Carbon Dioxide, Global Warming, and Michael Crichton’s “State of Fear”

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NIST North (820), Room 145
Abstract

In his recent novel, *State of Fear* (HarperCollins, 2004), Michael Crichton questioned the reality of global warming and its connection to increasing atmospheric carbon dioxide levels. He bolstered his arguments by including plots of historical temperature records and other environmental variables, together with footnotes and appendices that purport to document them. Although most of his arguments were flawed, he did introduce at least one legitimate question by pointing out that in the years 1940-1970, global temperatures were decreasing while atmospheric carbon dioxide was increasing. I resolve this apparent contradiction by constructing a suite of simple mathematical models for the temperature time series. Each model consists of an accelerating baseline plus a 64.7 year sinusoidal oscillation. This cycle, which was first reported by Schlesinger and Ramankutty [Nature, Vol 367 (1994) pp. 723-726], appears also, with its sign reversed, in the time series record of fossil fuel carbon dioxide emissions. This suggests a negative temperature feedback in fossil fuel production. The acceleration in the temperature baseline is demanded by the data, but the temperature record is not yet long enough to precisely specify both the form and the rate of the acceleration. The most interesting model has a baseline derived from a power law relation between temperature changes and changes in the atmospheric carbon dioxide level. And the increase in atmospheric carbon dioxide is easily modelled by the cumulative accretion of a fixed fraction of each year’s fossil fuel emissions, so the power law model posits a direct connection between the emissions and the warming. For all of the temperature models, the cycle was decreasing more rapidly than the baseline was rising in the years 1940-1970, and in 1880-1910. We have recently entered another declining phase of the cycle, but the temperature hiatus this time will be far less dramatic because the accelerating baseline is rising more rapidly now.
In Paris, a physicist dies after performing a laboratory experiment for a beautiful visitor.

In the jungles of Malaysia, a mysterious buyer purchases deadly cavitation technology, built to his specifications.

In Vancouver, a small research submarine is leased for use in the waters off New Guinea.

And in Tokyo, an intelligence agent tries to understand what it all means.
“So, if rising carbon dioxide is the cause of rising temperatures, why didn’t it cause temperatures to rise from 1940 to 1970?”
“Now I want to direct your attention to the period from 1940 to 1970. As you see, during that period the global temperature actually went down. You see that?”

“Yes ...”
Global Temperature 1880-2003

Source: giss.nasa.gov
Atmospheric CO$_2$ concentration data from CDIAC, Oak Ridge National Lab.

High precision Mauna Loa measurements by C. D. Keeling, et. al.
Charles David Keeling
April 1928 – June 2005

Mauna Loa Moun. Atmospheric CO₂ Conc.
Choose $t = 0$ at epoch 1856.0
\[ P(t) = P_0 e^{\alpha t} - A_1 e^{\alpha t} \sin \left( \frac{2\pi}{\tau} (t + \phi_1) \right) \]

\[ \hat{\alpha} = 0.02824 \pm 0.00029 \quad \hat{\tau} = 64.7 \pm 1.4 \]

\[ \hat{P}_0 = 132.7 \pm 4.4 \quad \hat{A}_1 = 25.1 \pm 1.1 \]

\[ \hat{\phi}_1 = -6.1 \pm 2.4 \]
\[ \omega \equiv \frac{2\pi}{\tau} = 0.0971 \ [\text{rad/yr}] \]

\[ P(t_i) = P_0 e^{\alpha t_i} - A_1 e^{\alpha t_i} \sin [\omega (t_i + \phi_1)] \]
Model for the Atmospheric CO$_2$ Concentration

\[ c(t) = c_0 + \gamma \int_0^t P(t')dt' + \delta S(t) \]

where

\[ P(t') = P_0 e^{\alpha t'} - A_1 e^{\alpha t'} \sin \left[ \omega (t' + \phi_1) \right] \]

\[ S(t) = \begin{cases} 
0 &, t \leq t_P \\
\frac{1}{2}(t - t_P) &, t_P < t < (t_P + 2) \\
1 &, (t_P + 2) \leq t
\end{cases} \]

\[ t_P = 1991.54 - 1856.0 = 135.54 \]

Mount Pinatubo erupted on June 15, 1991

\[ 1 \text{ [ppmv]} = 2130 \text{ [MtC]} \]

\[ c(t) = c_0 + \gamma \int_0^t P(t')dt' + \delta S(t) \]

Combined Atmospheric CO₂ Records

\[ \hat{c}_0 = 294.10 \pm 0.19 \text{ [ppmv]} \]

\[ \hat{\gamma} = 0.5926 \pm 0.0026 \]

\[ \hat{\delta} = -2.05 \pm 0.20 \text{ [ppmv]} \]
Extrapolating the Fit Backward

Combined Atmospheric CO₂ Records

- Law Dome Ice Cores
- Siple Sta. Ice Cores
- Mauna Loa Measurements

\[ c_0 + \gamma \int P(t') \, dt' + \delta S(t) \]

- Crichton CO₂ Curve
Fitting the Combined Data Set

Combined Atmospheric CO₂ Records

- Law Dome Ice Cores
- Siple Sta. Ice Cores
- Mauna Loa Measurements
- Fit to Mauna Loa Data Set
- Fit to Combined Data Set

Concentration [ppmv]

(1856.0 + t) [yr]
\[
P(t) = P_0 e^{\alpha t} - A_1 e^{\alpha t} \sin[\omega (t + \phi_1)]
\]

\[
c(t) = c_0 + \gamma \int_0^t P(t') dt' + \delta S(t)
\]
Temperature anomaly for year $t_i$ can be defined as:

$$\text{Temperature anomaly for year } t_i \equiv \text{Average Temperature in year } t_i - \text{Average Temp. for some reference period}$$
Crichton, GISS, and CRU Temperature Anomalies

\[
\begin{align*}
T &= T_{\text{Crichton}} \\
&= T_{\text{GISS}} - 0.066 \, ^\circ\text{C} \\
&= T_{\text{CRU}} + 0.013 \, ^\circ\text{C}
\end{align*}
\]
Improved and Corrected Crichton Plot

\[ T(t) = T_0 + \eta t \]

\[ T(t) = T_0 + \eta t^2 \]

<table>
<thead>
<tr>
<th>Stat.</th>
<th>( T(t) = T_0 + \eta t )</th>
<th>( T(t) = T_0 + \eta t^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSR</td>
<td>2.7572</td>
<td>1.9798</td>
</tr>
<tr>
<td>100R^2</td>
<td>67.89%</td>
<td>76.94%</td>
</tr>
</tbody>
</table>

\[ R^2 = 1 - \frac{SSR}{CTSS}, \quad CTSS = \sum_{i=1}^{m} (T_i - \bar{T})^2 \]
The data demand a concave upward baseline.

The warming is accelerating!
Residual Periodogram for $T(t) = T_0 + \eta t$

$\tau_1 = 63.5$ yr
$\tau_0 = 148$ yr

Residual Periodogram for $T(t) = T_0 + \eta t^2$

$\tau_1 = 62.5$ yr

“These oscillations have obscured the greenhouse warming signal…”

“...the oscillation arises from predictable internal variability of the ocean-atmosphere system.”
A Gaiaen Feedback?

\[ T(t) = T_0 + \eta t^2 + A_3 \sin \left[ \frac{2\pi}{\tau_1} (t + \phi_1) \right] \]

\[ P(t) = P_0 e^{\alpha t} - A_1 e^{\alpha t} \sin \left[ \frac{2\pi}{\tau_1} (t + \phi_1) \right] \]

Could the presence of the 65-year cycle in both records, with sign reversed, be caused by an inverse temperature feedback?

\[ \begin{cases} \text{cooler} \\ \text{warmer} \end{cases} T(t) \Rightarrow \begin{cases} \text{more} \\ \text{less} \end{cases} \text{demand for } P(t) \]


\[ \frac{dP}{dt} = \left( \alpha - \beta \frac{dT}{dt} \right) P , \quad P(0) = P_0 \]
\( P(t) = P_0 e^{\alpha t} - A_1 e^{\alpha t} \sin [\omega (t + \phi_1)] \)

\( c(t) = c_0 + \gamma \int_0^t P(t') dt' + \delta S(t) \)

\( T(t) = T_0 + \eta t + A_3 \sin [\omega (t + \phi_1)] \)

\( T(t) = T_0 + \eta t^2 + A_3 \sin [\omega (t + \phi_1)] \)

\( T(t) = T_0 + \eta \exp \left( \frac{3\alpha}{5} t \right) + A_3 \sin [\omega (t + \phi_1)] \)

\( T(t) = T_0 + \eta [\Delta c]^{2/3} + A_3 \sin [\omega (t + \phi_1)] \)

\( \Delta c \equiv c(t) - c_0 \)

\[ = \gamma \int_0^t P(t') dt' + \delta S(t) \]
\[ T = T_0 + \eta t + A_3 \sin(\omega(t + \phi_3)) \]

\[ T = T_0 + \eta t^2 + A_3 \sin(\omega(t + \phi_3)) \]

\[ T = T_0 + \eta e^{3\alpha t/5} + A_3 \sin(\omega(t + \phi_3)) \]

\[ T = T_0 + \eta \Delta c^{2/3} + A_3 \sin(\omega(t + \phi_3)) \]

<table>
<thead>
<tr>
<th>Stat.</th>
<th>( T_0 + \eta t )</th>
<th>( T_0 + \eta t^2 )</th>
<th>( T_0 + \eta e^{3\alpha t/5} )</th>
<th>( T_0 + \eta \Delta c^{2/3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSR</td>
<td>1.8965</td>
<td>1.2891</td>
<td>1.2604</td>
<td>1.2630</td>
</tr>
<tr>
<td>( 100R^2 )</td>
<td>77.91%</td>
<td>84.99%</td>
<td>85.32%</td>
<td>85.29%</td>
</tr>
</tbody>
</table>
Note concave upward pattern in straight-line residuals!

The warming is accelerating!
$T(t) = T_0 + \nu t + \eta t^2 + A_3 \sin[\omega(t + \phi_1)]$

![Graph showing annual global average temperature anomalies (1856-2004)](image)

$\hat{\nu} = (-1.08 \pm .73) \times 10^{-3} \quad \Rightarrow \quad H_0 : \nu = 0$

F-test accepts $H_0$ at the 95% level

The data demand a monotone increasing baseline
\[ T(t) = T_0 + \eta e^{\alpha t} + A_3 \sin(\omega(t + \phi_1)) \]

\begin{align*}
\text{Stat.} &\quad \alpha = 0.0168 & \frac{3\alpha}{5} = 0.0169 \\
\text{SSR} &\quad 1.46 & 1.26 \\
100R^2 &\quad 83.0\% & 85.3\%
\end{align*}
\[ T(t) = T_0 + \eta e^{\alpha t} + A_3 \sin[\omega(t + \phi_1)] \]
\[ T(t) = T_0 + \eta e^{\nu t} + A_3 \sin [\omega (t + \phi_1)] \]

\[ \hat{\eta} = 0.071 \pm 0.024 \quad \hat{\nu} = 0.0168 \pm 0.0022 \quad \hat{\rho}(\eta, \nu) = -0.995 \]

\[ \frac{3\alpha}{5} = 0.0169 \implies \eta = 0.0690 \pm 0.0024 \]

<table>
<thead>
<tr>
<th>Stat.</th>
<th>( \hat{\nu} = 0.0168 )</th>
<th>( \frac{3\alpha}{5} = 0.0169 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSR</td>
<td>1.260319</td>
<td>1.260355</td>
</tr>
<tr>
<td>100( R^2 )</td>
<td>85.3214%</td>
<td>85.3210%</td>
</tr>
</tbody>
</table>
\[ T(t) = T_0 + \eta [\Delta c]^\nu + A_3 \sin [\omega(t + \phi_1)] \]

\[ \hat{\eta} = (3.2 \pm 2.5) \times 10^{-4} \quad \hat{\nu} = 0.645 \pm 0.063 \quad \hat{\rho}(\eta, \nu) = -0.9989 \]

\[ \frac{2}{3} = 0.6666667 \implies \eta = (2.490 \pm 0.087) \times 10^{-4} \]

<table>
<thead>
<tr>
<th>Stat.</th>
<th>$\hat{\nu} = 0.645$</th>
<th>$\frac{2}{3} = 0.6666667$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSR</td>
<td>1.2620</td>
<td>1.2630</td>
</tr>
<tr>
<td>$100R^2$</td>
<td>85.302%</td>
<td>85.290%</td>
</tr>
</tbody>
</table>
The World’s Simplest Climate Model

(With apologies to Johannes Kepler)

“The third power of change in tropospheric temperature is proportional to the square of change in atmospheric CO$_2$ concentration”

$$[T(t) - T_0]^3 = \eta [c(t) - c_0]^2$$

$$T(t) = T_0 + \eta [c(t) - c_0]^{2/3}$$

“But an interaction between the oceans and the atmosphere imposes a cycle with period $\tau \approx 65$ year on the temperatures which is independent of the CO$_2$ concentration”

$$T(t) = T_0 + \eta [c(t) - c_0]^{2/3} + A_3 \sin \left[ \frac{2\pi}{\tau} (t + \phi_1) \right]$$
\[ T(t) = T_0 + \eta \left[ c(t) - c_0 \right]^{2/3} + A_3 \sin \left[ \frac{2\pi}{\tau} (t + \phi_1) \right] \]

\[ c(t) = c_0 + \gamma \int_0^t P(t') dt' + \delta S(t) \]

\[ T(t) = T_0 + \eta \left[ \gamma \int_0^t P(t') dt' + \delta S(t) \right]^{2/3} + A_3 \sin \left[ \frac{2\pi}{\tau} (t + \phi_1) \right] \]
\[ P(t) = P_0 e^{\alpha t} - A_1 e^{\alpha t} \sin \left[ \frac{2\pi}{\tau} (t + \phi_1) \right] \]

\[ T(t) = T_0 + \eta \left[ \gamma \int_0^t P(t') dt' + \delta S(t) \right]^{2/3} + A_3 \sin \left[ \frac{2\pi}{\tau} (t + \phi_1) \right] \]
\[ T(t) = T_0 + \eta \left[ \gamma \int_0^t P(t')dt' + \delta S(t) \right]^{2/3} + A_3 \sin \left[ \frac{2\pi}{\tau} (t + \phi) \right] \]

\[ T(t) = T_0 + \eta \left[ \gamma \int_0^t P(t')dt' + \delta S(t) \right]^{2/3} + A_3 \sin \left[ \frac{2\pi}{\tau} (t + \phi_1) \right] \]
\[ T(t) = T_0 + \eta \left[ \gamma \int_0^t P(t') dt' + \delta S(t) \right]^{2/3} + A_3 \sin \left[ \frac{2\pi}{\tau}(t + \phi_1) \right] \]

**Residual Periodogram**

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.000</td>
</tr>
<tr>
<td>0.05</td>
<td>0.000</td>
</tr>
<tr>
<td>0.10</td>
<td>0.001</td>
</tr>
<tr>
<td>0.15</td>
<td>0.001</td>
</tr>
<tr>
<td>0.20</td>
<td>0.002</td>
</tr>
<tr>
<td>0.25</td>
<td>0.002</td>
</tr>
<tr>
<td>0.30</td>
<td>0.003</td>
</tr>
<tr>
<td>0.35</td>
<td>0.003</td>
</tr>
<tr>
<td>0.40</td>
<td>0.004</td>
</tr>
<tr>
<td>0.45</td>
<td>0.004</td>
</tr>
<tr>
<td>0.50</td>
<td>0.005</td>
</tr>
</tbody>
</table>

\( \tau_2 = 20.4 \quad \tau_3 = 9.2 \quad \tau_4 = 3.54 \)

**Cumulative Periodogram**

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Cum. Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.000</td>
</tr>
<tr>
<td>0.05</td>
<td>0.020</td>
</tr>
<tr>
<td>0.10</td>
<td>0.040</td>
</tr>
<tr>
<td>0.15</td>
<td>0.060</td>
</tr>
<tr>
<td>0.20</td>
<td>0.080</td>
</tr>
<tr>
<td>0.25</td>
<td>0.100</td>
</tr>
<tr>
<td>0.30</td>
<td>0.120</td>
</tr>
<tr>
<td>0.35</td>
<td>0.140</td>
</tr>
<tr>
<td>0.40</td>
<td>0.160</td>
</tr>
<tr>
<td>0.45</td>
<td>0.180</td>
</tr>
<tr>
<td>0.50</td>
<td>0.200</td>
</tr>
</tbody>
</table>
\[ T(t) = T_0 + \eta \left[ \gamma \int_0^t P(t') \, dt' + \delta S(t) \right]^{2/3} + A_3 \sin \left[ \frac{2\pi}{\tau} (t + \phi_1) \right] \]
\[ T(t) = T_0 + \eta \left[ \gamma \int_0^t P(t') dt' + \delta S(t) \right]^{2/3} + A_3 \sin \left[ \frac{2\pi}{\tau} (t + \phi_1) \right] \]

\[ \Sigma r_i^2 = 1.263E+00 \]

Resid. Periodogram

Resid. Ampl. Spectrum

Cum. Periodogram

Length = 1.207
Extrapolating to epoch 2100.0 yields

\[ P(2100) \approx 140,000 \text{ [MtC/yr]} \approx 20 \times P(2002) \]
The next “cooling” period is
September 2007 – March 2040
“Here I define an index of the potential destructiveness of hurricanes based on the total dissipation of power, integrated over the lifetime of the cyclone, and show that this index has increased markedly since the mid-1970s. I find that the record of net hurricane power dissipation is highly correlated with tropical sea surface temperature, reflecting well-documented climate signals, including multi-decadal oscillations in the North Atlantic and North Pacific, and global warming.”
The diagram shows the annual global average temperature anomalies from 1856 to 2004. The data indicates a steady increase in temperature over time, with fluctuations that are less pronounced. The solid line represents HadISST data from 6°-18° N, 20°-60° W, and the dashed line represents the Atlantic PDI.
\[ \frac{dP}{dt} = \left( \alpha - \beta \frac{dT}{dt} \right) P , \quad P(0) = P_0 \]
\[ c(t) = c_0 + \gamma \int_{0}^{t} P(t') \, dt' \]
\[ T(t) = T_0 + \eta [c(t) - c_0] + A \sin \left[ \frac{2\pi}{\tau} (t + \phi) \right] \]

\[ \frac{dP}{dt} = \left( \alpha - \beta \frac{dT}{dt} \right) P \quad , \quad P(0) = P_0 \]
\[ \frac{dc}{dt} = \gamma P(t) \quad , \quad c(0) = c_0 \]
\[ \frac{dT}{dt} = \eta \frac{dc}{dt} + \frac{2\pi A}{\tau} \cos \left[ \frac{2\pi}{\tau} (t + \phi) \right] \quad , \quad T(0) = T_0 \]

\[ \frac{dP}{dt} = \alpha P - \beta \left\{ \eta' P + A' \cos \left[ \frac{2\pi}{\tau} (t + \phi) \right] \right\} P \quad , \quad P(0) = P_0 \]
\[ \frac{dc}{dt} = \gamma P \quad , \quad c(0) = c_0 \]
\[ \frac{dT}{dt} = \eta' P + A' \cos \left[ \frac{2\pi}{\tau} (t + \phi) \right] \quad , \quad T(0) = T_0 \]

\[ \eta' \equiv \gamma \eta \quad , \quad A' \equiv \frac{2\pi A}{\tau} \]