Uncertainty and Sensitivity Analysis: From Regulatory Requirements to Conceptual Structure and Computational Implementation

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Motivation for Talk (1)

- Regulatory requirements are often expressed in broad nonmathematical terms, e.g., following examples will be presented:

  - NRC requirements for Yucca Mountain (YM) repository
  - EPA requirements for Waste Isolation Pilot Plant (WIPP)
  - NRC reactor accident safety goals
  - NNSA mandate for quantification of margins and uncertainties (QMU)

- Yet, establishing compliance with these requirements must be based on extensive mathematical analysis

Motivation for Talk (2)

- Mathematical analyses supporting regulatory compliance need to be based on a clear conceptual structure to:

  - establish how nonmathematical requirements are transformed into the results of mathematical calculations
  - guide the computational structure and implementation of an analysis to establish compliance with regulatory requirements
  - support meaningful uncertainty and sensitivity analyses
  - support analysis verification and validation
  - provide a basis for clear and well organized analysis documentation
  - communicate to regulators and others exactly what was done in the analysis and how analysis results establish regulatory compliance

Message of Talk

- Given the nature of regulatory requirements for complex systems, the needed conceptual structure can be based on three fundamental entities:

  - EN1, a probability space ($\mathcal{A}$, $\mathbb{A}$, p_A) characterizing aleatory uncertainty
  - EN2, A model, or system of models, that predicts system behavior
  - EN3, A probability space ($\mathcal{E}$, $\mathbb{E}$, p_E) characterizing epistemic uncertainty

- Recognition and clear definition of the above three fundamental components provides the basis for the organization and presentation of an analysis of a complex system

- The previously indicated regulatory examples are now presented

Example: NRC Requirements for YM Repository (1)

- § 63.311 Individual protection standard after permanent closure

  (a) DOE must demonstrate, using performance assessment, that there is a reasonable expectation that the reasonably maximally exposed individual receives no more than the following annual dose from releases from the undisturbed Yucca Mountain disposal system: (1) 0.15 mSv (15 mrem) for 10,000 years following disposal; and (2) 1.0 mSv (100 mrem) after 10,000 years, but within the period of geologic stability.

  (b) DOE's performance assessment must include all potential environmental pathways of radionuclide transport and exposure.

- § 63.303 Implementation of Subpart L.

  (a) Compliance is based upon the arithmetic mean of the projected doses from DOE's performance assessments for the period within 1 million years after disposal, with: (1) Sections 63.311(a)(1) and 63.311(a)(2) ; and (2) Sections 63.321(b)(1), 63.321(b)(2) and 63.331, if performance assessment is used to demonstrate compliance with either or both of these sections.

Example: NRC Requirements for YM Repository (2)

- NRC definition for performance assessment

Performance assessment means an analysis that:

(1) Identifies the features, events, processes (except human intrusion), and sequences of events and processes (except human intrusion) that might affect the Yucca Mountain disposal system and their probabilities of occurring during 10,000 years after disposal;

(2) Examines the effects of those features, events, processes, and sequences of events and processes upon the performance of the Yucca Mountain disposal system; and

(3) Estimates the dose incurred by the reasonably maximally exposed individual, including the associated uncertainties, as a result of releases caused by all significant features, events, processes, and sequences of events and processes, weighted by their probability of occurrence.

Example: NRC Requirements for YM Repository (3)

- NRC definition for reasonable expectation

Reasonable expectation means that the Commission is satisfied that compliance will be achieved based upon the full record before it. Characteristics of reasonable expectation include that it:

(1) Requires less than absolute proof because absolute proof is impossible to attain for disposal due to the uncertainty of projecting long-term performance;

(2) Accounts for the inherently greater uncertainties in making long-term projections of the performance of the Yucca Mountain disposal system;

(3) Does not exclude important parameters from assessments and analyses simply because they are difficult to precisely quantify to a high degree of confidence; and

(4) Focuses performance assessments and analyses on the full range of defensible and reasonable parameter distributions rather than only upon extreme physical situations and parameter values.

Example: EPA Requirements for WIPP (1)

- § 191.13 Containment requirements:

- (a) Disposal systems for spent nuclear fuel or high-level or transuranic radioactive wastes shall be designed to provide a reasonable expectation, based upon performance assessments, that cumulative releases of radionuclides to the accessible environment for 10,000 years after disposal from all significant processes and events that may affect the disposal system shall:

  (1) Have a likelihood of less than one chance in 10 of exceeding the quantities calculated according to Table 1 (Appendix A); and

  (2) Have a likelihood of less than one chance in 1,000 of exceeding ten times the quantities calculated according to Table 1 (Appendix A).

- (b) Performance assessments need not provide complete assurance that the requirements of 191.13(a) will be met. Because of the long time period involved and the nature of the events and processes of interest, there will inevitably be substantial uncertainties in projecting disposal system performance. Proof of the future performance of a disposal system is not to be had in the ordinary sense of the word in situations that deal with much shorter time frames. Instead, what is required is a reasonable expectation, on the basis of the record before the implementing agency, that compliance with 191.13(a) will be achieved.

Example: EPA Requirements for WIPP (2)

- *§ 194.34 Results of performance assessments.*

(a) The results of performance assessments shall be assembled into "complementary, cumulative distribution functions" (CCDFs) that represent the probability of exceeding various levels of cumulative release caused by all significant processes and events.

(b) Probability distributions for uncertain disposal system parameter values used in performance assessments shall be developed and documented in any compliance application.

(c) Computational techniques, which draw random samples from across the entire range of the probability distributions developed pursuant to paragraph (b) of this section, shall be used in generating CCDFs and shall be documented in any compliance application.

(d) The number of CCDFs generated shall be large enough such that, at cumulative releases of 1 and 10, the maximum CCDF generated exceeds the 99th percentile of the population of CCDFs with at least a 0.95 probability.

(e) Any compliance application shall display the full range of CCDFs generated.

(f) Any compliance application shall provide information which demonstrates that there is at least a 95 percent level of statistical confidence that the mean of the population of CCDFs meet the containment requirements of § 191.13 of this chapter.

Example: NRC Reactor Accident Safety Goals

- *Individual early fatality risk*: The expected value for average individual early fatality risk in the region between the plant site boundary and 1609.3 m (1 mi) beyond this boundary will be less than 5×10^{-7}/ yr.

- *Individual latent cancer fatality risk*: The expected value for average individual latent cancer fatality risk in the region between the plant site boundary and 16,093 m (10 mi) beyond this boundary will be less than 2×10^{-6} / yr.

- *Severe accident frequency*: The expected value for the frequency of a severe accident will be less than 1×10^{-4} / yr.

- *Conditional probability of containment failure*: The expected value for the probability of containment failure given the occurrence of a severe accident will be less than 0.1.

- *Large release frequency*: The expected value for the frequency of a large release will be less than 1×10^{-6} / yr.

Example: NNSA's Mandate for Quantification of Margins and Uncertainties (QMU)

- Quantification of Margins and Uncertainties is a scientific methodology that identifies relevant nuclear-warhead parameters and quantifies, using available experimental and computational tools, the margin of that parameter relative to its failure point and the uncertainties associated with the parameter and the failure point. An assessment of the relationship between the margin and uncertainties facilitates stockpile management decisions, resource allocation prioritization, and informed judgments on the safety, reliability and performance of nuclear warheads.

- Uncertainty is a best estimate of the range of a particular metric which may derive from one or two broad sources. Uncertainties that reflect a lack of knowledge about the appropriate value to use for a quantity that is assumed to have (missing modifier: *a fixed?*) value in the context of a particular analysis are termed epistemic. Uncertainties that arise from an inherent randomness in the behavior of the system under study are termed aleatoric.

Recurrent Words/Phrases

- Reasonable expectation
- All significant processes and events
- Sequences of events and processes
- Likelihood of less than one chance in 10
- Performance assessment
  - Identifies features, events and processes
  - Examines effects of features, events and processes
  - Estimates dose caused by all features, events and processes
- Substantial uncertainties
- Complementary cumulative distribution functions (CCDFs)
- Probability distributions for uncertain system parameters
- Uncertainty, aleatory uncertainty, epistemic uncertainty
- Arithmetic mean of projected doses
- Risk
- Expected value of risk

Recurrent Ideas

- Occurrence of future events (aleatory uncertainty)
- Prediction of consequences (models for physical processes)
- Lack of knowledge w.r.t. parameter values (epistemic uncertainty)

Analyses with Aleatory and Epistemic Uncertainty

- Four basic questions underlie an analysis involving aleatory and epistemic uncertainty:

  - Q1, What can happen?
  - Q2, How likely is it to happen?
  - Q3, What are the consequences if it does happen?
  - Q4, How much confidence exists in the answers to the first three questions?

- Three basic entities underlie answering the preceding questions:

  - EN1, A probability space ($\mathcal{A}$, $\mathbb{A}$, p_A) characterizing aleatory uncertainty
  - EN2, A model, or system of models, that predicts system behavior
  - EN3, A probability space ($\mathcal{E}$, $\mathbb{E}$, p_E) characterizing epistemic uncertainty

Aleatory Uncertainty

- Arises from an inherent randomness in the properties or future behavior of the system under study

- Two possibilities:

  - Randomness in the properties of a population of “things” (e.g., manufactured equipment of a particular type)

  - randomness in what could happen in the future (e.g., weather conditions at the time of a reactor accident)

- Alternate descriptors: Variability, stochastic, reducible, type A

- Formally characterized by a probability space $(\mathcal{A}, \mathbb{A}, p_A)$

Epistemic Uncertainty

- Arises from a lack of knowledge about the appropriate value to use for a quantity that is assumed to have a fixed value in the context of a specific analysis

- Three possibilities:

  - Uncertainty w.r.t. a distribution that characterizes aleatory uncertainty (e.g., a poorly known seismic hazard curve)

  - Uncertainty w.r.t. a parameter in a model for a physical process (e.g., a poorly known value for a spatially averaged permeability)

  - Uncertainty w.r.t. the appropriate model for a physical process (e.g., a single or dual porosity model for fluid flow and contaminant transport)

- Alternate descriptors: state of knowledge, subjective, reducible, type B

- Formally characterized by a probability space ($\mathcal{E}$, $\mathbb{E}$, p_E)

Early History: Aleatory and Epistemic Uncertainty

- Formal development of probability began in the 1650-1700 time frame

- Dual nature of probability recognized from beginning:

  - Statistical, dealing with the stochastic laws of chance processes

  - Epistemological, Characterizing degrees of belief in propositions devoid of statistical background

- Excellent reference:

  Ian Hacking, 1975, *The Emergence of Probability*, Cambridge University Press.

- Also:

  F.H. Knight, 1921, Risk, Uncertainty, and Profit. Hart, Schaffner and Marx.

Recent History: Aleatory and Epistemic Uncertainty

- Number of papers in ISI data base with words “aleatory” and “epistemic” in title, key word list or abstract

  - 1980 – 1994: 0
  - 1995 – 1999: 15
  - 2000 – 2004: 38
  - 2005 – 2009: 104
  - 2010 – July 15, 2011: 62

- Example analyses:

  - PA for waste repository at Yucca Mountain (YM)
  - PA for Waste Isolation Pilot Plant (WIPP)
  - PRA for Surry Nuclear Power Station (NUREG-1150)

Additional Detail on EN1, EN2, EN3 (1)

- EN1, A probability space ($\mathcal{A}$, $\mathbb{A}$, p_A) characterizing aleatory uncertainty

  - $\mathbf{a} = [a_1, a_2, \ldots, a_m] \in \mathcal{A}$ one possible occurrence in universe under consideration

  - density function $d_{Ai}(a_i)$, $i = 1, 2, \ldots, m$, defines aleatory distribution for a_i on set $\mathcal{A}_i$

  - Collectively, the sets $\mathcal{A}_i$ and density functions $d_{Ai}(a_i)$ define the set $\mathcal{A}$ and the density function $d_A(\boldsymbol{a})$ for $\mathbf{a}$ on $\mathcal{A}$ and thus, in effect, define the probability space ($\mathcal{A}$, $\mathbb{A}$, p_A)

- EN2, A model, or system of models, that predicts system behavior

$$y(t|\mathbf{a}) = f(t|\mathbf{a}) \text{ for a single result with } t \sim \text{time}$$

$$\mathbf{y}(t|\mathbf{a}) = [y_1(t|\mathbf{a}), y_2(t|\mathbf{a}), \ldots] = \mathbf{f}(t|\mathbf{a}) \text{ for multiple resuults}$$

Additional Detail on EN1, EN2, EN3 (2)

- Results obtained from EN1 and EN2 for a single result $y(t|\mathbf{a})$:

  - Expected value:

$$E_A\left[y(t\,|\,\mathbf{a})\right] = \int_{\mathcal{A}} y(t\,|\,\mathbf{a})\, d_A(\mathbf{a})\, \mathrm{d}\mathcal{A}$$

$$\cong \begin{cases} \sum_{i=1}^{nS} y(t\,|\,\mathbf{a}_i)\,/\,nS & \text{Monte Carlo with } nS \text{ samples from } \mathcal{A} \\ \sum_{i=1}^{nS} y(t\,|\,\mathbf{a}_i)\, p_A(\mathcal{A}_i) & \text{Stratified sampling with } nS \text{ partitions of } \mathcal{A} \end{cases}$$

  - Exceedance probability:

$$prob_A\left[y(t\,|\,\mathbf{a}) > y\right] = \int_{\mathcal{A}} \bar{\delta}_y\left[y(t\,|\,\mathbf{a})\right] d_A(\mathbf{a})\, \mathrm{d}\mathcal{A}$$

$$\text{with } \bar{\delta}_y\left[y(t\,|\,\mathbf{a})\right] = \begin{cases} 1 \text{ if } y(t\,|\,\mathbf{a}) > y \\ 0 \text{ otherwise} \end{cases}$$

Additional Detail on EN1, EN2, EN3 (3)

- EN3, A probability space ($\mathcal{E}$, $\mathbb{E}$, p_E) characterizing epistemic uncertainty

  - $\mathbf{e} = [\mathbf{e}_A, \mathbf{e}_M] = [e_1, e_2, \ldots, e_n] \in \mathcal{E}$ one possible set of analysis inputs

  - $\mathbf{e}_A$ ~ uncertain quantities used in characterization of aleatory uncertainty

  - $\mathbf{e}_M$ ~ uncertain quantities used in models for physical processes

  - density function $d_{Ei}(e_i)$, $i = 1, 2, \ldots, n$, defines epistemic distribution for e_i *on* set $\mathcal{E}_i$

  - Collectively, the sets $\mathcal{E}_i$ and density functions $d_{Ei}(e_i)$ define the set $\mathcal{E}$ and the density function $d_E(\mathbf{e})$ for $\mathbf{e}$ on $\mathcal{E}$ and thus, in effect, define the probability space ($\mathcal{E}$, $\mathbb{E}$, p_E)

Additional Detail on EN1, EN2, EN3 (4)

- Results obtained from EN1, EN2 and EN3 for single result $y(t|\mathbf{a}, \mathbf{e}_M)$:

  - Expected value:

$$E_A\left[y(t\,|\,\mathbf{a},\mathbf{e}_M)\right] = \int_{\mathcal{A}} y(t\,|\,\mathbf{a},\mathbf{e}_M)\, d_A(\mathbf{a}\,|\,\mathbf{e}_A)\,\mathrm{d}\mathcal{A}$$

  - Exceedance probability:

$$prob_A\left[y(t\,|\,\mathbf{a},\mathbf{e}_M) > y\right] = \int_{\mathcal{A}} \bar{\delta}_y\left[y(t\,|\,\mathbf{a},\mathbf{e}_M)\right] d_A(\mathbf{a}\,|\,\mathbf{e}_A)\,\mathrm{d}\mathcal{A}$$

- Important point: Above integrals cannot be estimated when aleatory and epistemic uncertainty are sampled together (i.e., concurrently from $\mathcal{A}\times\mathcal{E}$)!

Example: Dose to RMEI from Seismic Ground Motion Events in 2008 YM PA (1)

- Aleatory uncertainty (EN1): Probability space ($\mathcal{A}$, $\mathbb{A}$, p_A)

$$\mathbf{a} = [n, t_1, v_1, \mathbf{p}_1, t_2, v_2, \mathbf{p}_2, \ldots, t_n, v_n, \mathbf{p}_n] \in \mathcal{A}$$

$n =$ number seismic GM events in 20,000 yr

$t_i =$ time (yr) event i

$v_i =$ peak ground velocity (m/s) event i

$\mathbf{p}_i =$ properties associated with event i (rockfall, WP damage, ...)

- Epistemic uncertainty (EN3): Probability space ($\mathcal{E}$, $\mathbb{E}$, p_E)

$$\mathbf{e} = [e_1, e_2, \ldots, e_{392}] \in \mathcal{E}$$

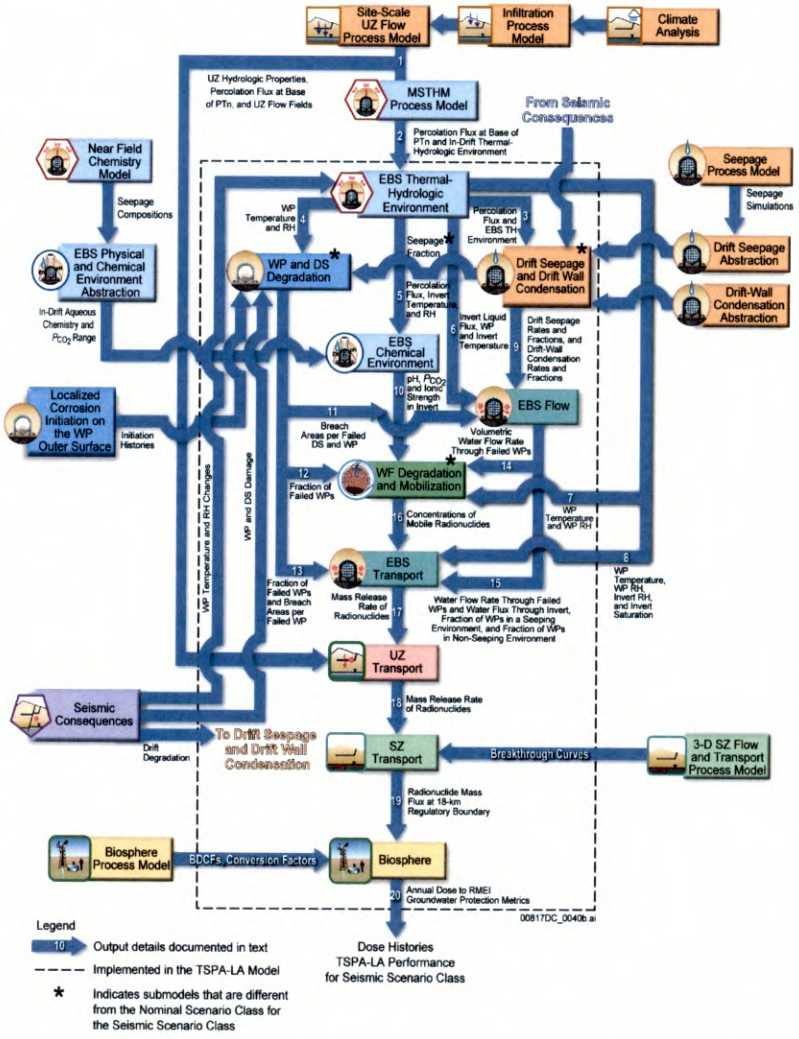
$e_i =$ SCCTHRP $=$ residual stress threshold (MPa) for stress corrosion cracking

$e_j =$ SZGWSPDM $=$ saturated zone groundwater specific discharge multiplier

$e_k =$ SZFIPOVO $=$ saturated zone flowing interval porosity in volcanic units

Example: Dose to RMEI from Seismic Ground Motion Events in 2008 YM PA (2)

- Models that predict system behavior (EN2)



Example: Dose to RMEI from Seismic Ground Motion Events in 2008 YM PA (3)

- Numerics

  - Integration over aleatory uncertainty (i.e., over $\mathcal{A}$)

    (i) Monte Carlo for distribution of dose over aleatory uncertainty

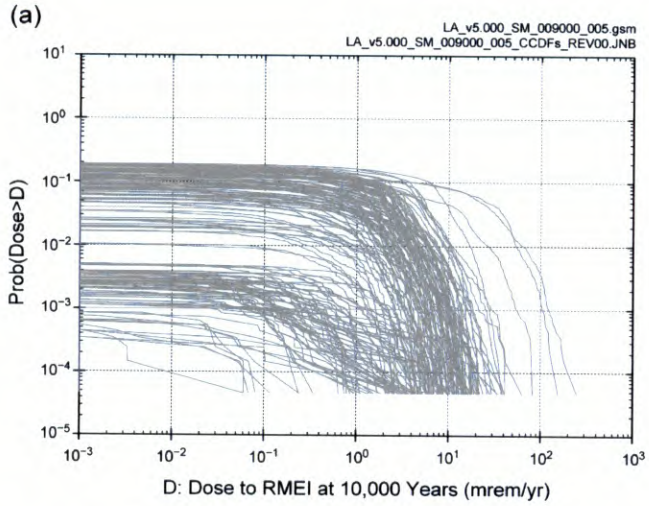
    (ii) Quadrature for time-dependent expected dose over aleatory uncertainty

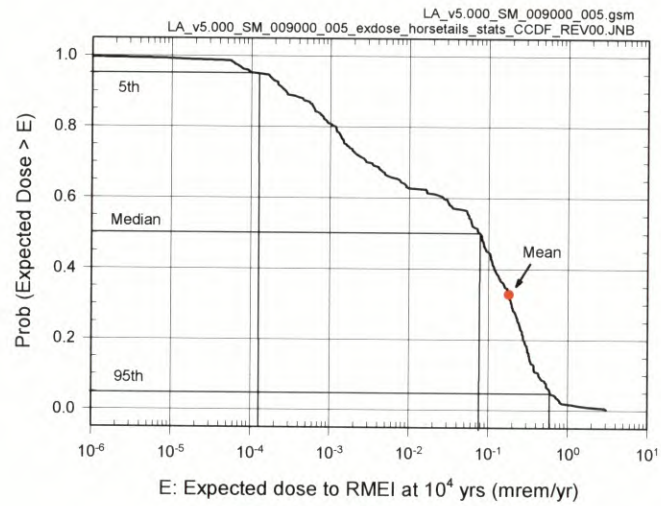
  - Integration over epistemic uncertainty (i.e., over $\mathcal{E}$)

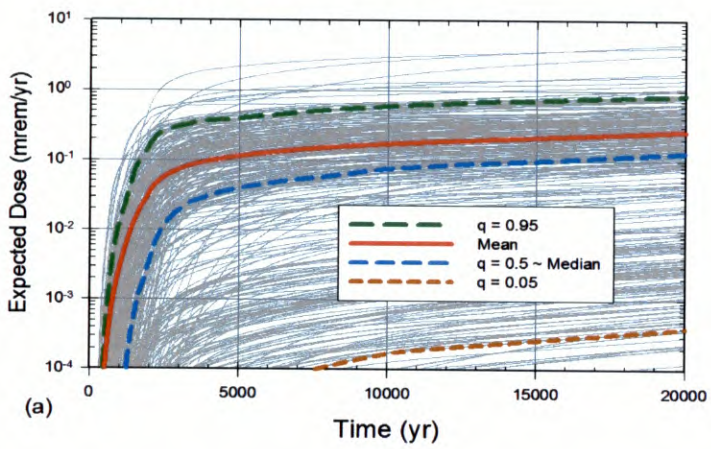
    (i) Latin hypercube sampling

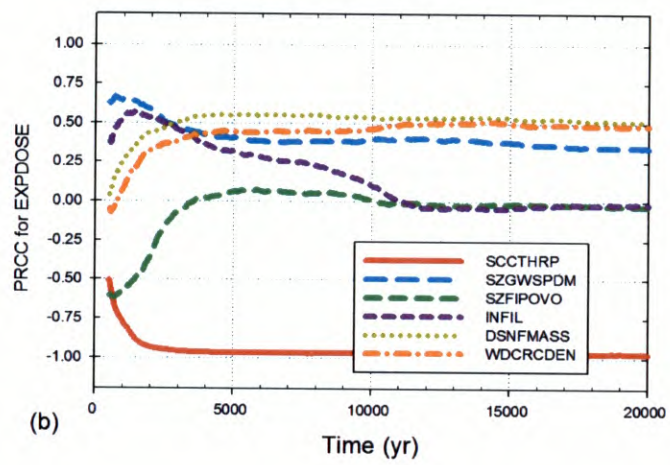
    (ii) Three independent samples of size 300 each

Example: Dose to RMEI from Seismic Ground Motion Events in 2008 YM PA (4)









Example: Dose to RMEI from All Events in 2008 YM PA (1)

- Aleatory uncertainty (EN1): Probability space ($\mathcal{A}$, $\mathbb{A}$, p_A)

$$\mathbf{a} = \left[nEW, nED, nII, nIE, nSG, nSF, \mathbf{a}_{EW}, \mathbf{a}_{ED}, \mathbf{a}_{II}, \mathbf{a}_{IE}, \mathbf{a}_{SG}, \mathbf{a}_{SF} \right]$$

where, for a time interval [*a*, *b*] (e.g., [0, 10^4 yr] or [0, 10^6 yr]),

nEW = number of early waste package (WP) failures,
nED = number of early drip shield (DS) failures,
nII = number of igneous intrusive (II) events,
nIE = number of igneous eruptive (IE) events,
nSG = number of seismic ground (SG) motion events,
nSF = number of seismic fault (SF) displacement events,
$\mathbf{a}_{EW}$ = vector defining the *nEW* early WP failures,
$\mathbf{a}_{ED}$ = vector defining the *nED* early DS failures,
$\mathbf{a}_{II}$ = vector defining the *nII* igneous intrusive events,
$\mathbf{a}_{IE}$ = vector defining the *nIE* igneous eruptive events,
$\mathbf{a}_{SG}$ = vector defining the *nSG* seismic ground motion events,
$\mathbf{a}_{SF}$ = vector defining the *nSF* seismic fault displacement events

Example: Dose to RMEI from All Events in 2008 YM PA (2)

- Epistemic uncertainty (EN3): Probability space ($\mathcal{E}$, $\mathbb{E}$, p_E)

  - Same as for seismic ground motion (GM) events

  - Individual sample space element: $\mathbf{e} = \left[e_1, e_2, \ldots, e_{392}\right] \in \mathcal{E}$

- Models that predict system behavior (EN2) - Similar to seismic ground motion (GM) events except for igneous eruptive events

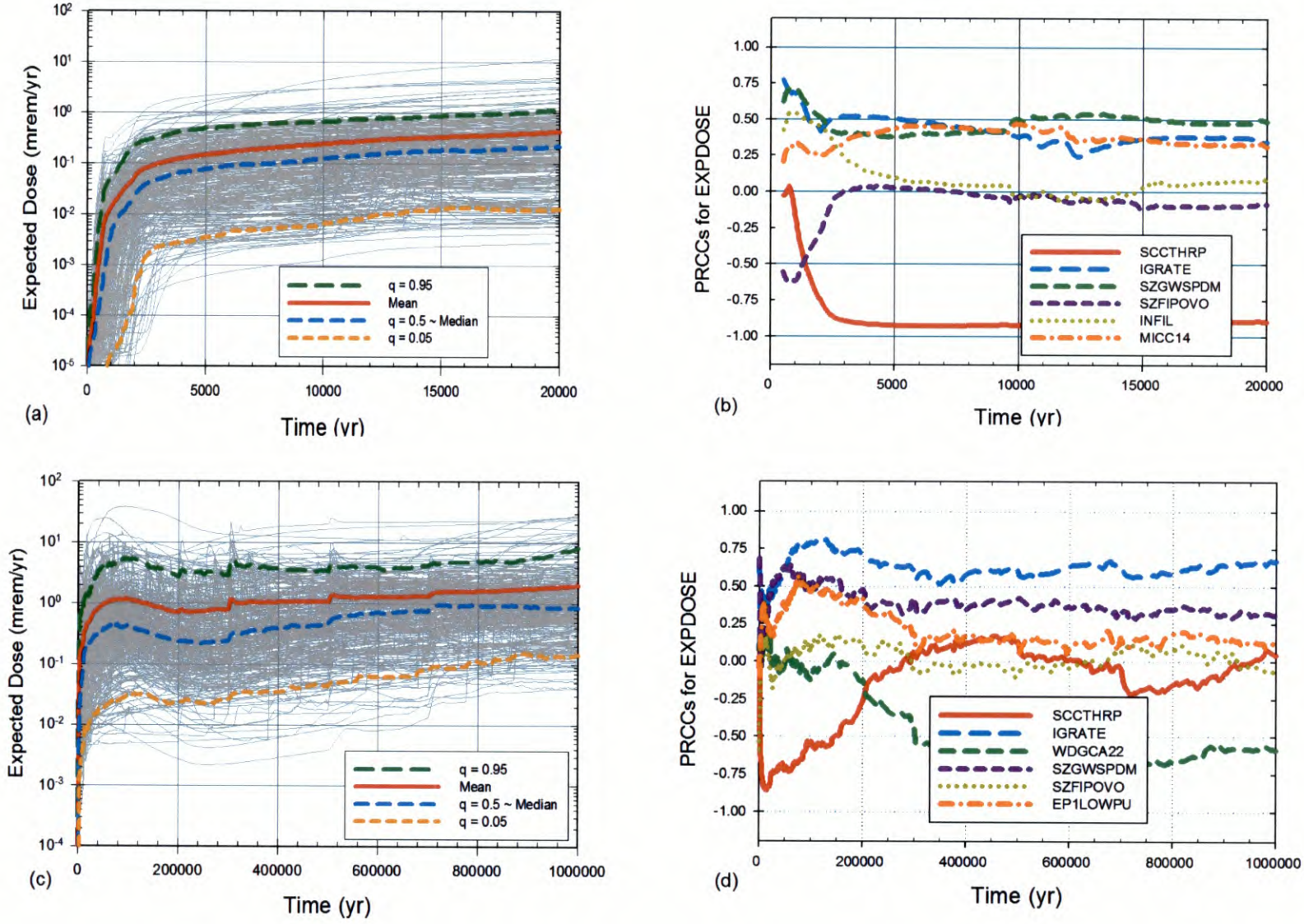
- Computational strategy

  - Calculate expected dose for individual scenario classes

  - Sum expected doses from individual scenario classes to obtain expected dose for all scenario classes

  - Strategy works because high dose scenarios have low probabilities and higher probability scenarios have low doses

Example: Dose to RMEI from All Events in 2008 YM PA (3)



Example: NRC Requirements for YM Repository (1)

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Example: NRC Requirements for YM Repository (2)

- Specifically, the NRC defines PA in 10 CFR Part 63.2 in the statement ([22], p. 55794):

  Performance assessment means an analysis that:

  (1) Identifies the features, events, processes (except human intrusion), and sequences of events and processes (except human intrusion) that might affect the Yucca Mountain disposal system and their probabilities of occurring during 10,000 years after disposal;

  (2) Examines the effects of those features, events, processes, and sequences of events and processes upon the performance of the Yucca Mountain disposal system; and

  (3) Estimates the dose incurred by the reasonably maximally exposed individual, including the associated uncertainties, as a result of releases caused by all significant features, events, processes, and sequences of events and processes, weighted by their probability of occurrence.

Example: NRC Requirements for YM Repository (3)

- NRC definition for reasonable expectation

Reasonable expectation means that the Commission is satisfied that compliance will be achieved based upon the full record before it. Characteristics of reasonable expectation include that it:

(1) Requires less than absolute proof because absolute proof is impossible to attain for disposal due to the uncertainty of projecting long-term performance;

(2) Accounts for the inherently greater uncertainties in making long-term projections of the performance of the Yucca Mountain disposal system;

(3) Does not exclude important parameters from assessments and analyses simply because they are difficult to precisely quantify to a high degree of confidence; and

(4) Focuses performance assessments and analyses on the full range of defensible and reasonable parameter distributions rather than only upon extreme physical situations and parameter values.

Concluding Message

- Everyone cannot be expected to agree on the details of an analysis of a complex system, but everyone should be able to known what those details are.

- Use of the following entities provides a coherent means of constructing, documenting and communicating an analysis performed to establish compliance with a regulatory requirement:

  - EN1, a probability space ($\mathcal{A}$, $\mathbb{A}$, p_A) characterizing aleatory uncertainty
  - EN2, A model, or system of models, that predicts system behavior
  - EN3, A probability space ($\mathcal{E}$, $\mathbb{E}$, p_E) characterizing epistemic uncertainty

- Quality documentation is essential but often not achieved – A clear conceptual structure is a necessary first step to quality documentation.

Additional Examples

- EPA requirements for Waste Isolation Pilot Plant (WIPP)
- NRC reactor accident safety goals based on NUREG-1150 PRAs

Example: Normalized Release in PA for WIPP (1)

- Aleatory uncertainty (EN1): Probability space ($\mathcal{A}$, $\mathbb{A}$, p_A)

$$\mathbf{a} = [n, t_1, l_1, e_1, b_1, p_1, \mathbf{a}_1, \ldots, t_n, l_n, e_n, b_n, p_n, \mathbf{a}_n, t_{mn}] \in \mathcal{A}$$

$n =$ number of drilling intrusions over 10,000 yr

$t_i =$ time (yr) of drilling intrusion i

$l_i =$ location of drilling intrusion i

$e_i =$ penetration of excavated repository area by drilling intrusion i

$b_i =$ penetration of pressurized brine by drilling intrusion i

$p_i =$ bore hole plugging procedure for drilling intrusion i

$\mathbf{a}_i =$ properties of waste penetrated by drilling intrusion i

$t_{mn} =$ time (yr) of potash mining

Example: Normalized Release in PA for WIPP (2)

- Epistemic uncertainty (EN3): Probability space ($\mathcal{E}$, $\mathbb{E}$, p_E)

$$\mathbf{e} = [e_1, e_2, \ldots, e_{57}] \in \mathcal{E}$$

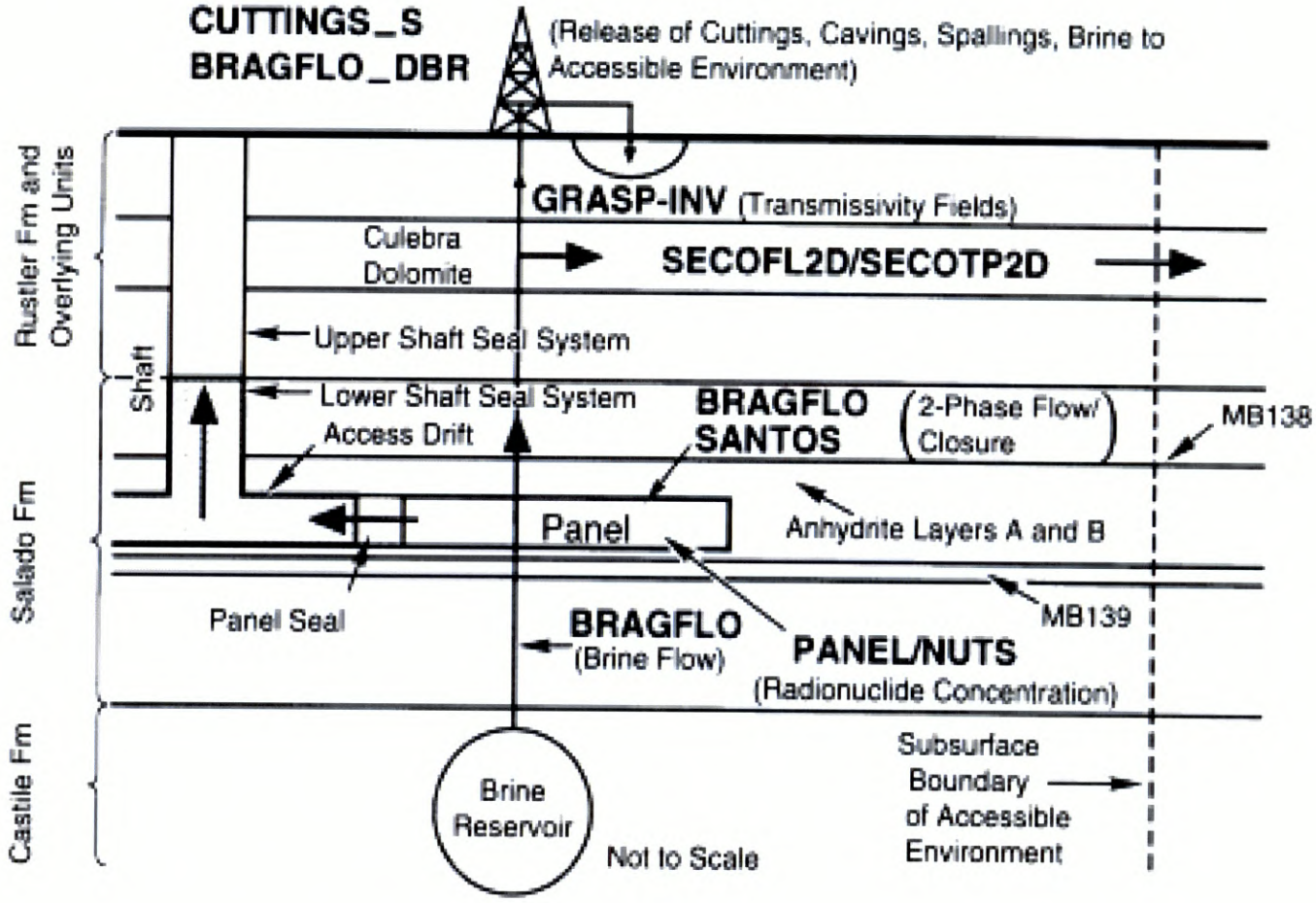
e_i = *WMICDFLF* = designator for microbial gas generation model

e_j = *WTAUFAIL* = waste shear strenth (Pa)

e_k = *WGRCOR* = corrosion rate (m/s) for steel under inundated conditions

Example: Normalized Release in PA for WIPP (3)

- Models for physical processes (EN2)



Example: Normalized Release in PA for WIPP (4)

- Numerics

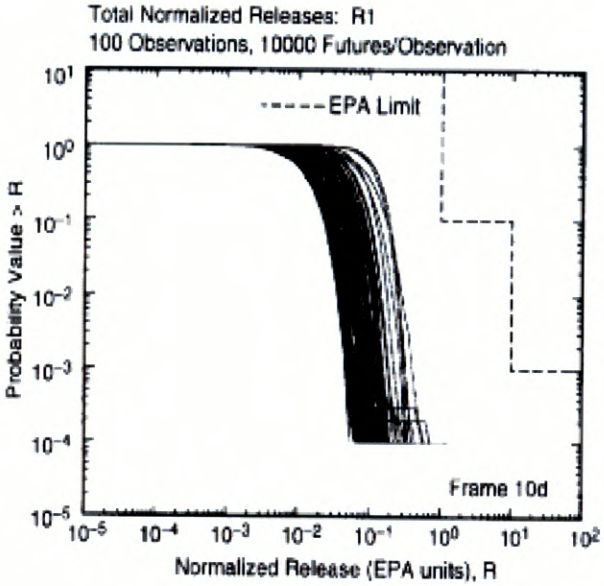
  - Integration over aleatory uncertainty (i.e., over $\mathcal{A}$): Monte Carlo with interpolation over results from physics-based models

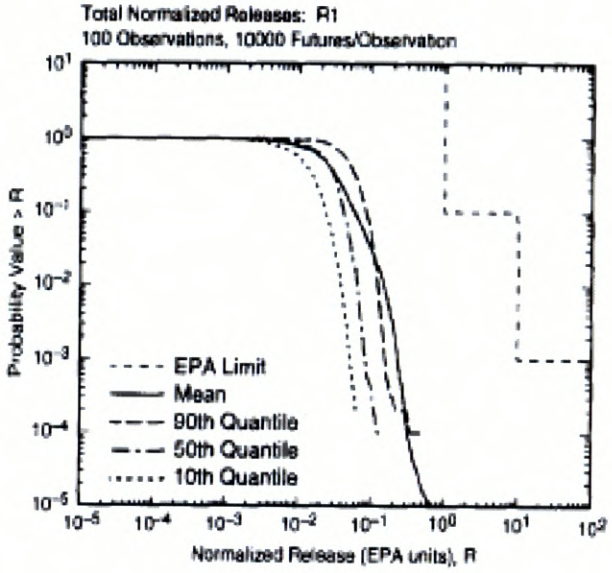
  - Integration over epistemic uncertainty (i.e., over $\mathcal{E}$):

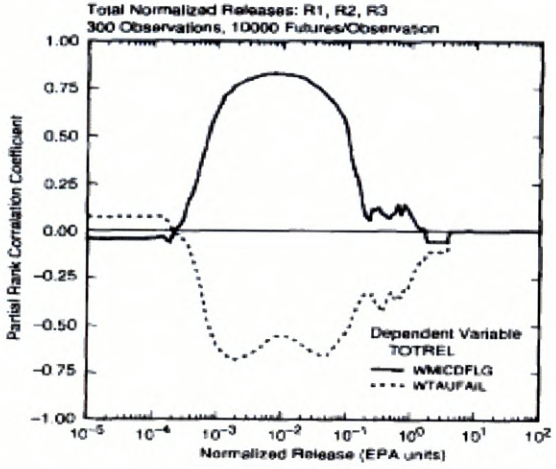
    (i) Latin hypercube sampling

    (ii) Three independent samples of size 100 each

Example: Normalized Release in PA for WIPP (5)







Total release

Variable	SRRC	R^2
WMICDFLG	0.60	0.40
WTAUFAIL	−0.39	0.55
WGRCOR	0.21	0.59
WPRTDIAM	−0.19	0.63
HALPOR	0.17	0.65
BHPRM	−0.17	0.68
HALPRM	0.16	0.71
WASTWICK	0.11	0.72
ANHPRM	0.09	0.73

Example: EPA Requirements for WIPP (1)

- § 191.13 Containment requirements:

- (a) Disposal systems for spent nuclear fuel or high-level or transuranic radioactive wastes shall be designed to provide a reasonable expectation, based upon performance assessments, that cumulative releases of radionuclides to the accessible environment for 10,000 years after disposal from all significant processes and events that may affect the disposal system shall:

  (1) Have a likelihood of less than one chance in 10 of exceeding the quantities calculated according to Table 1 (Appendix A); and

  (2) Have a likelihood of less than one chance in 1,000 of exceeding ten times the quantities calculated according to Table 1 (Appendix A).

- (b) Performance assessments need not provide complete assurance that the requirements of 191.13(a) will be met. Because of the long time period involved and the nature of the events and processes of interest, there will inevitably be substantial uncertainties in projecting disposal system performance. Proof of the future performance of a disposal system is not to be had in the ordinary sense of the word in situations that deal with much shorter time frames. Instead, what is required is a reasonable expectation, on the basis of the record before the implementing agency, that compliance with 191.13(a) will be achieved.

Example: EPA Requirements for WIPP (2)

- *§ 194.34 Results of performance assessments.*

(a) The results of performance assessments shall be assembled into "complementary, cumulative distribution functions" (CCDFs) that represent the probability of exceeding various levels of cumulative release caused by all significant processes and events.

(b) Probability distributions for uncertain disposal system parameter values used in performance assessments shall be developed and documented in any compliance application.

(c) Computational techniques, which draw random samples from across the entire range of the probability distributions developed pursuant to paragraph (b) of this section, shall be used in generating CCDFs and shall be documented in any compliance application.

(d) The number of CCDFs generated shall be large enough such that, at cumulative releases of 1 and 10, the maximum CCDF generated exceeds the 99th percentile of the population of CCDFs with at least a 0.95 probability.

(e) Any compliance application shall display the full range of CCDFs generated.

(f) Any compliance application shall provide information which demonstrates that there is at least a 95 percent level of statistical confidence that the mean of the population of CCDFs meet the containment requirements of § 191.13 of this chapter.

Example: Latent Cancer Fatalities in NUREG-1150 (1)

- Aleatory uncertainty (EN1): Probability space ($\mathcal{A}$, $\mathbb{A}$, p_A)

$$\mathbf{a} = [IE, AS, PDS, APB, STG, WT] \in \mathcal{A}$$

$IE =$ initiating event designator $(nIE = 11)$

$AS =$ accident sequence designator $(nAS = 28)$

$PDS =$ plant damage state designator $(nPDS = 25$ reduced to $nPDS = 7)$

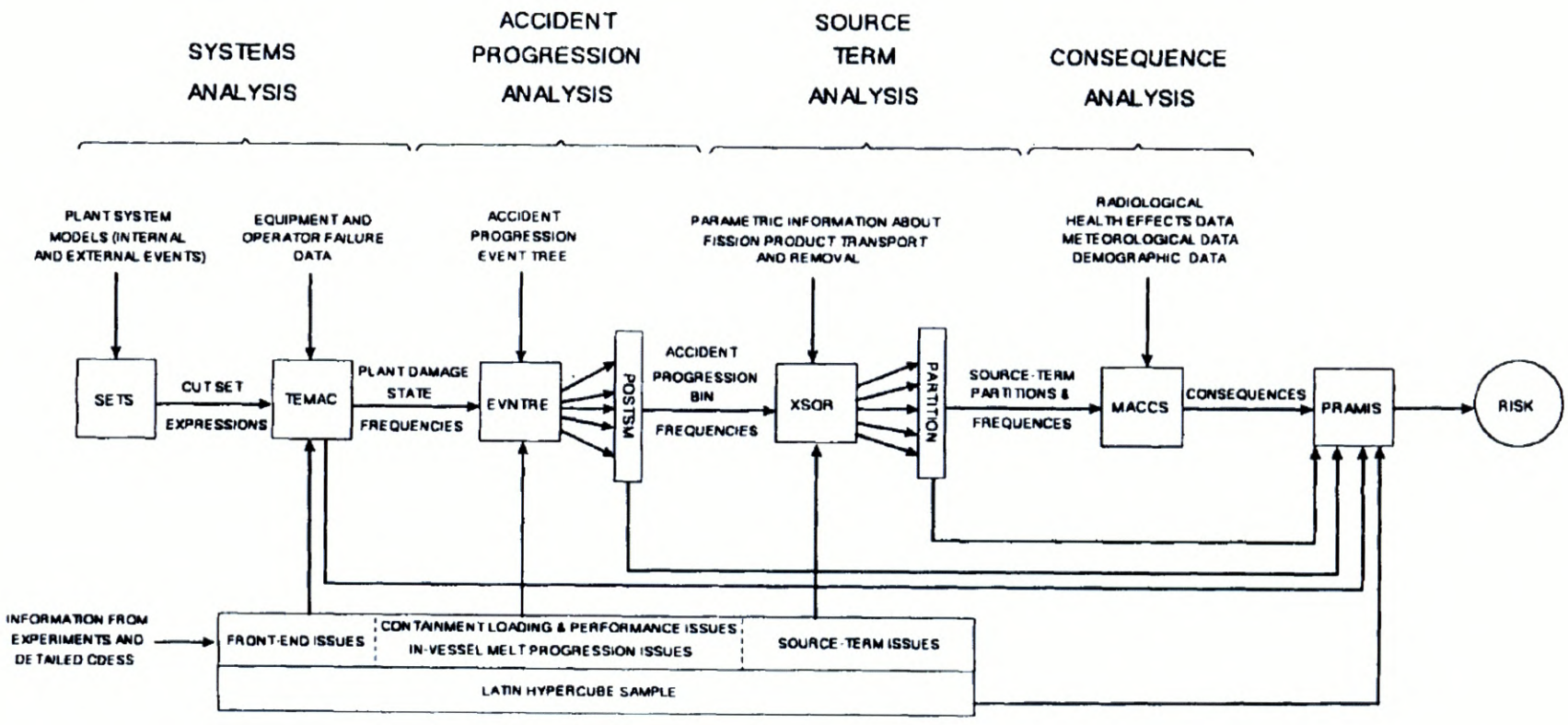
$APB =$ accident progression bin designator $(54 \leq nAPB \leq 157)$

$STG =$ source term group designator $(nSTG = 54)$

$WT =$ weather trial designator $(nWT = 2560)$

Example: Latent Cancer Fatalities in NUREG-1150 (3)

- System model (EN2)



Example: Latent Cancer Fatalities in NUREG-1150 (3)

- Epistemic uncertainty (EN3): Probability space ($\mathcal{E}$, $\mathbb{E}$, p_E)

$$\mathbf{e} = [e_1, e_1, \ldots, e_{130}] \in \mathcal{E}$$

e_i = *FISGFOSG* = pointer variable for SGTR fission product release

e_j = *FCOR* = pointer variable for fission product release from core to vessel

e_k = *DGFSTRT* = probability that diesel generator fails to start

Example: Latent Cancer Fatalities in NUREG-1150 (4)

- Numerics

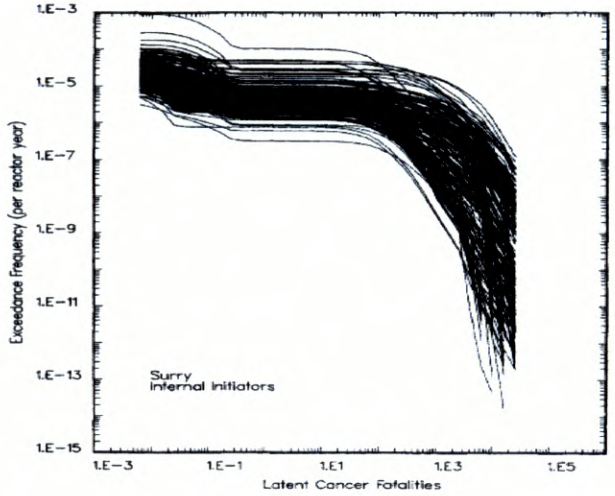
  - Integration over aleatory uncertainty (i.e., over $\mathcal{A}$): Complex use of event trees

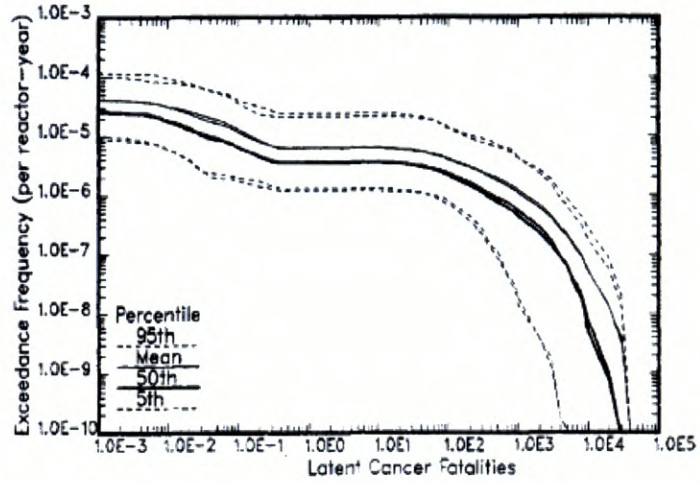
  - Integration over epistemic uncertainty (i.e., over $\mathcal{E}$):

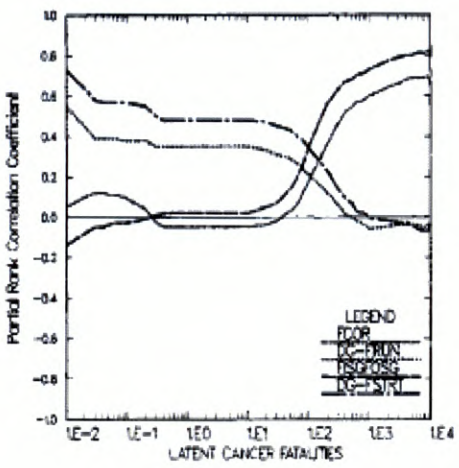
    (i) Latin hypercube sampling

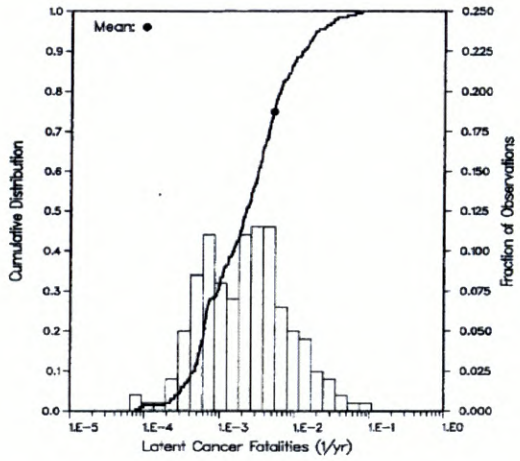
    (ii) Two independent samples of size 200 each

Example: Latent Cancer Fatalities in NUREG-1150 (5)



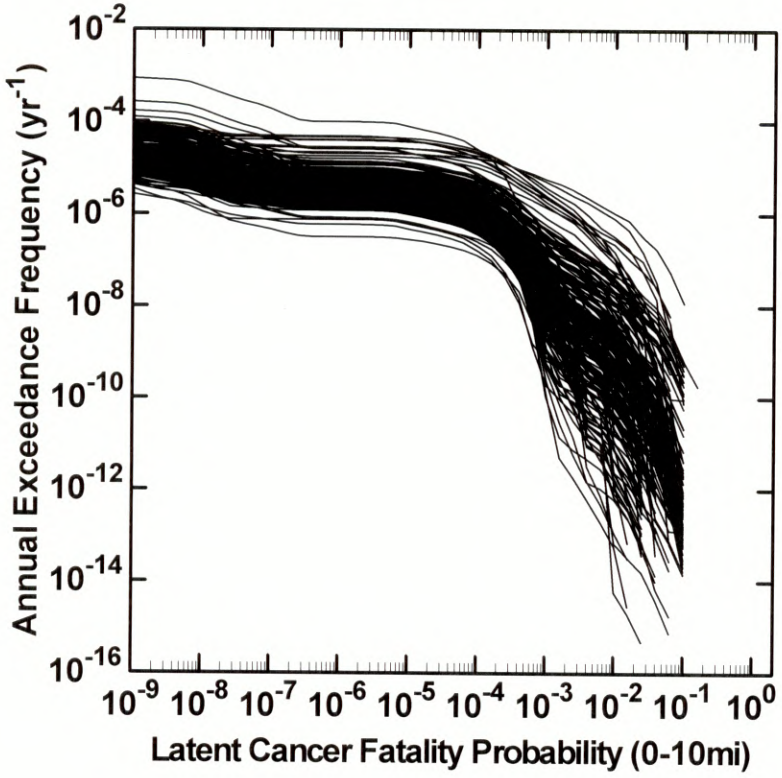


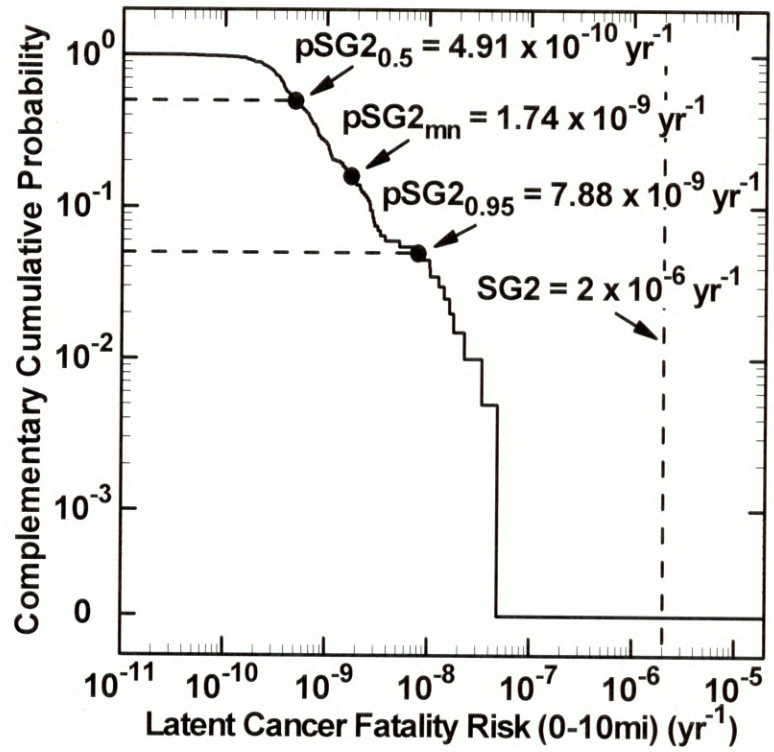




Variable	*SRC*	R^2
FISGFOSG	0.52	0.26
IE-SGTR	0.36	0.39
FCOR	0.33	0.50
V-TRAIN	0.30	0.59
DG-FSTRT	0.16	0.62
IE-LOSP	0.14	0.63

Example: Latent Cancer Fatality Risk in NUREG-1150





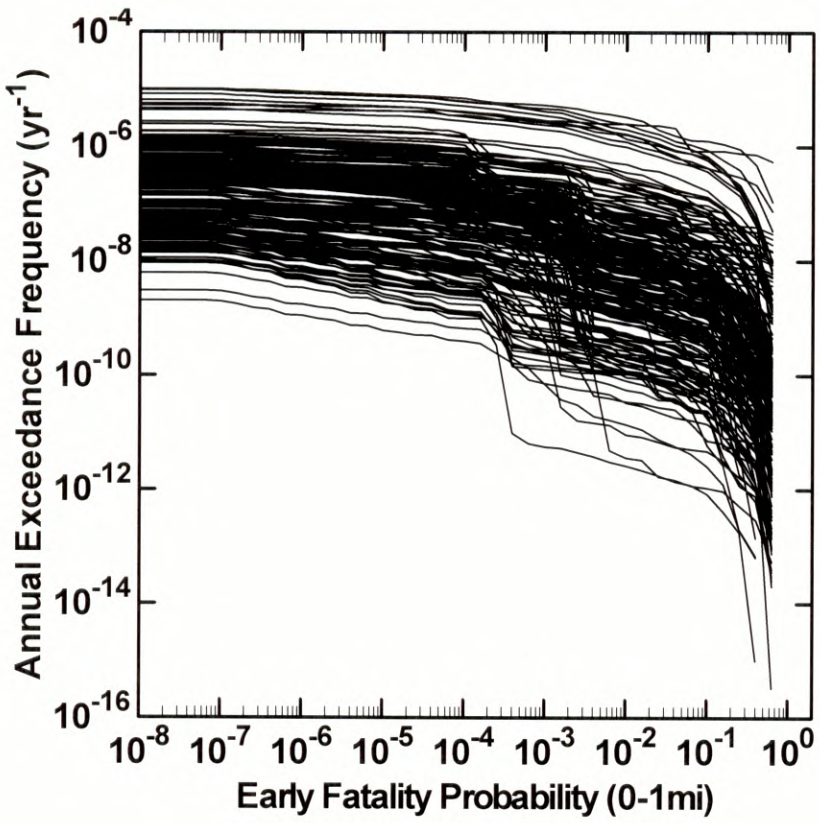
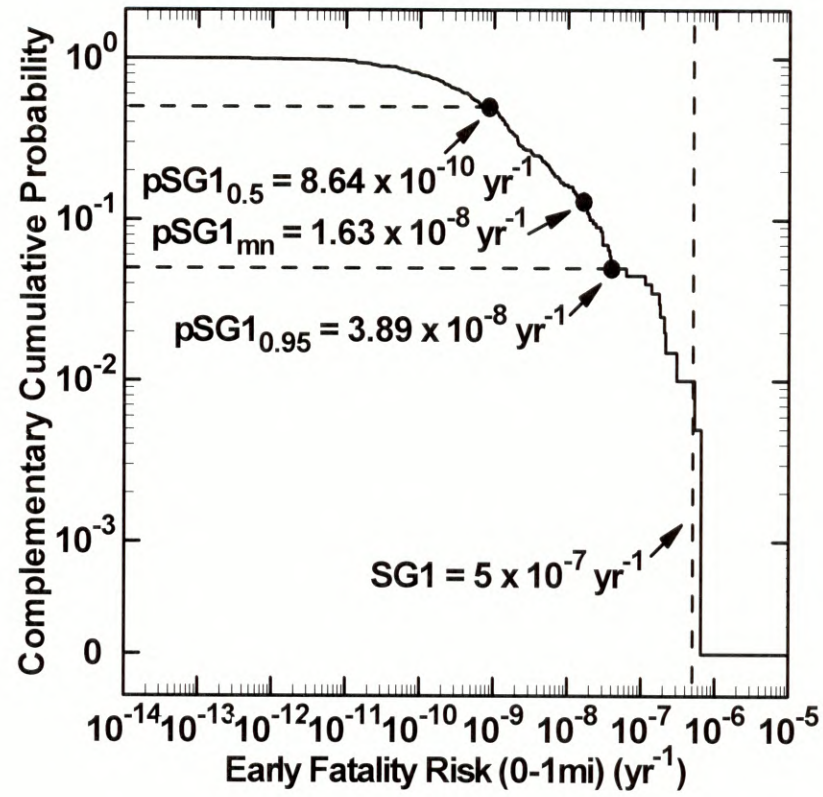
Example: NRC Reactor Accident Safety Goals (1)

- *Individual early fatality risk*: The expected value for average individual early fatality risk in the region between the plant site boundary and 1609.3 m (1 mi) beyond this boundary will be less than 5×10^{-7}/ yr.

- *Individual latent cancer fatality risk*: The expected value for average individual latent cancer fatality risk in the region between the plant site boundary and 16,093 m (10 mi) beyond this boundary will be less than 2×10^{-6} / yr.

- *Severe accident frequency*: The expected value for the frequency of a severe accident will be less than 1×10^{-4} / yr.

- *Conditional probability of containment failure*: The expected value for the probability of containment failure given the occurrence of a severe accident will be less than 0.1.

- *Large release frequency*: The expected value for the frequency of a large release will be less than 1×10^{-6} / yr.

Example: Early Fatality Risk in NUREG-1150