NIST AMR Benchmark Reference Solution File Format
Version 1.2
August 18, 2015
William F. Mitchell

For some problems in the NIST adaptive mesh refinement benchmark suite for which the exact solution is not known, a reference solution is provided. The reference solution is a C^0 piecewise polynomial function over a mesh of simplicies. This reference solution can be used as the "true" solution of the differential equation to compute norms of the error of some computed solution, provided the computed solution is not more accurate than the reference solution.

Software to evaluate the reference solution at a given point is available on the NIST Adaptive Mesh Refinement Benchmark Problems web page, http://math.nist.gov/amr-benchmark.

The reference solution files should have the suffix nrs, for NIST Reference Solution. The file format is:

NIST AMR Benchmark Reference Solution
version
comments
error-estimates
solution-norms
dimension degree number-of-solutions
number-of-vertices number-of-other-nodes
coordinates solutions
...
solutions
...
number-of-elements
vertices nodes
...

The first line identifies this file as a NIST AMR benchmark reference solution.

version is the version number of the reference solution format, 1.2

comments is an arbitrary number of comment lines. The first non-blank character of a coment line is the symbol #. Comment lines can only appear at this location of the file.

error-estimates is four real numbers giving estimates of the error in the energy, L_∞, L_2, and H_1 norms, in that order. The value -1.0 indicates that an estimate is not available.

solution-norms is four real numbers giving the energy, L_∞, L_2, and H_1 norms of the reference solution, in that order. The value -1.0 indicates that the norm is not available.

dimension is the number of spacial dimensions, 1, 2 or 3.

degree is a positive integer giving the degree of the polynomials in the piecewise polynomial solution.

number-of-solutions is the number of solutions given at each vertex. This can be more than one if the solution comes from a coupled system of differential equations, or if more than one eigenfunction is given for an eigenvalue problem.

number-of-vertices is a positive integer giving the number of vertices in the mesh. *number-of-other-nodes* is the number of nodes in the mesh that are not vertices.

Each of the next *number-of-vertices* lines contains information for one vertex. The i^{th} such line will be referred to as *vertex i*, starting with vertex 1. *coordinates* is *dimension* real numbers with the coordinates of the vertex. *solutions* is *number-of-solutions* real numbers with the values of the solutions at the vertex.

Each of the next *number-of-other-nodes* lines contains the solutions for one node. The i^{th} such line will be referred to as node *number-of-vertices* + *i*. *solutions* is *number-of-solutions* real numbers with the values of the solutions at the node.

number-of-elements is the number of elements in the mesh.

Each of the next *number-of-elements* lines contains information for one element. The i^{th} such line is referred to as *element i*, begining with element 1. *vertices* is *dimension*+1 integers with the vertices of the element. *nodes* is a list of the other nodes in the element.

If *dimension*=1, there are *degree*-1 other nodes, which are equally spaced going from the one closest to the first vertex to the one closest to the second vertex.

If *dimension*=2, there are (*degree*+1)(*degree*+2)/2 - 3 other nodes, which are uniformly spaced in the triangle. The order is: nodes on the edge from vertex 1 to vertex 2, then edge 1-3, and edge 2-3; nodes in the interior on lines from vertex 1 to edge 2-3, going from the 1-2 edge to the 1-3 edge. For example, with degree=4 the order of the vertices and nodes is shown in Figure 1.

If *dimension*=3, there are (*degree*+1)(*degree*+2)(*degree*+3)/6 - 4 other nodes, which are uniformly spaced over the tetrahedron. The order is: nodes on the edge from vertex 1 to vertex 2, then edge 1-3, edge 1-4, edge 2-3, edge 2-4 and edge 3-4; nodes on face 1-2-3 on lines from vertex 1 to edge 2-3, going from the 1-2 edge to the 1-3 edge, then face 1-2-4, then face 1-3-4, and face 2-3-4; nodes in the interior on planes (triangles) parallel to face 2-3-4, progressing from vertex 1 to face 2-3-4. In each plane, they are on lines parallel to face 1-3-4, progressing from a point on edge 1-2 to a line on face 1-3-4, with the nodes on the line going from plane 1-2-3 to plane 1-2-4.

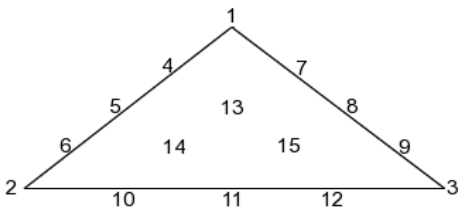

Figure 1. Order of vertices and nodes in a triangle.