Recovering Circles and Spheres From LADAR Data

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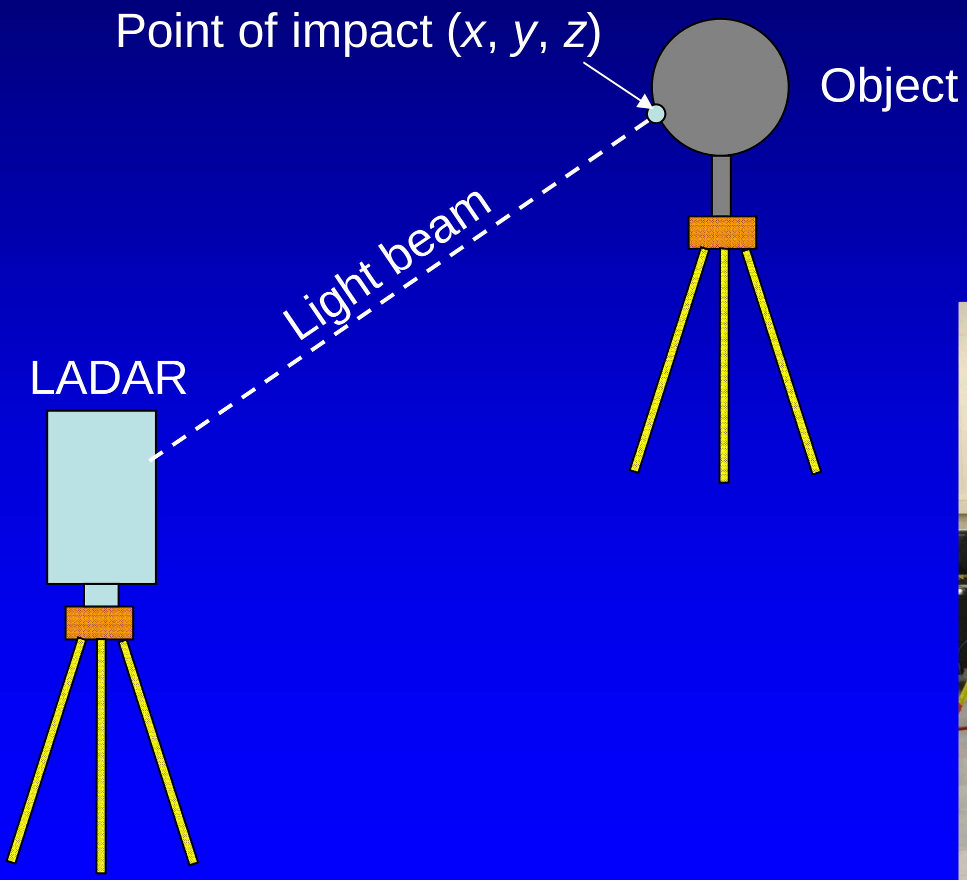
LADAR

LADAR – Laser Detection and Ranging

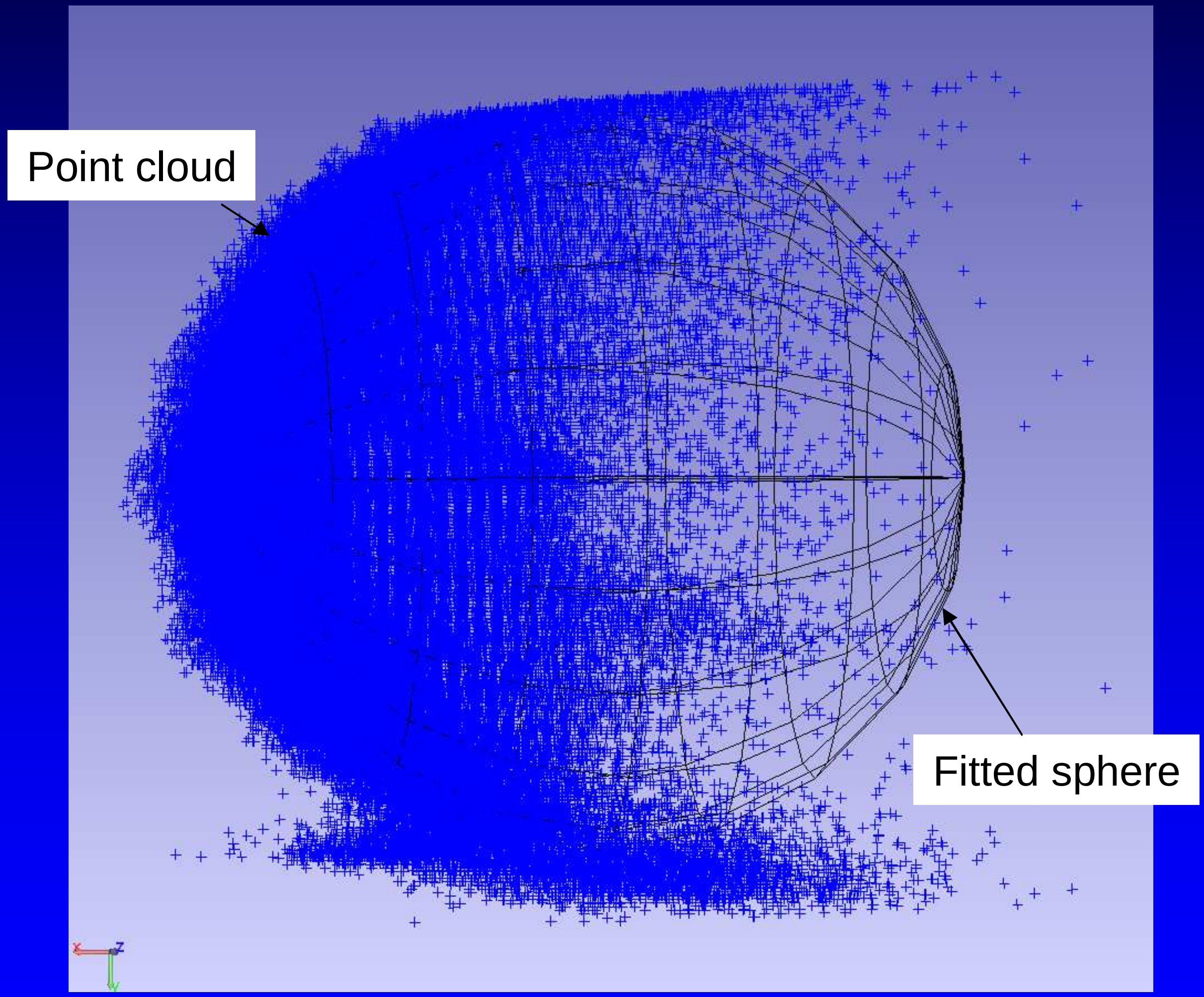


LADAR Facility at NIST

LADAR Imaging

This is not a photograph!

Point Cloud & Sphere



Fitting

- One way of recovering spheres from point clouds is by “fitting”
  - Select a “gauge” function = measure of deviation
  - Find the sphere that minimizes that gauge function
- Fitting spheres comes in two flavors:
  - Fixed radius
  - Variable radius

Gauge Functions

- **Define a gauge function**
  - Specify deviation concept:

$$\Delta_i = \Delta\left(\text{point } i \text{ from surface}\right)$$

  - Select norm $\|*\|$ for vector of deviation

$$\text{gauge function} = \left\|\Delta_1, \Delta_2, \ \ldots \ , \Delta_n\right\|$$

- **Common norms**
  - $\text{Max}\,|\,\Delta_i\,| \quad (L_\infty) \quad \text{"Chebychev"}$
  - $\sqrt{\sum \Delta_i^2} \quad (L_2) \quad \text{"Least squares"}$
  - $\sum |\,\Delta_i\,| \quad (L_1) \quad \text{"Least absolutes"}$

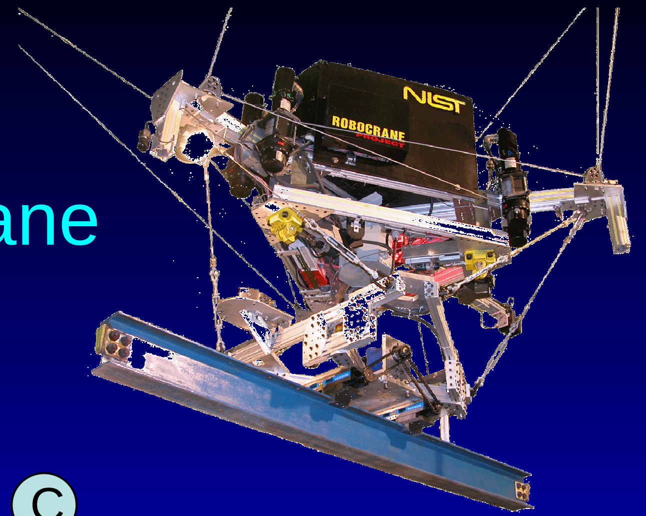
How well does fitting work?

or rather

What gauge function is appropriate?

Locating an I-Beam

For pick-up by an automated crane



The spheres are targets for “registration”.

Registration

Those coordinates are based on the instrument. They need to be registered to the real world coordinates.

The coordinates of the sphere centers, when determined both in real and instrument coordinates.

$\Rightarrow$ “Rosetta Stone”

Relating the two different coordinate systems.

A Red Flag

Repeated measurements (n ~ 90)

$\Rightarrow$ Trouble at Sphere C

Radius	Algebraic Fitting	Geometric Fitting
Average (cm)	5.4	6.9
σ (cm)	0.1	0.3
True (cm)	7.6	7.6

Circles vs. Spheres

Fitting methods for spheres are

Analogous to

Fitting methods for circles

Discussion of fitting methods will be in terms of circles

Algebraic Fitting

- Measure-of-deviation is compliance with equation

$$x^2 + y^2 + Ax + By + C = 0$$

- Use least squares for gauge function

$$\text{Minimize} \ \sum_{\mathrm{i}} \left(x_i^2 + y_i^2 + Ax_i + By_i + C \right)^2$$

$\Rightarrow$Ordinary linear regression

$$-\left(x_i^2 + y_i^2 \right) \sim Ax_i + By_i + C$$

$\Rightarrow$Unique solution

$$A^*, B^*, C^*$$

Completing the Square

$$x^2 + y^2 + A^* x + B^* y + C^* = 0$$

can be written as

$$\left(x + \frac{A^*}{2}\right)^2 + \left(y + \frac{B^*}{2}\right)^2 - \left[\left(\frac{A^*}{2}\right)^2 + \left(\frac{B^*}{2}\right) - C^*\right] = 0$$

with

$$x^* = -\frac{A^*}{2} \quad , \quad y^* = -\frac{B^*}{2} \quad , \quad \left(r^*\right)^2 = \left[\left(\frac{A^*}{2}\right)^2 + \left(\frac{B^*}{2}\right)^2 - C^*\right]$$

this becomes

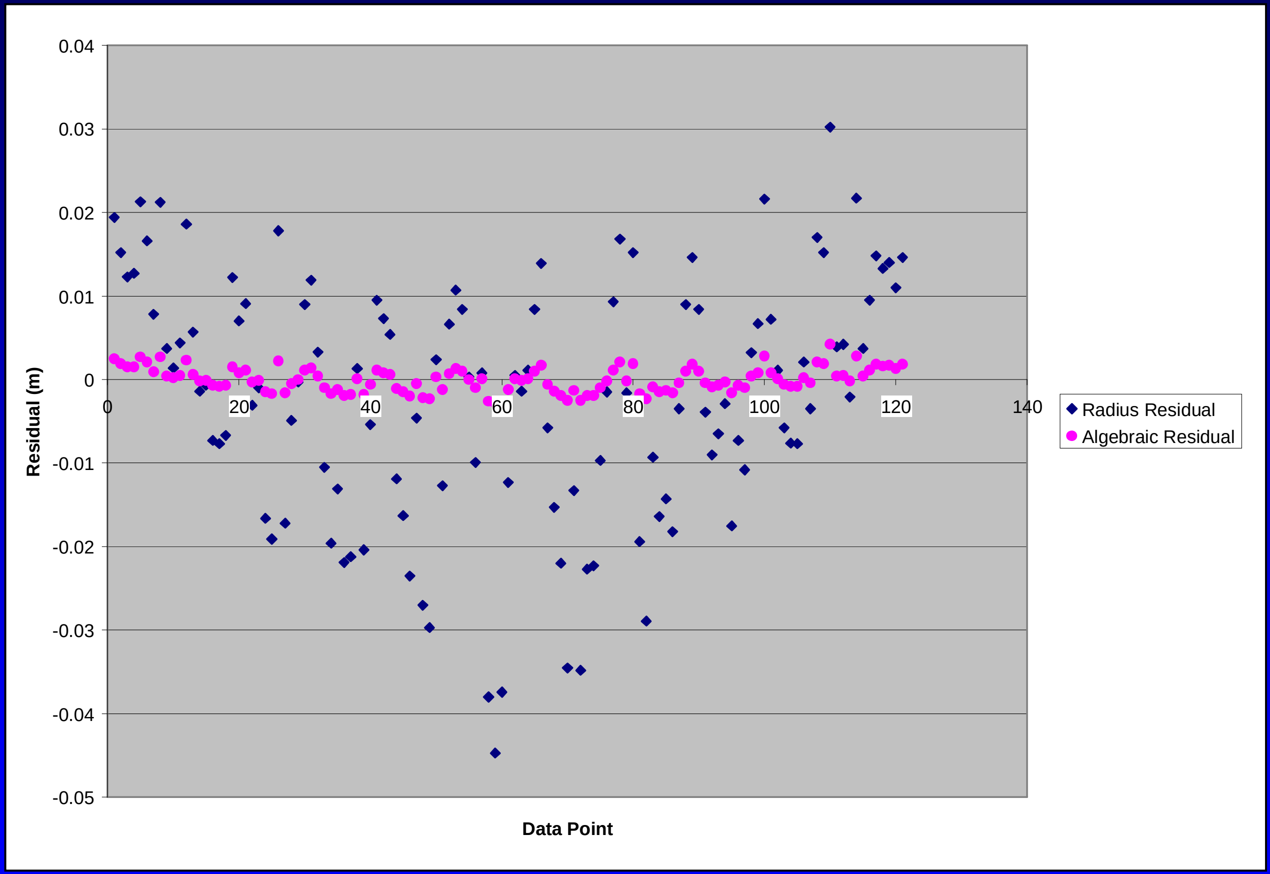
$$\left(x - x^*\right)^2 + \left(y - y^*\right)^2 - \left(r^*\right)^2 = 0$$

$\Rightarrow \left(x^*, y^*\right) = \text{optimal center}, \; r^* = \text{optimal radius}$

$$\text{THEOREM}: \left(\frac{A^*}{2}\right)^2 + \left(\frac{B^*}{2}\right)^2 - C^* \geq 0$$

How Good Is It?

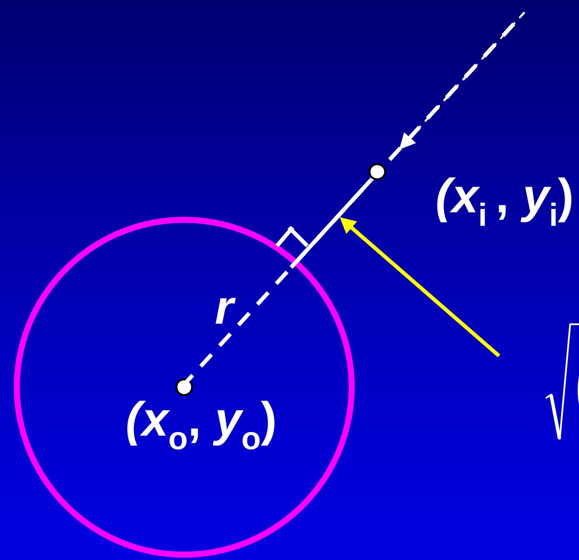
Example of an algebraic fit



Plots actual distance of data from fitted sphere

Geometric Fitting

The gauge function is based on the orthogonal distances of data points to a circle (sphere)

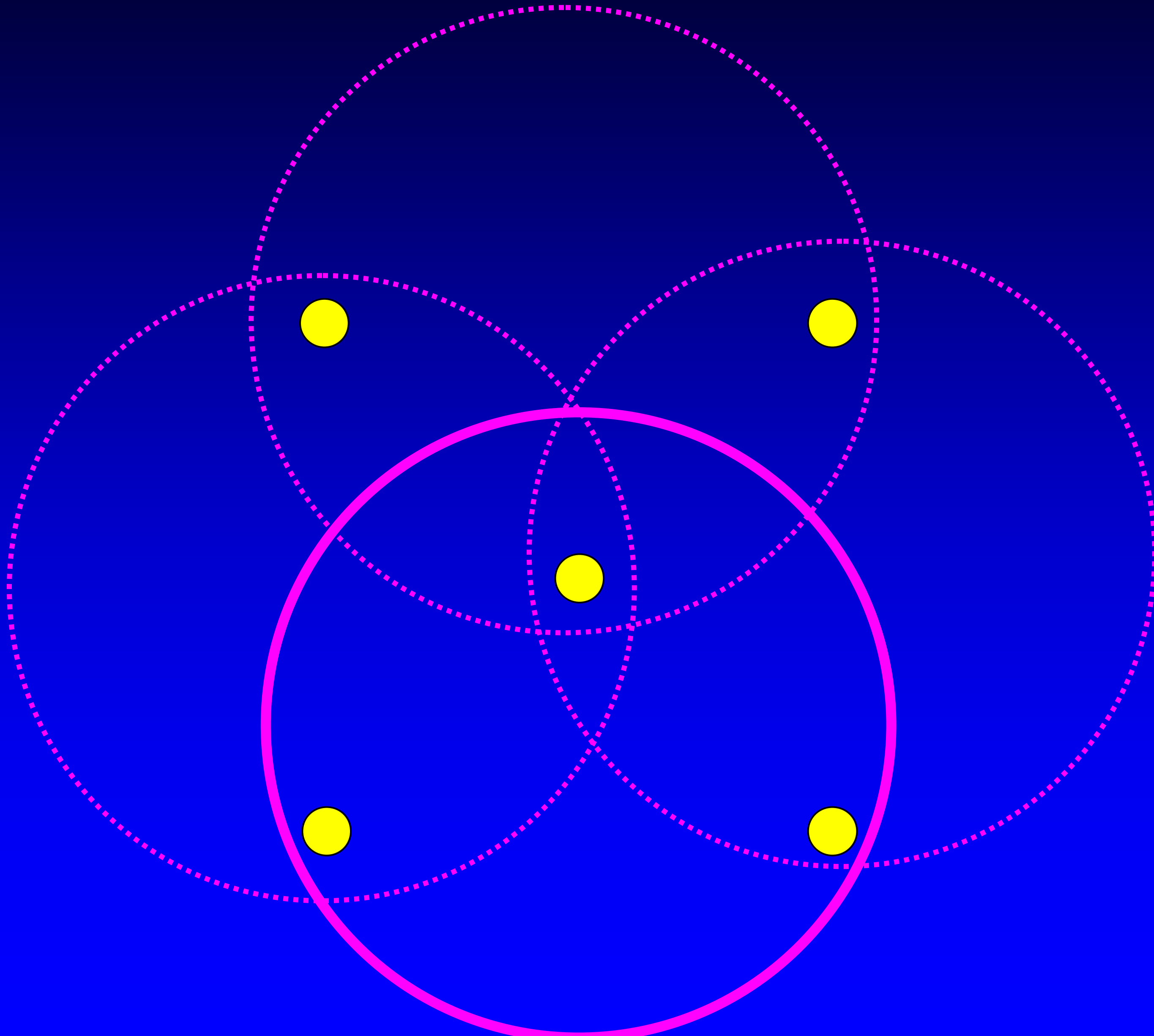


$$\sqrt{\left(x_i - x_0\right)^2 + \left(y_i - y_0\right)^2} - r$$

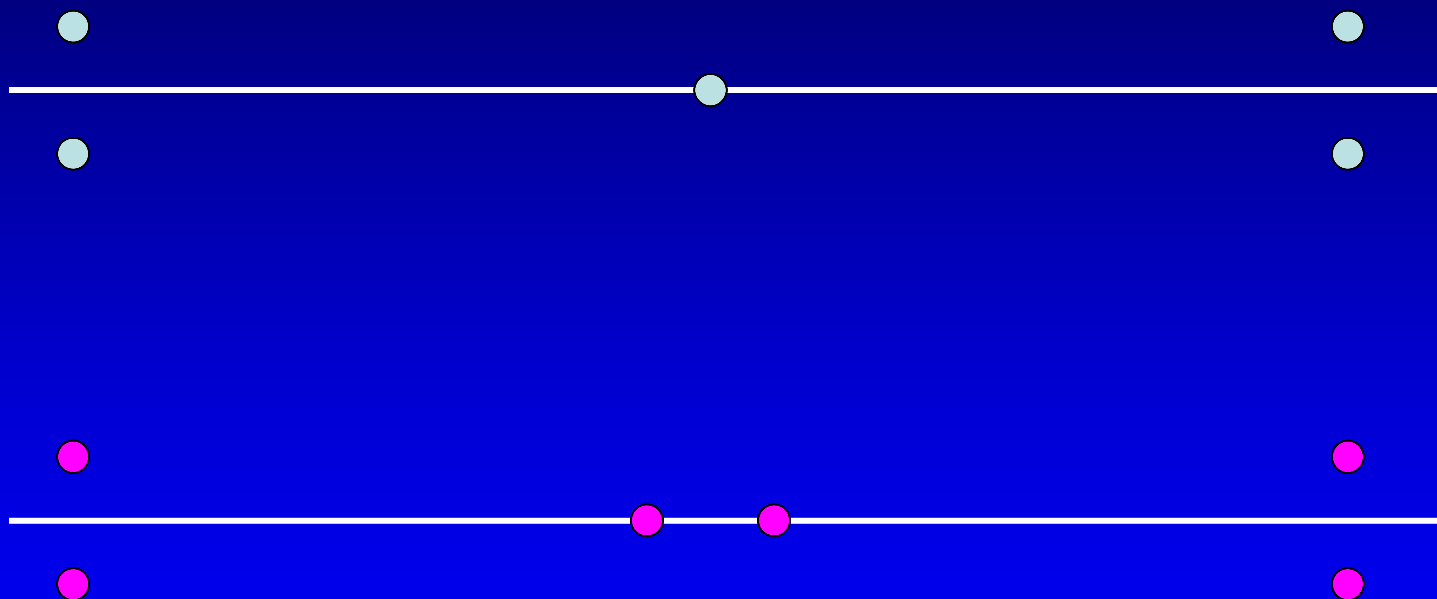
$$\Rightarrow \quad \sum_i \left(\sqrt{\left(x_i - x_0\right)^2 + \left(y_i - y_0\right)^2} - r\right)^2 \quad \text{for } x_0, y_0, r$$

- **Does the minimum always exist?**
- **Is the minimum uniquely defined?**

Non-Uniqueness

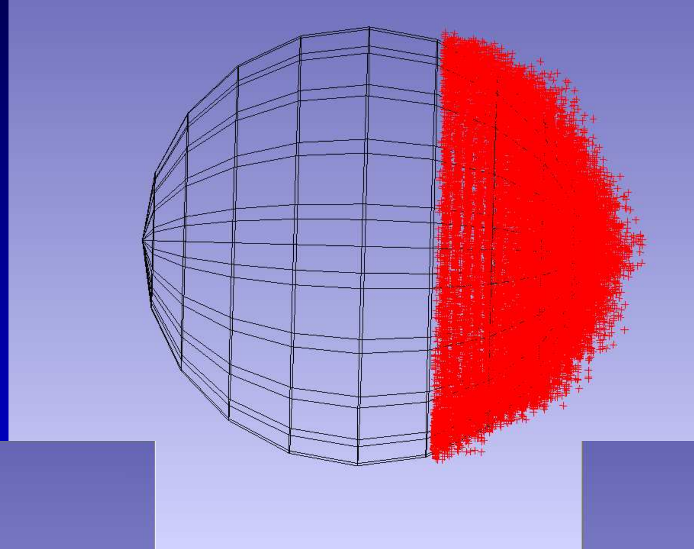


No Solution

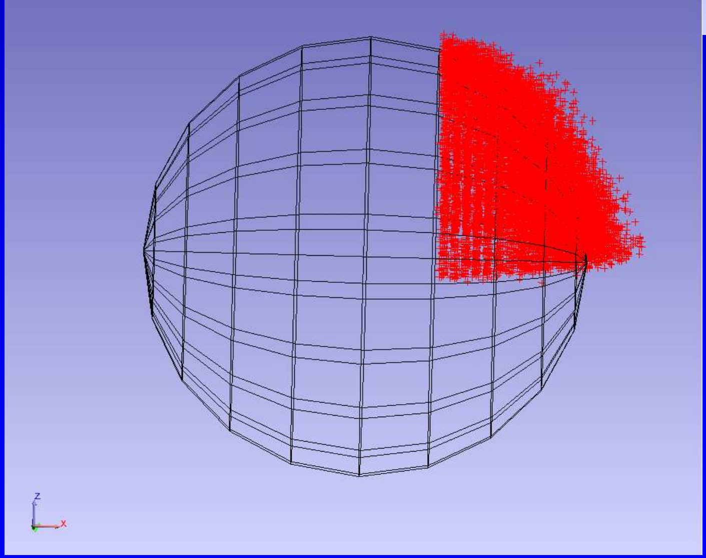


What is a Reasonable Data Set?

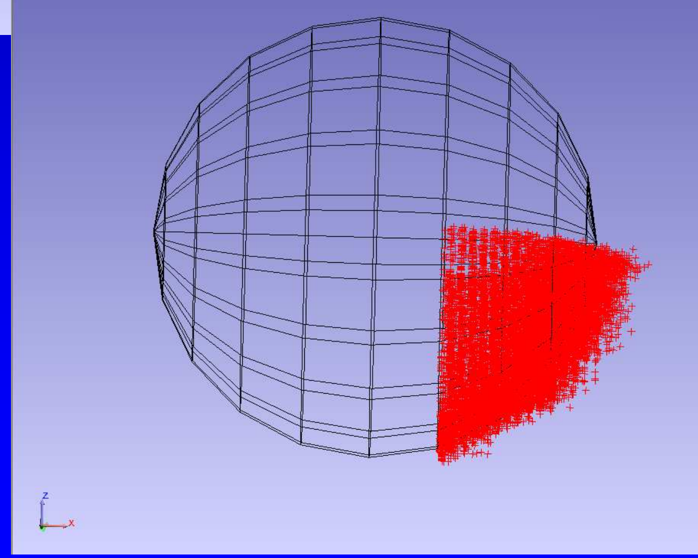
#1 “Full”



#2 “Upper”



#2 “Lower”

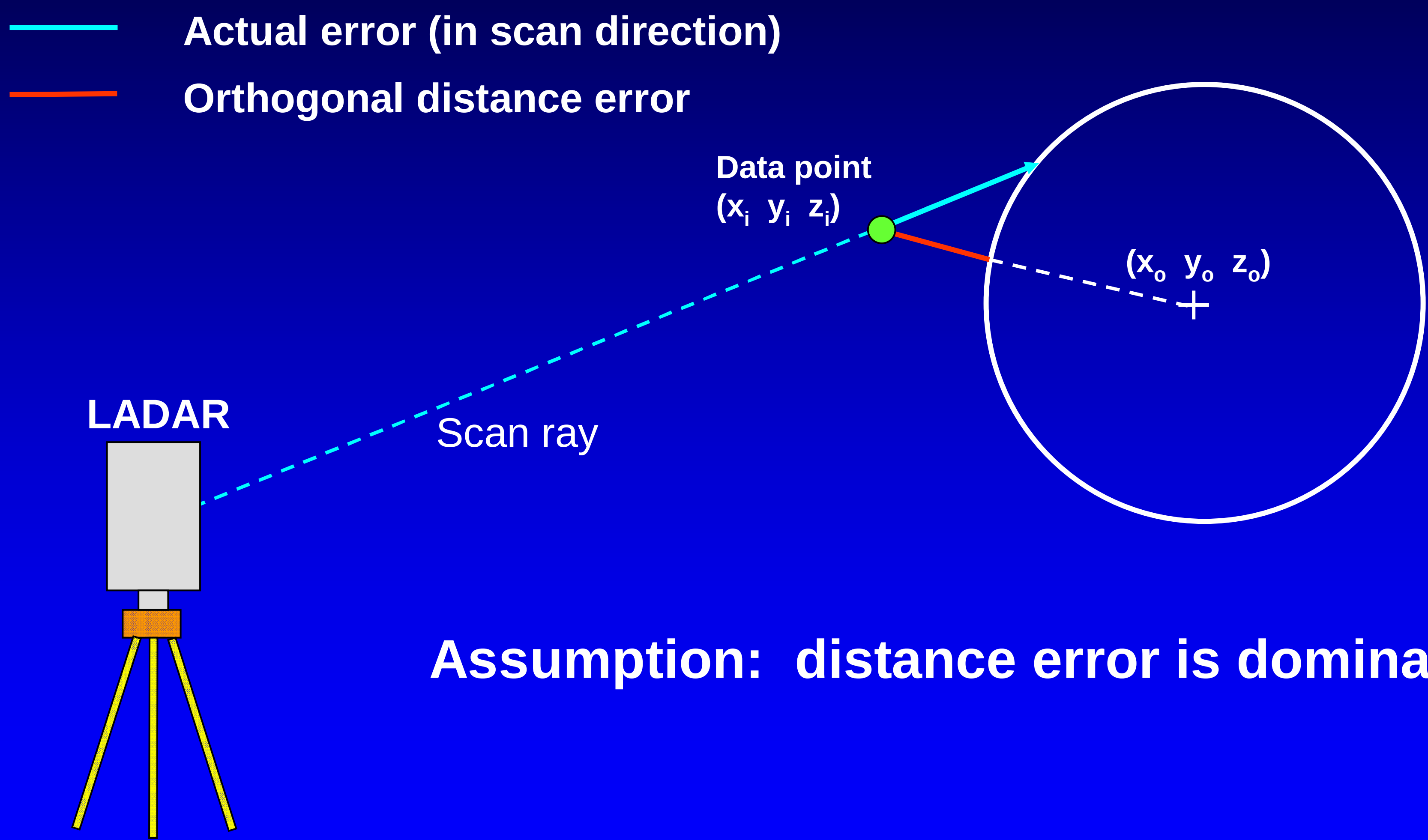


Geometric Fitting Results

VARIABLE RADIUS				
	X (mm)	**Y (mm)**	**Z (mm)**	**R (mm)**
Full	-6254.99	-196.51	-78.85	98.41
Upper	-6258.27	-196.37	-83.02	102.36
Lower	-6258.61	-196.82	-72.61	103.66

FIXED RADIUS				
	X (mm)	**Y (mm)**	**Z (mm)**	**R (mm)**
Full	-6259.19	-196.58	-78.87	101.6
Upper	-6257.52	-196.36	-82.55	101.6
Lower	-6256.59	-196.77	-73.98	101.6

Actual Error



Assumption: distance error is dominant

How about minimizing deviation in scan direction?

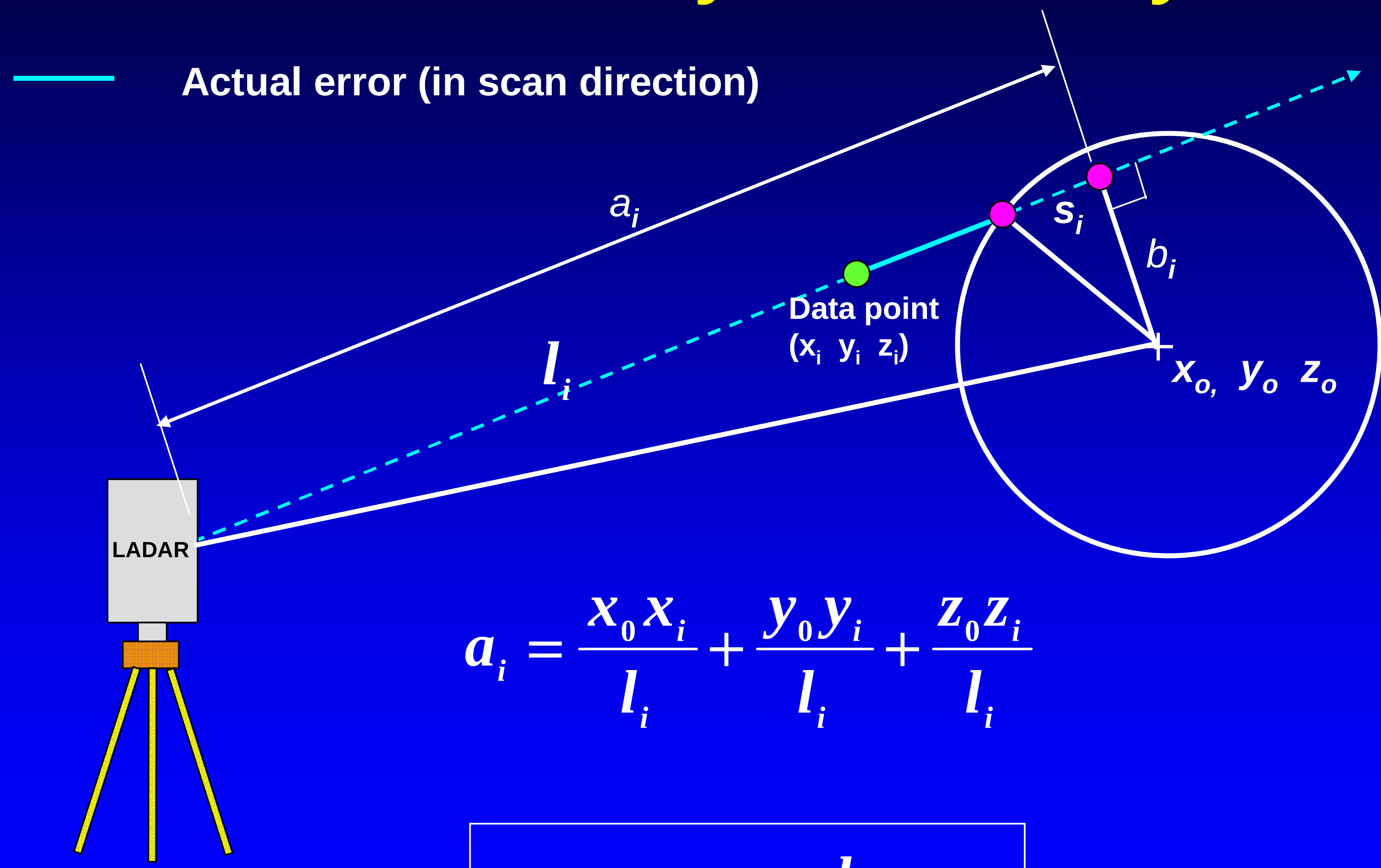
Fitting in Scan Direction

Gauge function = Least squares of scan errors

	X	Y	Z	r
Full	-6258.98	-198.07	-79.18	101.29
Upper	-6259.06	-198.15	-78.90	101.22
Lower	-6259.38	-198.01	-79.12	101.60

Compare to Geometric Fit				
	X	Y	Z	r
Full	-6254.99	-196.51	-78.85	98.41
Upper	-6258.27	-196.37	-83.02	102.36
Lower	-6258.61	-196.82	-72.61	103.66

Scan Ray Geometry



$$a_i = \frac{x_0 x_i}{l_i} + \frac{y_0 y_i}{l_i} + \frac{z_0 z_i}{l_i}$$

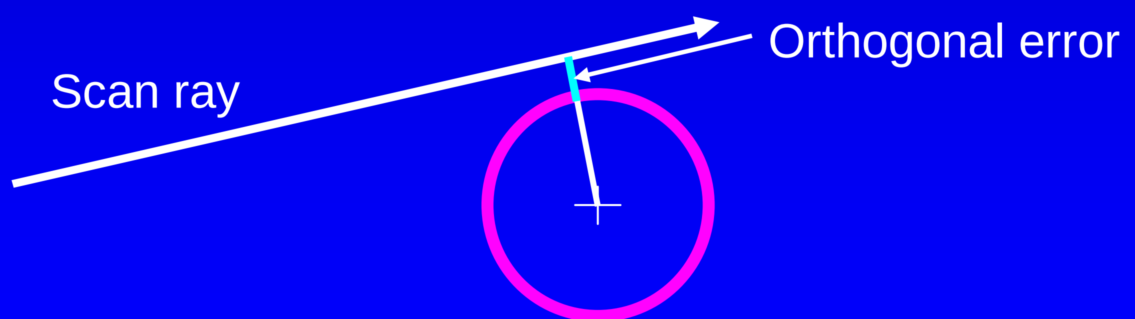
$$\boxed{error = a_i - l_i - s_i}$$

How to Compute - 1

- Difficulty fitting in scan direction
  - Errors incurred only if the

    Tentative sphere

    is actually hit by the scan ray
  - Gauge function is minimized by simply moving it out of the way
- Remedy
  - Define deviation error for scan rays missing the sphere



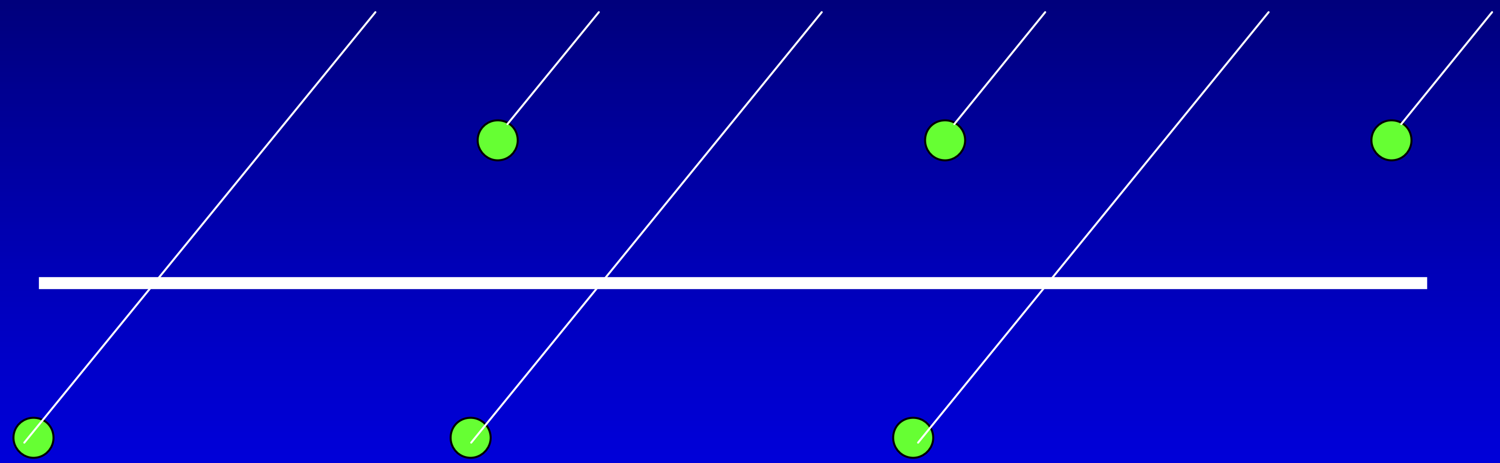
How to Compute - 2

- Difficulty with proposed remedy
  - Appended gauge function not differentiable
  - Orthogonal errors cause distortion
- Solution to non-differentiability problem
  - Use optimizer which handles
    - Non-differentiability
    - Multiply local minima
  - Kearsley's modification of the BFGS algorithm
    - BFGS = Broyden Fletcher Goldfarb Shanno method
    - Hybrid algorithm combines aspects of BFGS and Nelder-Mead type approach improving on both

How to Compute - 3

- Solution to distortion problem
  - Iterative procedure
    - Solve with appended orthogonal error
      - Sphere
    - Temporarily delete data points missing the sphere
    - Repeat until set of misses stabilizes
    - Actually delete points missing the sphere
  - Final optimization
    - To determine a fragile local minimum

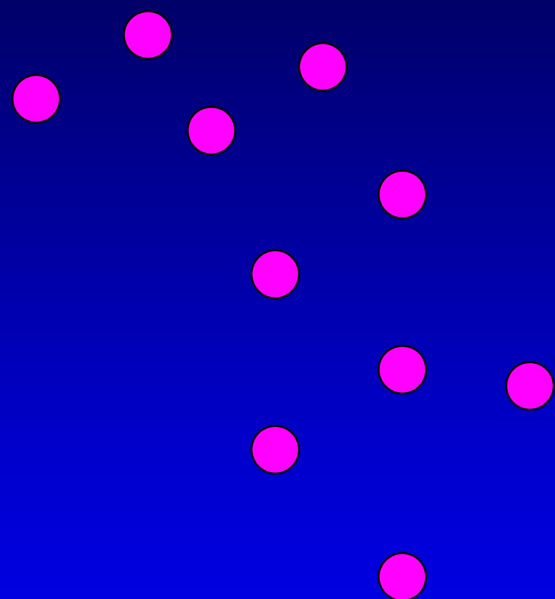
Plane Fitting



Message

When fitting a curve or a surface, it may not be sufficient to provide the data coordinates only. If there are directions in which the individual data points have been obtained, then those directions need to be taken into account.

Not



But