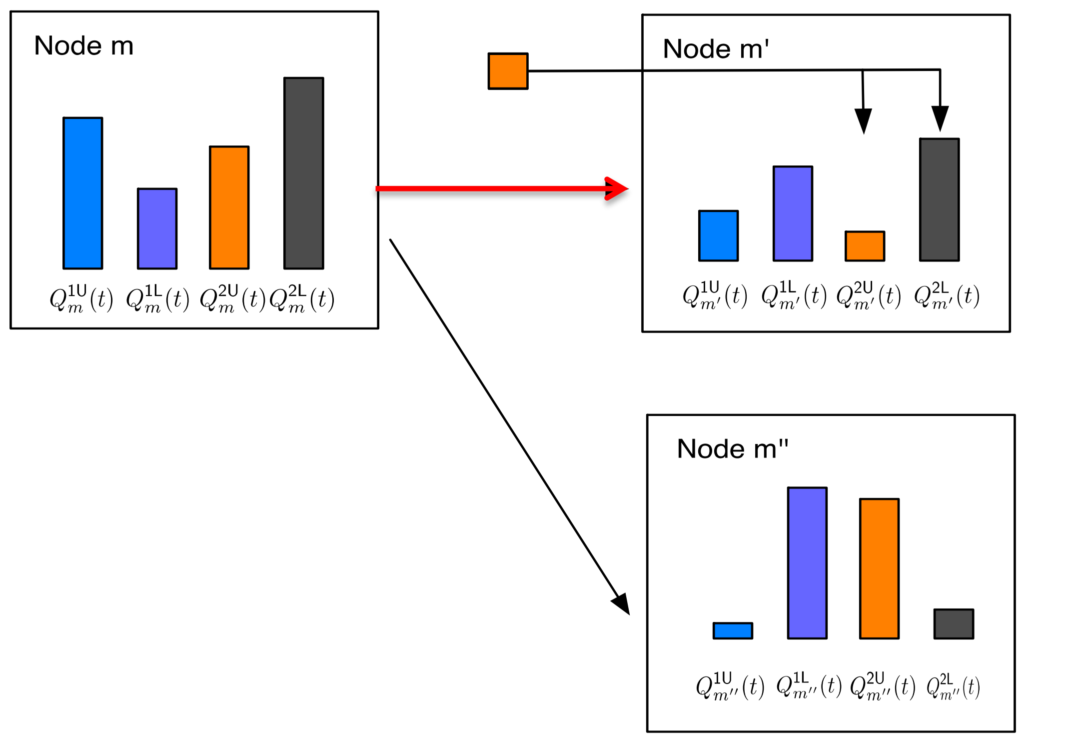
- If $W^{1U} > W^{2U}$ & $W^{1U} > 0$, send 1U packets over link [m, m’]
- At m’, *randomly* put the arrival into 1U or 1L

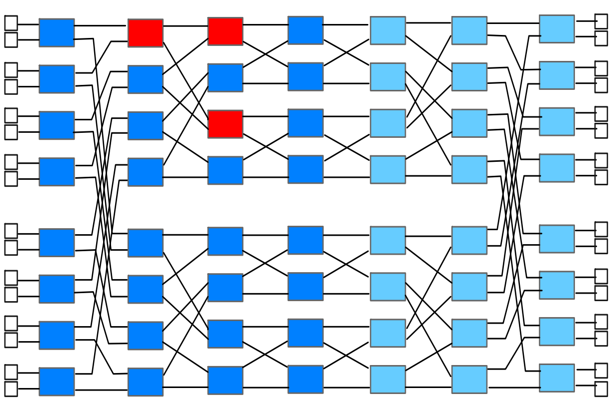


G-BP Component 4 – Dynamic Scheduling



- If $W^{1U} < W^{2U}$ & $W^{2U} > 0$, send 2U packets over link [m, m’]
- At m’, *randomly* put the arrival into 2U or 2L





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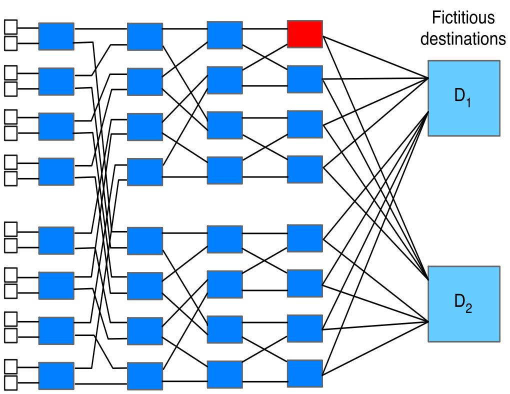
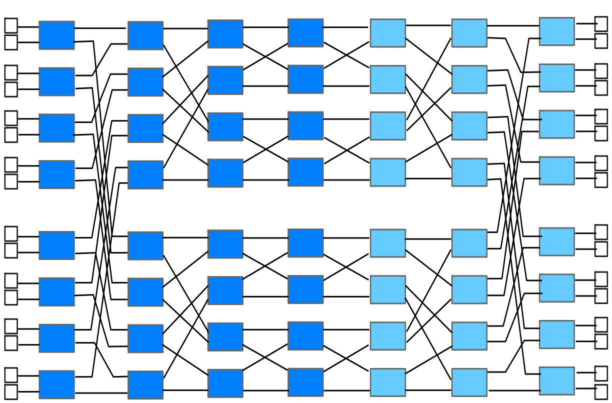
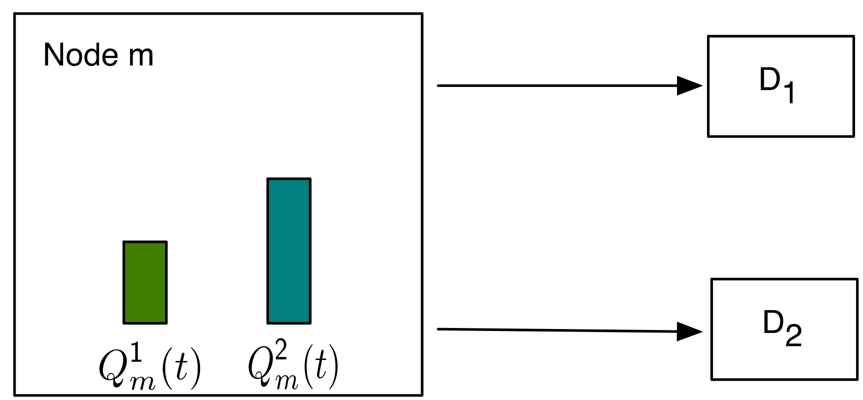
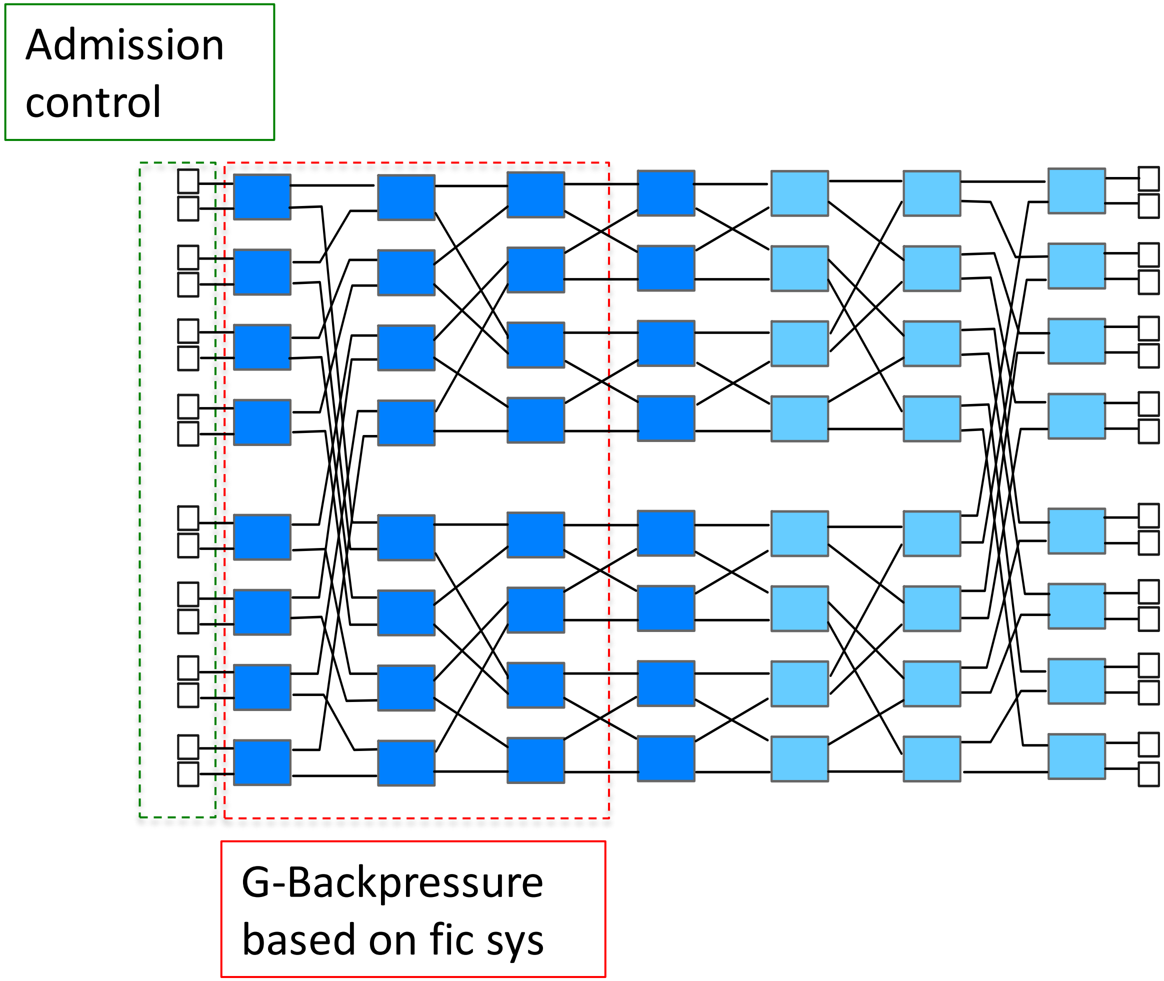


- If queue is not empty, transmit packet
- Else remain idle





Grouped-Backpressure



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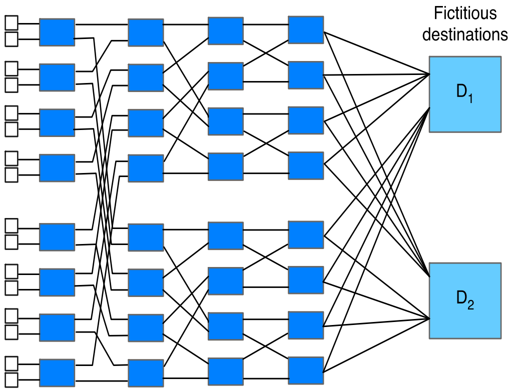
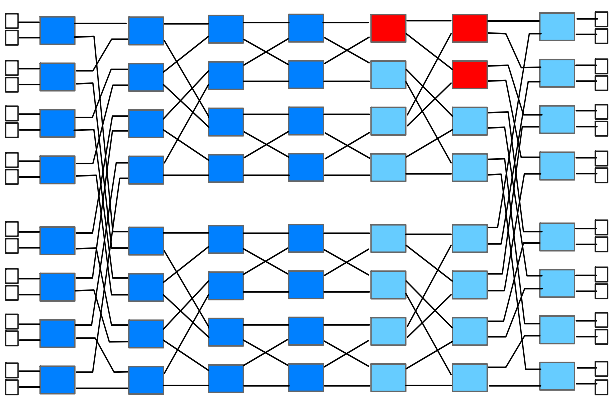
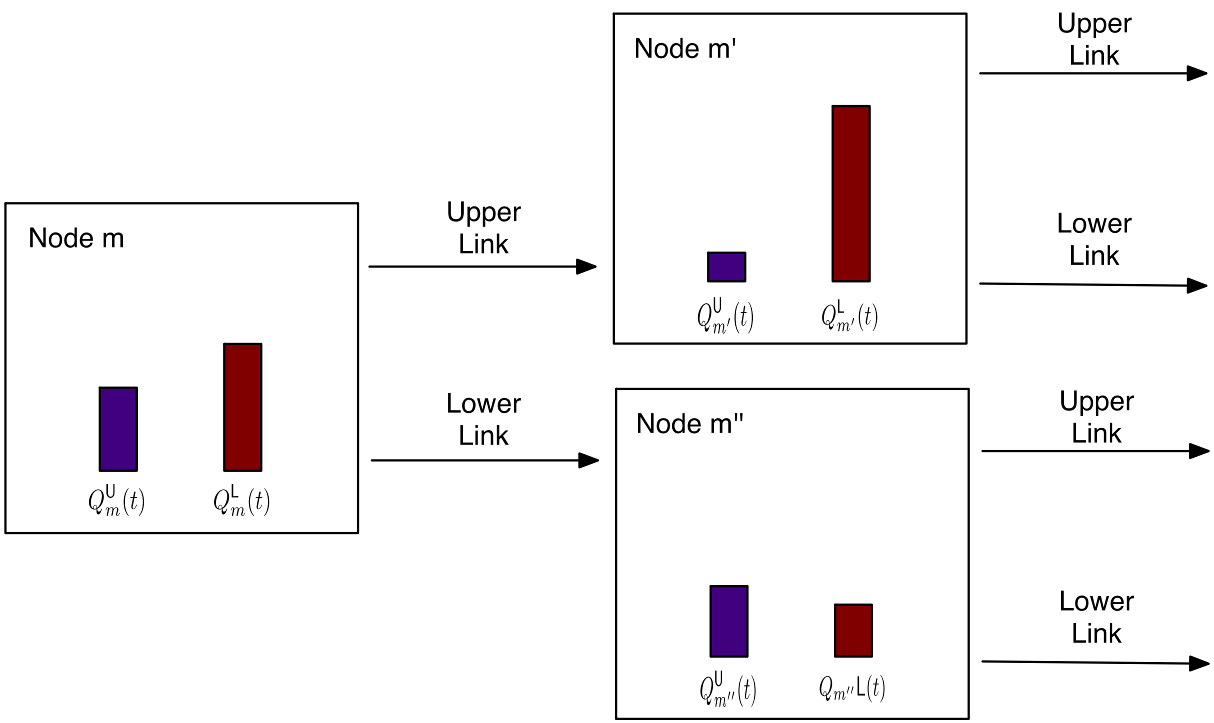


- If queue is not empty, transmit packet
- Place packets into corresponding queues

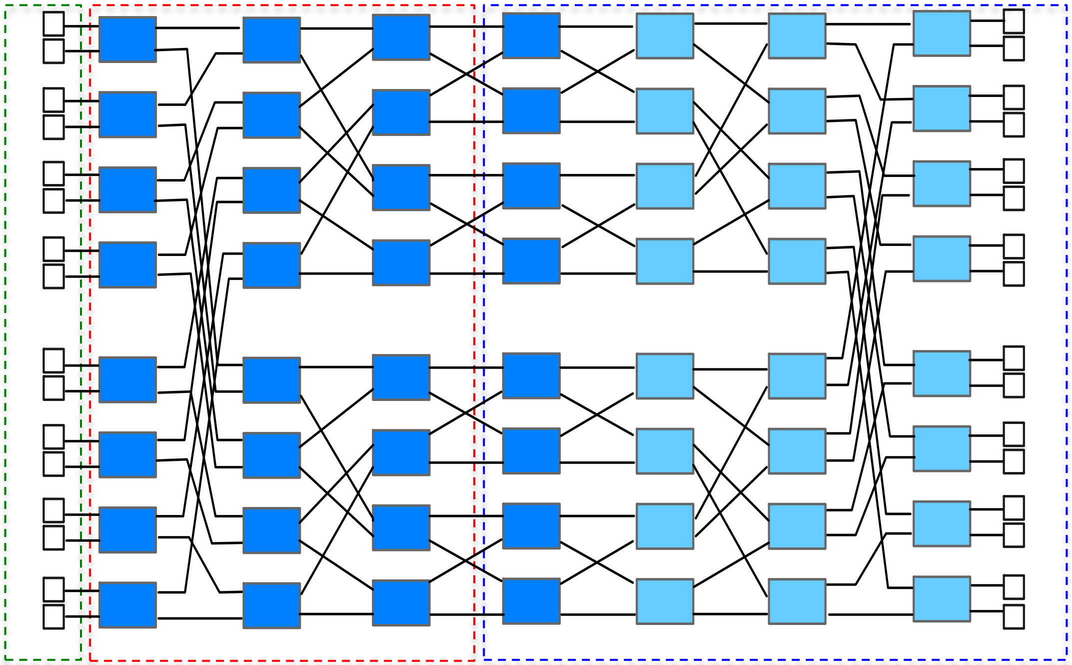




Grouped-Backpressure

Admission control

Free-flow forwarding



G-Backpressure based on fic sys

Grouped-Backpressure – Performance

Theorem: Under the G-BP* algorithm, (i) both physical & fictitious networks are stable, and (ii) we achieve:

$$U(\boldsymbol{r}^{\text{G-BP}}) \geq U(\boldsymbol{r}^{\text{opt}}) - O(\frac{1}{V} + \epsilon)$$

* This is the idealized algorithm ….

Grouped-Backpressure – Performance

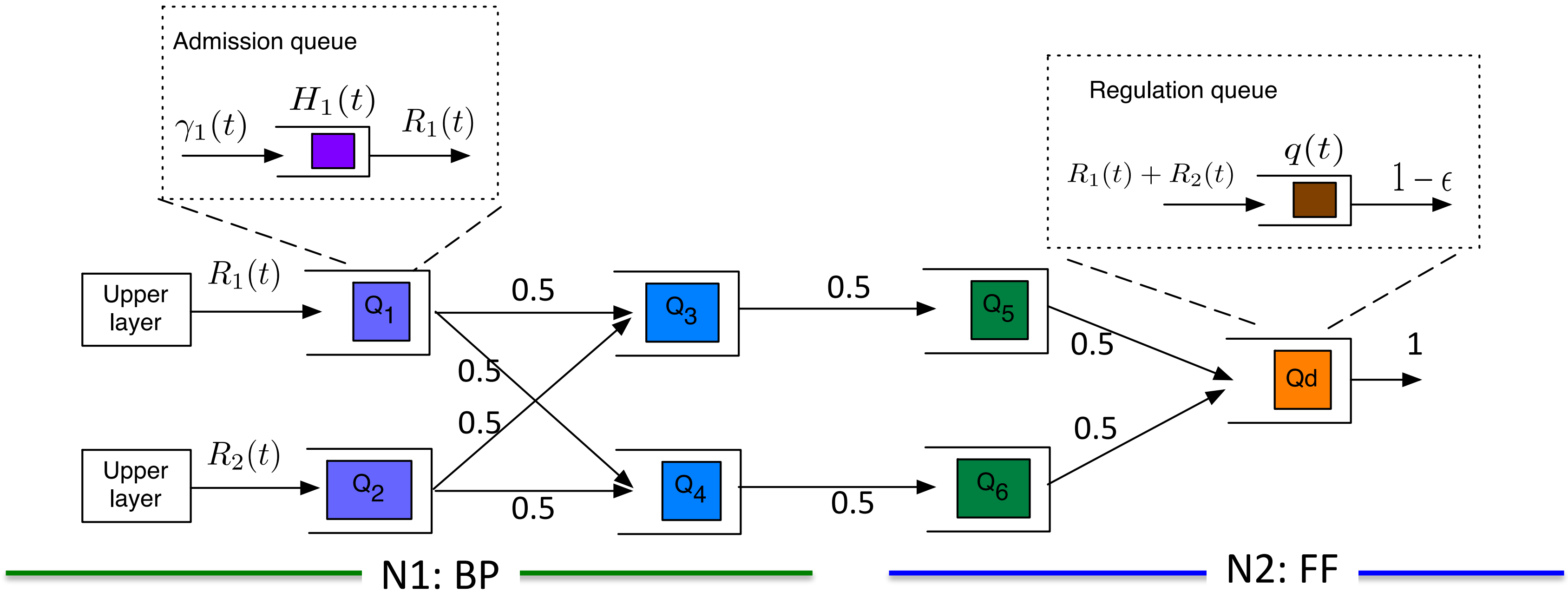
Theorem: Under the G-BP algorithm, (i) both physical & fictitious networks are stable, and (ii) we achieve:

$$U(\boldsymbol{r}^{\text{G-BP}}) \geq U(\boldsymbol{r}^{\text{opt}}) - O(\frac{1}{V} + \epsilon)$$

Remarks:

- No statistical info is needed
- Distributed hop-by-hop routing & scheduling
- Four queues per node (BP needs 2^n)

Grouped-Backpressure – Analysis Idea

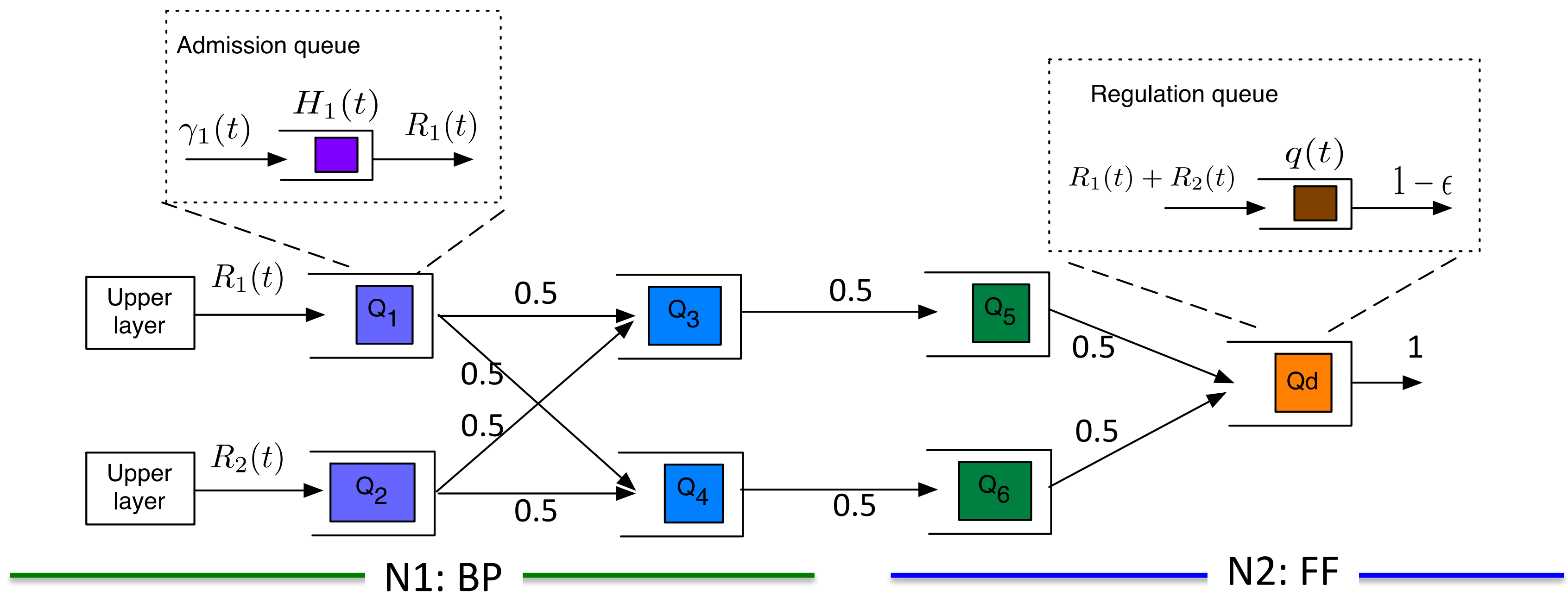


- Update $\gamma_i(t)$

$$\max: \quad VU(\gamma_i) - H_i(t)\gamma_i$$
$$\text{s.t.} \quad \gamma_i \in [0, A_{\max}]$$

- Admit packets:
    - If $H_i(t)>Q_i(t)+q(t)$, admit arrivals
    - Else, do not admit

Grouped-Backpressure – Analysis Idea



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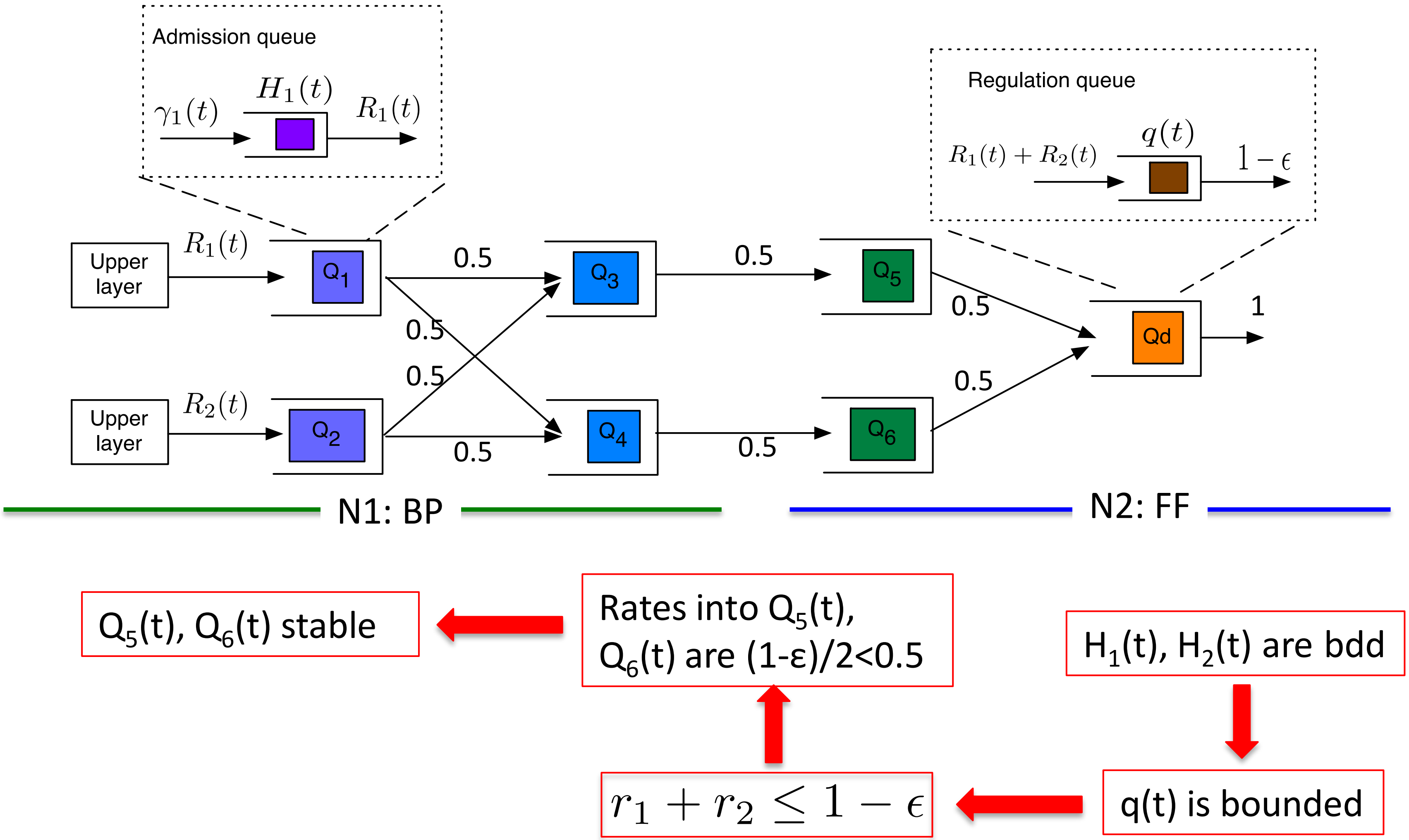
→ $H_1(t)$, $H_2(t)$ are bdd

↓

- Admit packets:
  - If $H_i(t) > Q_i(t) + q(t)$, admit arrivals
  - Else, do not admit

→ q(t) is bounded

Grouped-Backpressure – Analysis Idea



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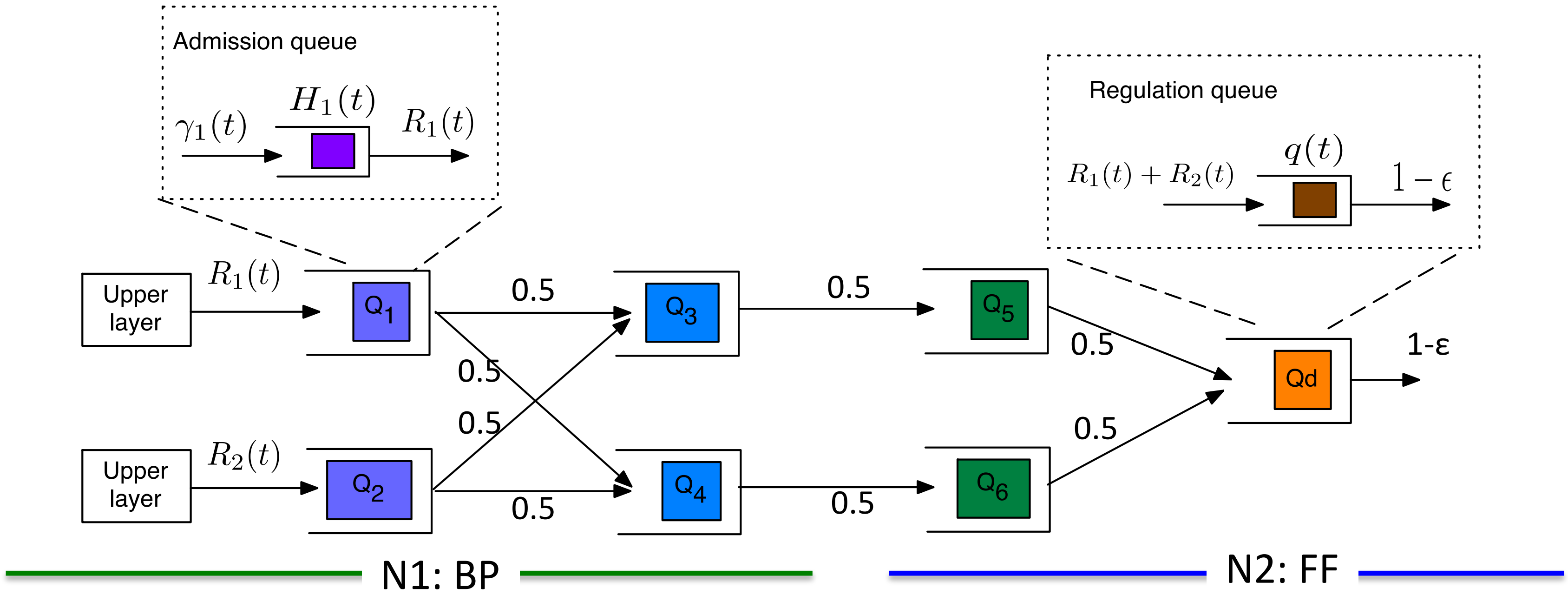


$Q_5(t)$, $Q_6(t)$ stable

$Q_1(t)$ – $Q_4(t)$ stable by Backpressure

Network stability

Grouped-Backpressure – Intuition

The flow optimization problem:

$$\begin{aligned} \max: \quad & VU(r) \\ \text{s.t.} \quad & r \leq 1 \end{aligned}$$

Due to the random arrival

The augmented & relaxed flow opt problem:

$$\begin{aligned} \max: \quad & VU(\gamma) \\ \text{s.t.} \quad & \gamma \leq r \\ & r \leq 1 - \epsilon \end{aligned}$$

Taking the dual decomposition

The dual form:

$$\begin{aligned} g(H, q) &= \sup_{\gamma, r} \left\{ VU(\gamma) - H(\gamma - r) - q(r - (1 - \epsilon)) \right\} \\ &= \sup_{\gamma, r} \left\{ VU(\gamma) - H\gamma + (H - q)r + q(1 - \epsilon) \right\} \end{aligned}$$

Grouped-Backpressure – Intuition

The flow optimization problem:

$$\max: \quad VU(r)$$
$$\text{s.t.} \quad r \le 1$$

Due to the random arrival

The augmented & relaxed flow opt problem:

$$\max: \quad VU(\gamma)$$
$$\text{s.t.} \quad \gamma \le r$$
$$r \le 1 - \epsilon$$

Taking the dual decomposition

Admission queue

Data queue

The dual form:

$$g(H,q) = \sup_{\gamma, r} \left\{ VU(\gamma) - H(\gamma - r) + q(r - (1-\epsilon)) \right\}$$
$$= \sup_{\gamma, r} \left\{ VU(\gamma) - H\gamma + (H-q)r + q(1-\epsilon) \right\}$$

Grouped-Backpressure – Proof Steps

Step 1 - Define a Lyapnov function:

$$L(t) \triangleq \frac{1}{2}H^2(t) + \frac{1}{2}q^2(t)$$

Step 2 - Compute a Lyapunov drift Δ(t)=E{ L(t+1) - L(t) | X(t) }

$$\begin{aligned}\Delta(t) - V\mathbb{E}\{U(\gamma(t)) \mid X(t)\} \\ \leq\ & B - \mathbb{E}\{VU(\gamma(t)) + H(t)[R(t) - \gamma(t)] + q(t)[1 - \epsilon - R(t)] \mid X(t)\} \\ =\ & B - \mathbb{E}\{VU(\gamma(t)) - H(t)\gamma(t) + [H(t) - q(t)]R(t) + q(t)(1-\epsilon) \mid X(t)\}\end{aligned}$$

Step 3 - Plug in the opt solution of the relaxed problem, $\gamma_\varepsilon^* = r_\varepsilon^*$

$$\begin{aligned}\Delta(t) - V\mathbb{E}\{U(\gamma^{\text{GBP}}(t)) \mid X(t)\} \\ \leq\ & B - \mathbb{E}\{VU(\gamma_\epsilon^*) + H(t)[R_\epsilon^* - \gamma_\epsilon^*] + q(t)[1 - \epsilon - R_\epsilon^*] \mid X(t)\} \\ \leq\ & B - VU(\gamma_\epsilon^*)\end{aligned}$$

Step 4 - Do a telescoping sum

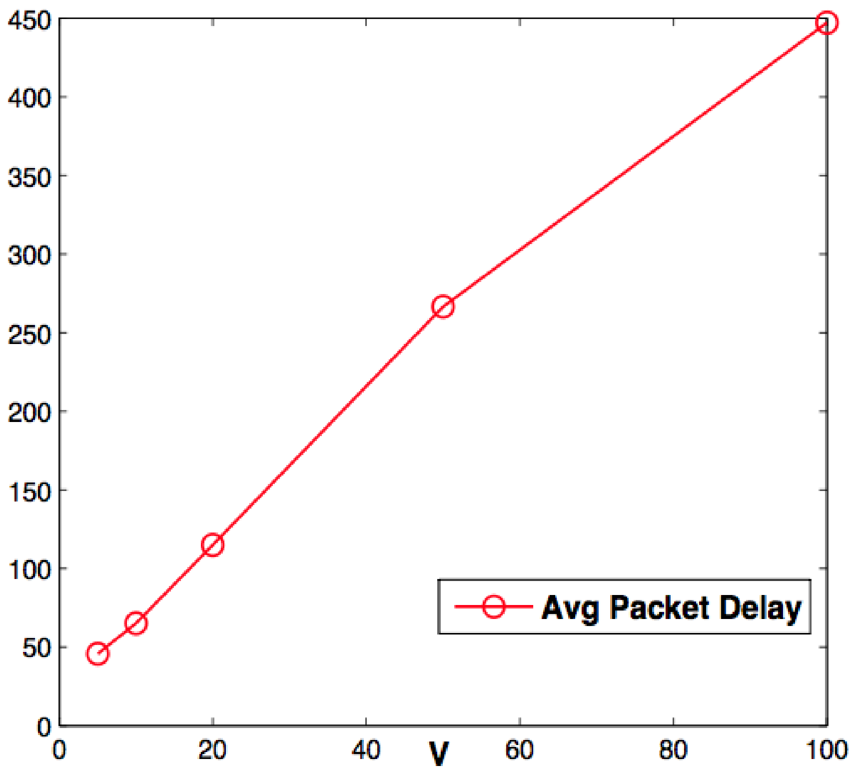
$$U(\overline{\gamma}^{\text{GBP}}) \geq U(r^*) - \frac{B}{V} - O(\epsilon)$$

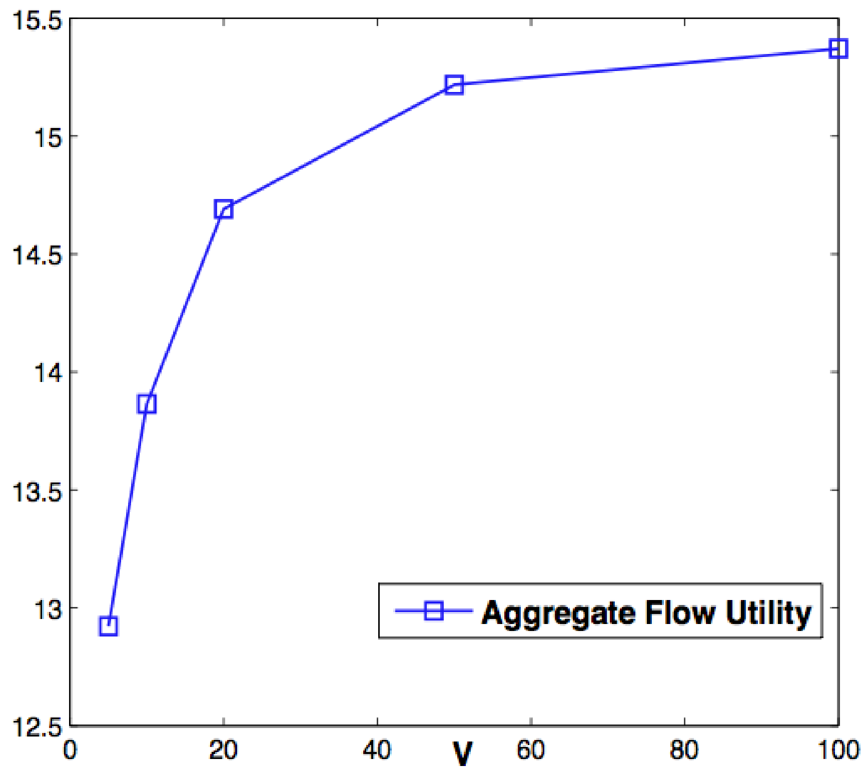
Step 5 - H(t) is stable

$$\overline{\gamma}^{\text{GBP}} \leq \overline{r}^{\text{GBP}} \Rightarrow U(\overline{r}^{\text{GBP}}) \geq U(r^*) - \frac{B}{V} - O(\epsilon)$$

Grouped-Backpressure – Simulation*

Setting: 16x16 Benes network, ε=0.01, utility=log(1+r)





* This is the idealized algorithm ….

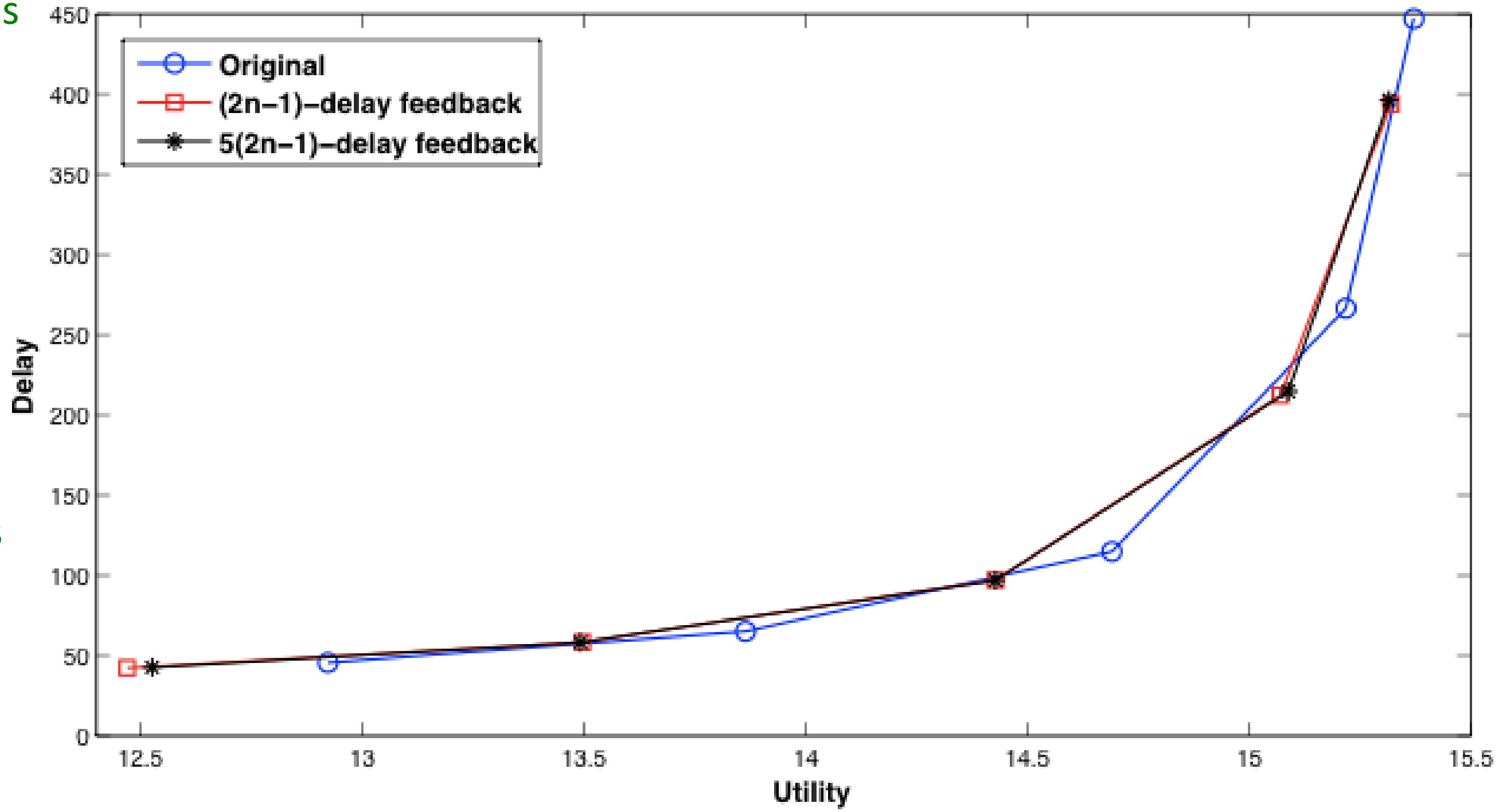
Grouped-Backpressure – Simulation

Setting: 16x16 Benes network, ε=0.01, utility=log(1+r)

1.8ms

1ms

0.5ms

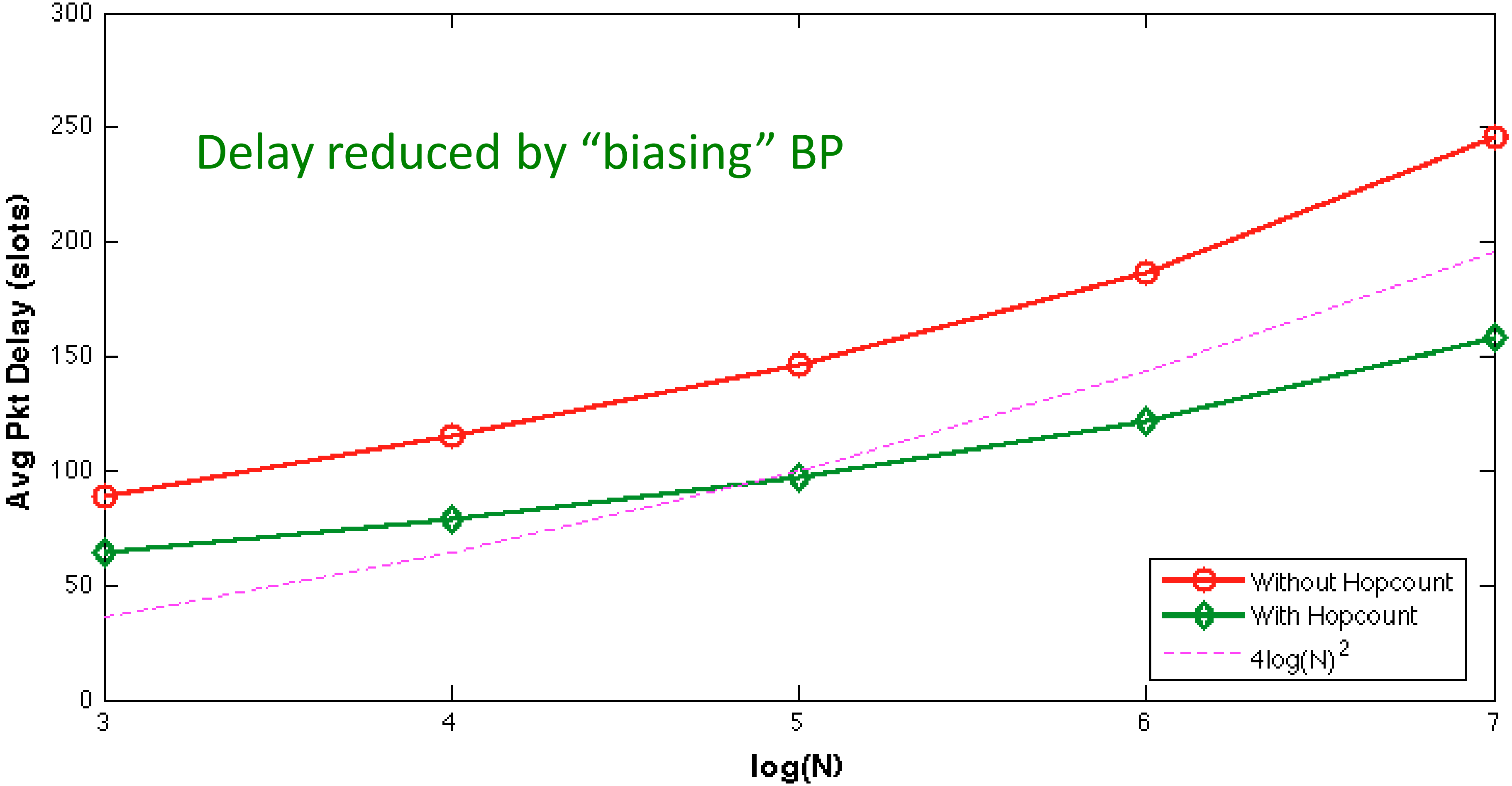


Note: For 1Gbps links and 500-Byte packets

Grouped-Backpressure – Simulation

Delay versus network size – logarithmic growth

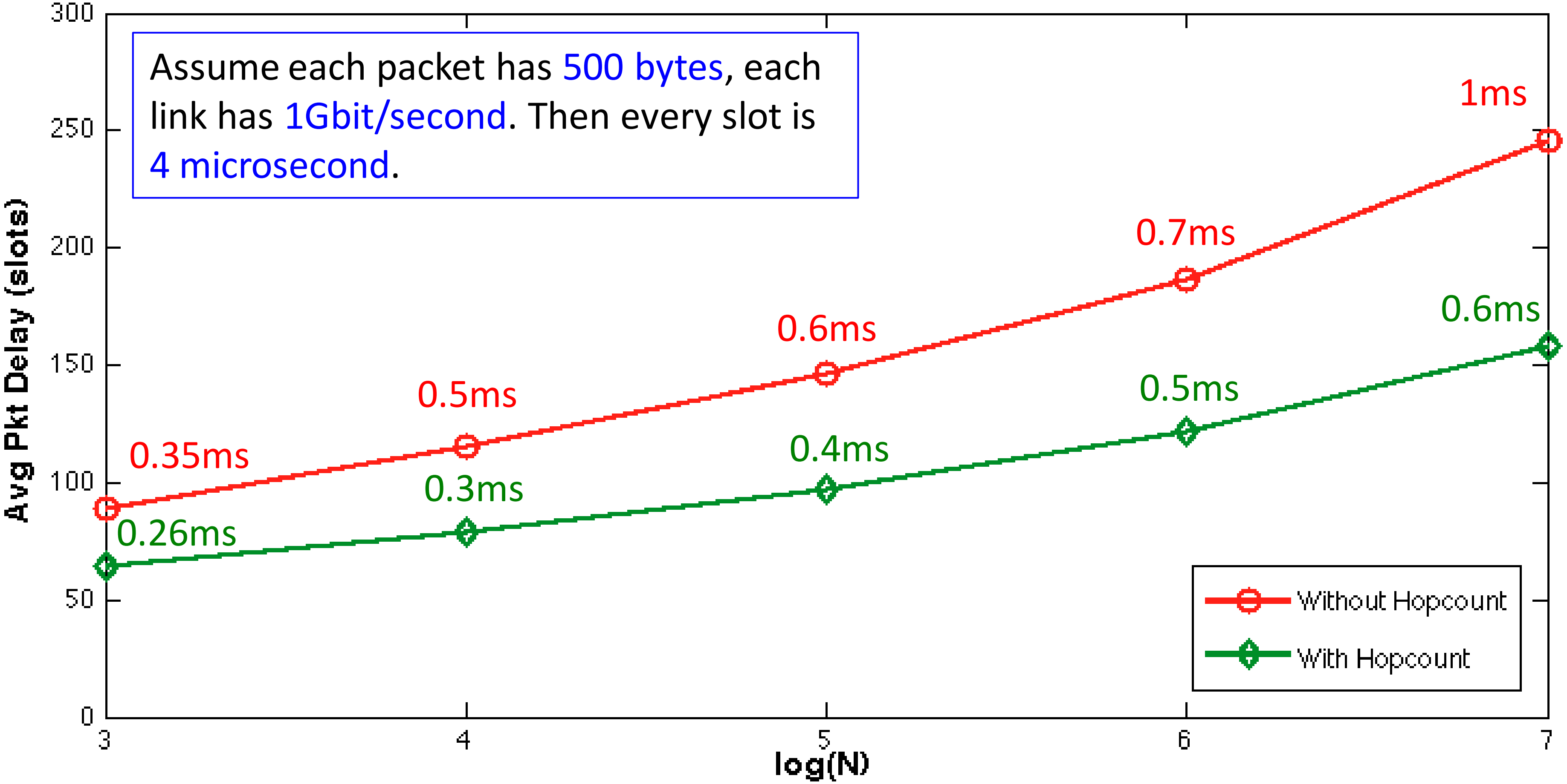
V=20, ε=0.01



Grouped-Backpressure – Simulation

Delay versus network size – logarithmic growth

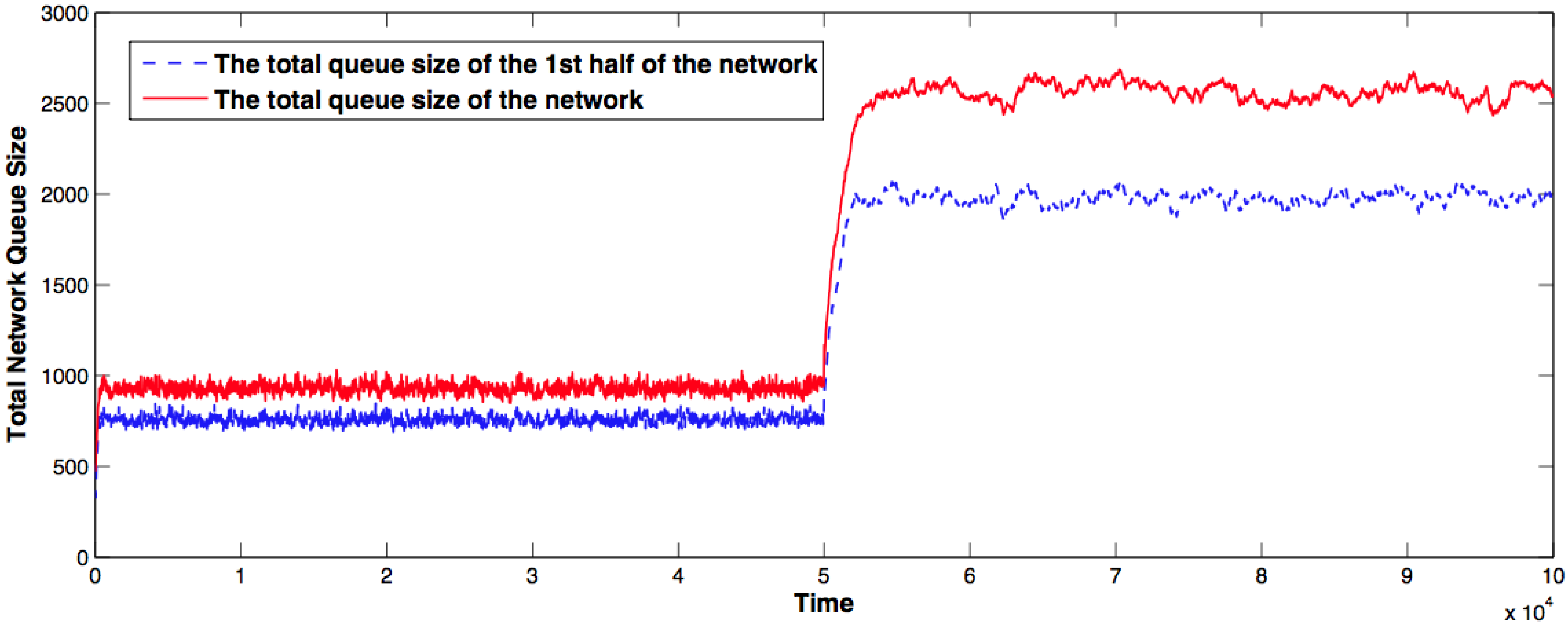
V=20, ε=0.01



Grouped-Backpressure – Simulation

Setting: 16x16 Benes network, ε=0.01, utility=wlog(1+r)

Adaptation to change of traffic – At time 5, weights w_{sd} change



Summary

- Using Benes network and Backpressure for data center networking
    - Scalable: built with basic switch modules
    - Simple: four queues per node
    - Small delay: logarithmic in network size
    - High throughput: supports all rates in capacity region
    - Distributed: hop-by-hop routing and scheduling

- Future research: Implementation issues

Thank you very much !

More info: www.eecs.berkeley.edu/~huang